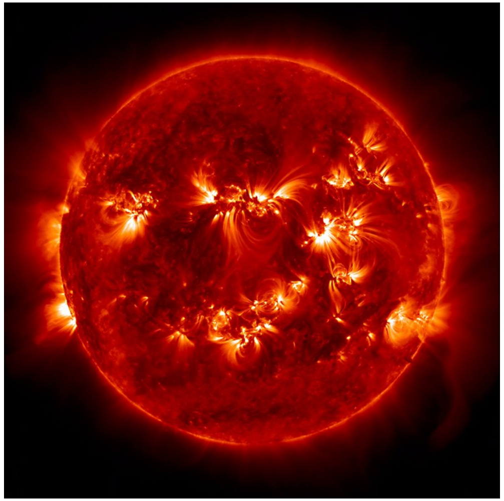
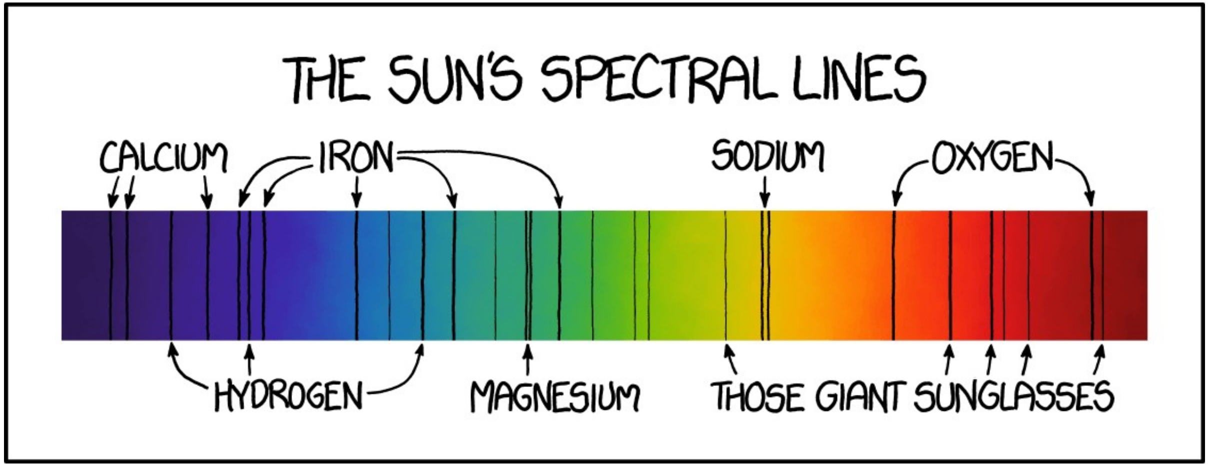
PHYS 7810 / COLLAGE2021:

Solar Spectral Line Diagnostics





Lecture 17: Magnetic Field Diagnostics

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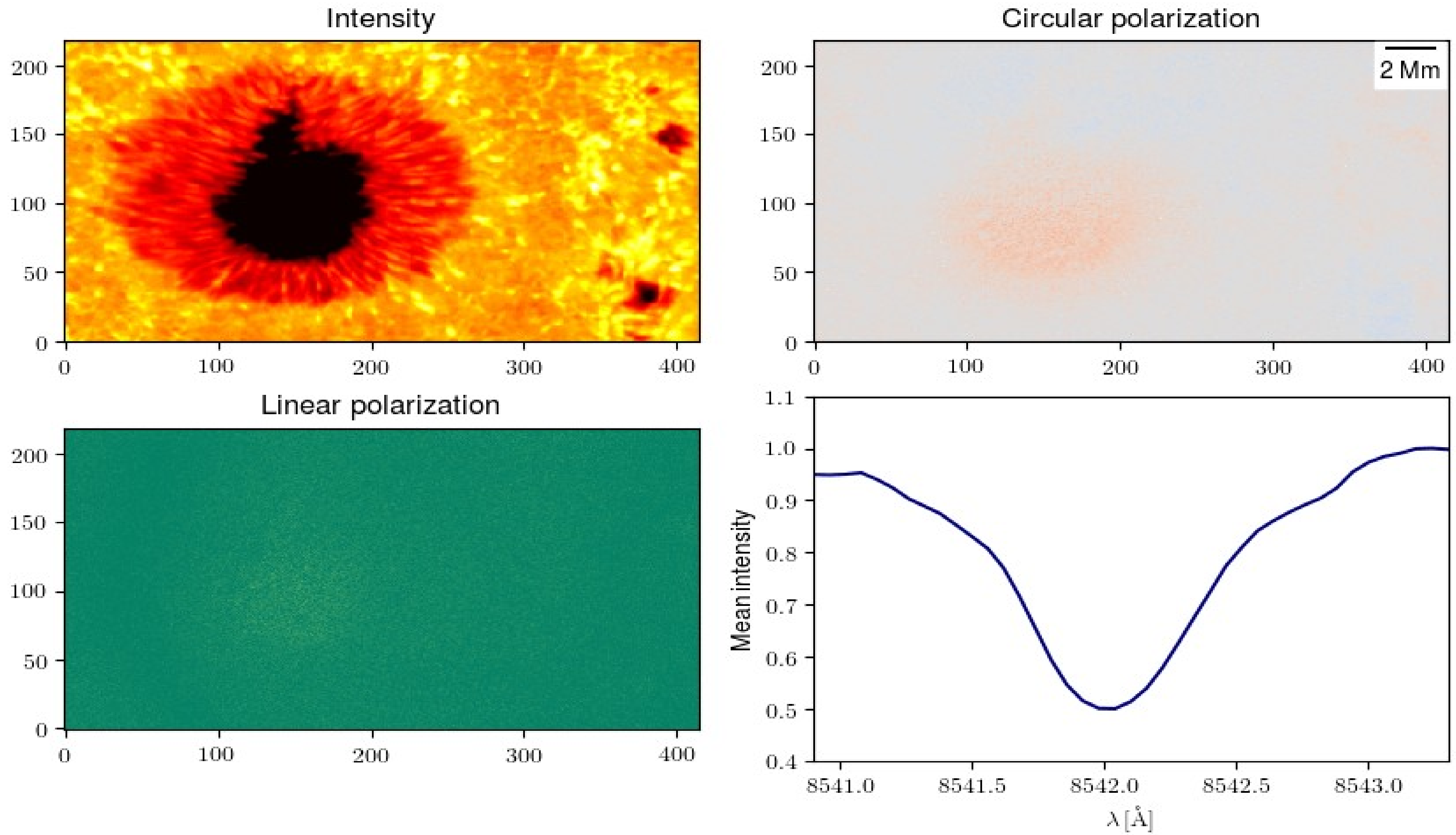
Previous class

- Last time we saw what is the structure of the atomic levels and what magnetic field does do the levels (breaks degeneracy in m, and thus levels split into 2J+1 components)
- Today we we will see how we can use that to ~~measure~~ estimate magnetic fields in the Sun

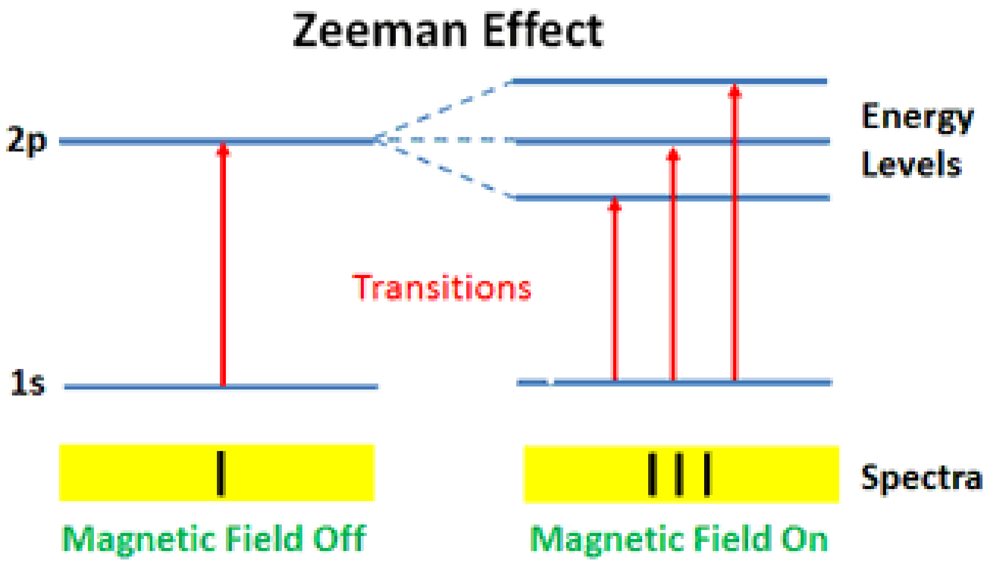
Let's look at a so-called "Stokes cube"



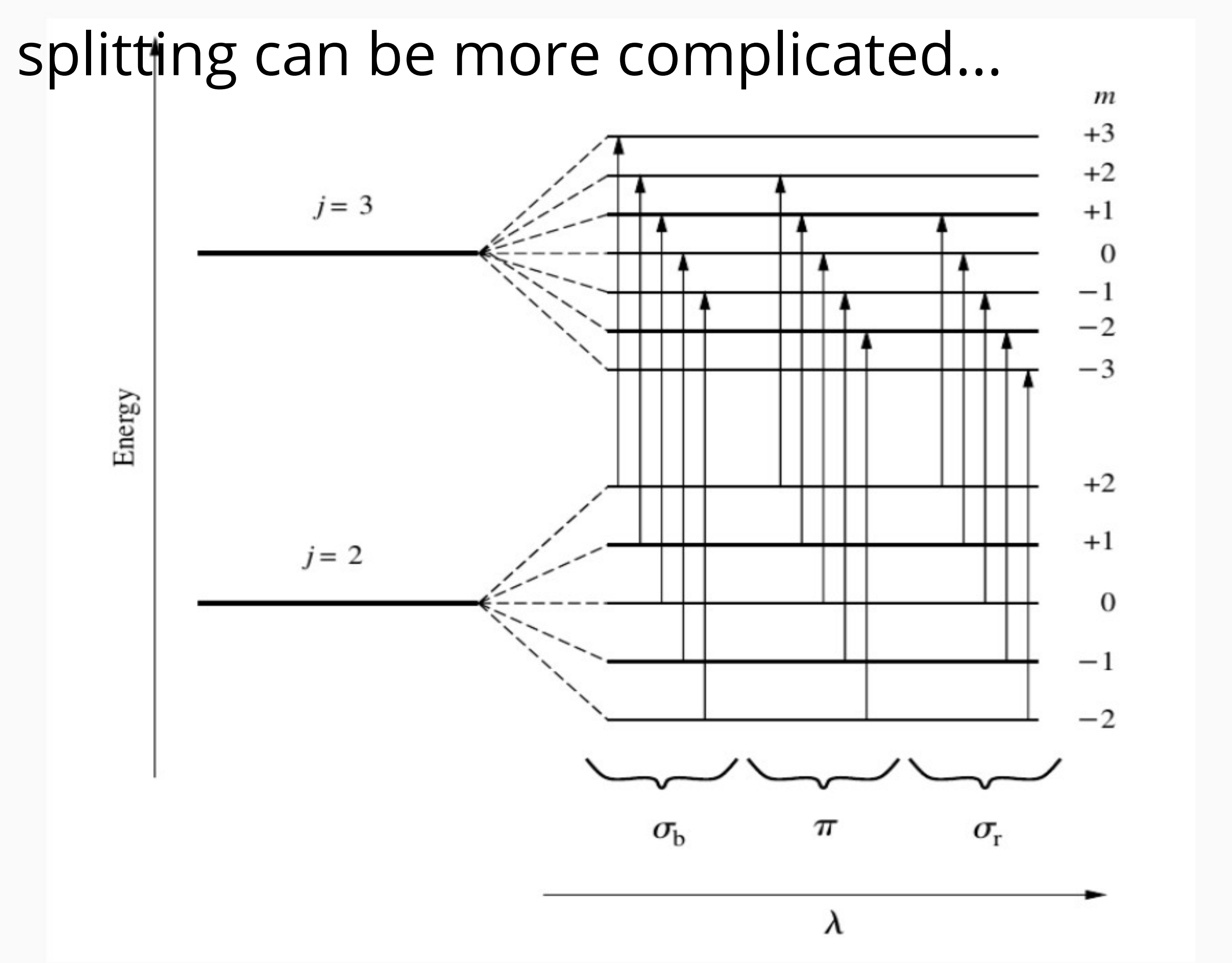
Time → Wavelength; Sunspot observations with IBIS at the NSO/Dunn Solar Telescope.
Courtesy of L. Kleint, K. Reardon, and A. Tritschler

Zeeman effect

- In the presence of the magnetic field. Levels with nonzero J **split** and we can see the multiple sub-transitions. For the so called, normal Zeeman triplet, it looks like this:



Of course, splitting can be more complicated...



Zeeman splitting for the transition where upper level has J = 3 and lower J = 2. There are total of 15 sub-transitions. I sometimes call different *m* values – Zeeman sublevels.

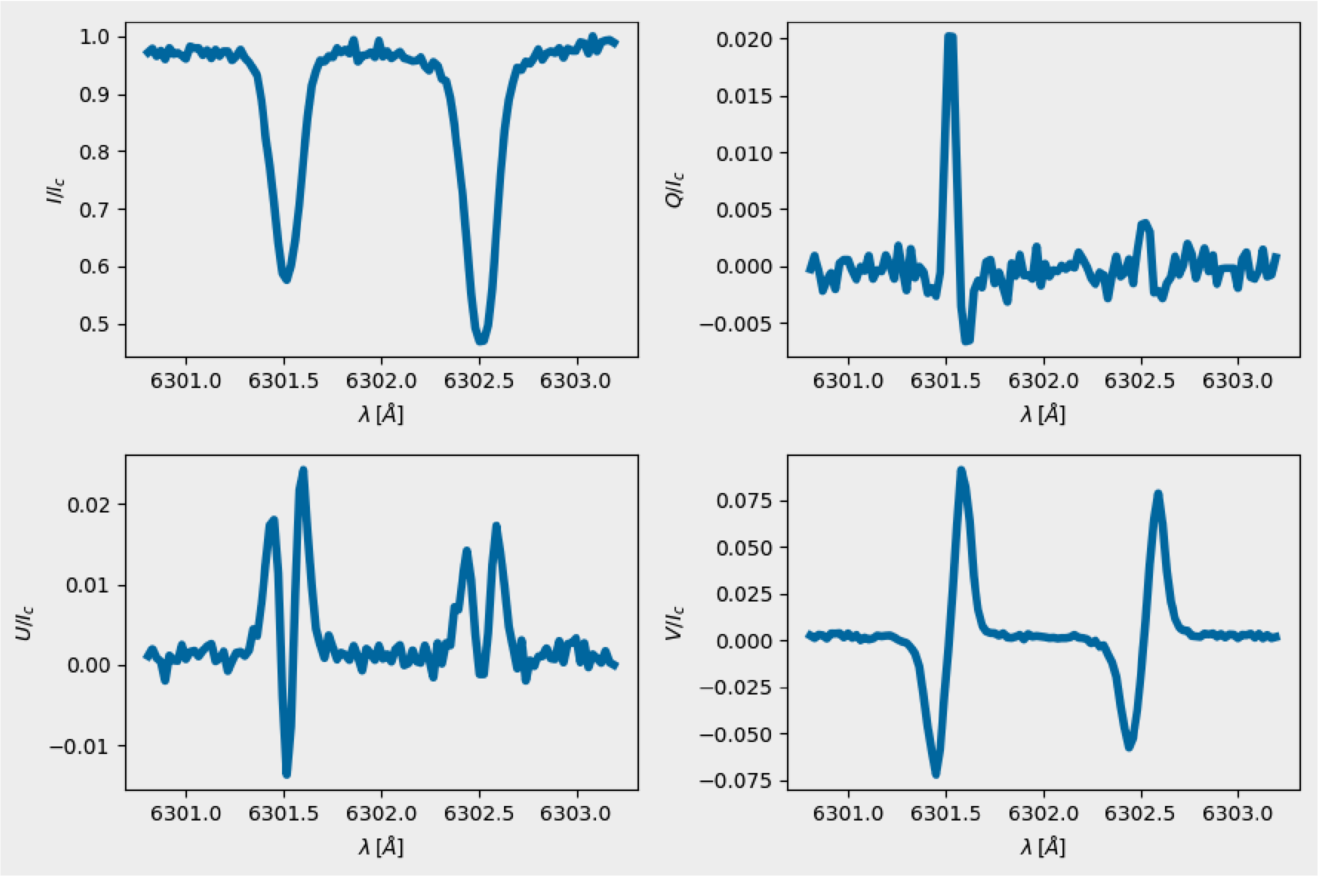
Zeeman splitting

- We can express this splitting in terms of wavelengths. For each m, the wavelength changes by:

$$\Delta\lambda_B = 4.67 \times 10^{-13} g_L \lambda_0^2 B$$

- Where wavelengths are in angstrom and magnetic field in Gauss.
- This means that infrared lines split more than the visible ones! (They do, but in practice it's not quadratic, for the reasons that will become clear soon).

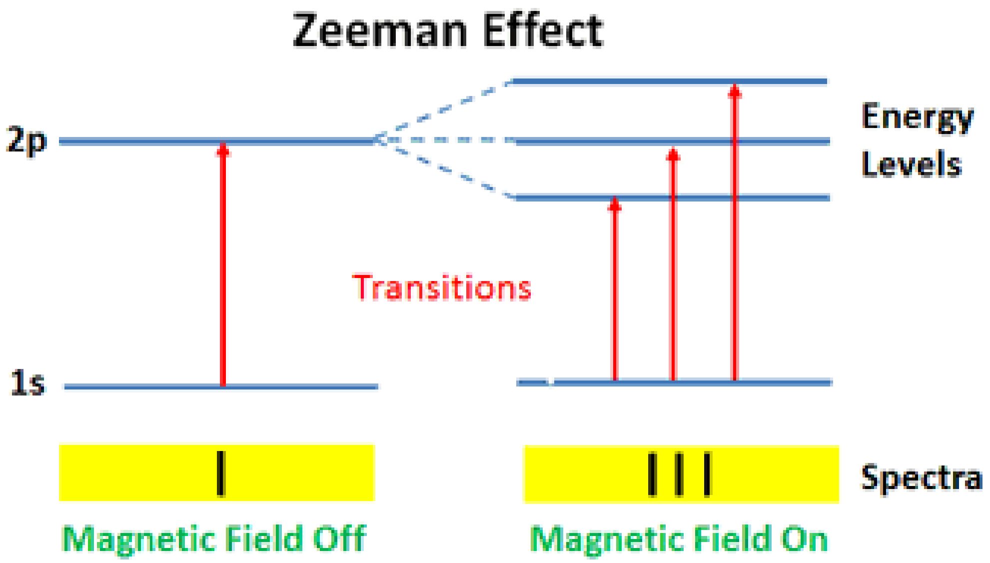
But then, there is also the polarization in the spectral lines



"Quiet-ish" Sun observed with HINODE/SOT SP

Zeeman effect – polarization

- Individual photons are always 100% polarized. Spin of the photon is it's polarization. Depending on the basis we have completely left/right circular or completely x/y linear polarization.



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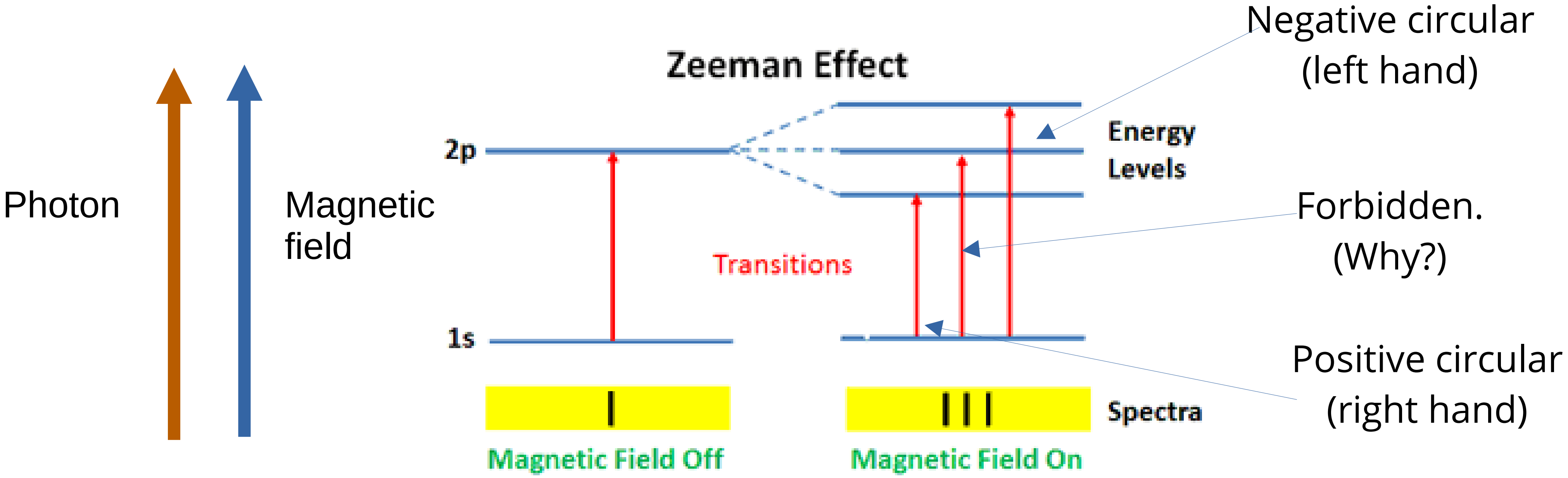

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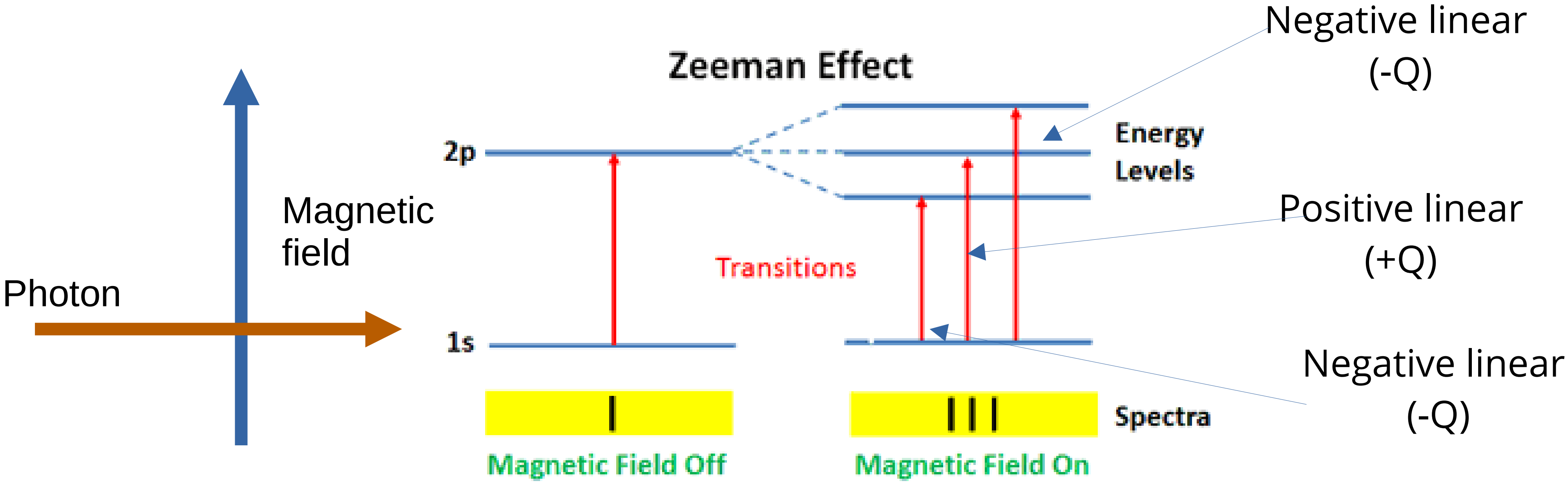
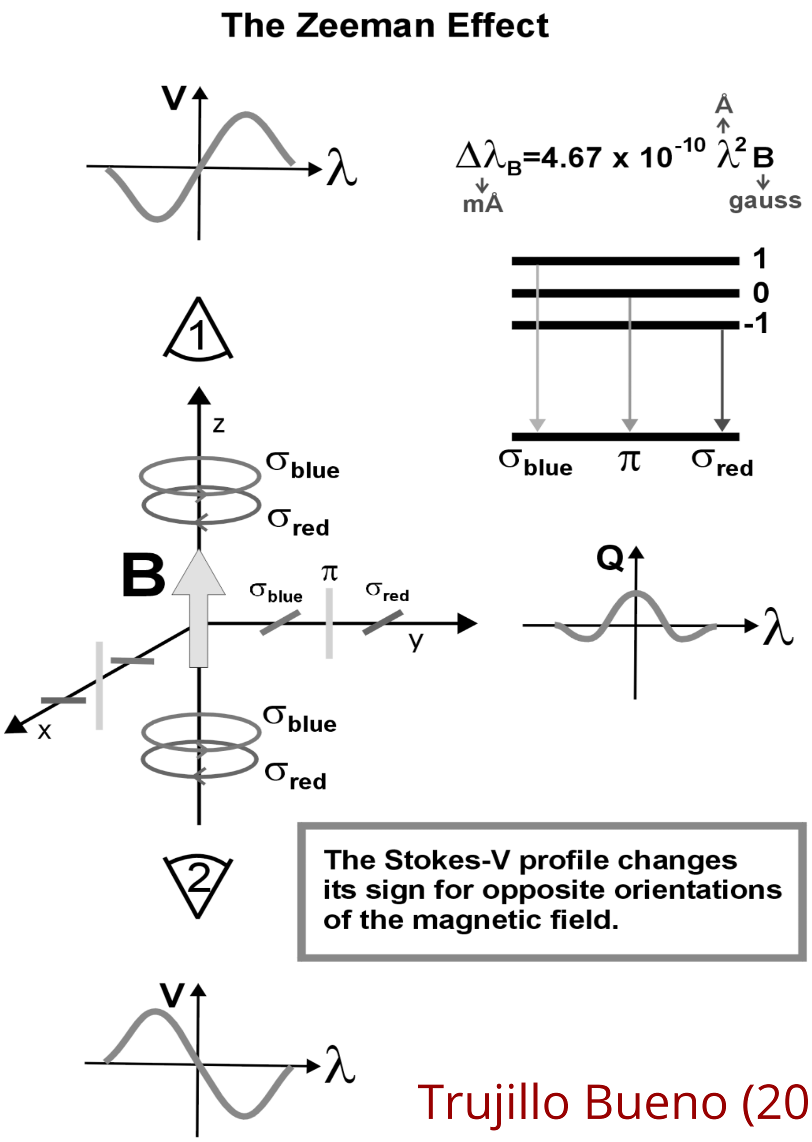


So to summarize

- Photons have spin one and two possible states, -1 and 1 (massless)
- Quantization axis for the angular momentum is this case is chosen to be the magnetic field director
- Projection of the angular momentum of the photon on the mag field can be -1,0,1
- However the angular momentum has to be conserved. If the photon travels paralelly to the mag field – only -1 and 1 transitions in *m* are allowed
- If it travels perpendicularly, then all three are allowed.

Now, wavelengths…

- These polarizations always exist, even if there is no magnetic field. But they all have the same energy/wavelength, so polarization cancels.
- However, when we introduce magnetic field, sublevels split, their wavelength changes and they do not overlap any more.
- In a way **It is not the magnetic field that causes the polarization, it just reveals it to us!**



Trujillo Bueno (2006)

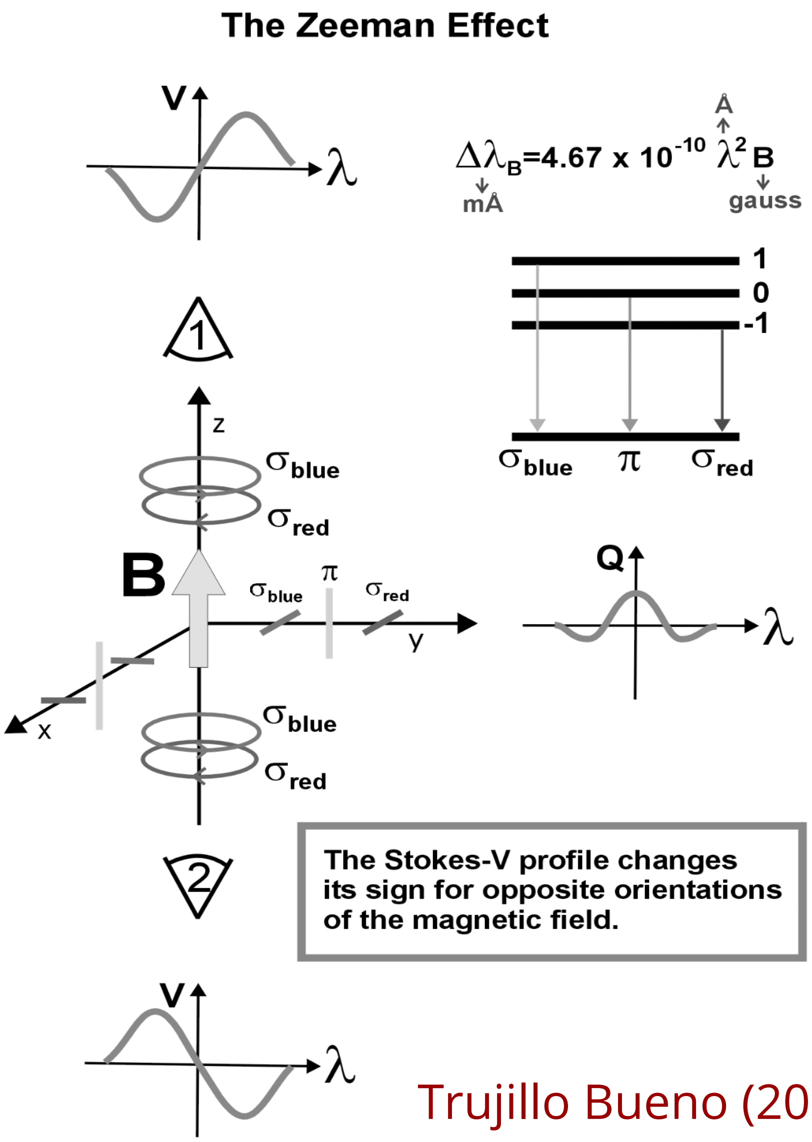
Why the polarization?

Individual photons are 100% polarized.

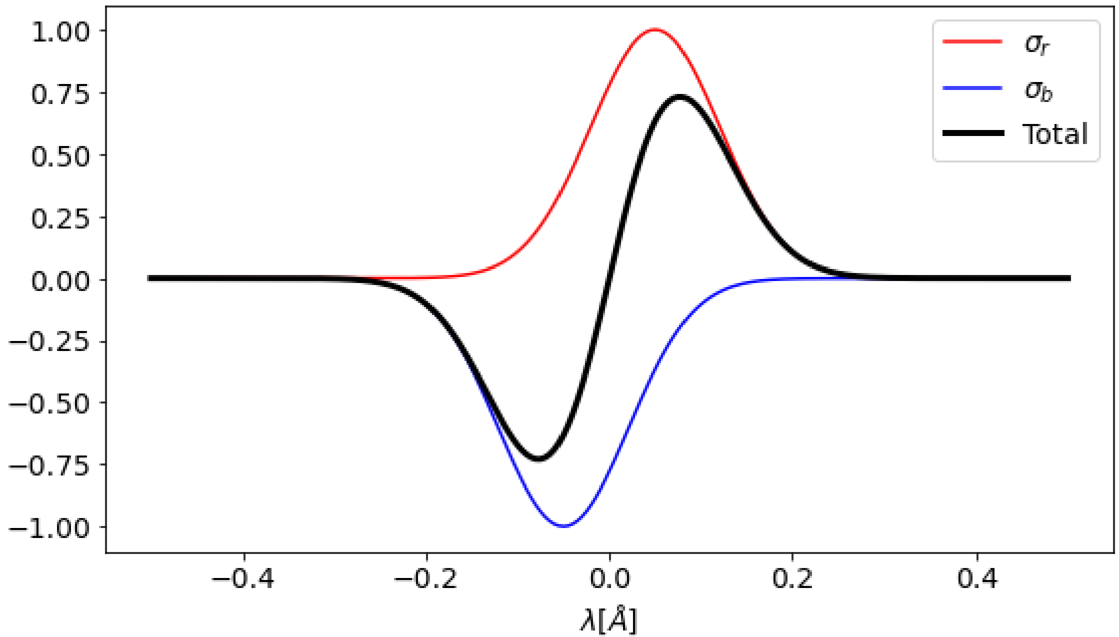
Different Δm transitions – different polarizations!

Parallel with B: only positive and negative circular polarization (σ_{blue},σ_{red})

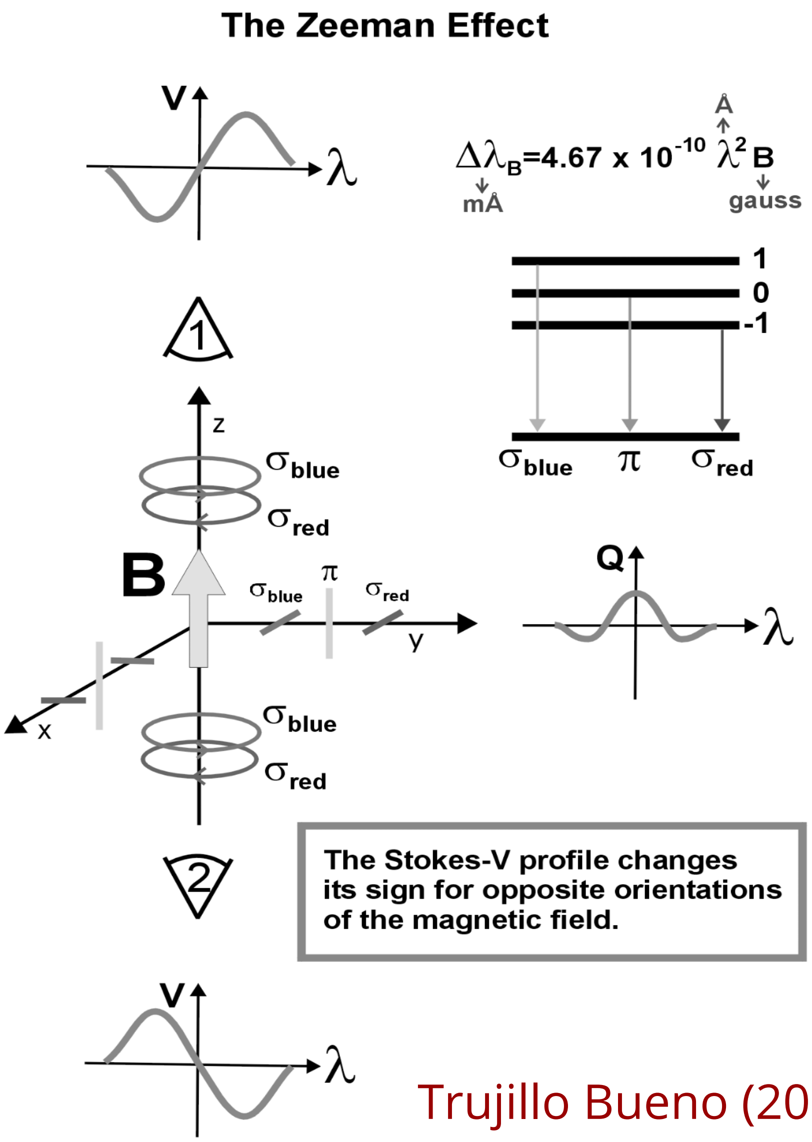
Perpendicular to B: σ_{blue},σ_{red} seen as negative linear polarization, π as positive linear polarization



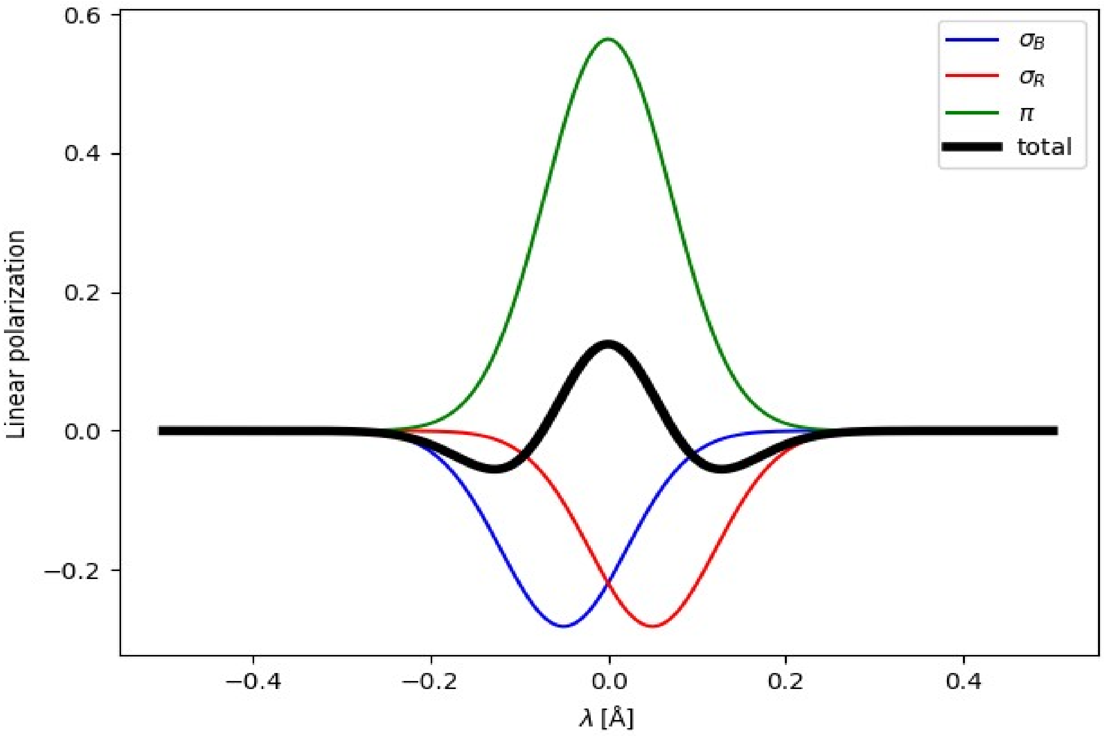
Parallel to **B**



Trujillo Bueno (2006)

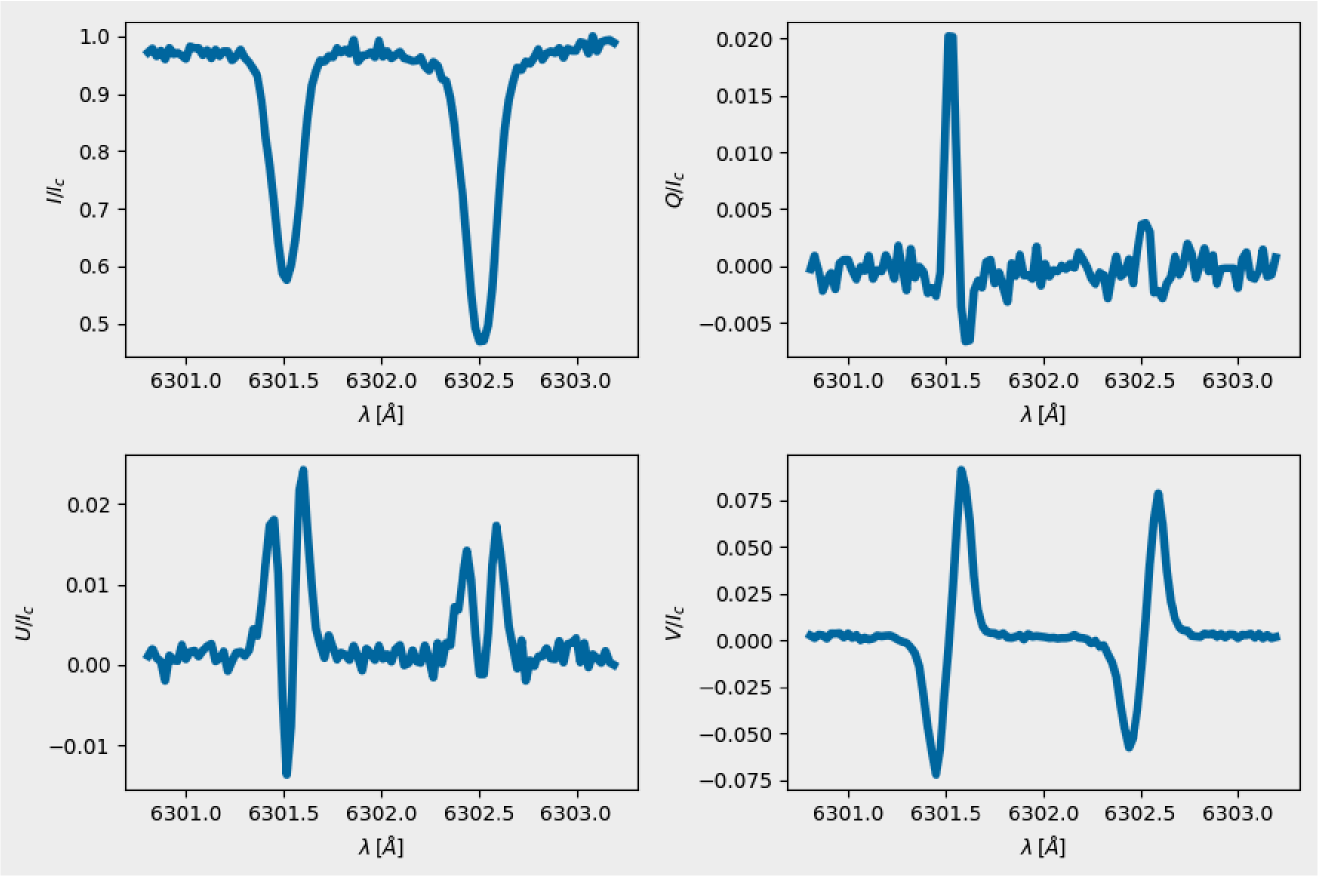


Perpendicular to **B**



Trujillo Bueno (2006)

Does it make more sense now?



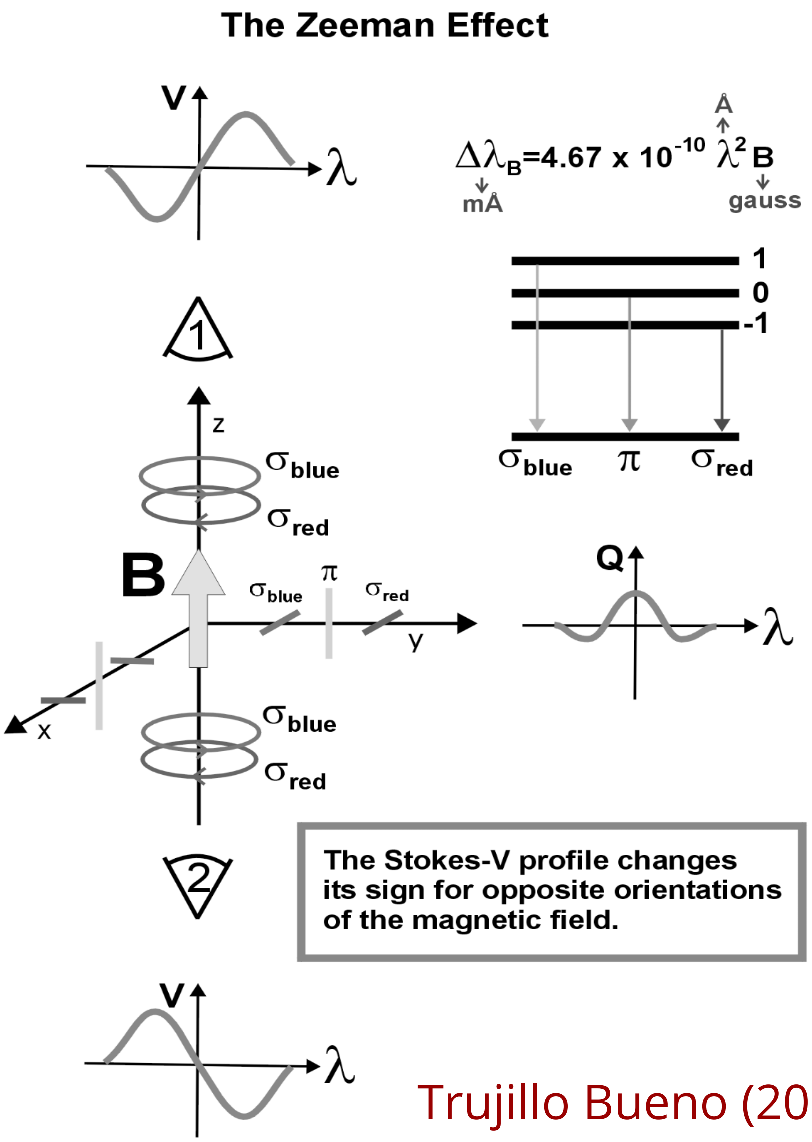
“Quiet-ish” Sun observed with HINODE/SOT SP

How to use Zeeman effect to measure the magnetic field?

- We don't always see splitting in the spectral line (as in the previous figure)
- In that case we need to extract the magnetic field from the polarization
- Let's focus on Stokes V for start
- And, let's agree that the same way we have emission of circularly polarized photons, we can have the **absorption** of circularly polarized photons
- And, let's look at our Stokes vector.

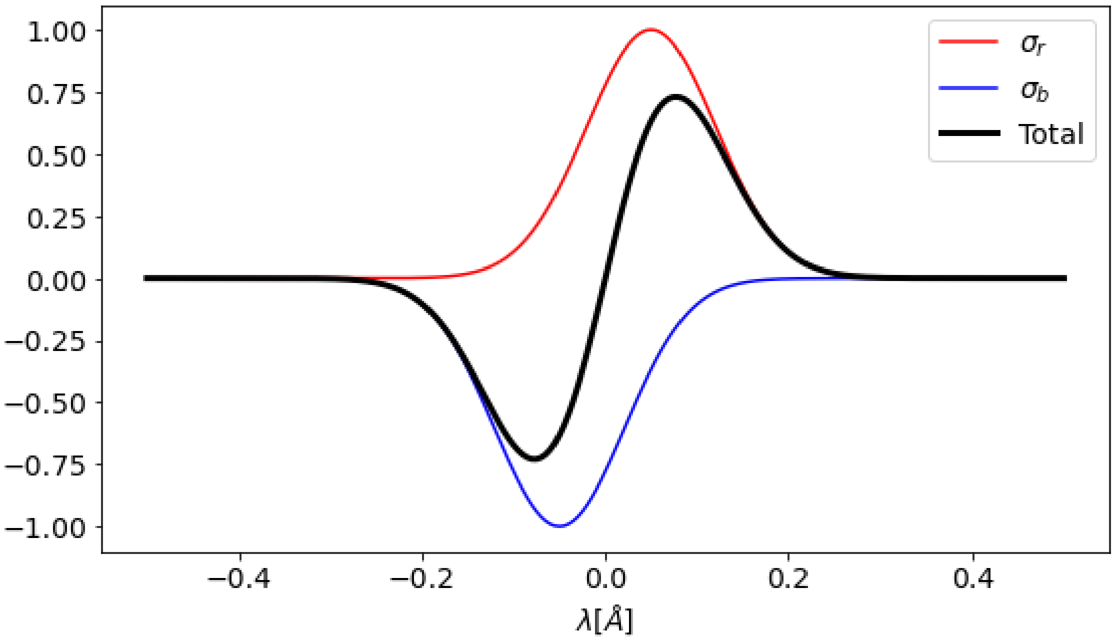
$$\hat{I} = (I, Q, U, V)^T$$

- And think how I and V get absorbed as they travel through the atmosphere

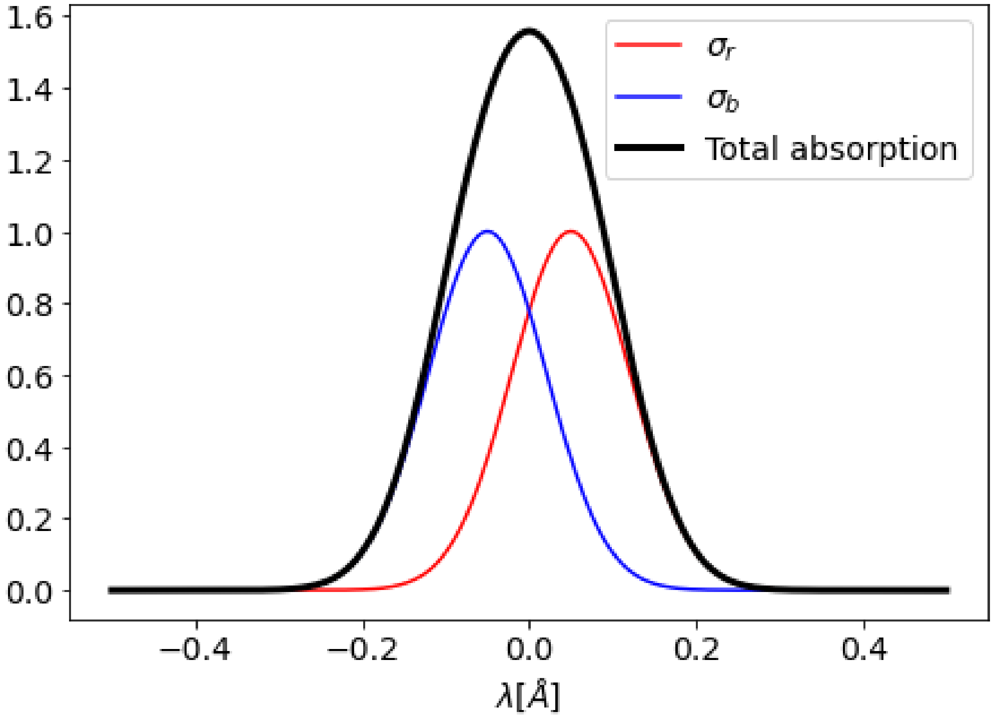


Trujillo Bueno (2006)

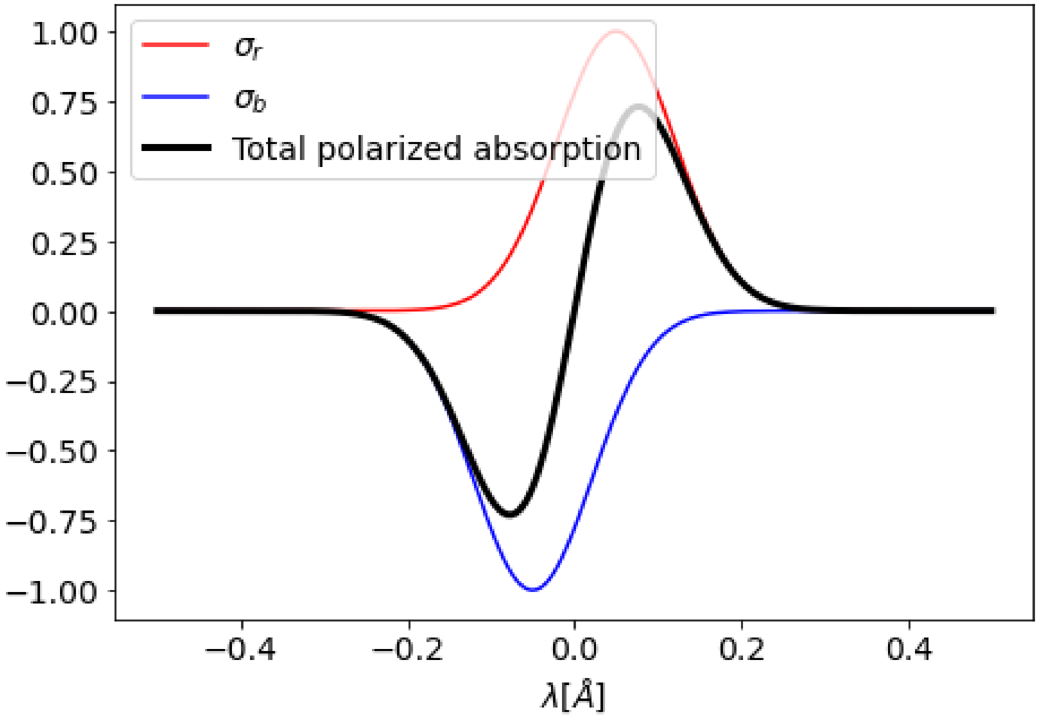
Parallel to **B**



So our absorption for total intensity and the polarization will look like this:



$$\chi_{I,\lambda} = \chi_0 \left(\phi_b(\lambda) + \phi_r(\lambda)\right)$$



$$\chi_{V,\lambda} = \chi_0 \left(\phi_r(\lambda) - \phi_b(\lambda)\right)$$

Now, a big step

- Let’s try and write a Radiative Transfer Equation that describes the propagation of both I and V
- Let’s discuss it a little bit :-)

Now, a big step

- Let's try and write a Radiative Transfer Equation that describes the propagation of both I and V
- It will be two, coupled, differential equations, something like this:

$$\frac{dI_\lambda}{dz} = -\chi_{I,\lambda} I_\lambda - \chi_{V,\lambda} V_\lambda + j_{I,\lambda}$$
$$\frac{dV_\lambda}{dz} = -\chi_{V,\lambda} I_\lambda - \chi_{I,\lambda} V_\lambda + j_{V,\lambda}$$

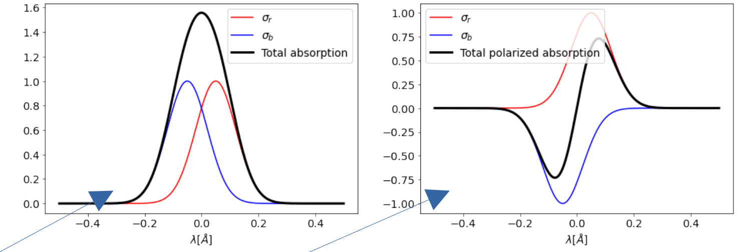
Or as we more commonly write it:

$$\frac{dI_\lambda}{d\tau} = \eta_I I_\lambda + \eta_V V_\lambda - \eta_I B$$

$$\frac{dV_\lambda}{d\tau} = \eta_I V_\lambda + \eta_V I_\lambda - \eta_V B$$

- Where “etas” are the ratios w.r.t. some referent opacity

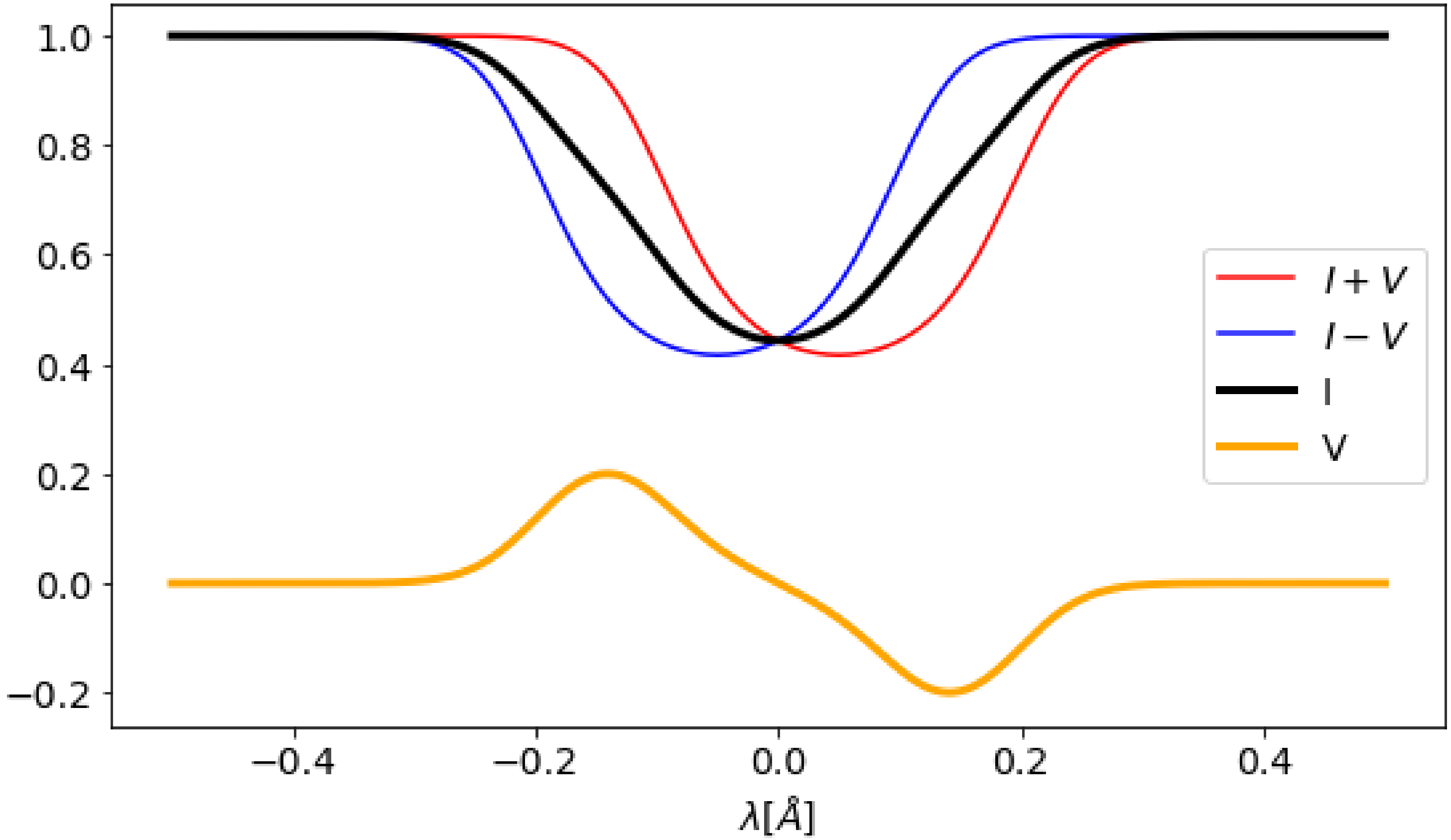
- We neatly add and subtract these two equations and get:



$$\frac{d(I \pm V)}{d\tau} = (\eta_I \pm \eta_V)(I \pm V) - (\eta_I \pm \eta_V)B$$

And, finally:

- We solve RTE for I +/- V and from that we get I and V

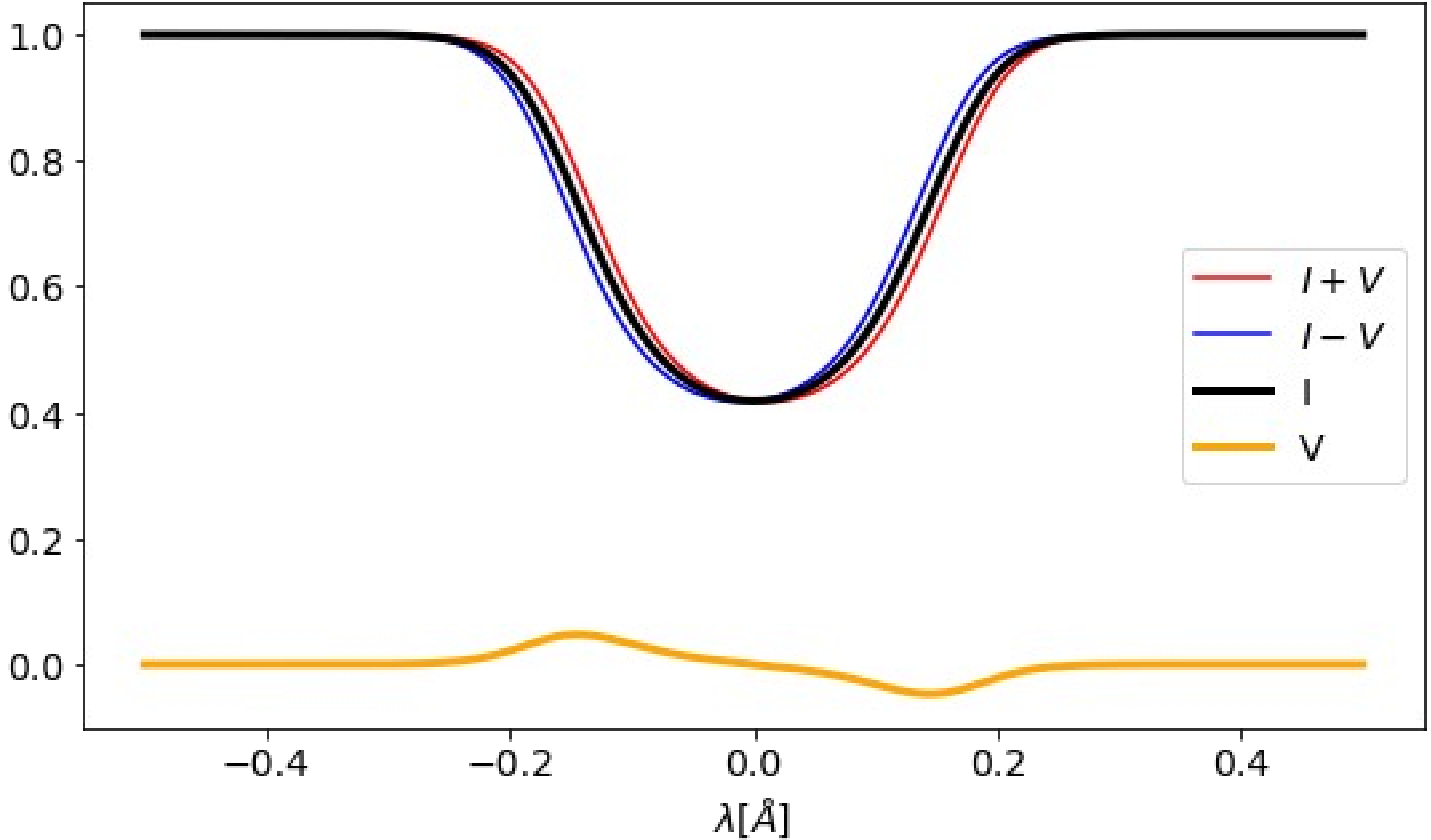


So let’s summarize

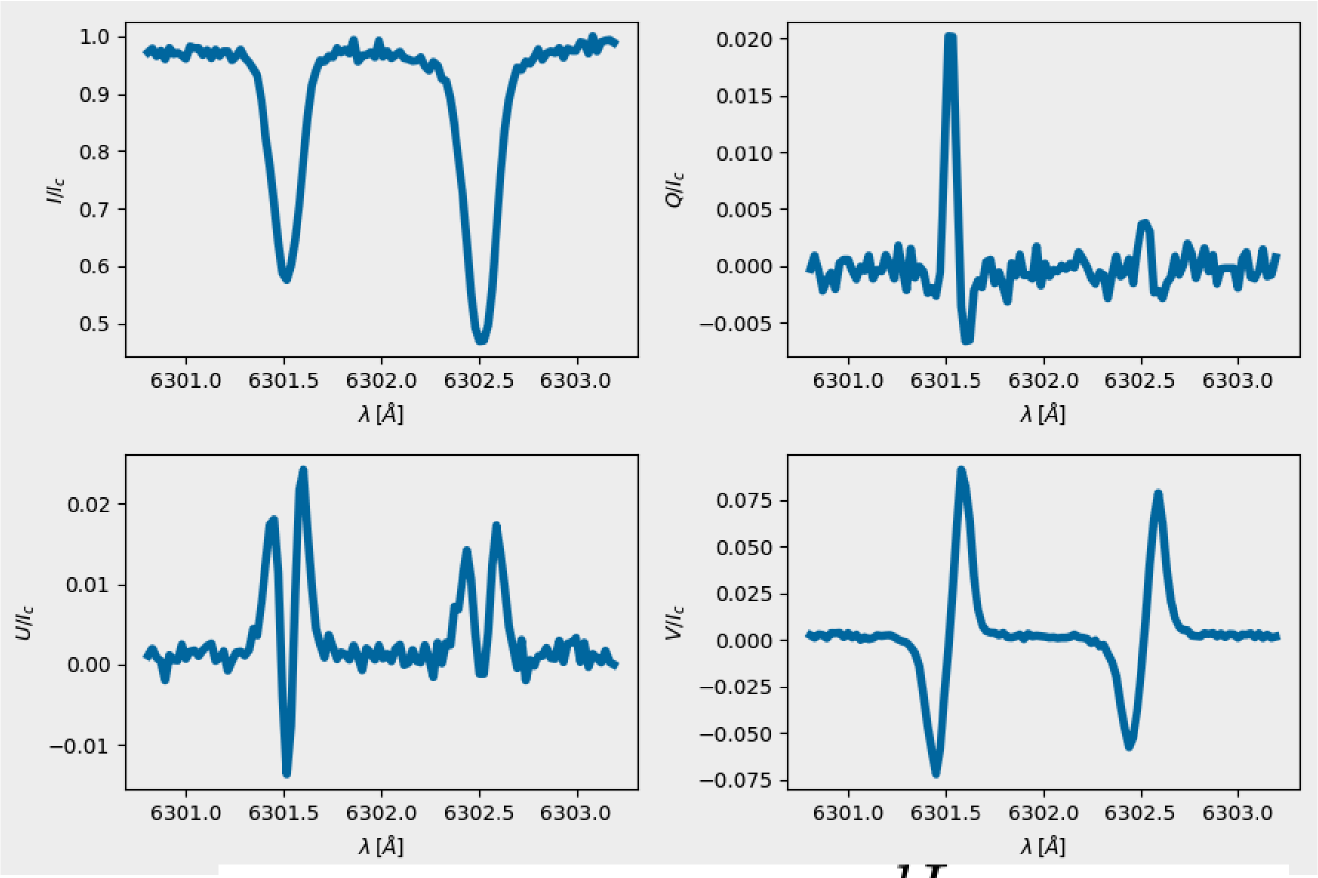
- In this very simple case (uniform magnetic field, oriented along z, normal Zeeman triplet), we will only have Stokes I and V that are non-zero
- Stokes I will be broadened, because of the splitting
- Stokes V will have typical antisymmetric shape due to selective absorption of the medium
- This is not super-hard to calculate (model), provided you follow the steps

Weak field approximation, very small B

$$V = -4.67 \times 10^{-13} \frac{dI}{d\lambda} g_L \lambda_0^2 B_{LOS}$$



Tell me what is the sign of B_los in this pixel?



$$V = -4.67 \times 10^{-13} \frac{dI}{d\lambda} g_L \lambda_0^2 B_{LOS}$$

Polarized RTE

- In a more general case (non-constant B, transversal B exists, etc), we have to write RTE for all 4 Stokes parameters.
- It becomes, well, scary:

$$\frac{d}{d\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - S \\ Q \\ U \\ V \end{pmatrix}$$

Where the coefficients are also kinda scary

- These are all the absorption matrix elements.
- Each of the three profiles counts all the contributions (if we have multiple transitions with same change of *m*)
- Explicit dependency on the orientation of **B**
- Magnitude of **B** appears in the profiles (shifts them)

$$\eta_I = 1 + \frac{\eta_0}{2}\left\{\phi_\mathrm{p}\sin^2\theta + \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\left(1 + \cos^2\theta\right)\right\}$$

$$\eta_Q = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

$$\eta_U = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

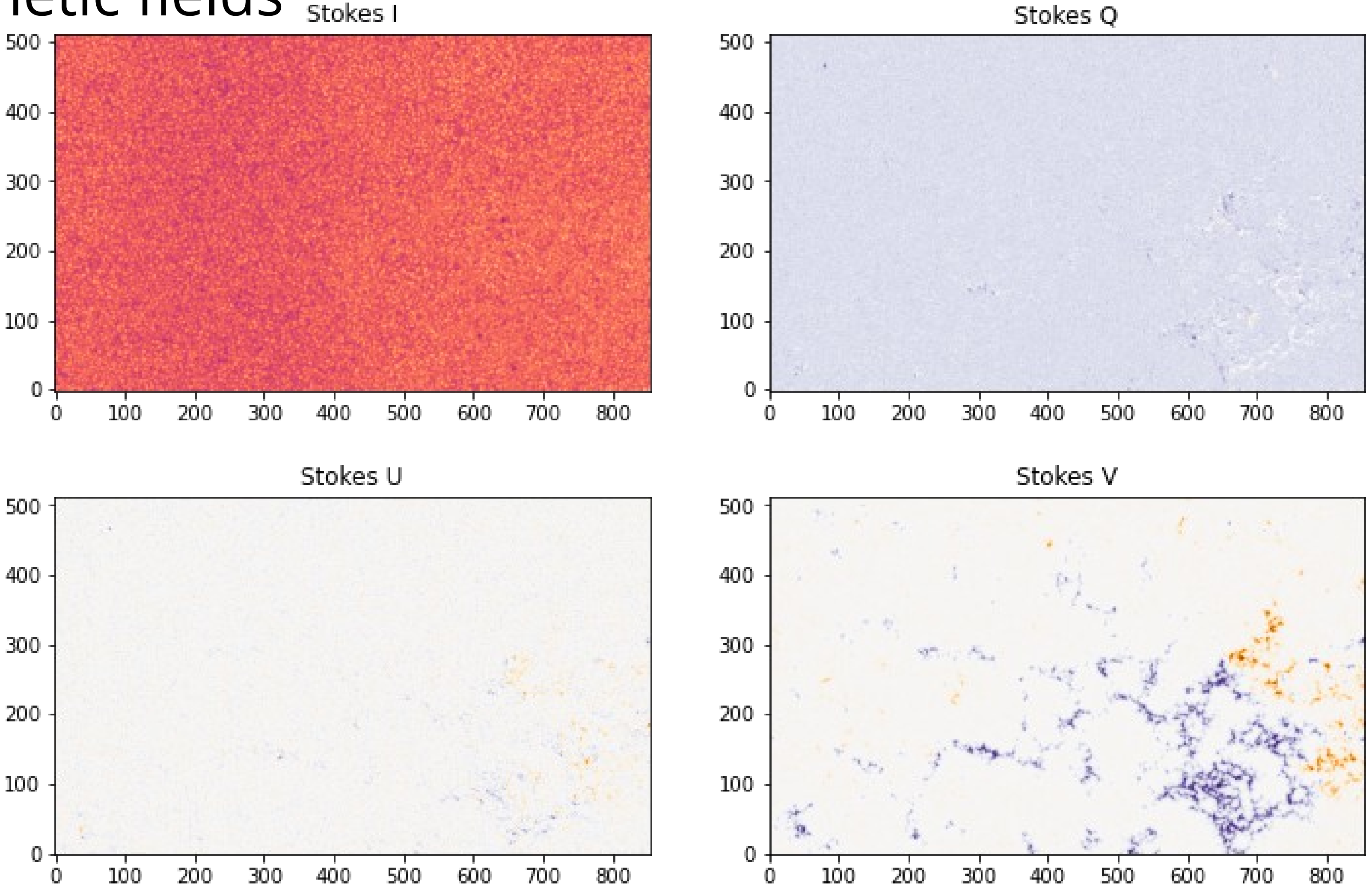
$$\eta_V = \frac{\eta_0}{2}\left[\phi_\mathrm{r} - \phi_\mathrm{b}\right]\cos\theta$$

$$\rho_Q = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

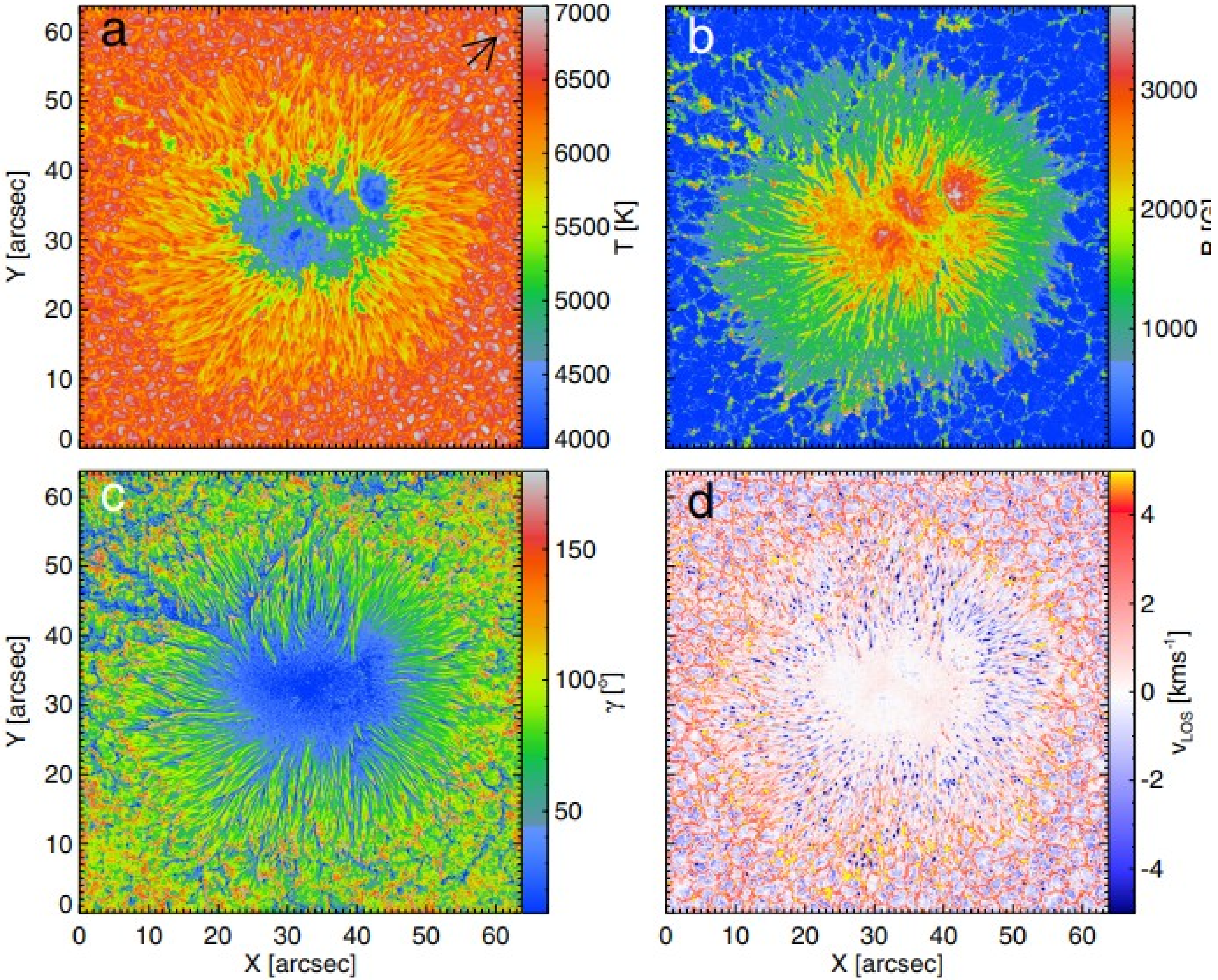
$$\rho_U = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

$$\rho_V = \frac{\eta_0}{2}\left[\psi_\mathrm{r} - \psi_\mathrm{b}\right]\cos\theta$$

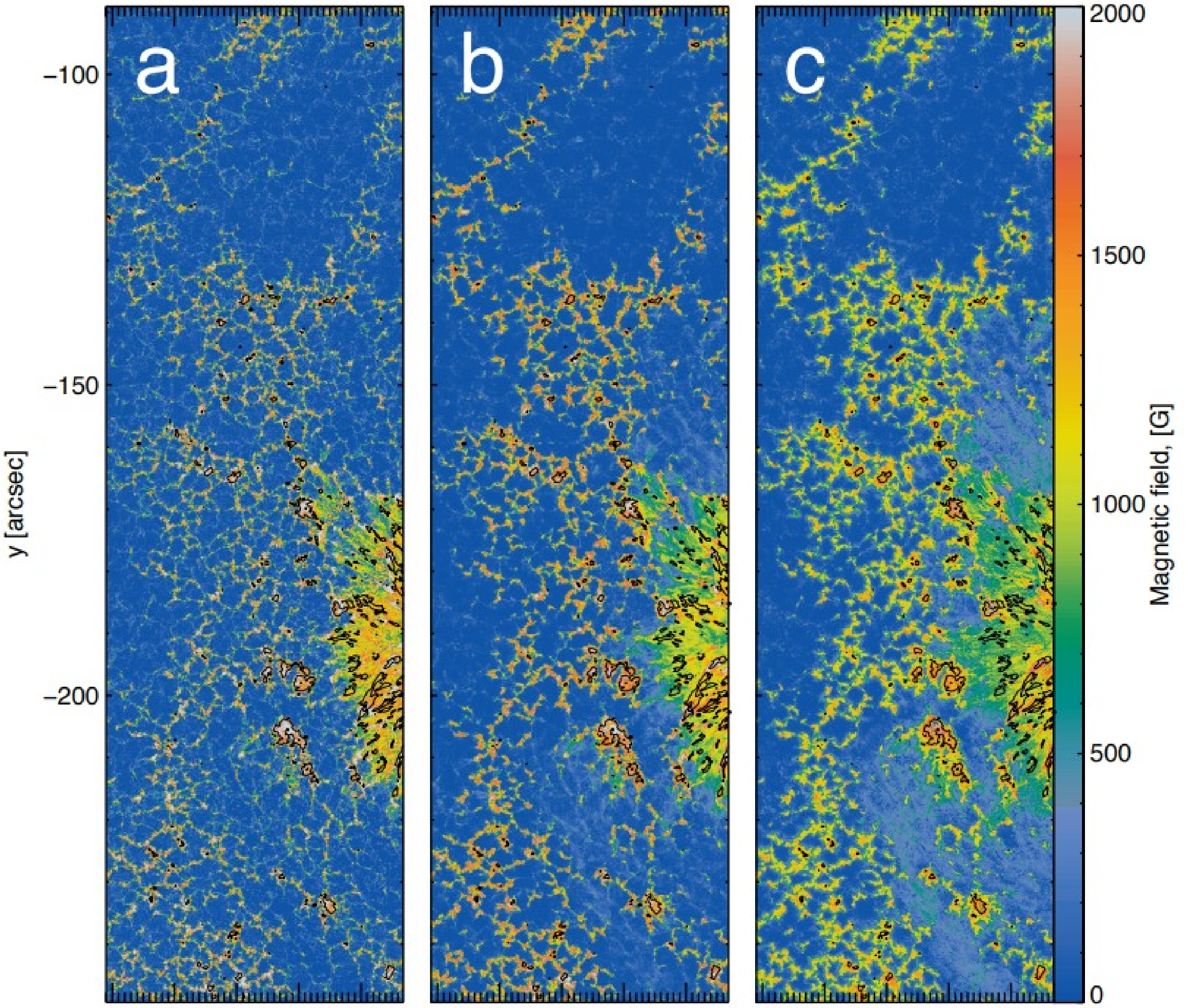
Anyway, all that together, gives us possibility of mapping magnetic fields



- Tiwari et al. (2015)
- Hinode data, two rather magnetically sensitive lines
- Sunspot – strong fiels, we can see Q,U,V
- Detailed “inversions”, so we get temperatures, velocities, magnetic field and inclination

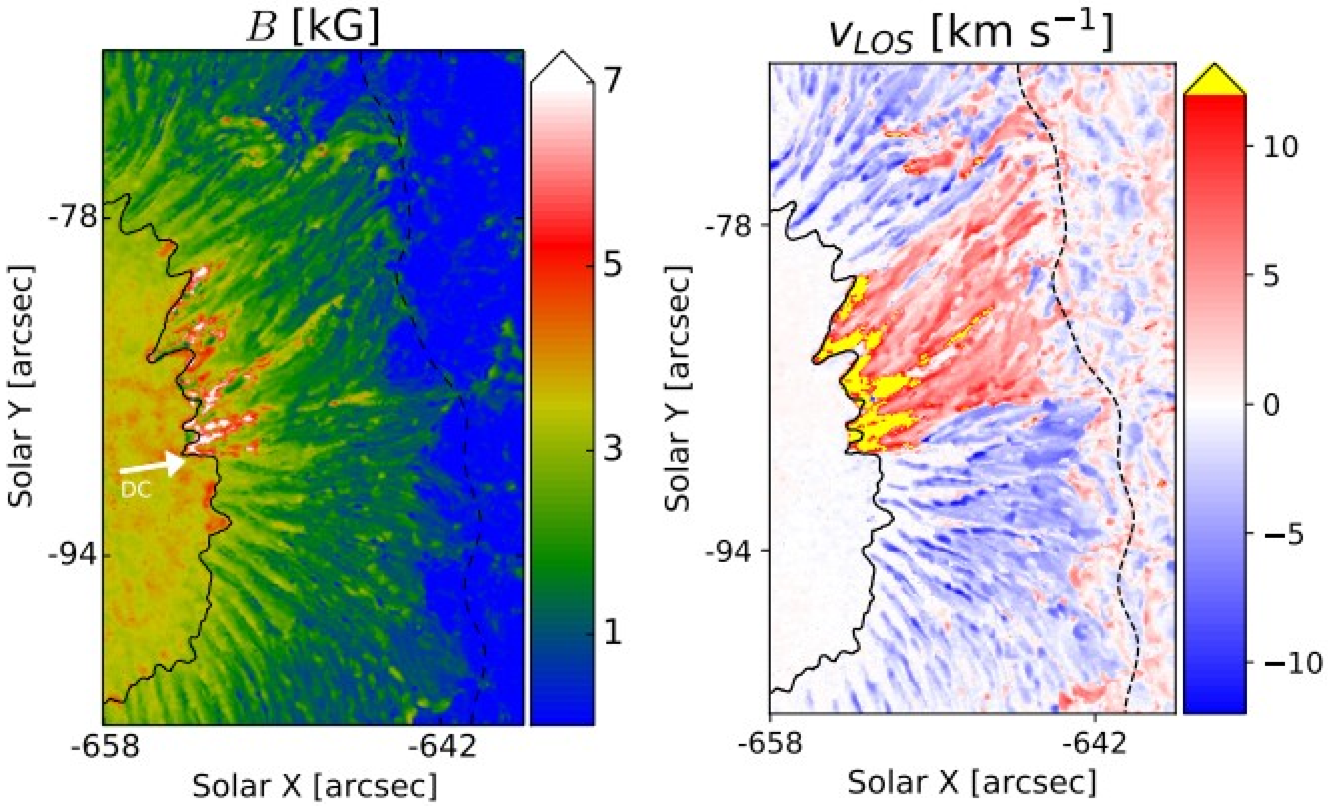


- Bueheler et al. (2015), look at depth dependent magnetic field in a plage
- Look at that beauty!
- Does the structure of the magnetic field make sense?



- Siu Tapia et al. (2019), magnetic fields at the umbra/penumbra boundary

- Do you see the magnetic fields and the velocities there?



For the end

- To better understand behavior of spectral lines and the way these maps have been created, let’s go and play with:
- http://research.iac.es/proyecto/magnetism//pages/codes/milne-eddington-simulator.php
- Enjoy!