Collage 2021 : Homework 3

April 11, 2021

To respect your other out-of-class commitments, a flexible deadline for this homework is 05/01/2021. However, the sooner you get to it, the easier it will be. Homework yields 25 points in total

Problem 1: This problem builds on the hands-on dedicated to estimating magnetic fields from HINODE SOT/SP spectropolarimetric observations.

Start by making sure what was done in the main part of the hands-on on 04/01 makes sense and that once you run the notebook you get results consistent with our class. In this problem we will try to estimate the uncertainties for our magnetic field.

- First, we estimate the level of noise in our data. Pick roughly 100 pixels and make a sample of continuum signals in Stokes V (circular polarization) that consists of 10-20 wavelength points that can be classified as continuum. In total you should have 1000-2000 points. Plot a histogram of these continuum signals. It should be centered at zero and its standard deviation ca be taken to be our "level of noise" for Stokes V. Let's take this level of noise to be our uncertainty for V. Reasoning behind this is that the continuum has to be unpolarized and that thus signal in these points is just Gaussian noise.
- We have seen that weak field approximation results in a linear relationship between Stokes V and derivative of Stokes I with respect to wavelength:

$$V(\lambda) = -4.67 \times 10^{-13} \frac{dI}{d\lambda} \lambda^2 g_{\mathrm{L}} B_{\mathrm{los}}. \tag{1}$$

  Where magnetic field is Gauss, and wavelengths are in Angstrom. Stokes I and V should be in same units. Lande factors are 1.67 for 6301.5 line and 2.5 for 6302.5 line. We can fit this linear relationship with a straight line ($kx + n$) to find the slope and thus derive the magnetic field. This fit can be solved as an overdetermined system of linear equations:

$$k(\frac{dI}{d\lambda})_i + n_i = V_i \tag{2}$$

Either starting from equation 2 and "propagating errors", or by using a convenient python package, perform the linear fit and find uncertainty in $B_{\rm los}$.

- Repeat the estimation for a few pixels with different signals in Stokes V (in the original google drive, there is an additional dataset with some stronger magnetic fields). How do magnetic field uncertainties in different pixels compare? How about relative uncertainties? (Ratio of the uncertainty and the inferred magnetic field).

Problem 2: This problem deals with calculating mean intensity and anisotropy of radiation in a given "blob" of gas situated above the solar atmosphere, and representing a part of prominence. Let's define mean intensity and the anisotropy of the radiation field as (here we are assuming incident radiation is wavelength independent, so integration over wavelength will disappear):

$$J_0^0 = J = \frac{1}{2}\int_{-1}^{1} I(\mu')d\mu'$$
$$J_0^2 = \frac{1}{4\sqrt{2}}\int_{-1}^{1} (3\mu'^2 - 1)I(\mu')d\mu' \qquad (3)$$

where μ' is the cosine of the angle between the incoming "ray" and the z axis, from the prominence system of reference.

- Sketch the situation and outline clearly the difference between the μ's for the radiation impinging on the blob and the radiation leaving the solar surface (latter one should be in the coordinate system of the point where the ray leaves solar surface).
- Assuming that the radiation field leaving the Sun is completely isotropic (we know it's not, see below), calculate the variation of J_0^0 and J_0^2 with the height of the blob above the Solar surface. Reasonable values for this height range from 10 000 to 200 000 km.
- How does the situation complicate if we consider limb-darkening of the radiation leaving solar surface. You do not have to carry out calculations completely but outline the solution. If it is easier for you to work with numerical quantities, you can assume the outgoing radiation from the solar surface varies with the μ as : $I(\mu) = 0.3 + 0.7\mu$.

Problem 3: Go back to the jupyter notebook from the hands-on on solar flare simulations. (03/30, see the website), and, using the accompanying sheet, answer the questions 13,14,15,16,17. Professor Adam Kowalski will also be available for the discussion on this part.

Good luck, do not hesitate to reach out and discuss with me or among yourselves if you have any questions!