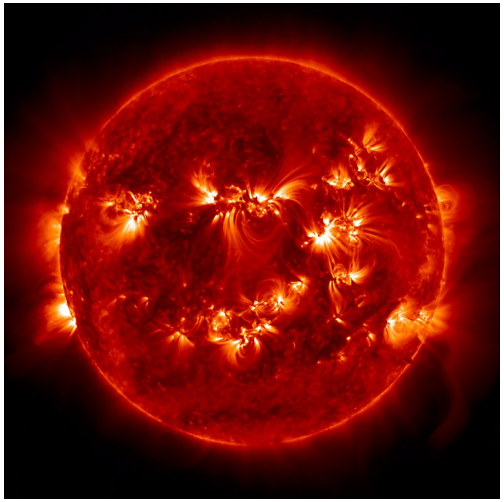
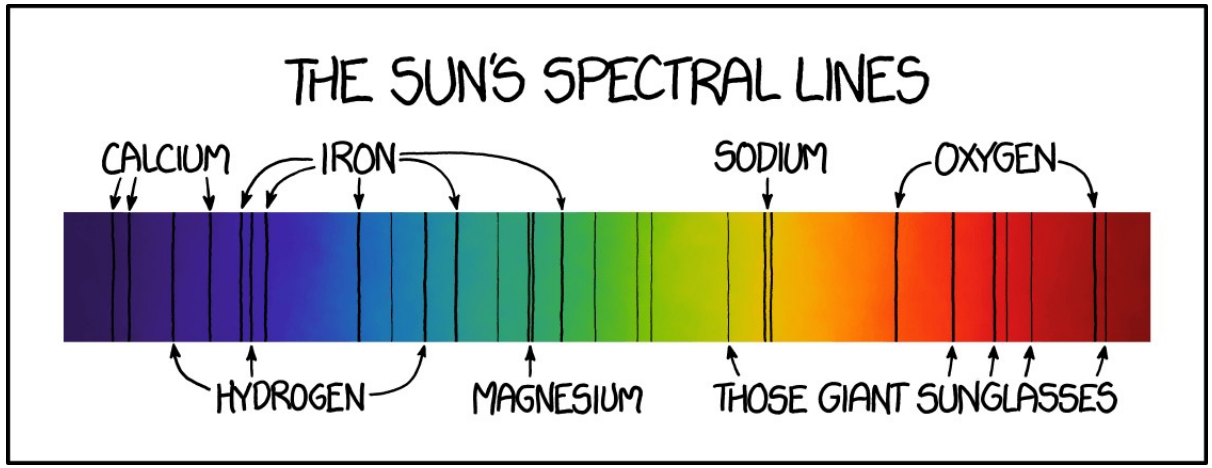
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 27: Spectropolarimetric Inversions

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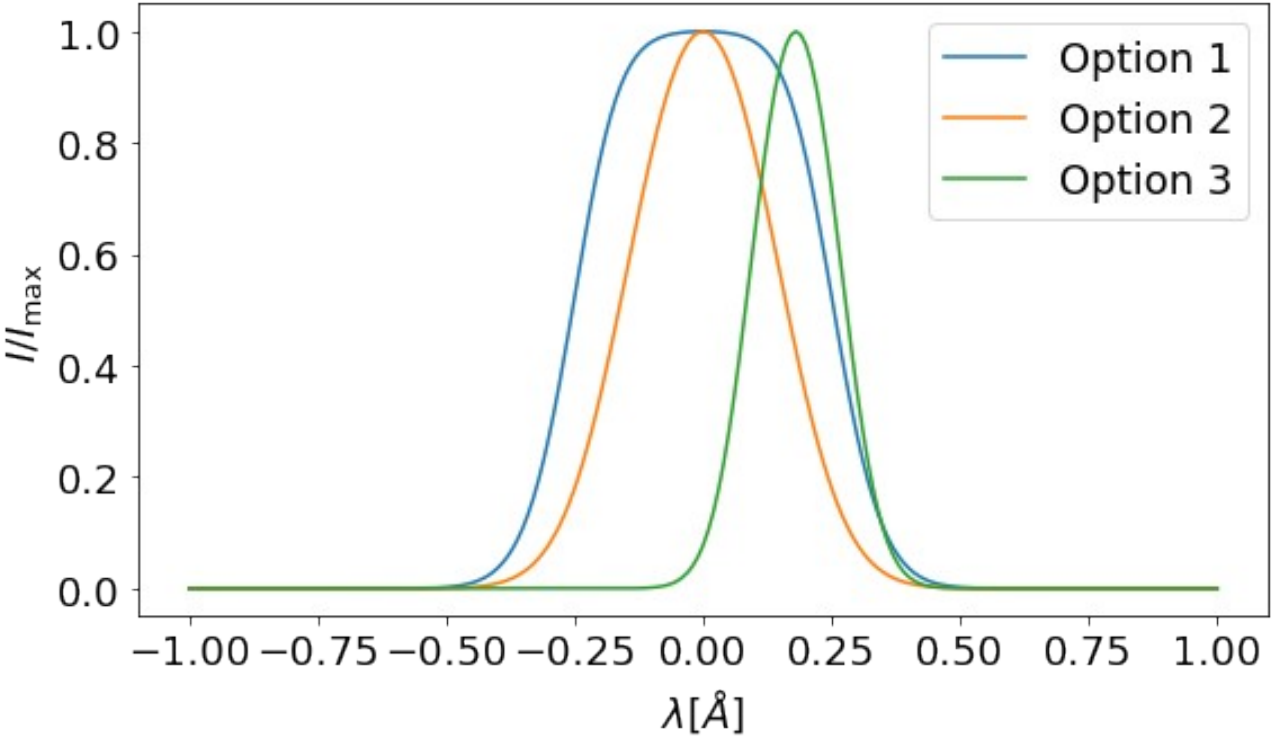
Today

- Last class we saw how magnetic field as well as other parameters influence the spectrum (and the polarization) formed in a simple slab model of a prominence (filament)
- If we consider this to be a **model** of spectrum formation, we can, instead of toying around like during the last class **fit this model to our observed data** in order to try and infer values of some relevant physical quantities
- In solar spectropolarimetry this fitting is called **Spectropolarimetric Inversion**
- Today we talk about the inversions :-)

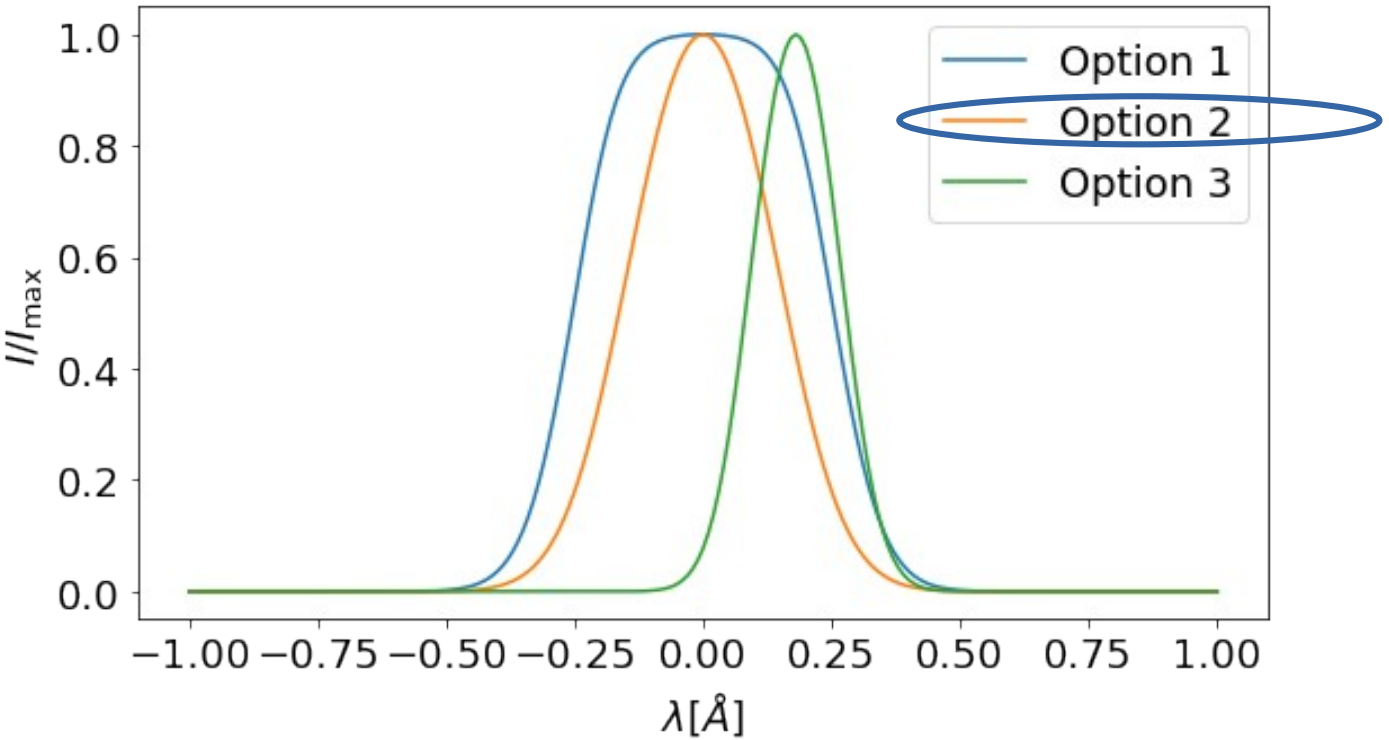
To kick it off, a question!

- Which of the three plotted prominence spectra corresponds to a model with **line center optical depth of 0.5, no los velocity and line thermal width of 5km/s? (2 mins)**



To kick it off a question!

- Which of the three plotted prominence spectra corresponds to a model with **line center optical depth of 0.5, no los velocity and line thermal width of 5km/s?**

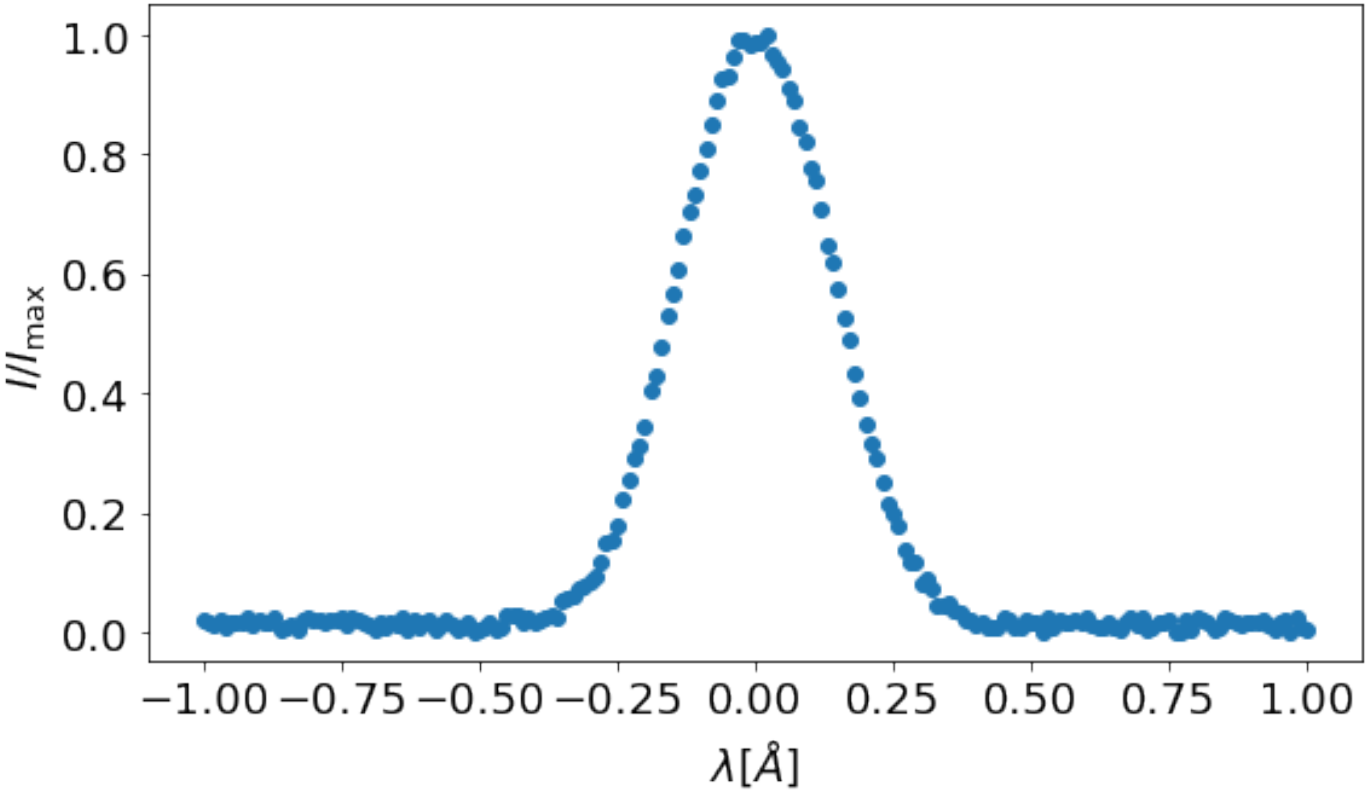


Obviously, for scientific applications this guessing won't do

- For starters, often we deal with up to millions of spectra
- Some dependencies are not obvious to quantify (i.e. dependence on temperature can be quite non-linear)
- Or there are "degeneracies" between different parameters
- Or we want exact numbers and difference between say, 200G and 400G makes a difference!
- **But you should always trust your eyes. If the fit or diagnostics does not make sense, it is more likely the diagnostic technique failed than that your eyes are playing tricks. (e.g. magnetic field of 15000 Gauss is to be quadriple - checked)**

Anyway, back to the question:

- If I gave you this spectrum, and told you that the model that generated it, how would you find the optical depth, line center position and broadening?



$$I_\lambda = S(1 - e^{-\tau\phi_\lambda})$$

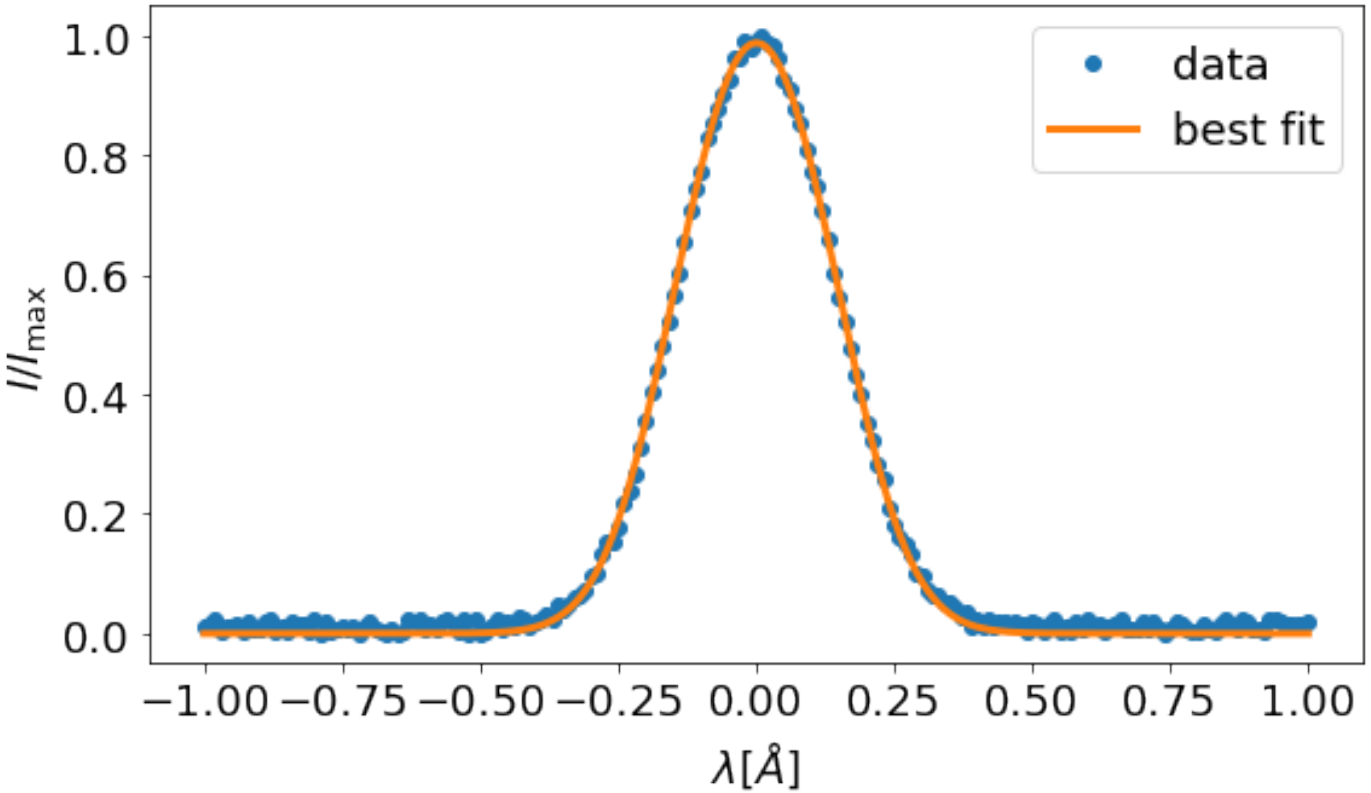
$$\phi_\lambda = e^{-(\lambda-\lambda_0)^2/\Delta\lambda_D^2}$$

That’s right, you would fit the **model to the data**

- What are my “observables”? What do I know? **Intensities at corresponding wavelengths**
- What are my parameters? What do I want to find out? **Optical depth, line center position (los velocity), line broadening (thermal velocity).**
- What does “fit” mean?
- Well, we are finding the **parameter values** that, when substituted in the model, provide the “synthetic spectrum” that agrees the best with the observed one.

Well, let's fit the data:

- Fit does indeed yield: **line center optical depth of 0.5, no los velocity and line thermal width of 5km/s**



Questions that we can ask now

- How do we know what is “the best” fit?
- How certain are we about the inferred parameters (as there is noise in the data)
- Related to the two above: Is there a different combination of the parameters that would fit equally well?
- And, hardest of all: **How do we know or even, can we know if our model really generated the data?**

“All the models are wrong, but some are useful”

George Box (1976), copied from Wikipedia:

Now it would be very remarkable if any system existing in the real world could be exactly represented by any simple model. However, cunningly chosen parsimonious models often do provide remarkably useful approximations. For example, the law PV = RT relating pressure P, volume V and temperature T of an "ideal" gas via a constant R is not exactly true for any real gas, but it frequently provides a useful approximation and furthermore its structure is informative since it springs from a physical view of the behavior of gas molecules. For such a model there is no need to ask the question "Is the model true?". If "truth" is to be the "whole truth" the answer must be "No". The only question of interest is "Is the model illuminating and useful?".

What am I trying to say here?

- Leaving philosophy aside (do real life objects actually behave *exactly* according to the laws of physics / astrophysics?).
- It is worthwhile asking ourselves: What do the parameters of our model mean in the context of the object we are observing?
- **Example:**

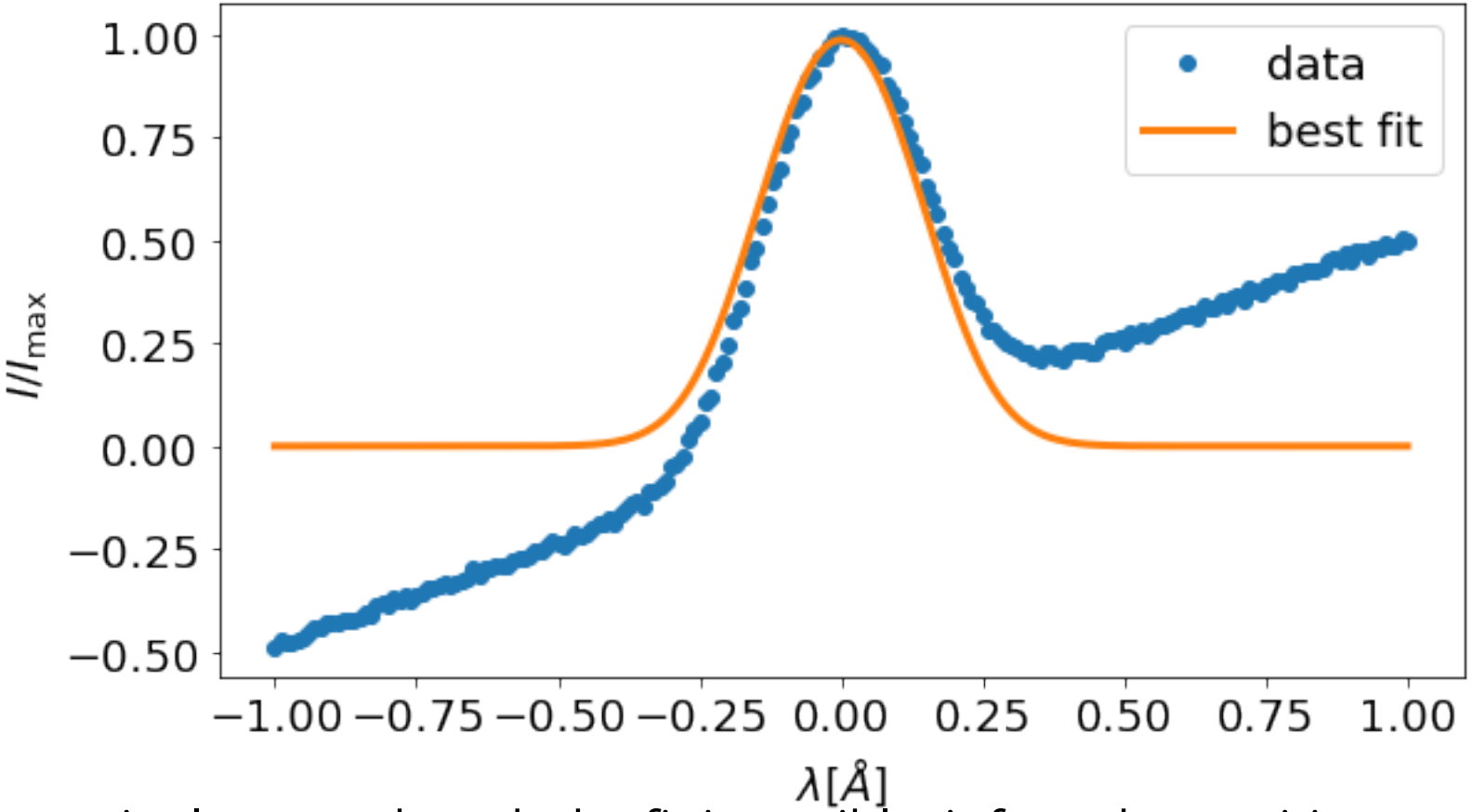
  In our magnetometry hands-on we found the line-of-sight magnetic field by fitting weak-field approximation to the observed Stokes V. What does this magnetic field mean?
- If our inferred quantity is, for any reason not representative of the reality, we are making a systematic error.

Systematic errors

- I will argue (we can disagree), that **systematic errors are essentially result of our incomplete knowledge of the model that generated the data.**

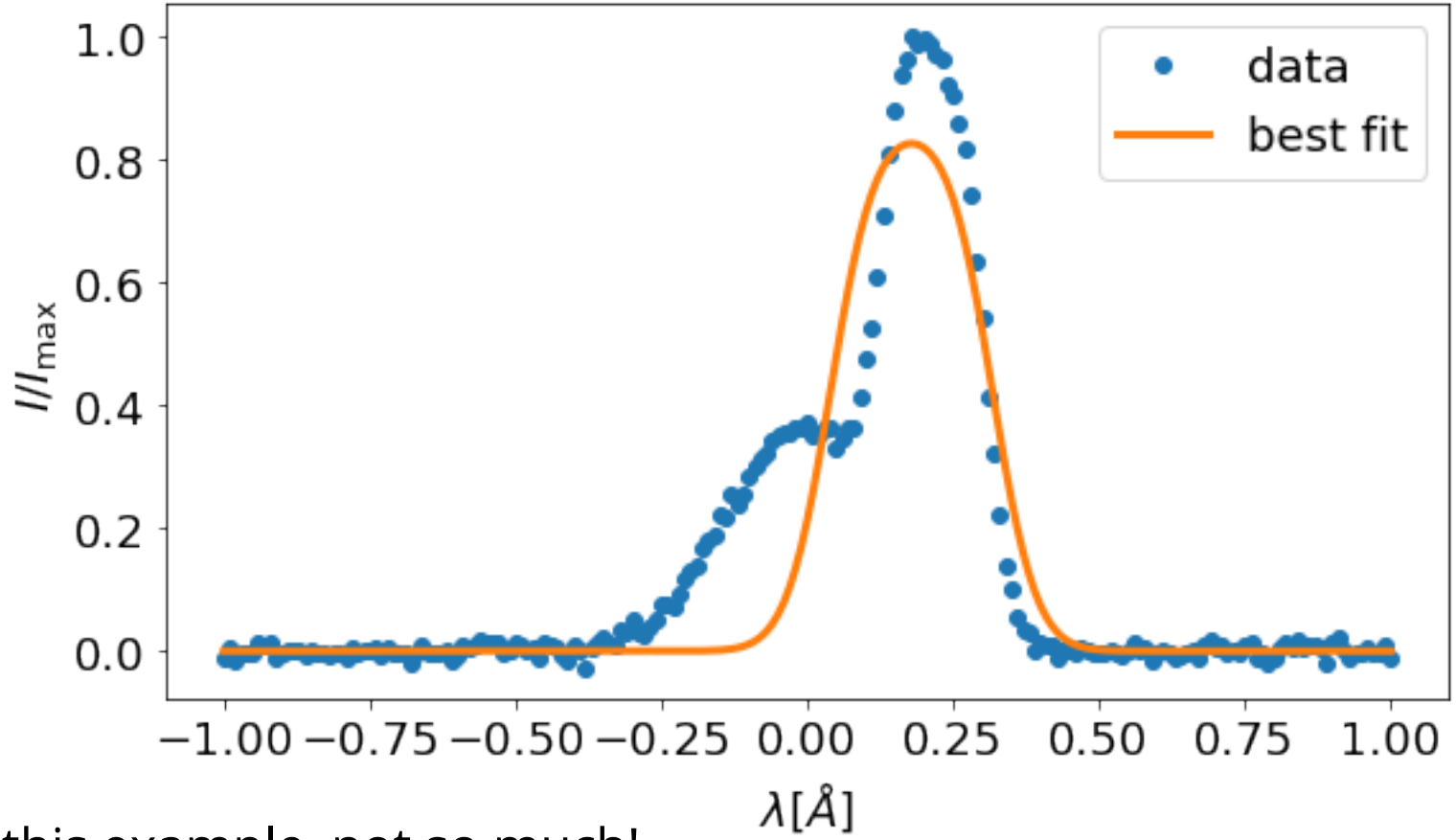
- This means that we can’t really ever remove all the systematics, we can only make them so small so they are insignificant with respect to the random uncertainties.

For example:



- Interestingly, even though the fit is terrible, inferred quantities are actually pretty close to what we cared about.

For example:



- In this example, not so much!

What was this excursion about

- To warn you that in spectropolarimetry, we are basically forced to use simplified models
- There are no homogeneous things, in neither of three coordinates
- Nothing, ever, is in equilibrium
- All wavelengths influence all other wavelengths
- There is no “one magnetic field”, “one velocity”, “one temperature”
- You almost certainly don’t know your true measurement uncertainties and systematics

But we still try!

- We analyze the spectra in as much detail we can to try and learn something about the solar atmosphere
- It gives us insight in the physical processes
- It gives us numbers!
- It’s fun!
- Let’s learn more about inversions then :-)
- (Going to talk more about photosphere and chromosphere now)

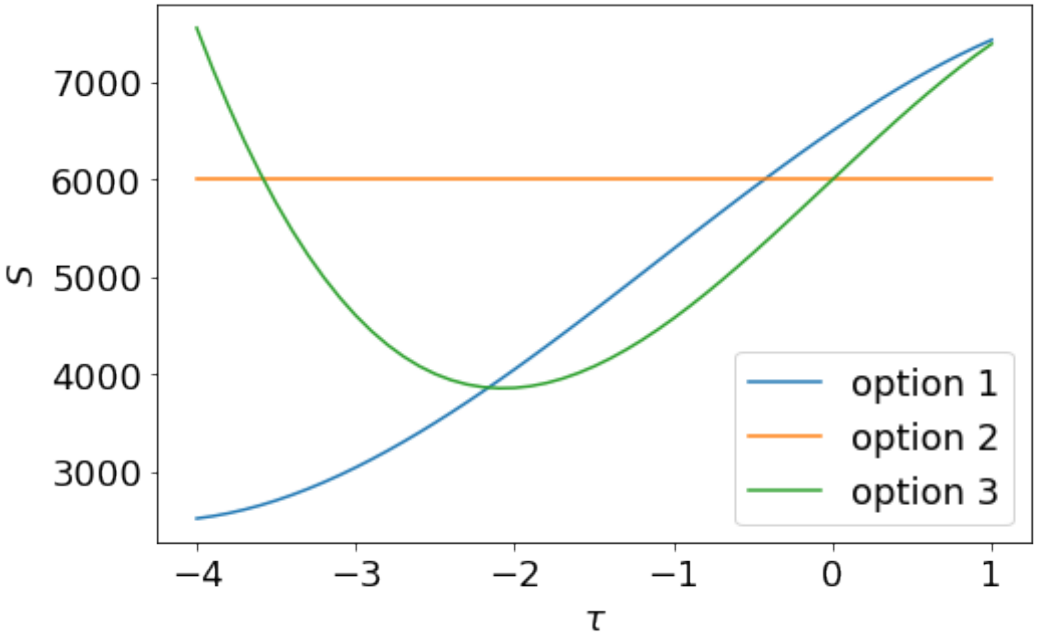
Another question, a bit more complicated one.

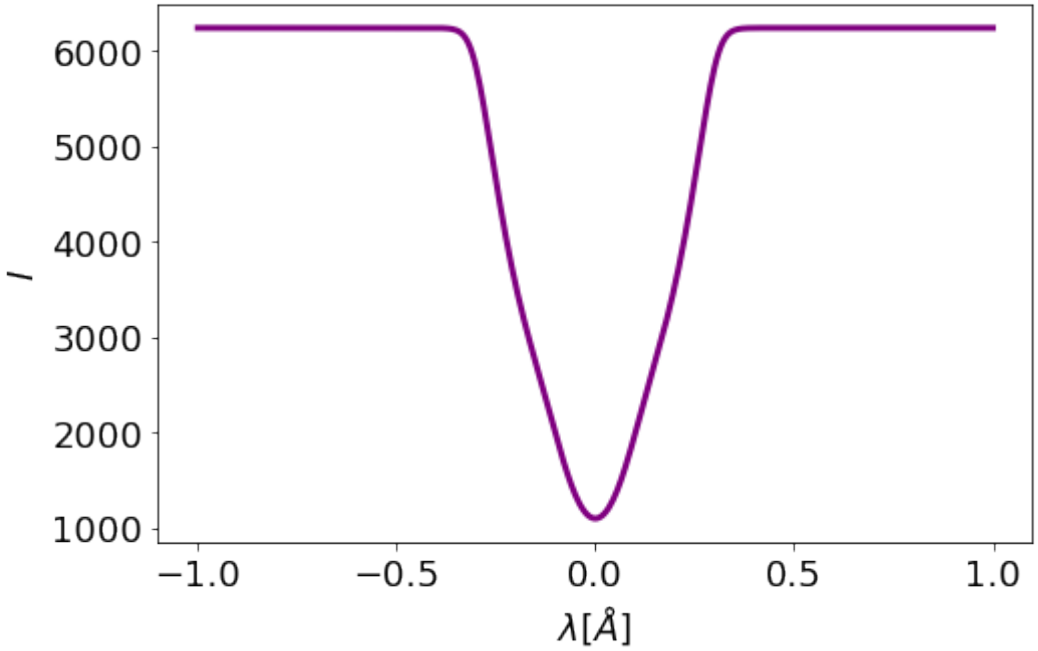
- Say our emergent intensity is generated by the formal solution of the RTE, where source function varies with depth:

$$I_\lambda = \int_0^\infty S(\tau_\lambda) e^{-\tau_\lambda} d\tau_\lambda$$
$$\tau_\lambda = \tau(1 + r\phi_\lambda)$$

- Let's say that the profile and r are fixed and the only "unknown" is the **function S(t).** Let's say we have a reason to believe r is large.
- **This is weird, let's talk about this setup.**

Which of the functions produces the profile on the right?





This was not **that** hard (it could have been)

- But what if I asked you to infer **exact** stratification of the source function.
- That would have been an inversion.
- **Inversion** is a mathematical term for finding an unknown function from it's integral. (We are inverting an integral)
- How do we do that?
- Well, for starters, all the stuff we do is discretized, so there **is** a limited (although large) number of unknowns.

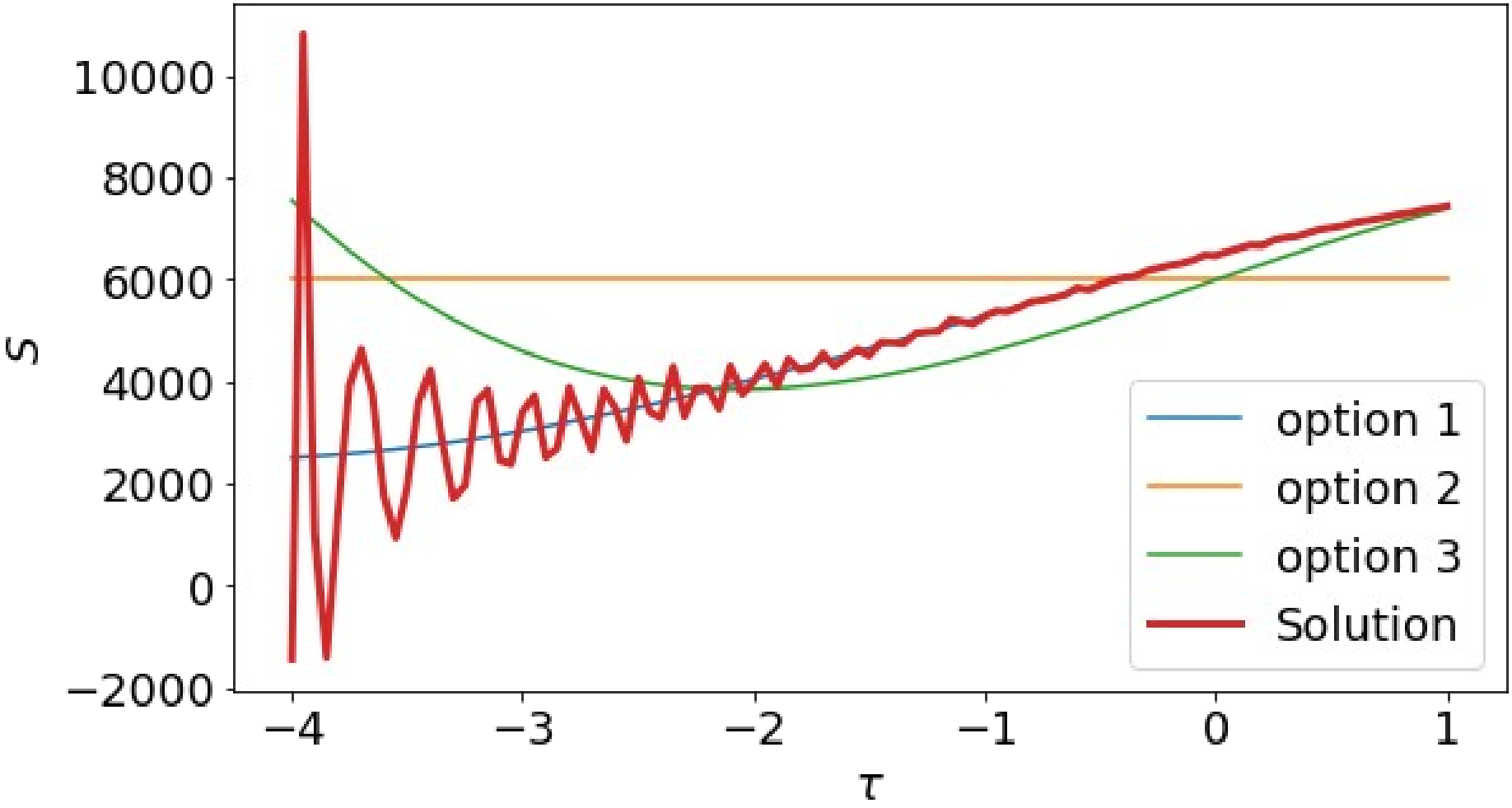
Remember this from the start of the course?

- In a numerical manner we can write down the formal solution as:

$$I_{0,\lambda} = \sum_{d} w_{0,d,\lambda} S_d$$

- In this simplified case we know the weights, we know the Intensity at a set of wavelengths, we only don't know S_d "vector".
- All we need to do now is solve system of equations and we done. Right?
- Right?

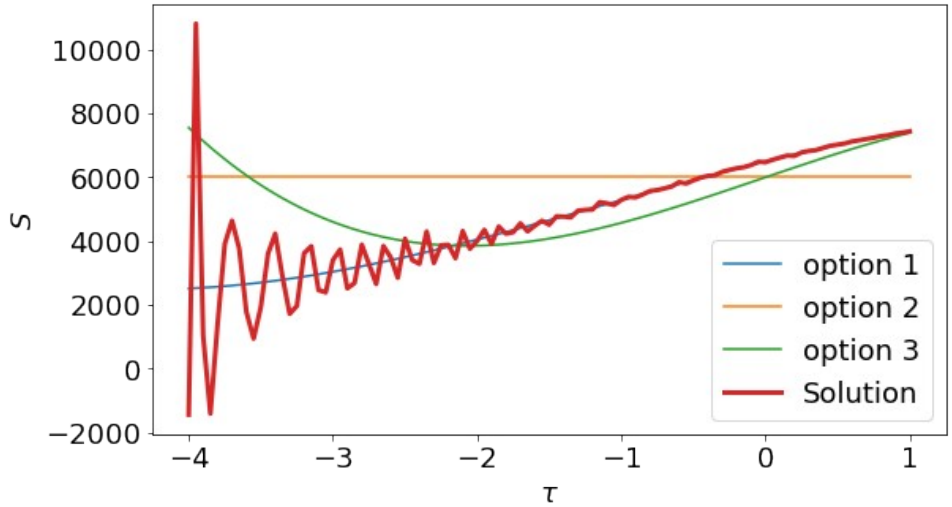
Wrong (well, out of the box - wrong)



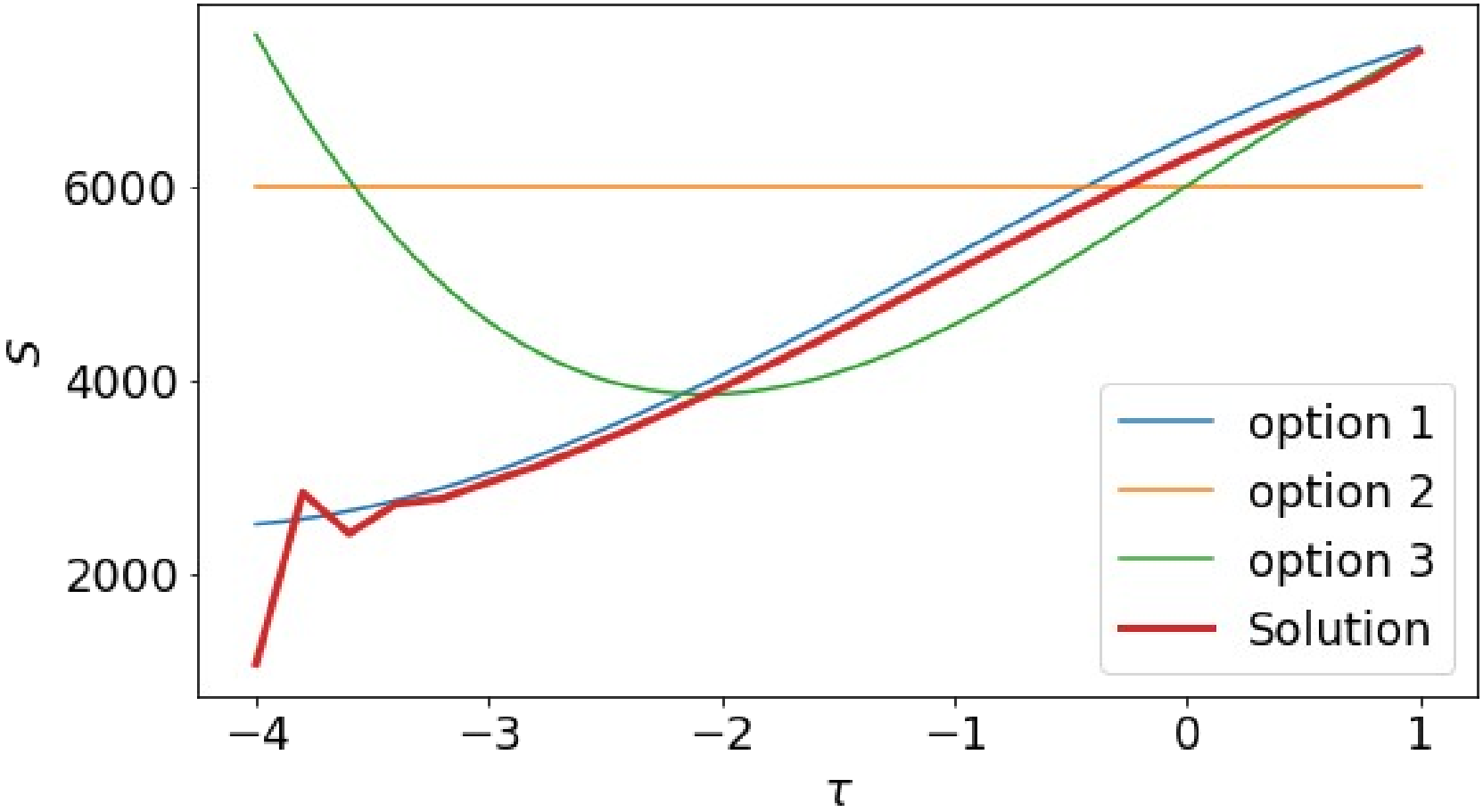
What did just happen?

- The system of the equation is what we call “ill posed”.
- You can think about it in the following way: too many equations are very similar and thus it is hard to find exact unknowns
- Solution?
- Decrease the amount of unknowns!

$$I_{0,\lambda} = \sum_d w_{0,d,\lambda} S_d$$



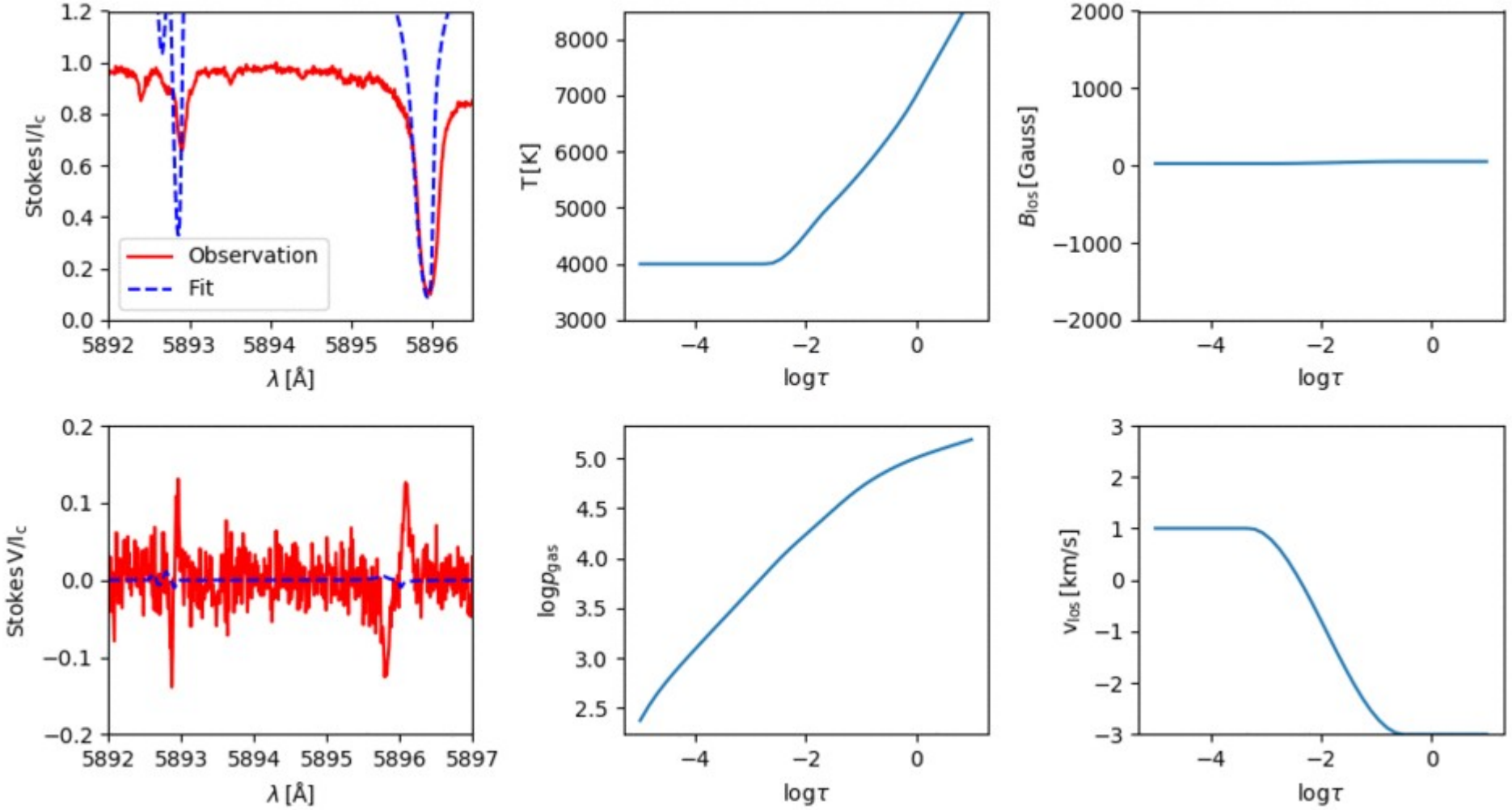
Now it kinda works



What happens in the “real world”

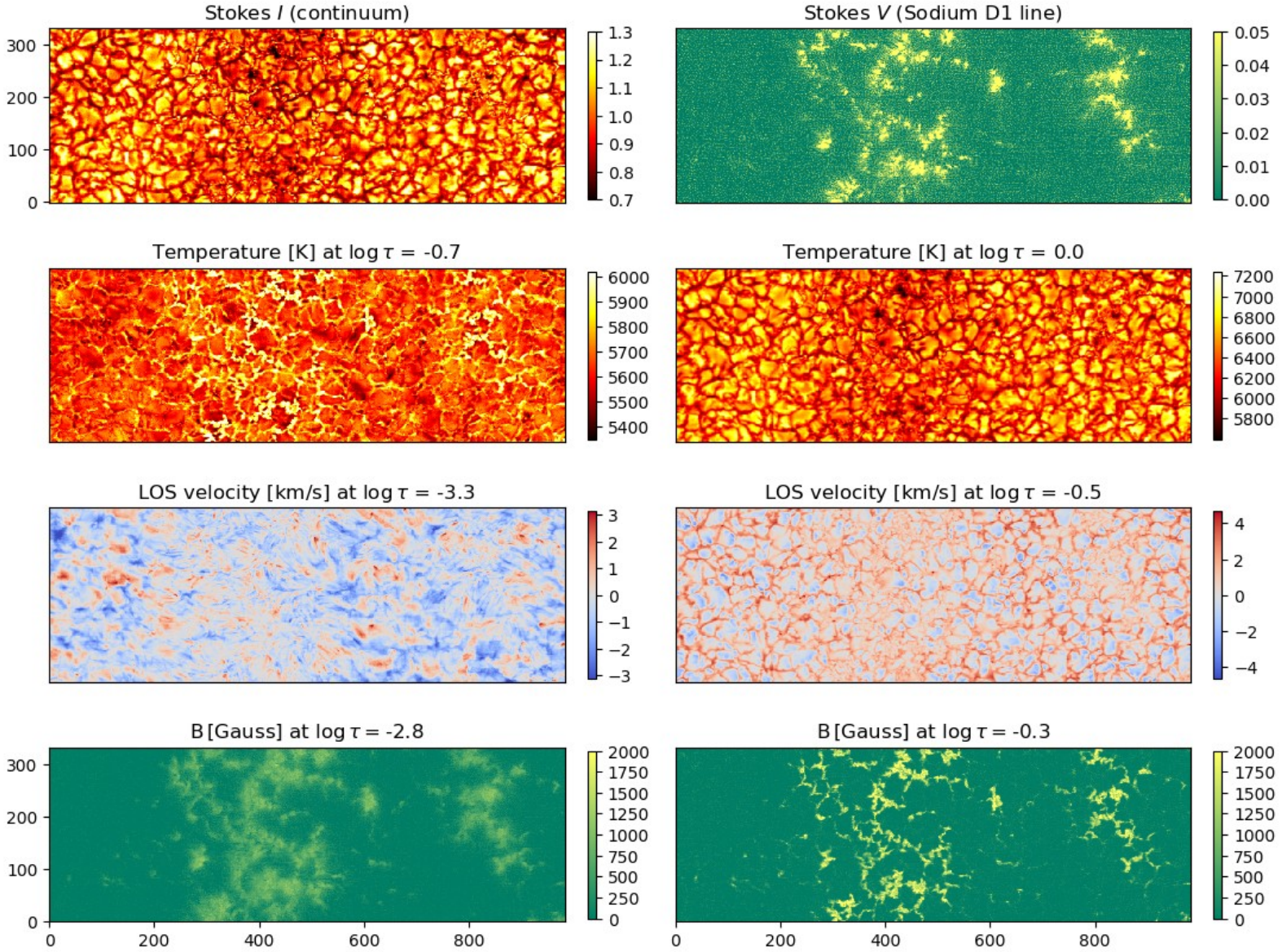
- It’s not the source function that is considered unknown, but temperatures, velocities, magnetic fields, pressures, etc…
- These influence the spectrum via various processes: ionization, excitation, broadening, collisional damping, Zeeman effect…
- And (don’t forget), in NLTE, radiation “influences itself”, that is **radiation and state of matter are coupled, so we have to account for that as well**
- Generally the situation is very complicated, but can be solved (see the list of references later)

This is how real life inversion looks like

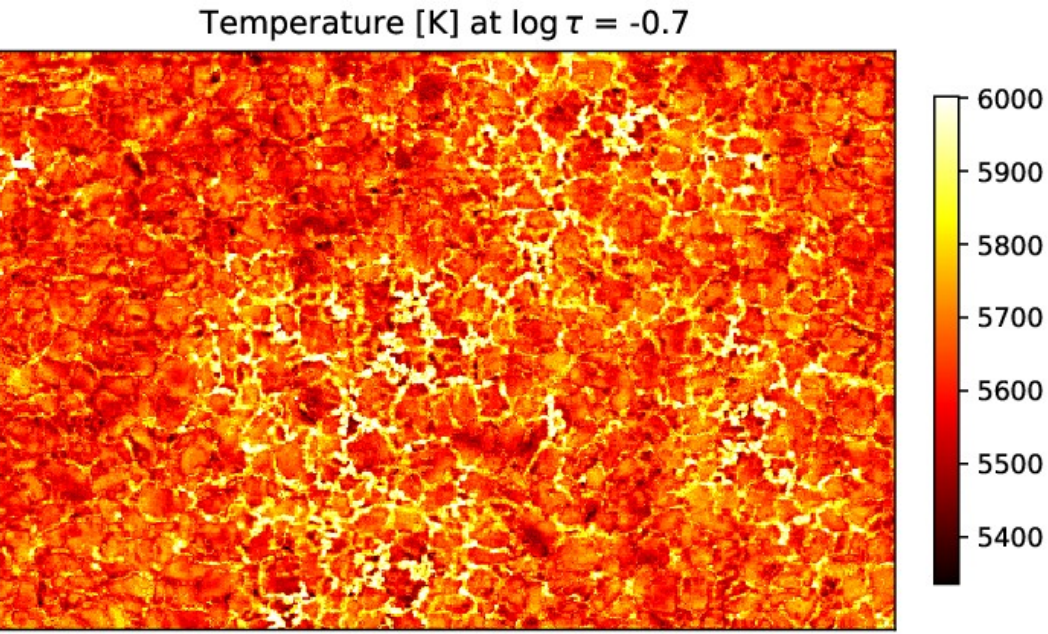


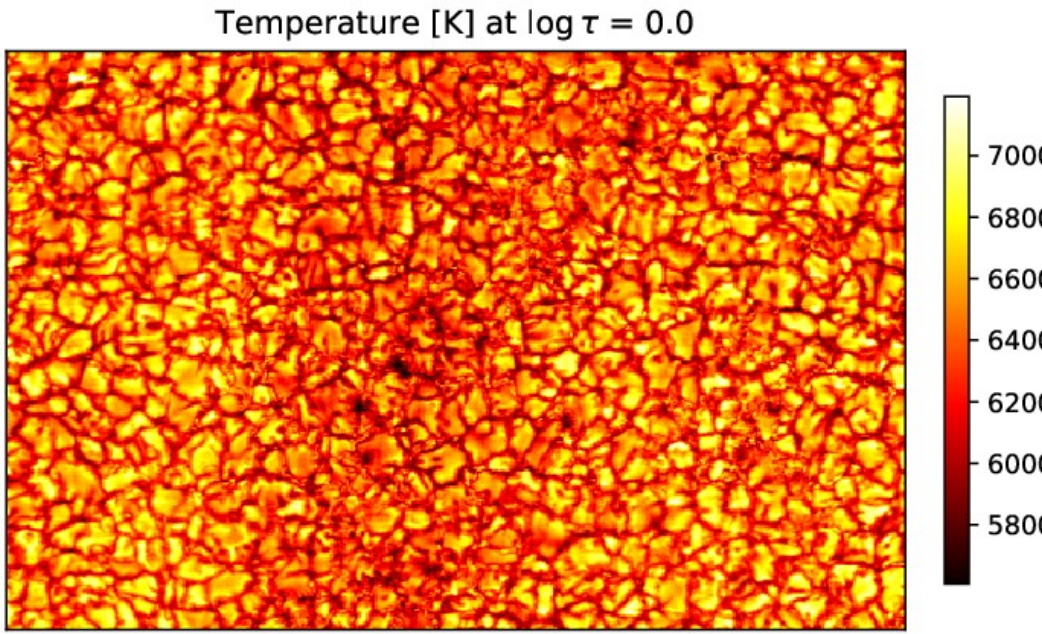
And this is what we get!

- I like to call this: ”Inversion process in one slide”
- First two panels are (examples of) observables, other 6 are parameter maps



I got some bigger maps that look nice





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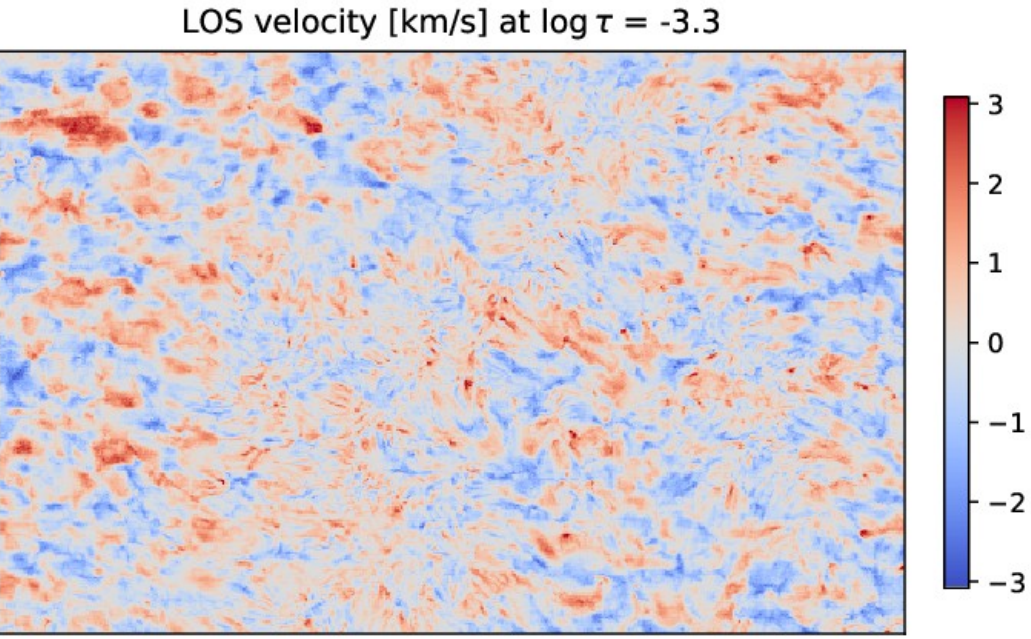

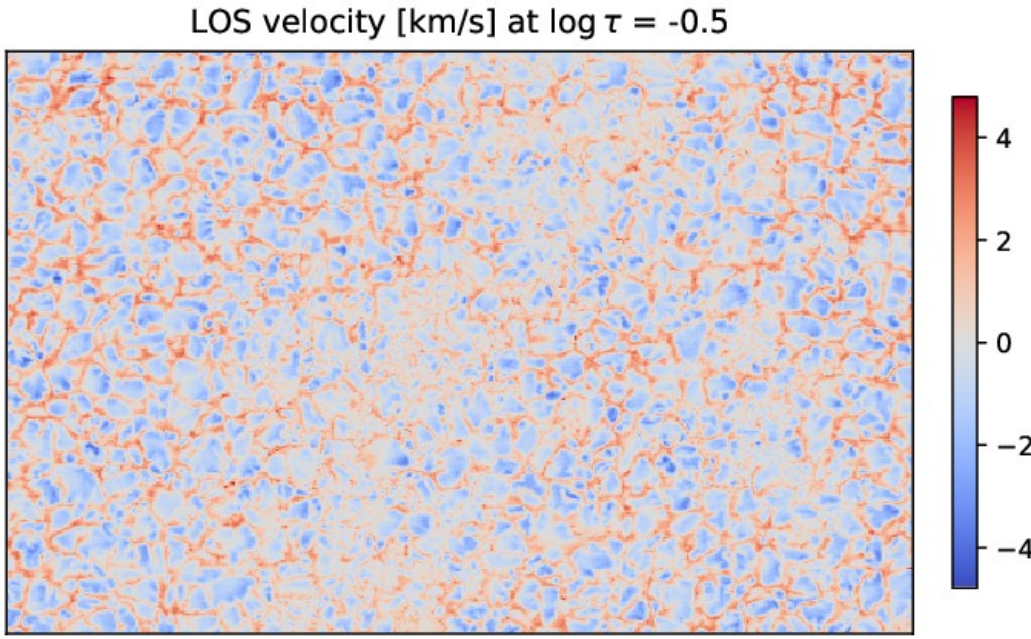

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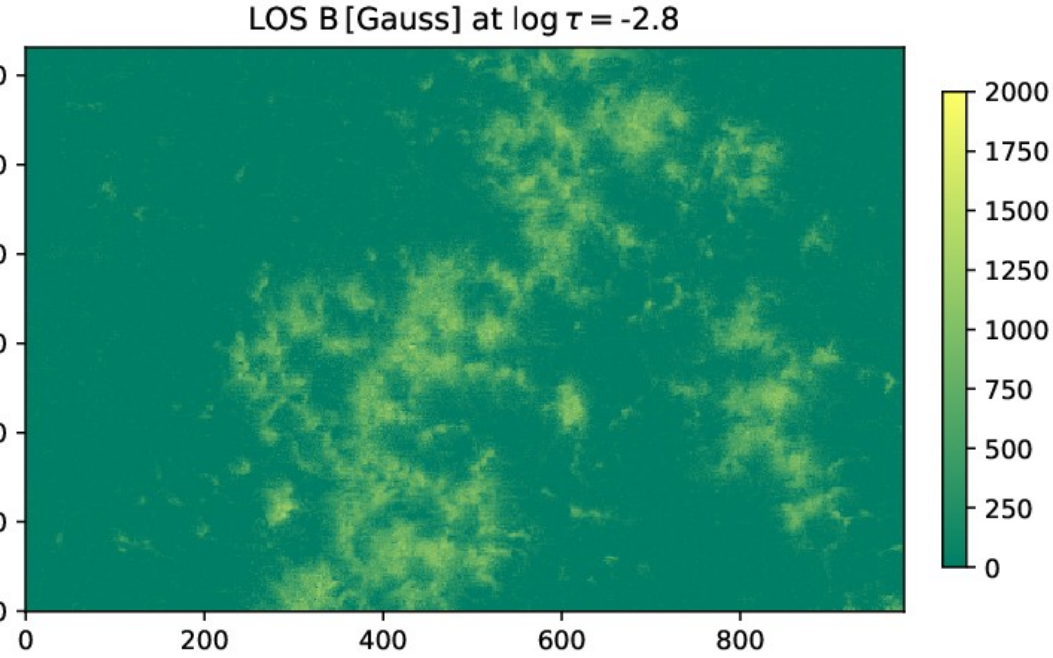
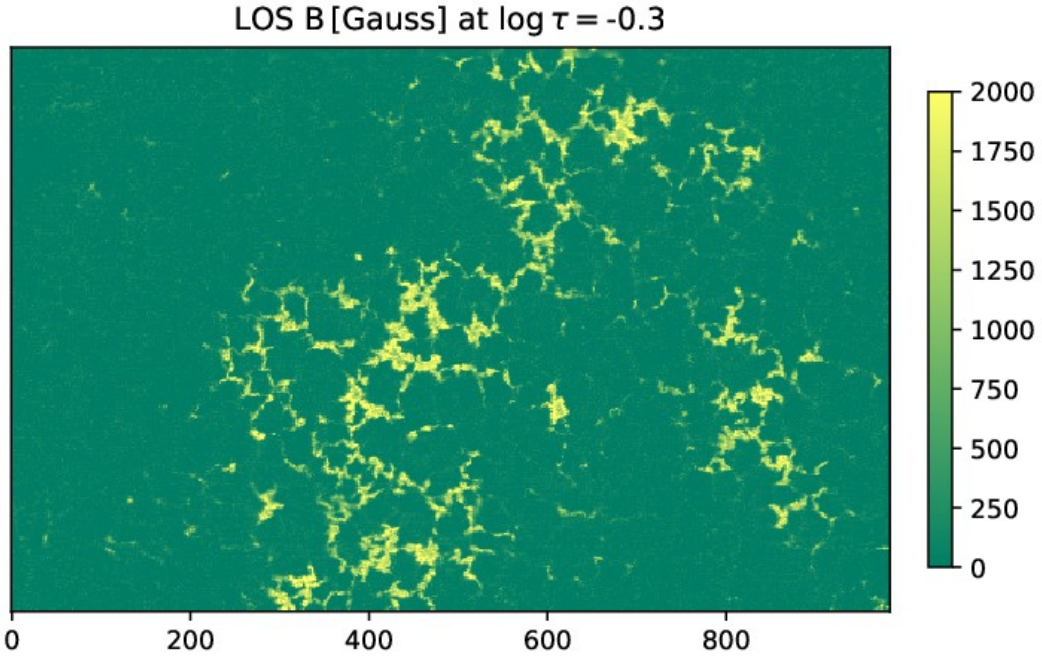




So, how would we describe inversions

- Methods that aim to constrain a model atmosphere so that once RTE is solved from that model, it yields the spectrum that fits best the observed spectrum.

$$\frac{dI_\lambda}{dz} = -\chi_\lambda I_\lambda + j_\lambda$$

- Physics hides in the absorption and emission coefficients, numerics in the integration of this beast.
- Still the question remains : *"What is the best fit?"*

A little bit of probability

- Focus on one specific measurement (this means one wavelength)
- Let's say that we know the model that **generates** the data perfectly (meaning both the equations as well as values of model parameters)
- However, let's say that there is a random number, called **noise** (measurement uncertainty) superimposed on top of our value
- Assume that random number is drawn from a Gaussian distribution with mean **0** and some width **sigma.**
- Let's write down the probability of measuring some value **y**

So, here we are:

$$y_{\text{true}} = f(\mathbf{M})$$

$$y = y_{\text{true}} + \epsilon$$

$$p(\epsilon) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\epsilon^2/2\sigma^2}$$

$$p(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(y-f(\mathbf{M}))^2/2\sigma^2}$$

Then for the whole set of points:

$$p(y_0, y_1 \ldots) = \prod_i \frac{1}{\sqrt{2\pi}\sigma} e^{-(y_i - f(\mathbf{M}, x_i))^2/2\sigma^2}$$

- This is what we call **likelihood.** It tells us how likely we are to get a set of measurements, given a set of parameters **M.**
- Chi-squared is just log likelihood, up to a constant.

$$\boxed{\chi^2(\mathbf{M}) = \sum_i \frac{(y_i - f(\mathbf{M}, x_i))^2}{\sigma^2}}$$

Chi-squared minimization

- This is just maximizing likelihood. So whenever you see “max likelihood” method, it is basically this.
- There is more to inference, but about it, some other time.

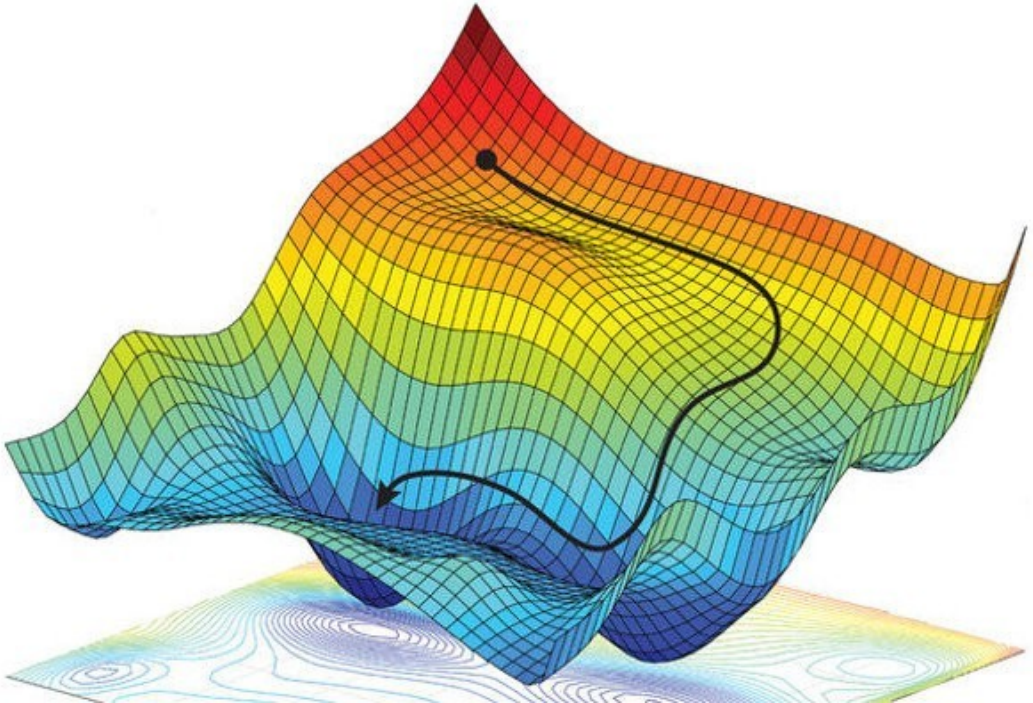
Probability of B occurring given evidence A has already occurred

Probability of A occurring

$$P(\mathrm{A}|\mathrm{B}) = \frac{P(\mathrm{B}|A) \cdot P(A)}{P(B)}$$

Probability of A occurring given evidence B has already occurred

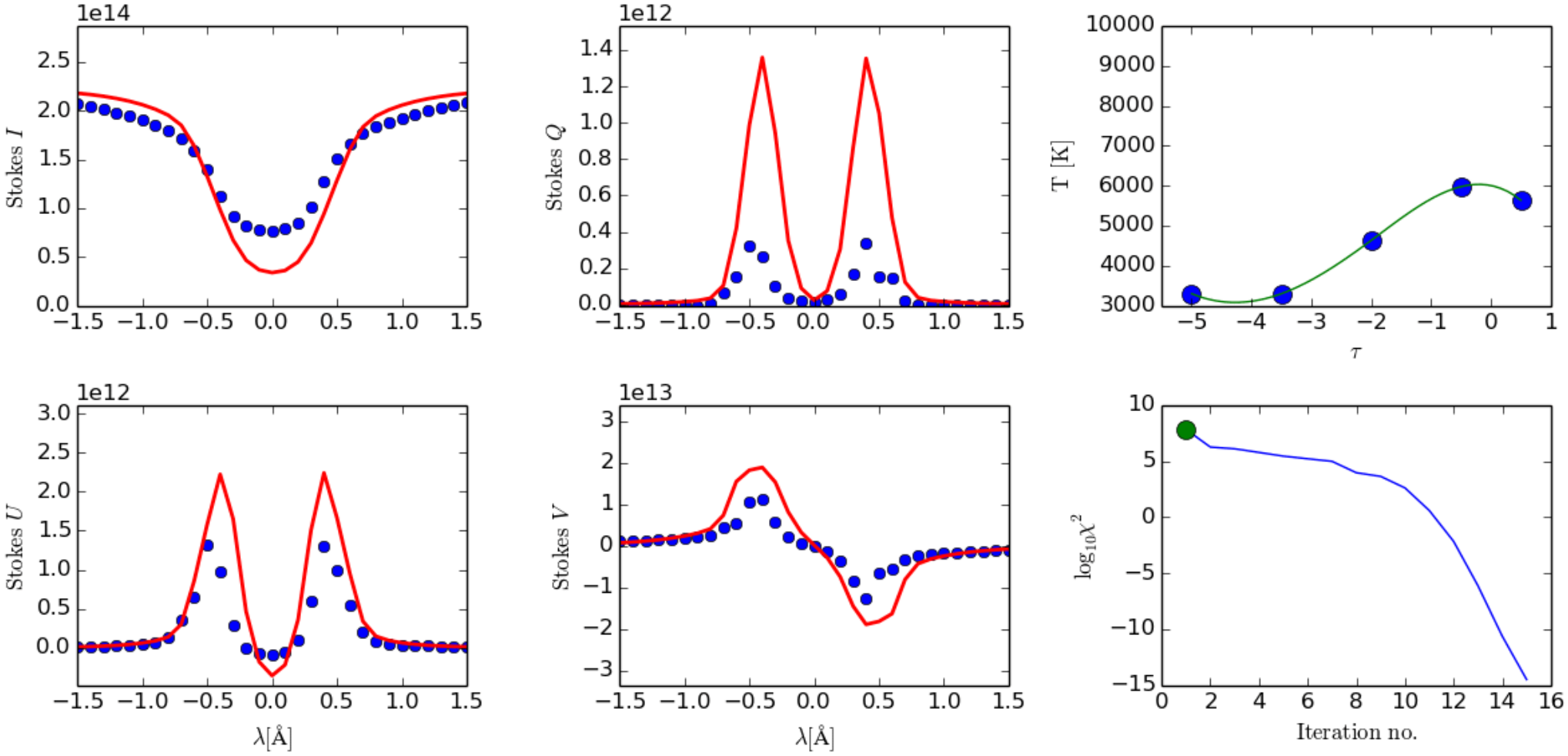
Probability of B occurring



What does it mean for spectropolarimetry?

- We are always fitting the spectra with a model, but the models have a range of complexities!
- **Slab + magnetic field** (As in Hazel)
- **Constant properties with linear source function** (Milne-Eddington, recall HW1)
- **Stratified atmosphere, with depth dependent physical parameters** (node-based LTE and NLTE inversions)
- **Coronal optically thin models** (various “global” inversions that try to reconstruct corona in 3D)

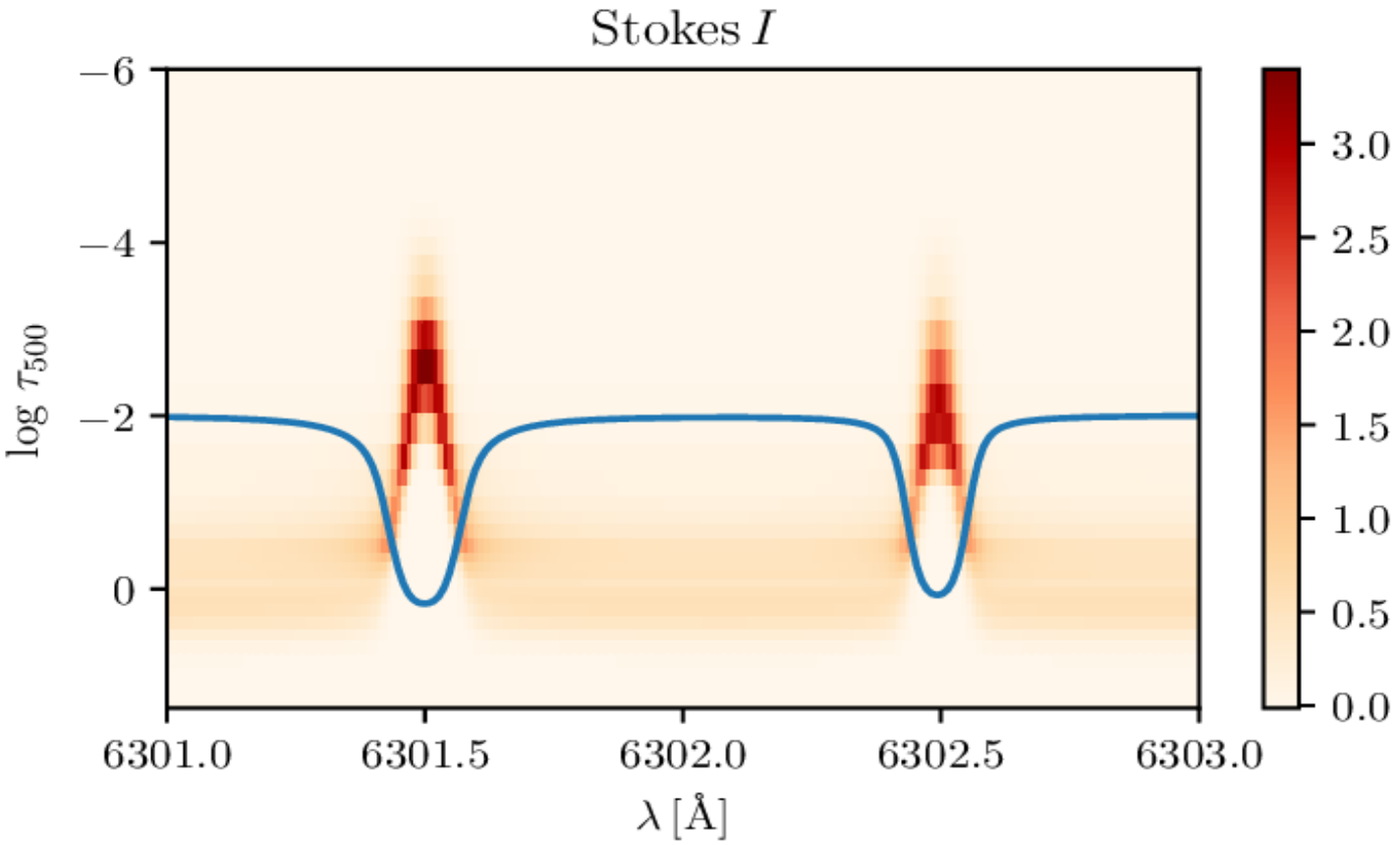
A common thing is that a metric is minimized



My specialty are “atmospheric” inversions, so here is a list

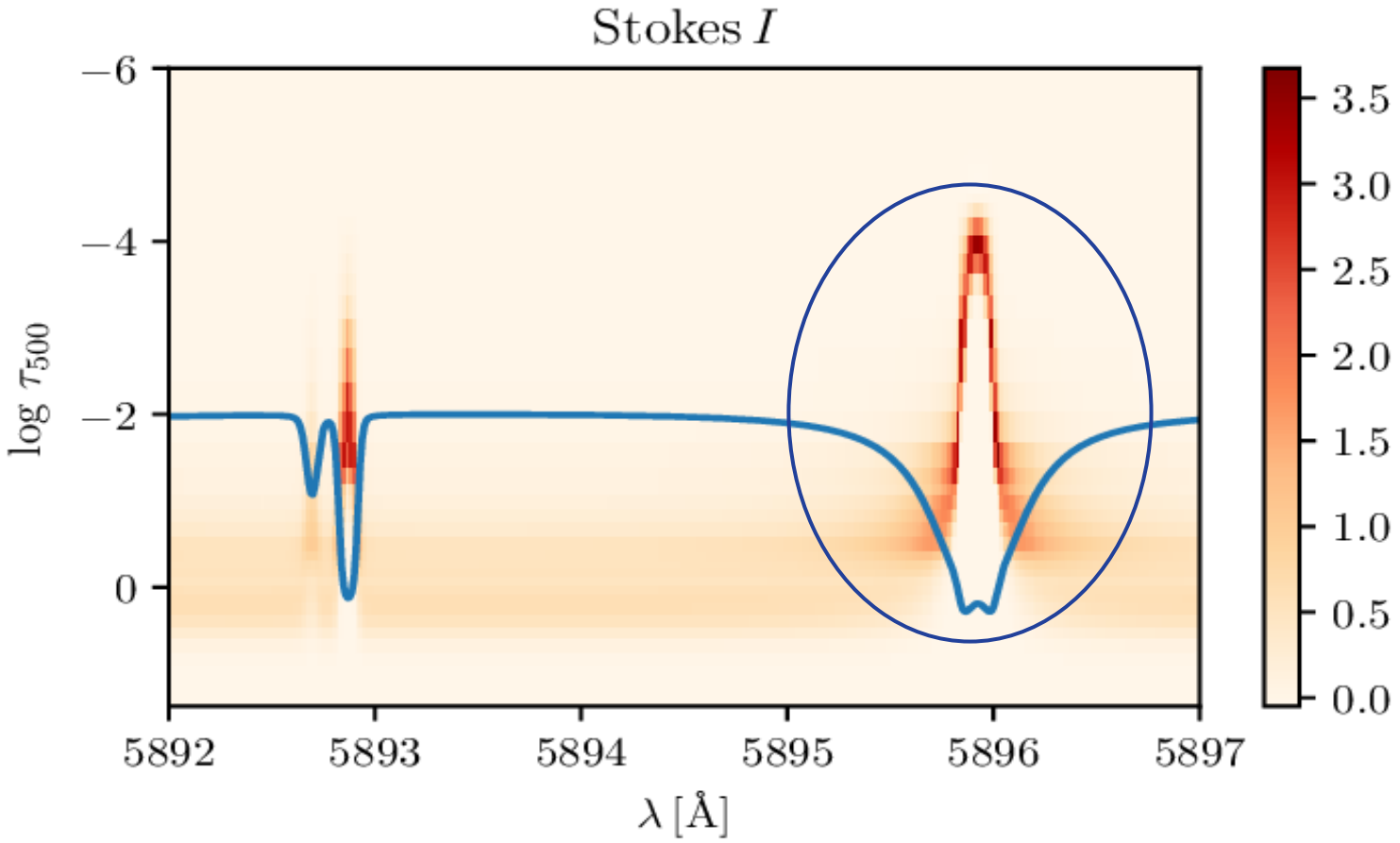
- SIR code – Ruiz Cobo & Del Toro Iniesta, 1992
- SPINOR code – Frutiger 2000 (phd thesis)
- NICOLE nlte code – Soccas Navaro et al. 2000 +
- STiC code (de la Cruz Rodriguez et al. 2019+)
- SNAPI code (Milic & van Noort 2018+)
- DESIRE code (Ruiz Cobo, Uitenbroek, Orozco Suarez, Gafeira et al, 2021)
- *Actual links to the papers will be posted at the website*

And all these codes use **response functions**



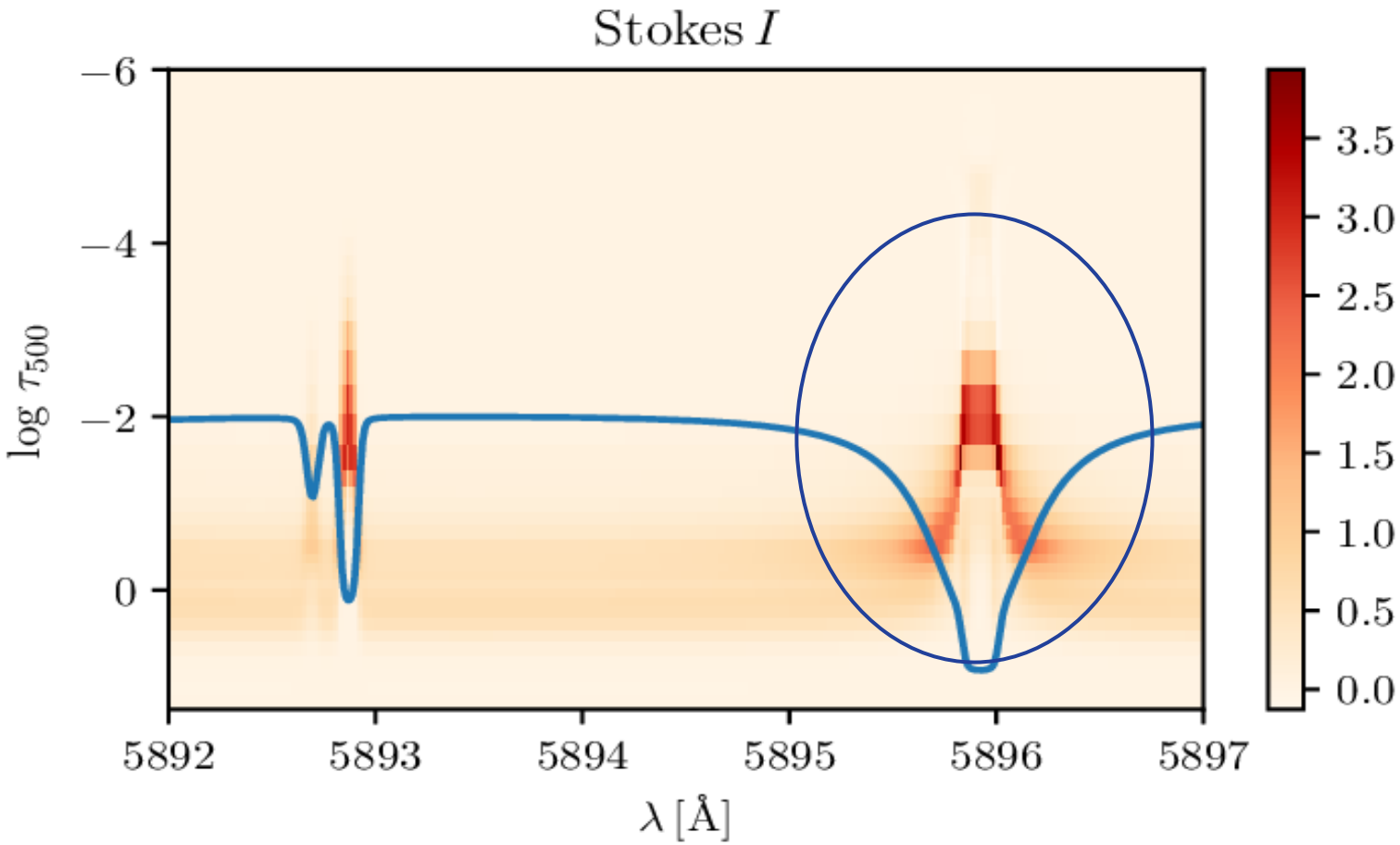
$$\frac{\partial I(\lambda)}{\partial T_d}$$

Other spectral lines – Na I D



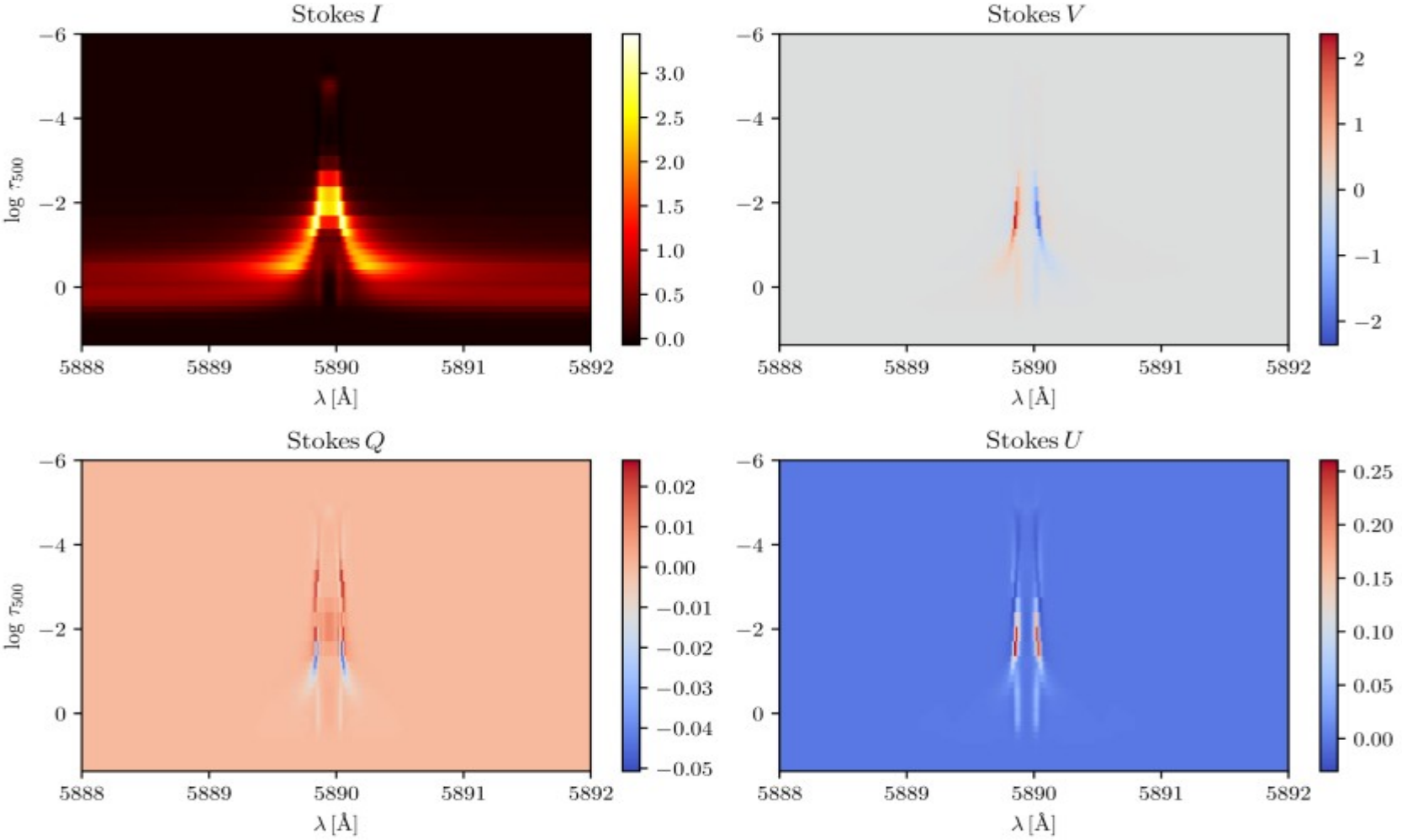
$$\frac{\partial I(\lambda)}{\partial T_d}$$

Ah sweet, this NLTE stuff actually is important

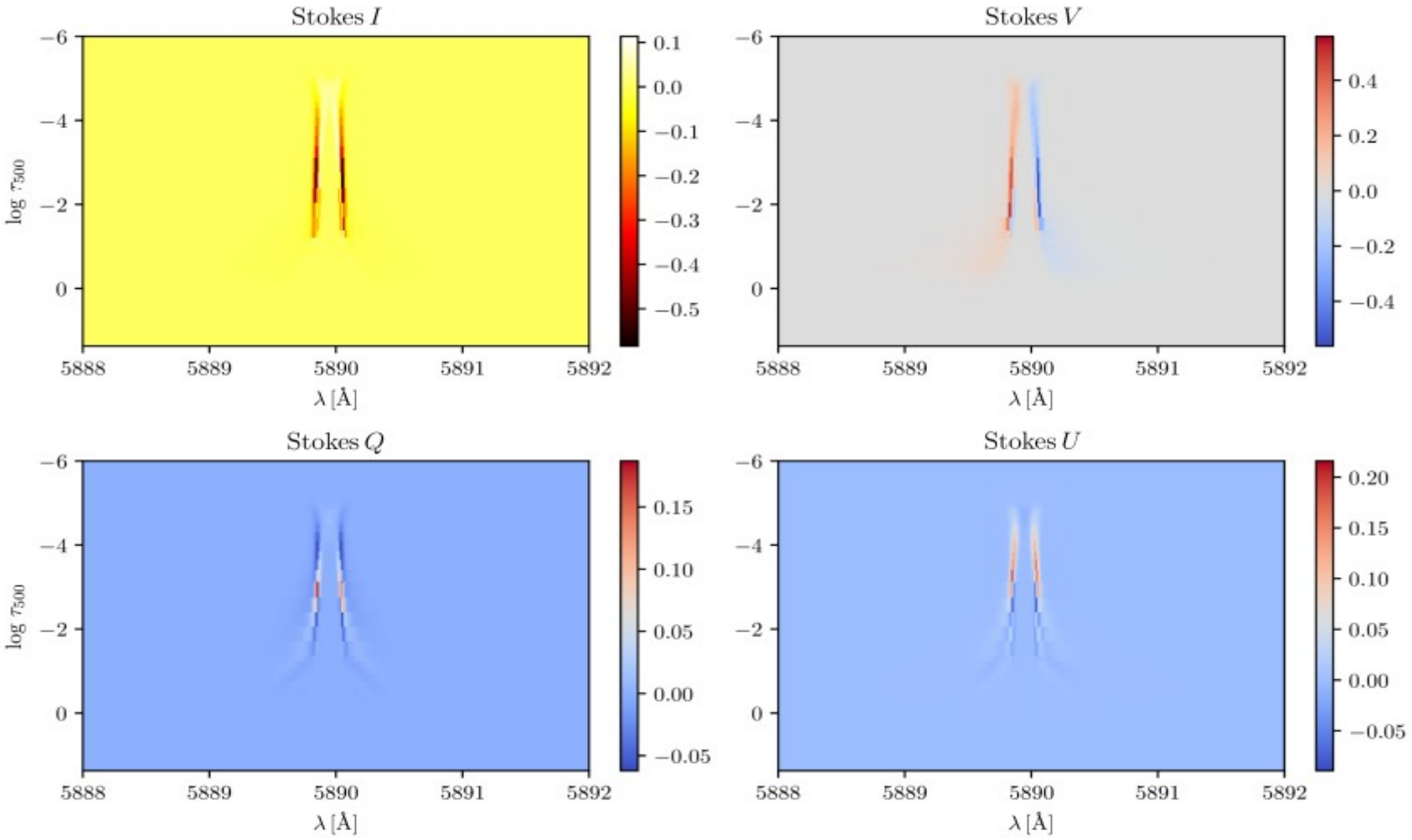


$$\frac{\partial I(\lambda)}{\partial T_d}$$

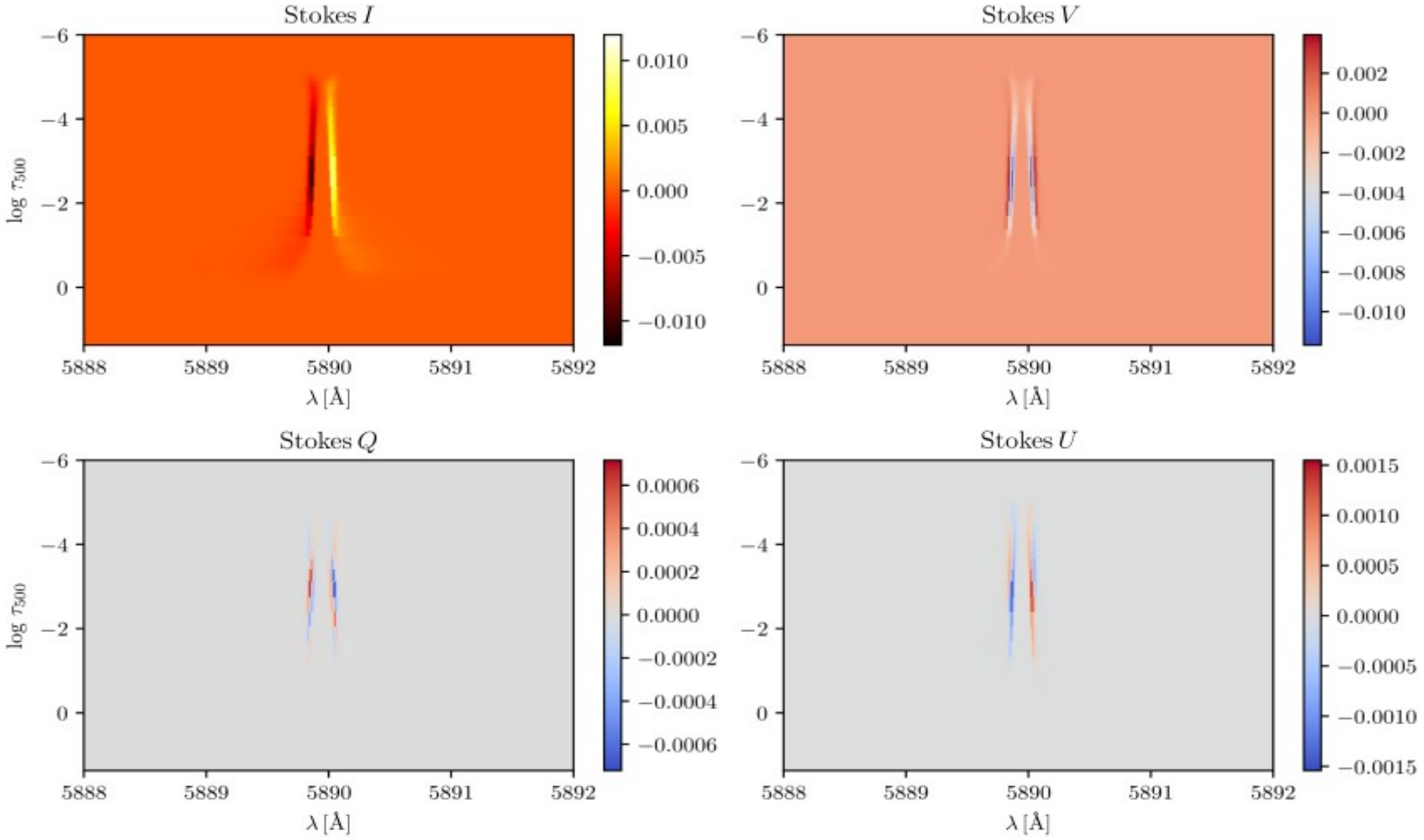
We can plot these for all four Stokes parameters:



And for other physical parameters – e.g. magnetic field

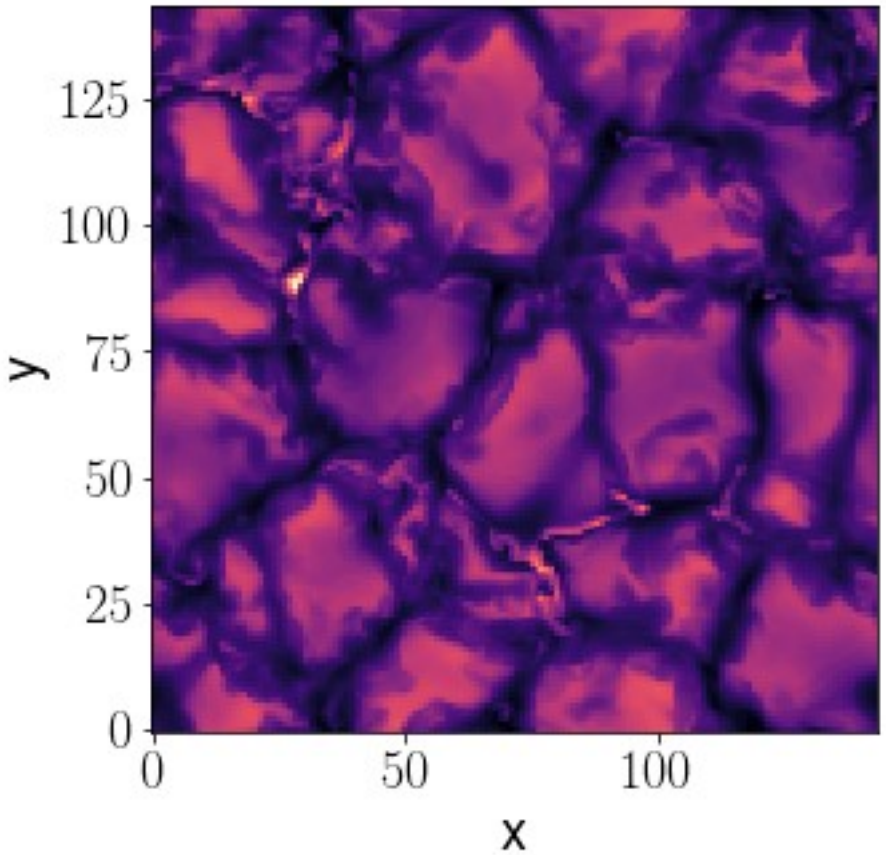


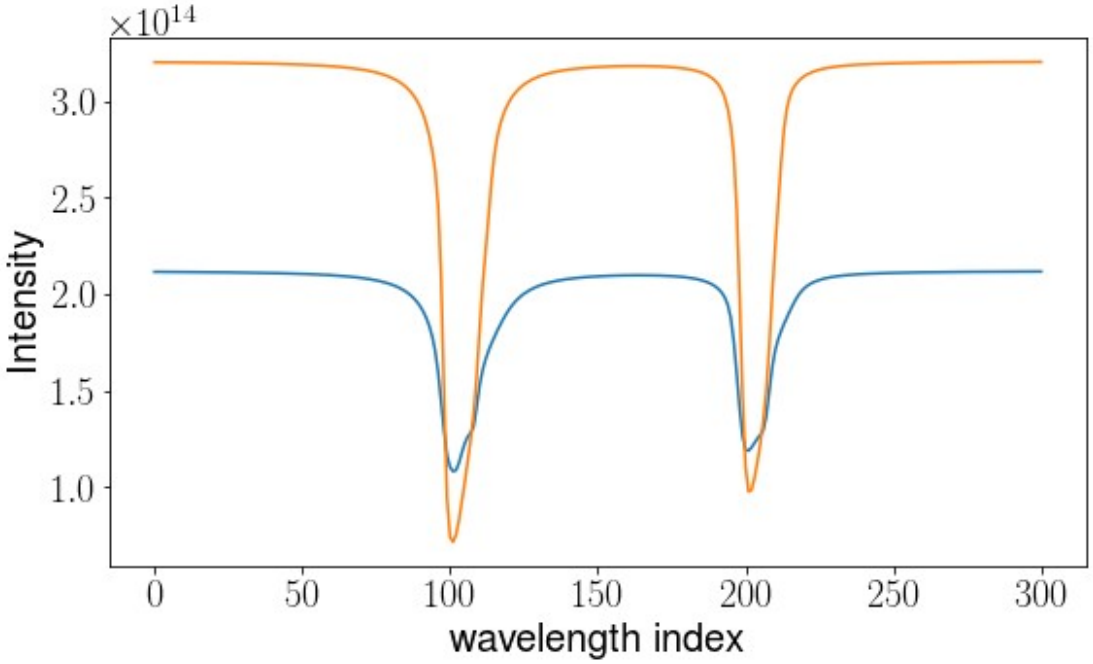
Or line of sight velocity:



Remember, **wavelength → depth**

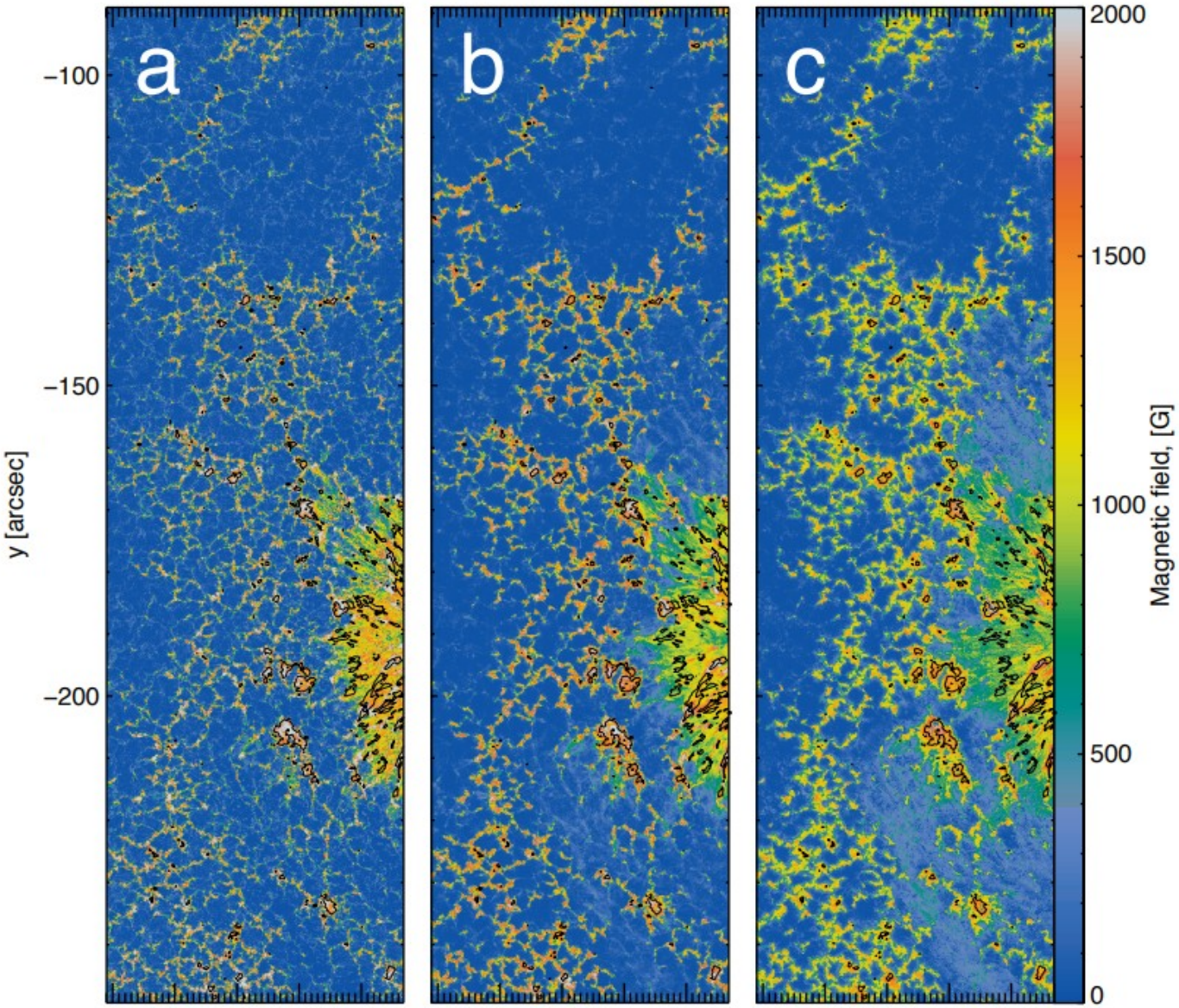
- Spectra yields information on the depth stratification, not only on one layer!

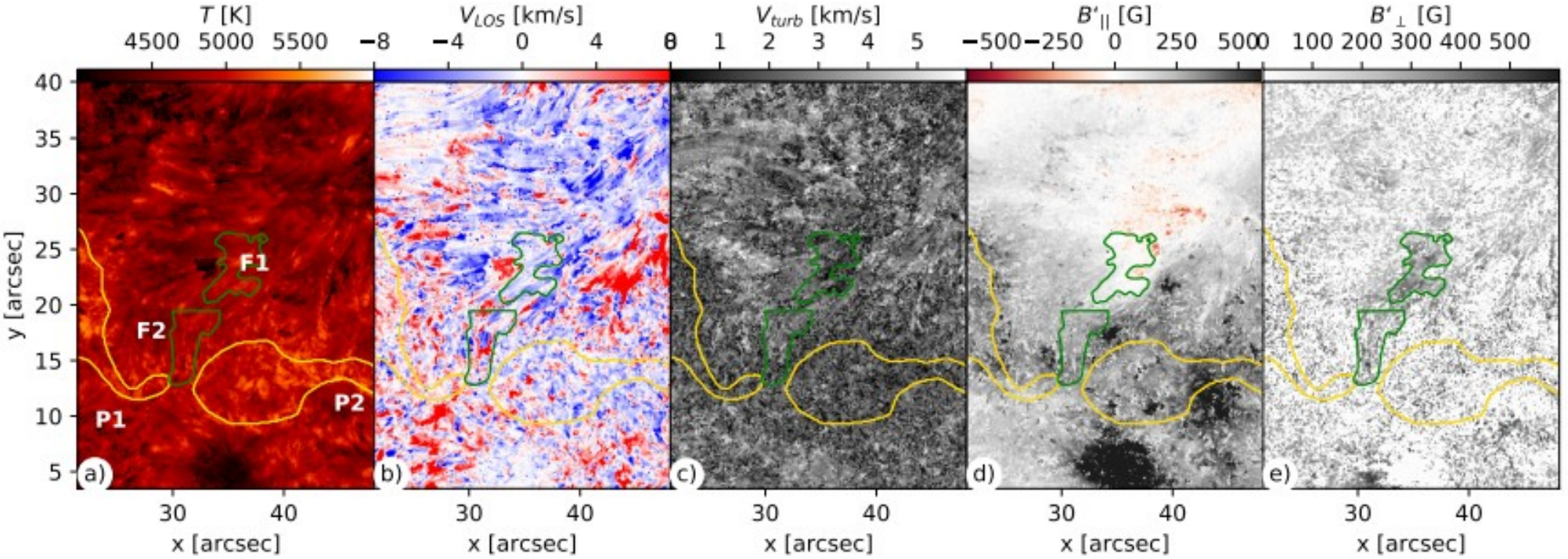


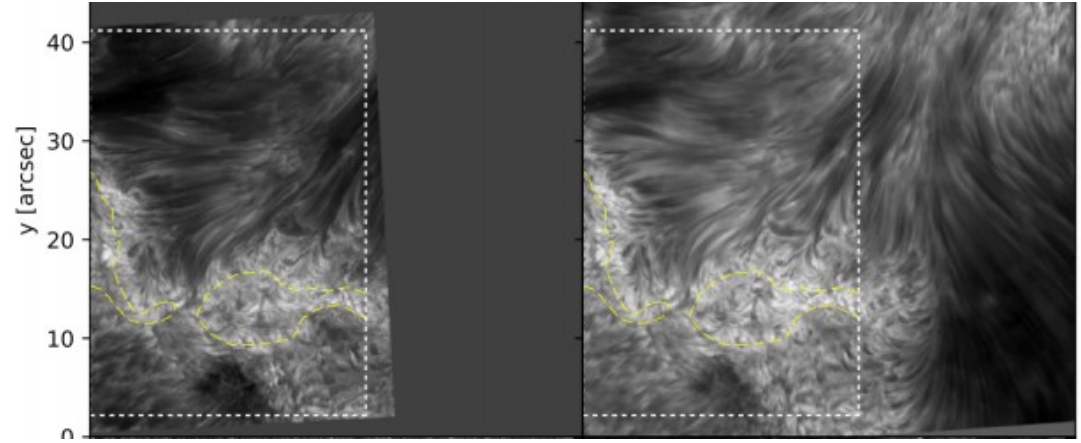


These are spectra of a bright and a dark pixel.

- Now we can maybe understand these better
- Bueheler et al. (2015), look at depth dependent magnetic field in a plage
- Look at that beauty!
- Does the structure of the magnetic field make sense?







From Pietrow et al. 2020

Summary

- Inversions are physically motivated models that are fit to the observed data using some non-linear optimization
- I like to describe them as “mini-simulations” that you cleverly tune so they would agree with your data
- State-of-the-art inversions allow us to infer depth stratification of the physical parameters and thus map the solar atmosphere in 3D
- Inversions are complicated, tricky, powerful, frustrating, but **fun**