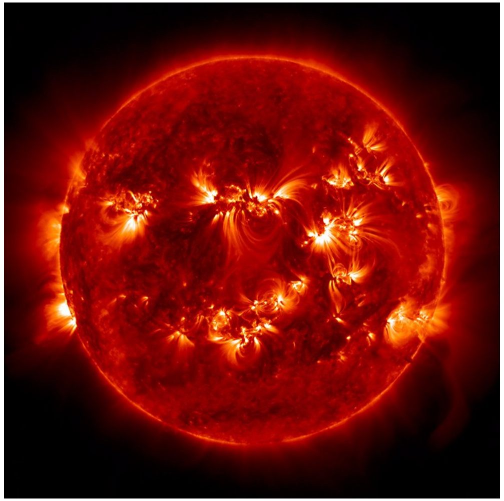
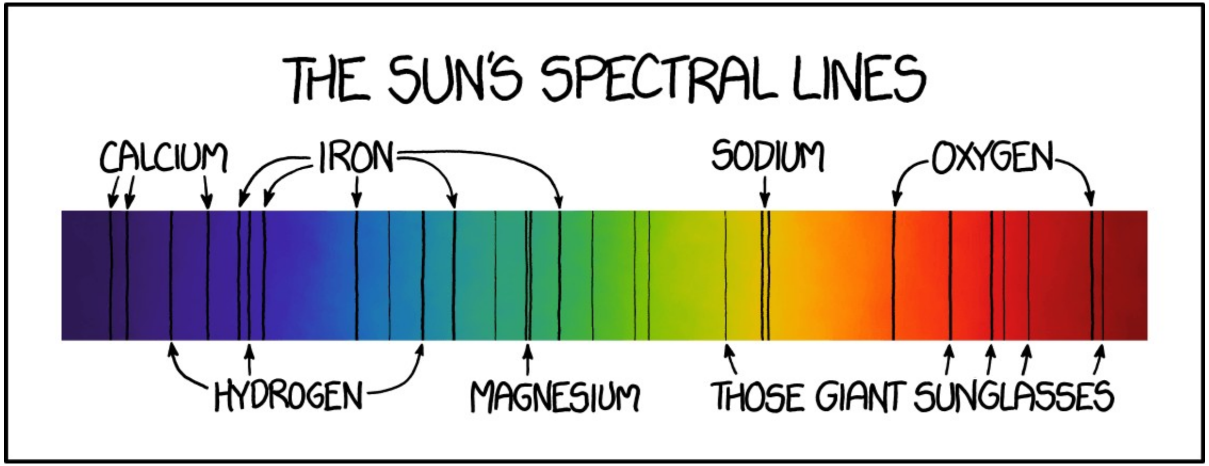
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 16: Zeeman effect

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A brief summary of what we did so far

- We figured out that intensity coming out of the atmosphere is solution of RTE at given wavelength (*everything* depends on the wavelength).
- For RTE we need opacity and emissivity (optical depth and the source function), so we also figured out them
- We found some solutions
- We then saw that scattering creates some problems, so we involved NLTE
- We studies a few particularly interesting spectral lines: H alpha and Ca II 8542, both strong, NLTE, chromospheric lines
- Let’s test what we learned so far with some quick questions!

Question 1

Spectral lines arise in the transitions between:

A) Two bound states (e.g. two atomic bound levels)

B) A bound and a free state (e.g. an atomic level and a continuum)

C) Two free states (electron is free before and after the transition)

D) All of the above

Question 2

Which one of the following is true for **Local Thermodynamic Equilibrium**

A) Source function and the intensity follow Planck function

B) Only the source function follows the Planck function

C) Neither the intensity nor source function follow Planck Function

Question 3

What is **Spectral Line Scattering**

A) A phenomena that occurs when a photon scatters on an electron that is occupying an excited state

B) Absorption of a photon, followed immediately by an emission of a similar photon

C) Continuum scattering of wavelength dependent radiation on free electrons, molecules or atoms, that results in a line-shaped spectrum

D) None of the above

Question 4

What happens when line scattering becomes more important than collisional processes?

A) We have more absorption than we would usually have, but nothing else changes

B) Medium cools much more efficiently because photons are escaping

C) Source function (emissivity) depends on the radiation field and not only on temperature

D) All of the above

Question 5 (still not covered)

How many quantum numbers we use to fully describe an atomic energy level (state)?

A) Only 1

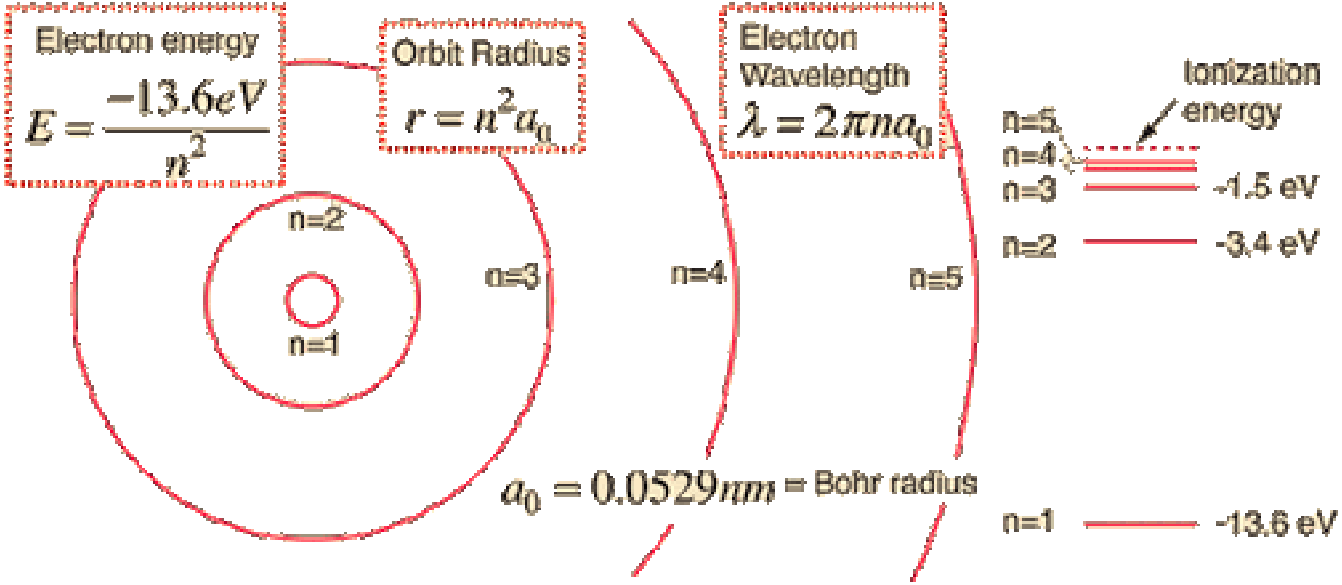
B) 2

C) 3

D) 4

Let's look in more detail at the atomic levels!

- For simplicity let's think of Hydrogen and then wave our hands and generalize ;)
- In the very simple solutions of the Schrodinger equation you will see that the energy is mostly influenced by the so called **principle quantum number, n**



There are more quantum numbers however

- For some reason, the higher the quantum number, the more electrons I can fit in
- What is that about?

Orbital and magnetic quantum number

- It turns out that there is an additional quantum number **l,** that measures the total orbital angular momentum
- This means that there are different ways to put an electron in to same energy level as there are different sublevels ("orbitals")
- Turns out that **l** can take values from **0 to n-1**
- **Magnetic quantum number m,** measures the projection of the total angular momentum on a given axis, has values from **-l, to l**
- Oh, but it means that my first energy level has one orbital, so it would sound reasonable I can fit one electron there
- Wait a second, I think we remember it was **2 electrons!**

Spin quantum number and spin-orbit interaction

- In addition to it's orbital angular momentum, electrons also posses their own angular momentum – **spin**
- Absolute value of that spin is $\hbar/2$, with possible values -$\hbar/2$ and $\hbar/2$ (down and up)
- Oh neat, now I can fit two electrons in one orbital!
- This is the so called Pauli exclusion principle, we can't have two electrons with all the same quantum numbers

Cool, so what happens in the hydrogen in the end?

Principal quantum number	Angular quantum number	Magnetic quantum number	Spin (Absolute value always ½ in case of H)	Statistical weigth
n=1	l=0	m=0	s = +/- ½	2
n=2	l=0, 1	m=0 m=-1,0,1	s = +/- ½	8
n=3	l=0, 1, 2	m=0 m=-1, 0, 1 m=-2, -1, 0, 1, 2	s = +/- ½	18

Are all these levels important?

Principal quantum number	Angular quantum number	Magnetic quantum number	Spin	Statistical weigth
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In the absence of the magnetic field, no! Magnetic sub-levels are degenerate, we cannot distinguish between them.

Spin-Orbit Interaction

- It turns out that it is convenient to add together **orbital quantum number** and **spin quantum number** to get the **total angular momentum quantum number, J**
- In the so called S-L coupling we simply add our orbital and spin angular moment and we get allowed values for angular momentum quantum number:

$$|l-s| \leq j \leq |l+s|$$

Configuration	Term	J	Level (eV)
1*s*	^2S	1/2	0.00000000000000
2*p*	$^2P^\circ$	1/2	10.19880615024
2*s*	^2S	1/2	10.19881052514816
2			10.1988358
2*p*	$^2P^\circ$	3/2	10.19885151459
3*p*	$^2P^\circ$	1/2	12.0874936591
3*s*	^2S	1/2	12.0874949611
3			12.0875052
3*d*	^2D	3/2	12.0875070783
3*p*	$^2P^\circ$	3/2	12.0875071004
3*d*	^2D	5/2	12.0875115582

Make sure to check NIST webpage for these cool tables

Ok, what happens in say... Helium then?

- Ground level has to have J =0 (both electrons in ground state, so their spins have to be opposite). Also mind that we use capital J for quantum numbers describing multiple electrons
- First excited level though, can have J = 0 or 1, even for L =0. Why?
- For L = 1, it's even more complicated, J = 0,1,2!

Configuration	Term	*J*	Level (eV)
$1s^2$	^1S	0	0.00000000
1s2s	^3S	1	[19.81961484203]
1s2s	^1S	0	[20.6157751334]
1s2p	$^3P°$	2	[20.96408720675]
		1	[20.96409668230]
		0	[20.96421916817]
1s2p	$^1P°$	1	[21.2180230218]
1s3s	^3S	1	[22.718466742]
1s3s	^1S	0	[22.920317682]
1s3p	$^3P°$	2	[23.0070734673]
		1	[23.0070761918]
		0	[23.0071097475]

To finish this sweet torture

- In multi-electron atoms, the energy level is going to be described by:

  Principal quantum number n

  Orbital quantum number L

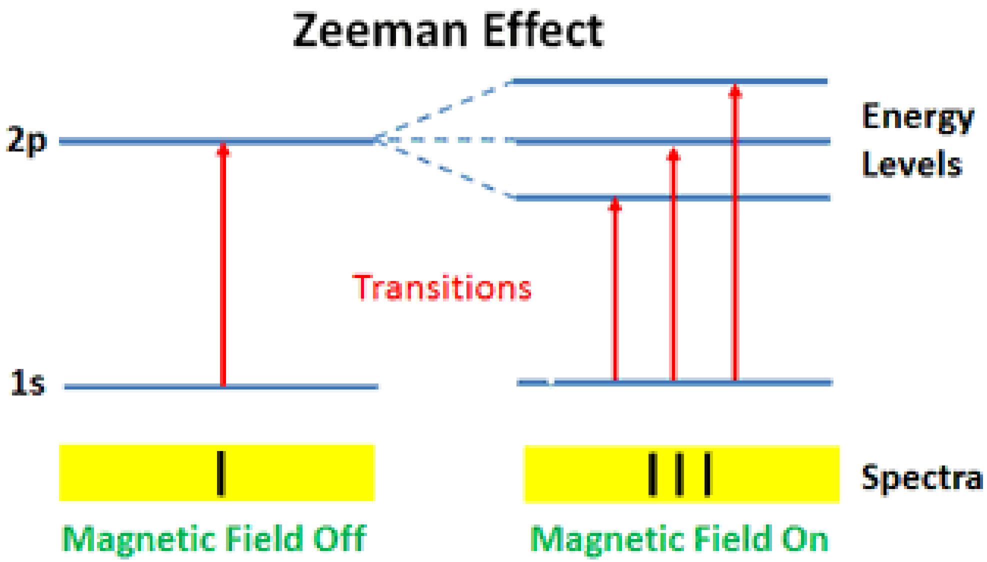
  Total angular quantum number J

- Now, this allows for *really really very many* combinations and possible atomic levels. No wonder Iron has so many of them!

- Thankfully, in practice we only need to know the energy of the level and it’s J to understand what comes next.

Zeeman effect

- In the presence of the magnetic field. Levels with nonzero J **split** and we can see the multiple sub-transitions. For the so called, normal Zeeman triplet, it looks like this:



Why do the levels split?

- Well, the magnetic field influences the energy. In the language of Quantum Mechanics that means our Hamiltonian operator and it's eigenvalues change.
- Hamiltonian is no longer degenerate in m, the levels are separated
- More pragmatically, each level splits into 2J+1 sublevels, each shifted by:

Magnetic quantum number

Bohr magneton (constant)

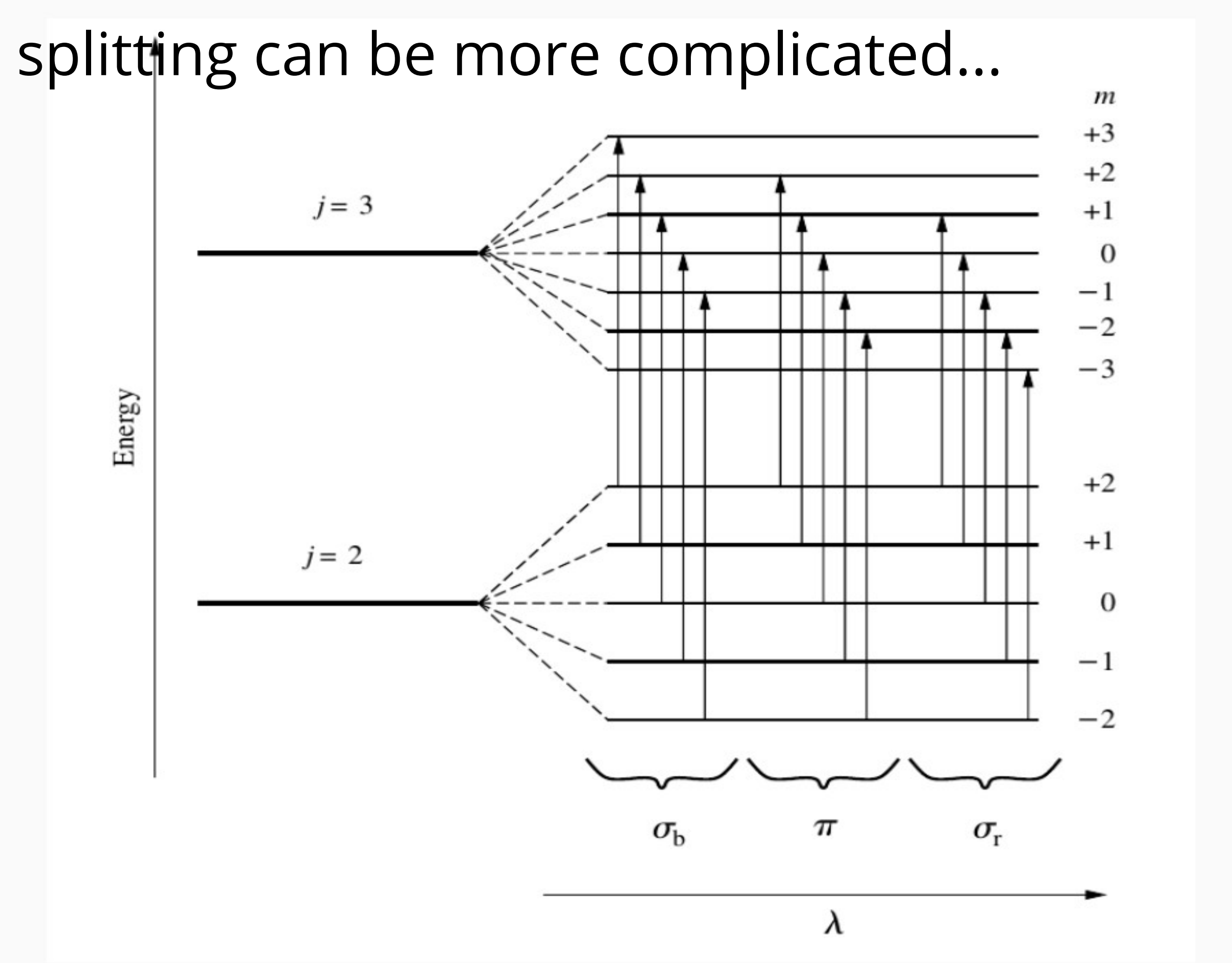
$$\Delta E = m g_L \mu_B B$$

Magnetic field

Energy shift

Lande factor

Of course, splitting can be more complicated...



Zeeman splitting for the transition where upper level has J = 3 and lower J = 2. There are total of 15 sub-transitions. I sometimes call different *m* values – Zeeman sublevels.

How big is this splitting in practice?

- Well, you tell me. Let’s assume a 1000 G magnetic field (some typical strong-ish magnetic field, that is 0.1 Tesla), m=1, Lande factor equal to 1 and we know that:

$$\mu_B = 5.79 \times 10^{-5} \mathrm{ev/T}$$

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$$\mu_B = 5.79 \times 10^{-5} \mathrm{ev}/\mathrm{T}$$

- Well this is quite easy:

$$\Delta E = 5.79 \times 10^{-6} \mathrm{eV}$$

- Is this a lot compared to the transition energies of the lines we observe?

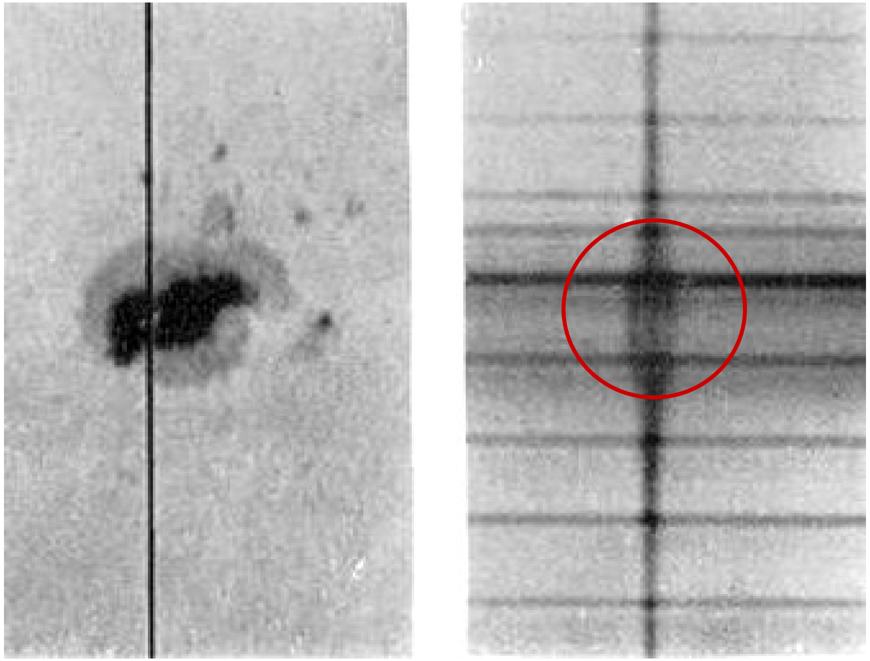
Zeeman splitting

- We can express this splitting in terms of wavelengths. For each m, the wavelength changes by:

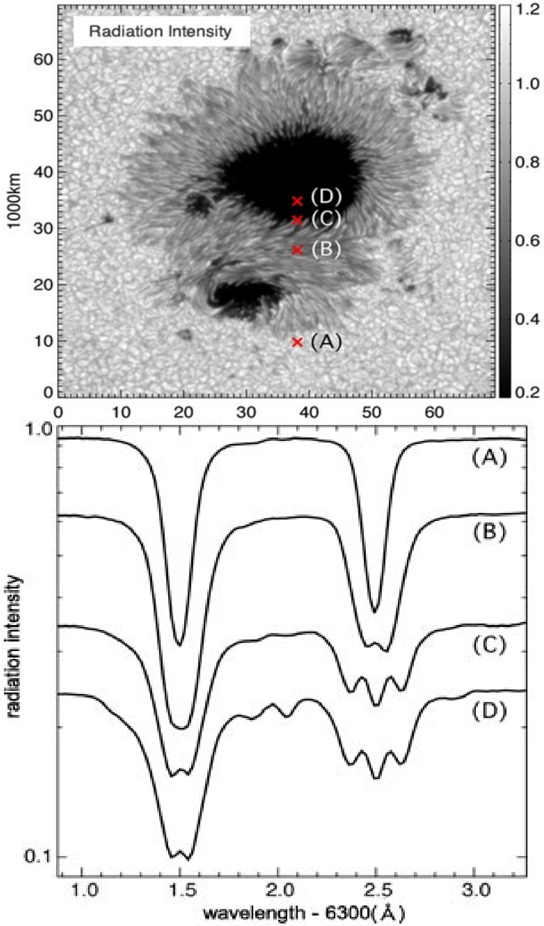
$$\Delta\lambda_B = 4.67 \times 10^{-13}\, g_L\, \lambda_0^2\, B$$

- Where wavelengths are in angstrom and magnetic field in Gauss.
- This means that infrared lines split more than the visible ones! (They do, but in practice it's not quadratic, for the reasons that will become clear soon).

We know the Zeeman effect splits spectral lines in the Sun

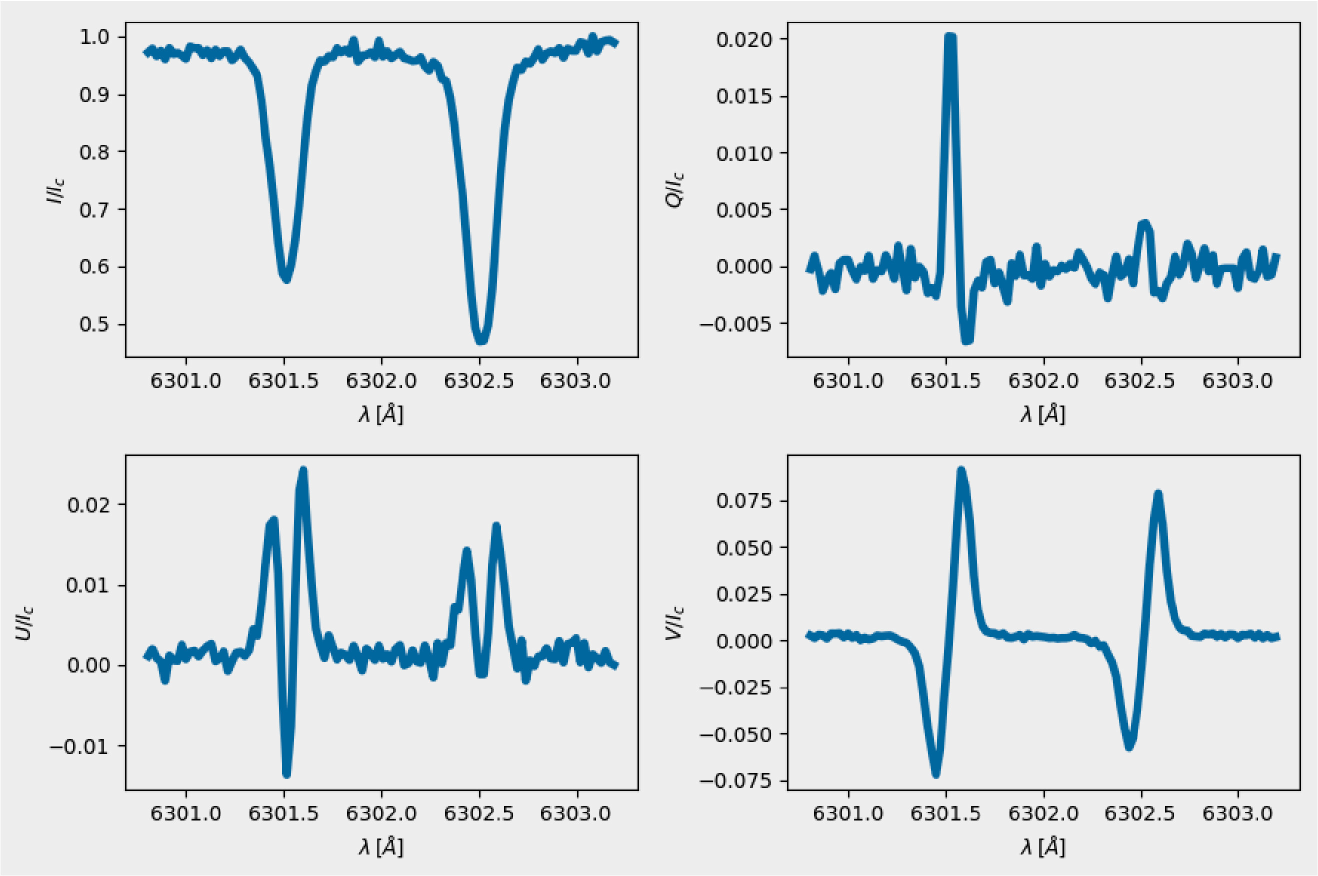


G.E. Hale, F. Ellerman, S.B. Nicholson, and A.H. Joy (ApJ, 1919)



Credits: Yukio Katsukawa

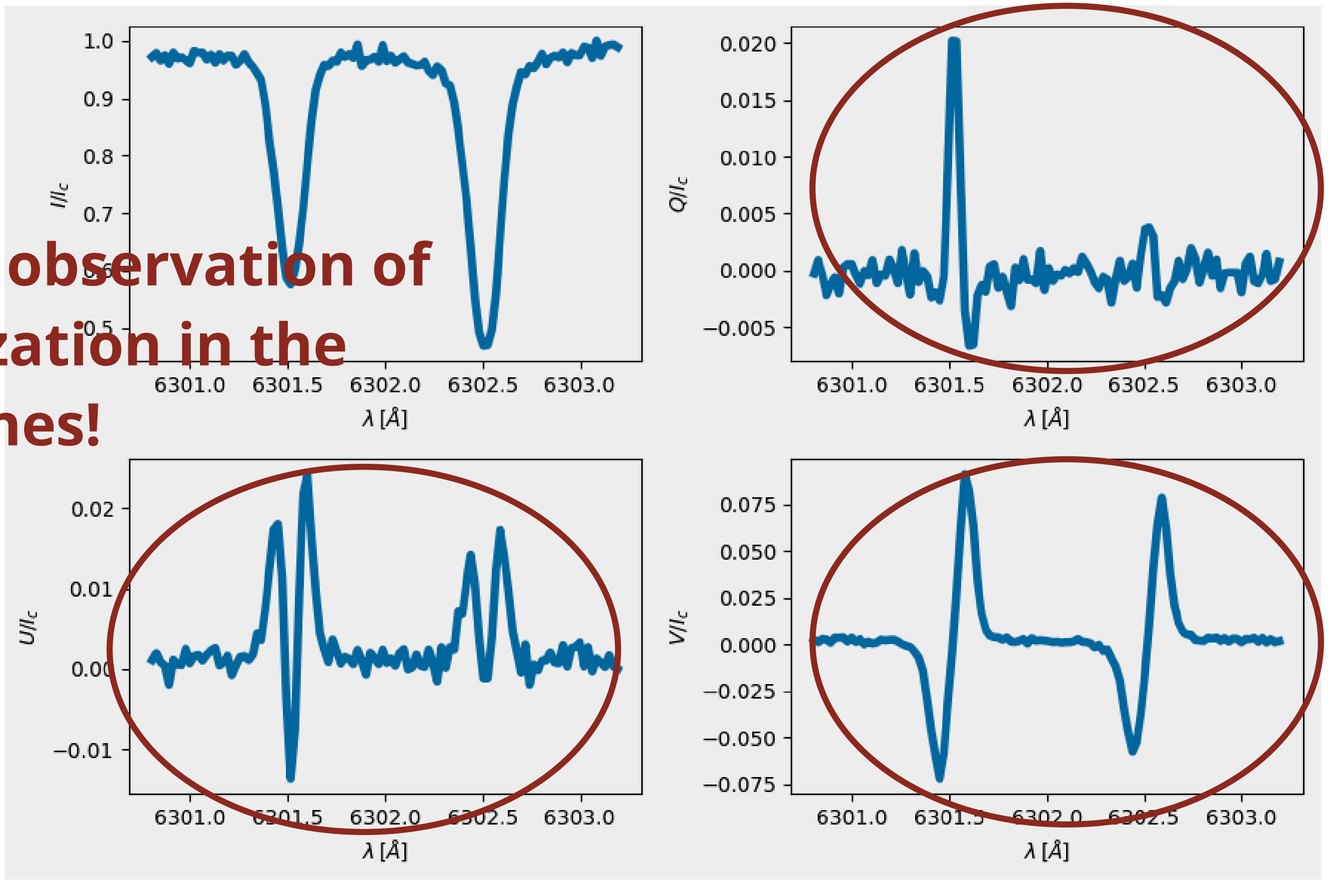
Hinode SP (image on the right), also measures something else



“Quiet-ish” Sun observed with HINODE/SOT SP

Hinode SP (image on the right), also measures something else

This is the observation of the polarization in the spectral lines!



"Quiet-ish" Sun observed with HINODE/SOT SP

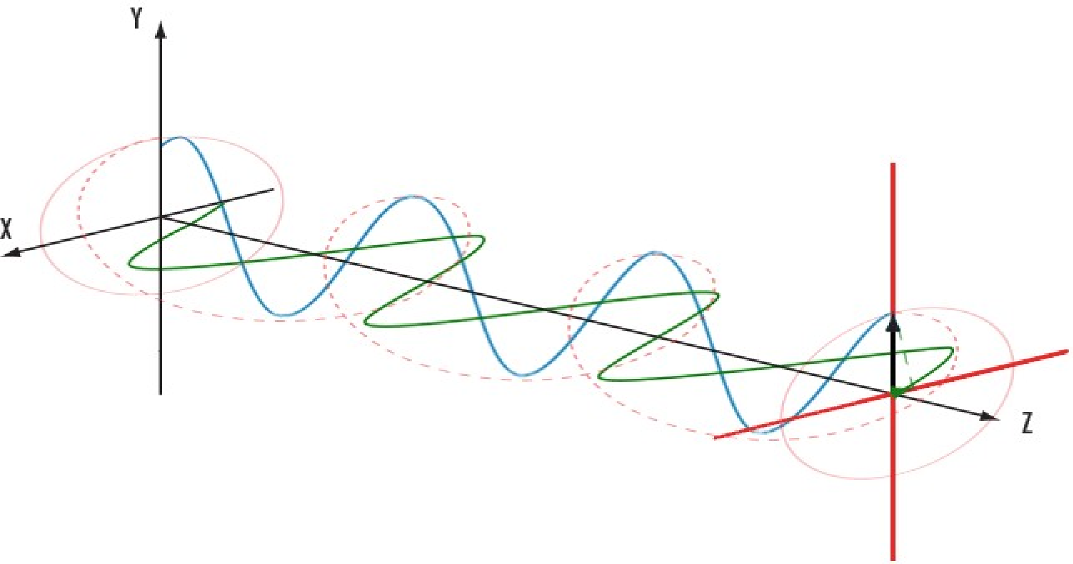
So the two questions we are posing now are:

What is polarization?

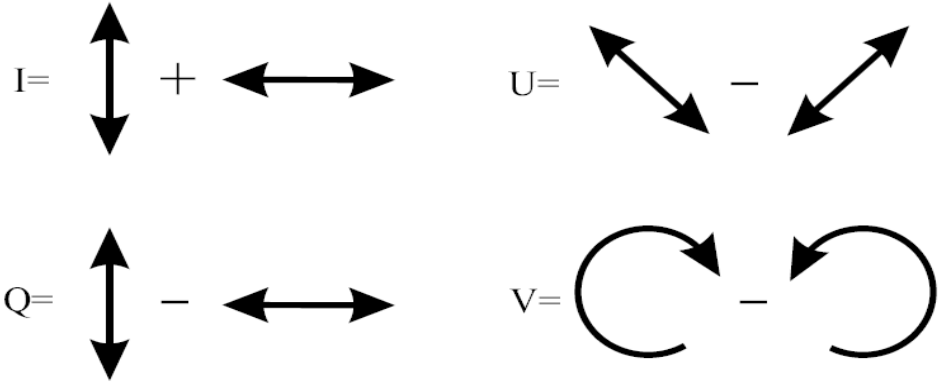
and

Why does it have such wavelength dependence?

Polarization of the light



Credits: www.edmundoptics.com



What is this about? Let's take it slowly

- Focus on one wavelength / frequency. Recall that Maxwells equations allow for the propagation of two electromagnetic waves with the electric fields perpendicular to each other.

$$E_x(z,t) = E_{0,x}e^{i(kz-\omega t+\phi_x)}$$
$$E_y(z,t) = E_{0,y}e^{i(kz-\omega t+\phi_y)}$$

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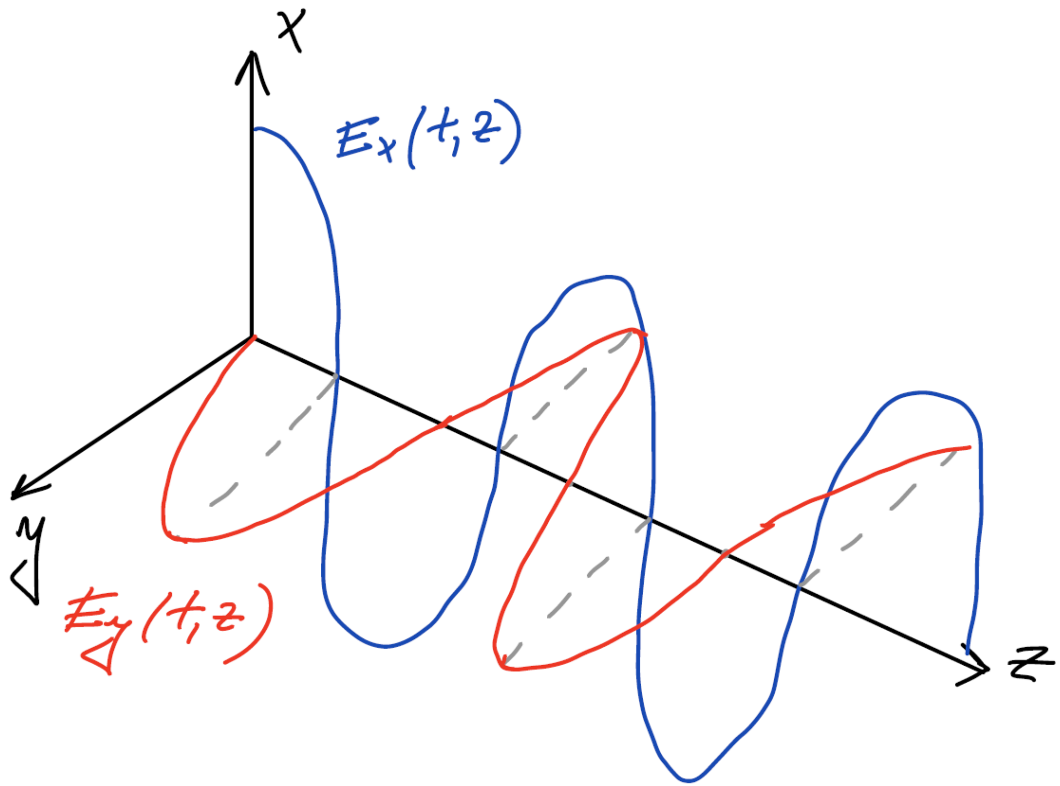
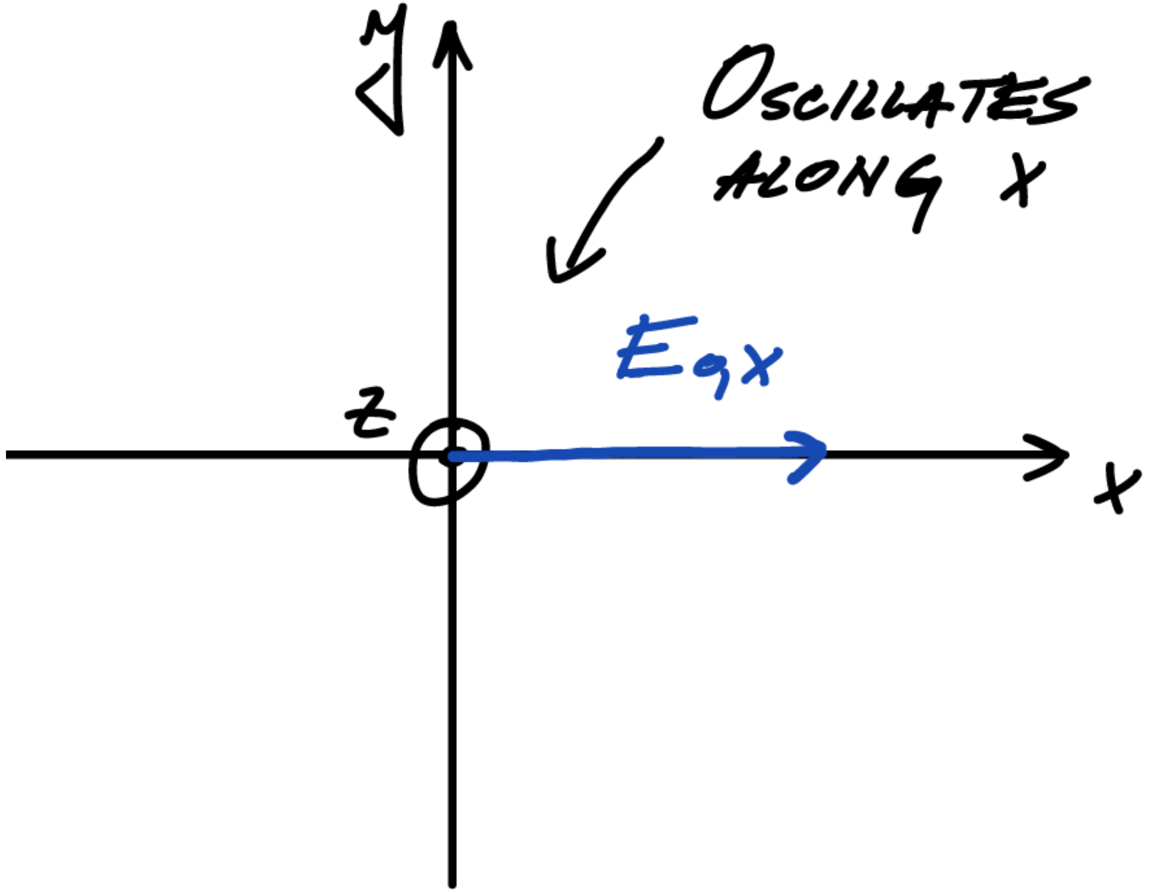


This bad sketch is supposed to show you that Ex and Ey don't have to be in phase!

Let’s look at some examples shall we?

$$E_x(z,t) = E_{0,x} e^{i(kz-\omega t)}$$
$$E_y(z,t) = 0$$



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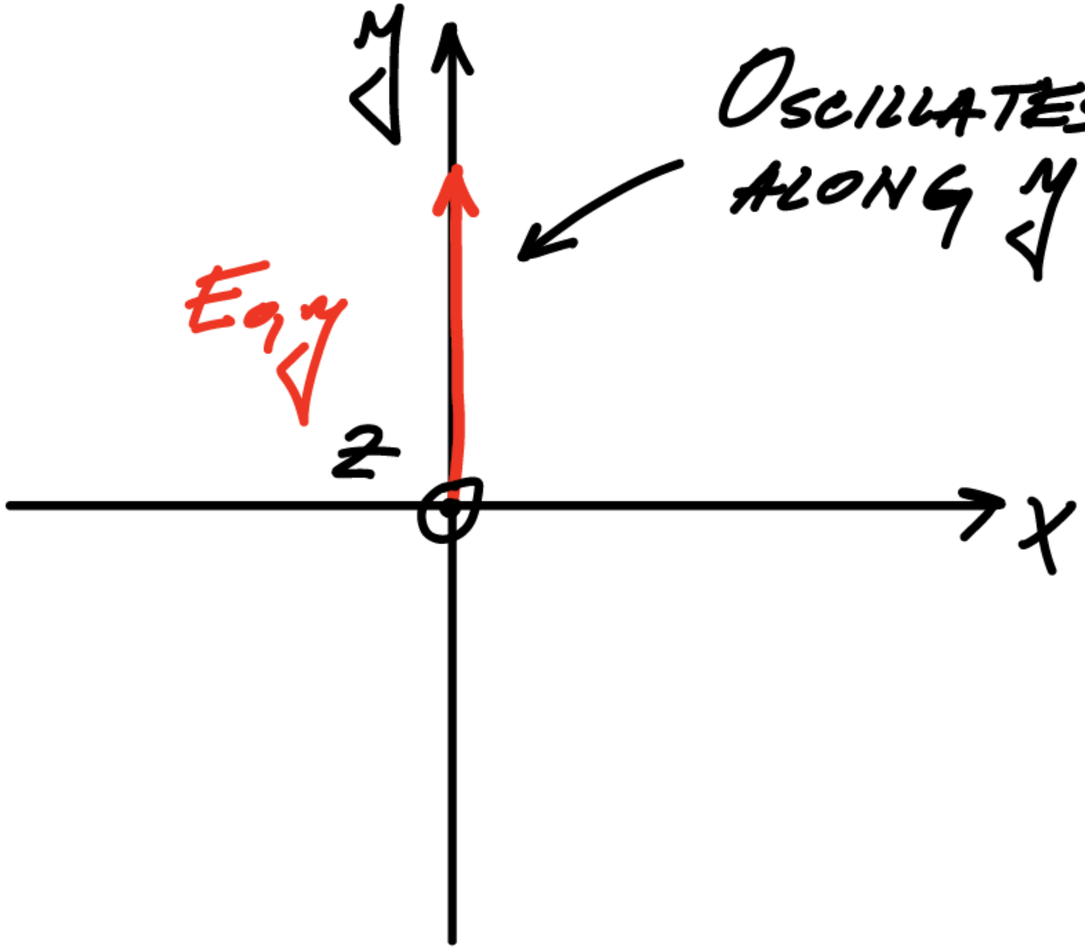

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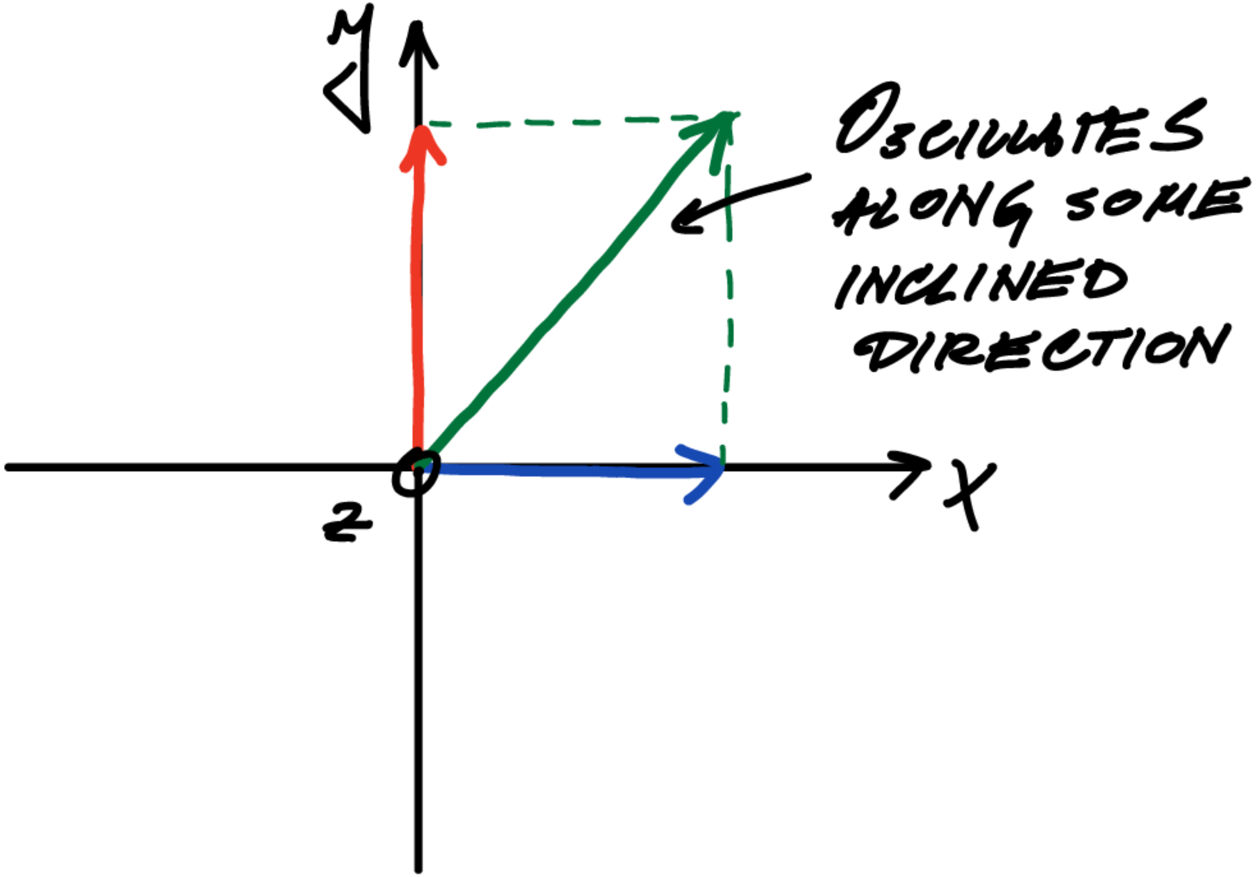

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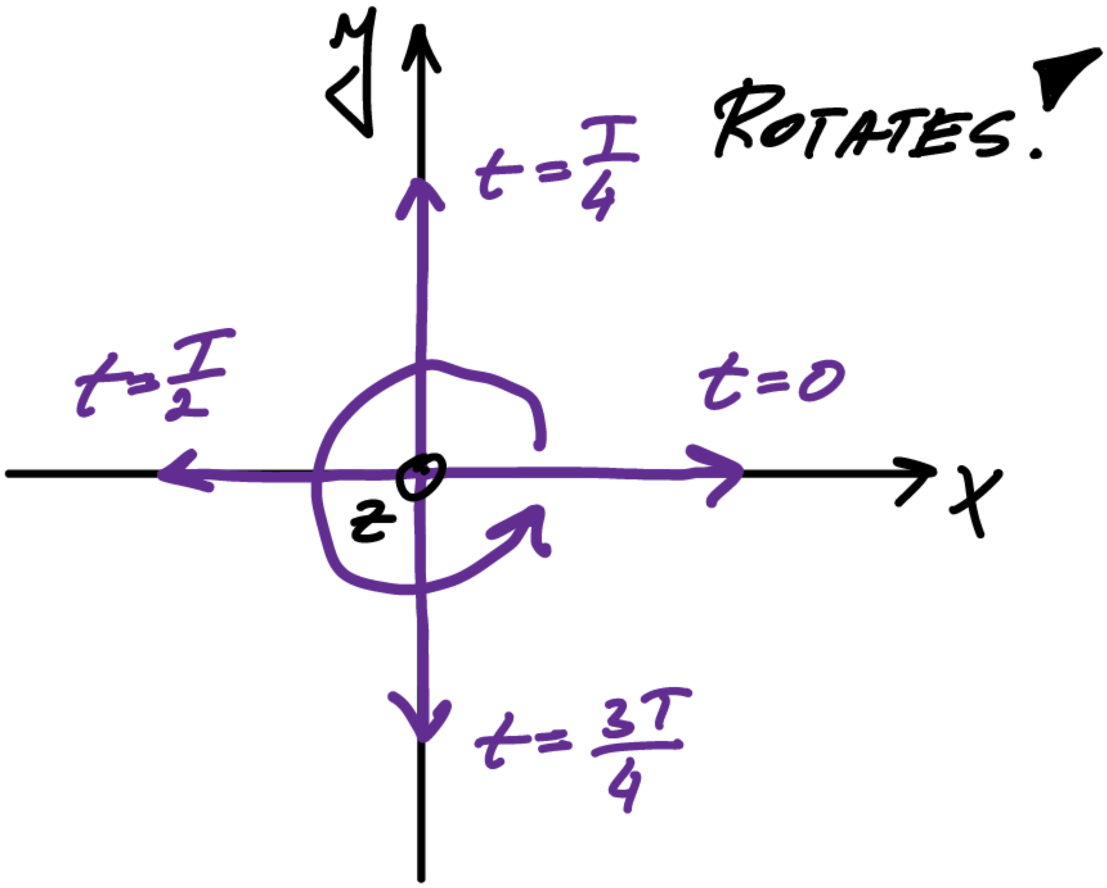
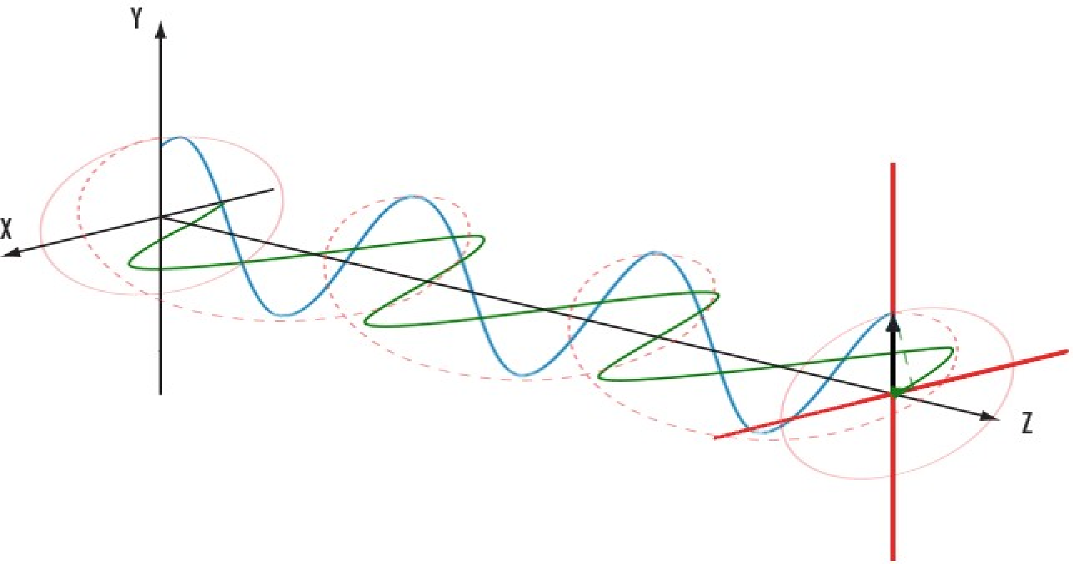
$$E_x(z,t) = E_{0,x} e^{i(kz-\omega t)}$$
$$E_y(z,t) = E_{0,y} e^{i(kz-\omega t+\pi/2)}$$



Polarization of the light



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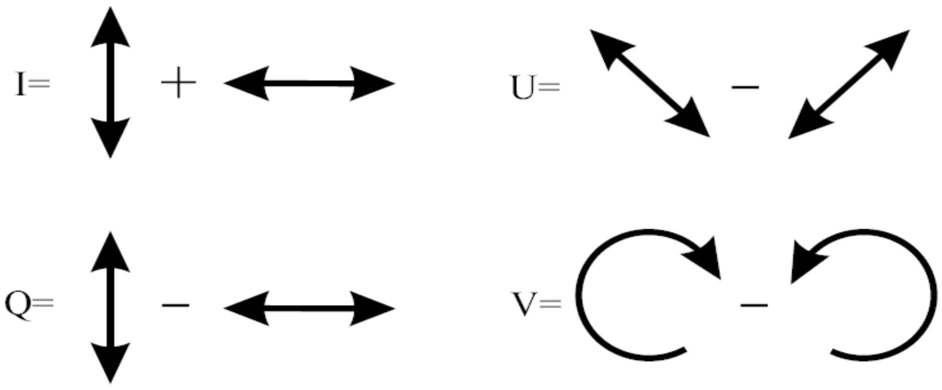
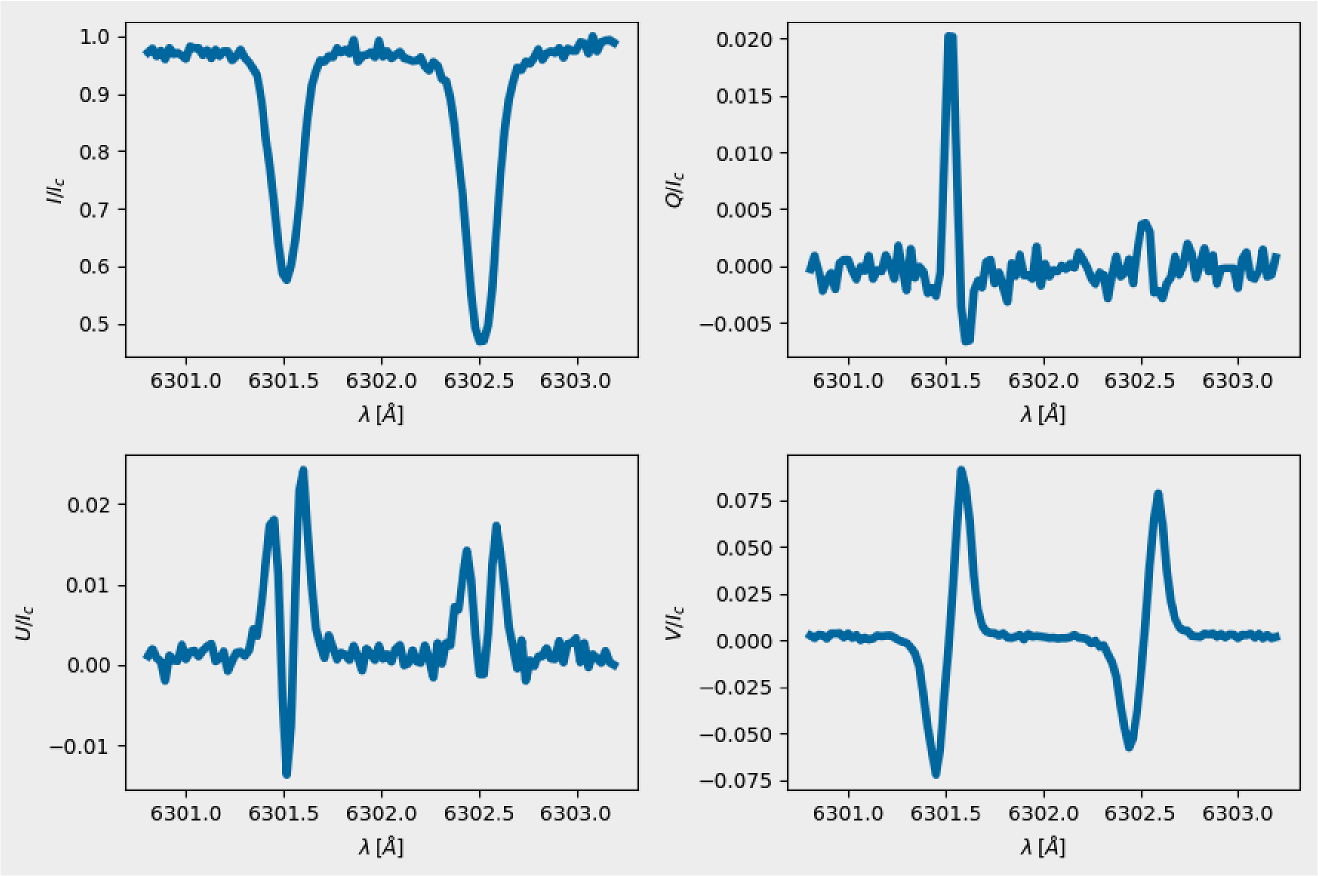
We can describe our polarization state completely using the Stokes parameters, I,Q,U,V



Keep in mind that we don’t observe these ideal EM waves!

- What we actually see is an ensemble of photons (wave packets), that are similar to ideal harmonic monochromatic plane waves, except they are time-limited.
- (This also means they are not completely monochromatic)
- Each wave-packed has different polarization so when we add them together, if they are completely randomly polarized, their polarization cancels
- We have completely unpolarized light.
- There are, however, mechanisms that polarize the light!

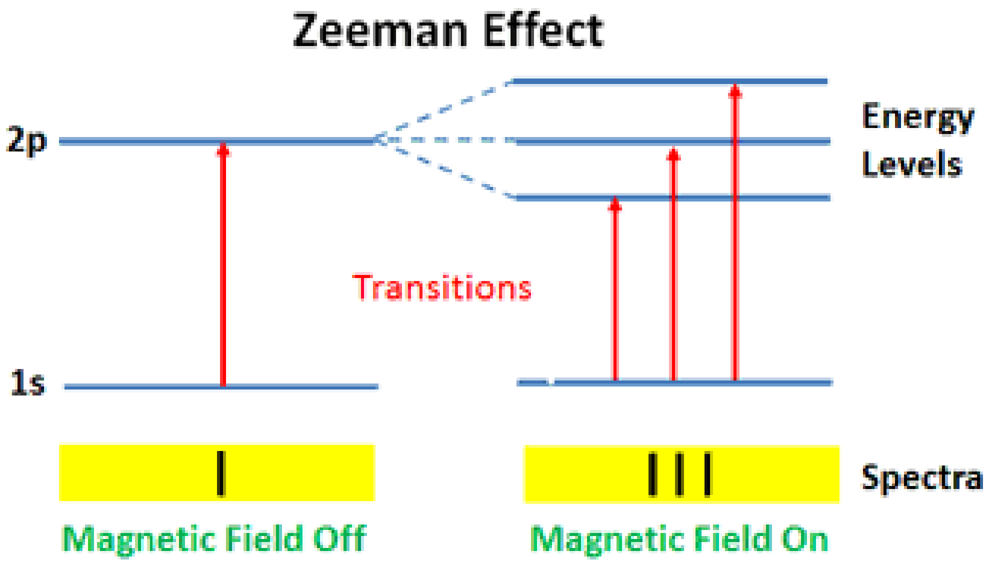
These mechanisms are **wavelength dependent**



"Quiet-ish" Sun observed with HINODE/SOT SP

Zeeman effect – polarization

- Individual photons are always 100% polarized. Spin of the photon is it's polarization. Depending on the basis we have completely left/right circular or completely x/y linear polarization.



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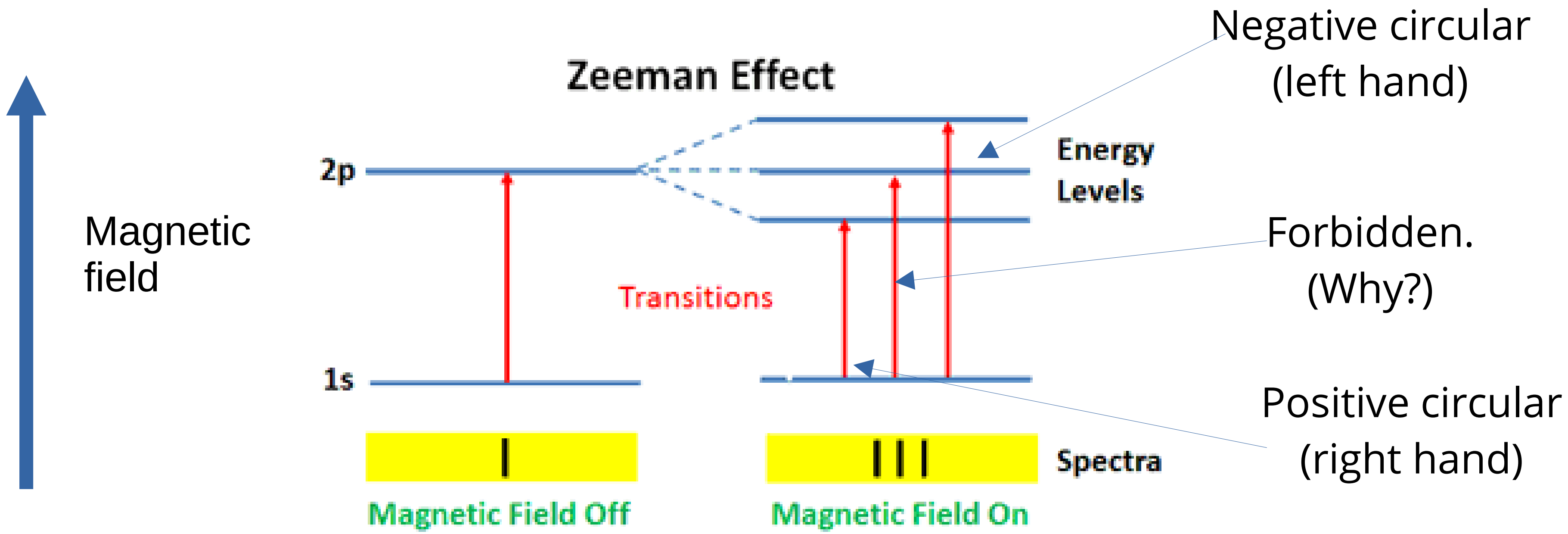

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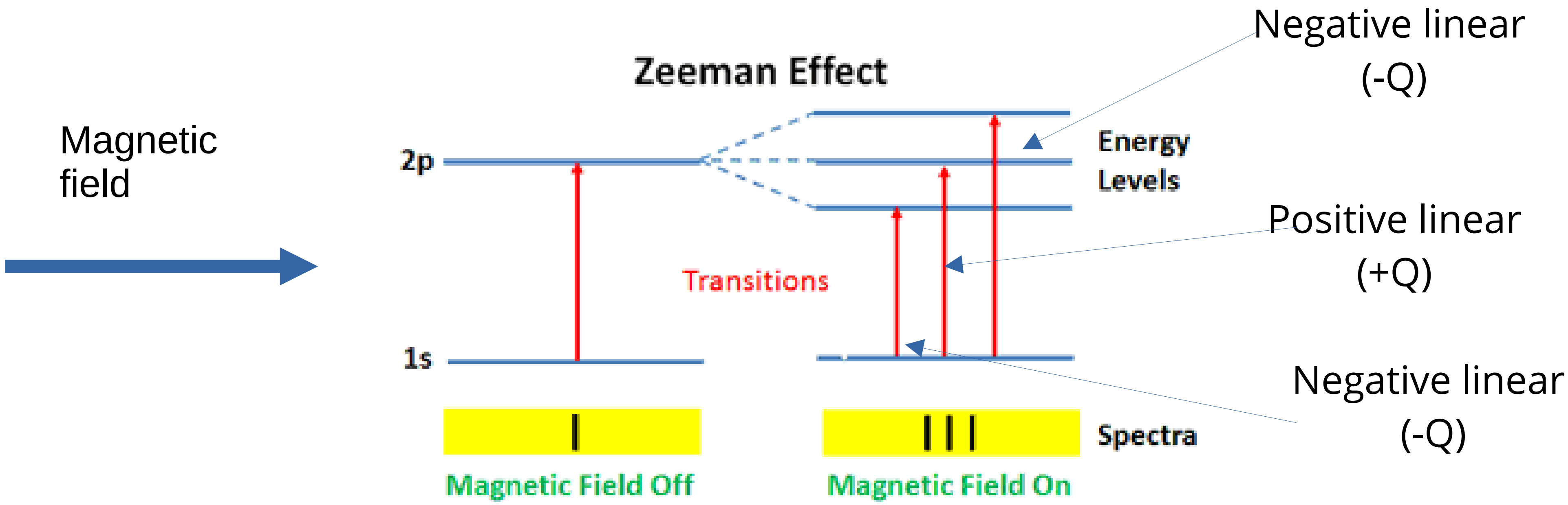
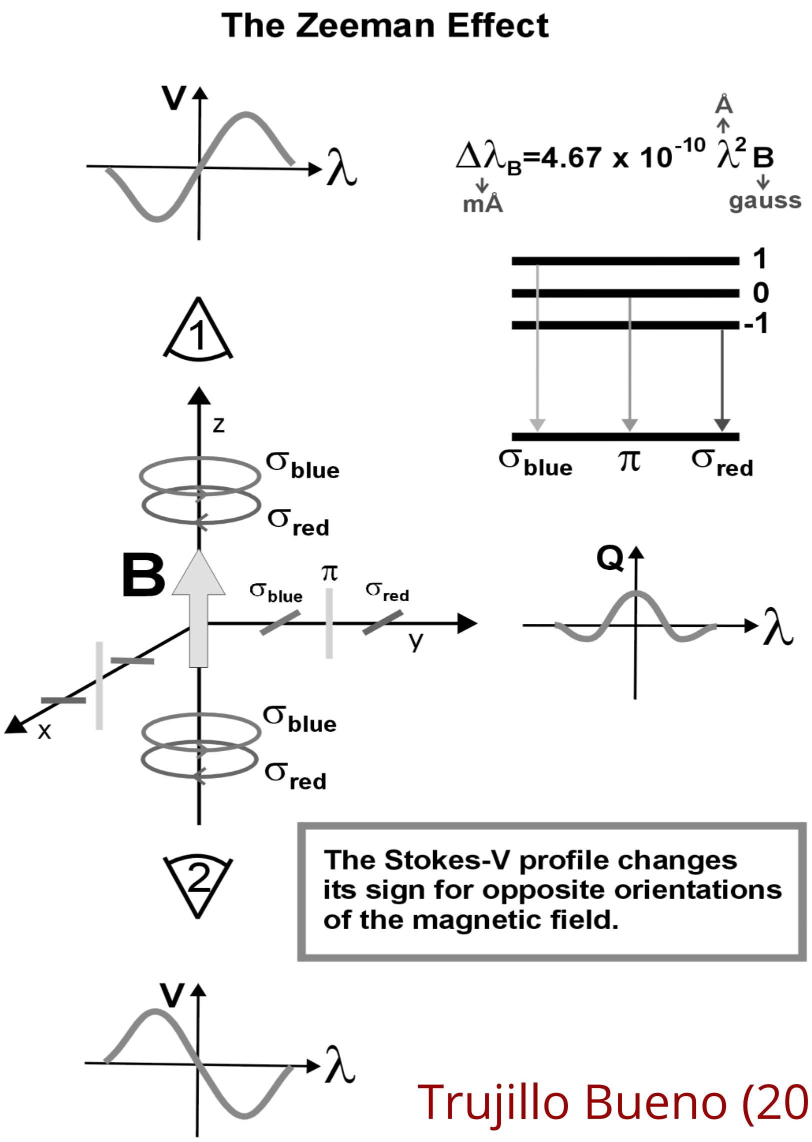


So to summarize

- Photons have spin one and two possible states, -1 and 1 (massless)
- Quantization axis for the angular momentum is this case is chosen to be the magnetic field director
- Projection of the angular momentum of the photon on the mag field can be -1,0,1
- However the angular momentum has to be conserved. If the photon travels paralelly to the mag field – only -1 and 1 transitions in *m* are allowd
- If it travels perpendicularly, then all three are allowed.

Now, wavelengths…

- These polarizations always exist, even if there is no magnetic field. But they all have the same energy/wavelength, so polarization cancels.
- However, when we introduce magnetic field, sublevels split, their wavelength changes and they do not overlap any more.
- In a way **It is not the magnetic field that causes the polarization, it just reveals it to us!**



Trujillo Bueno (2006)

Why the polarization?

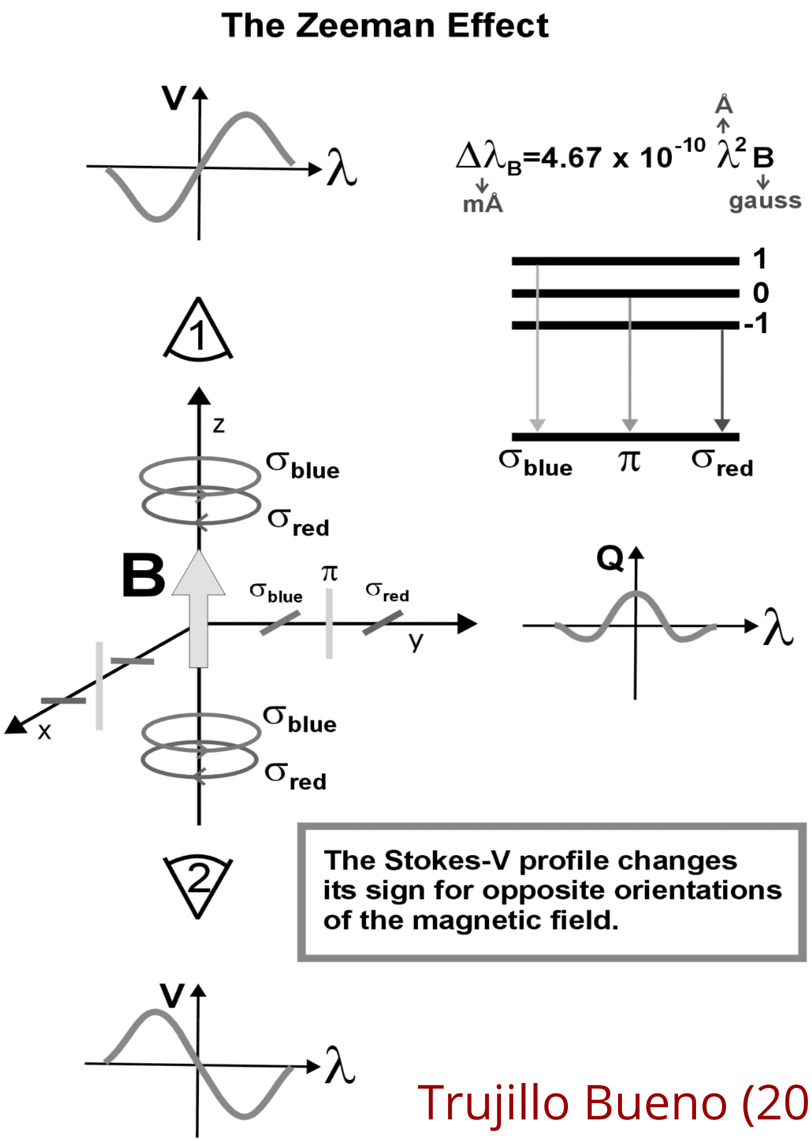
Individual photons are 100% polarized.

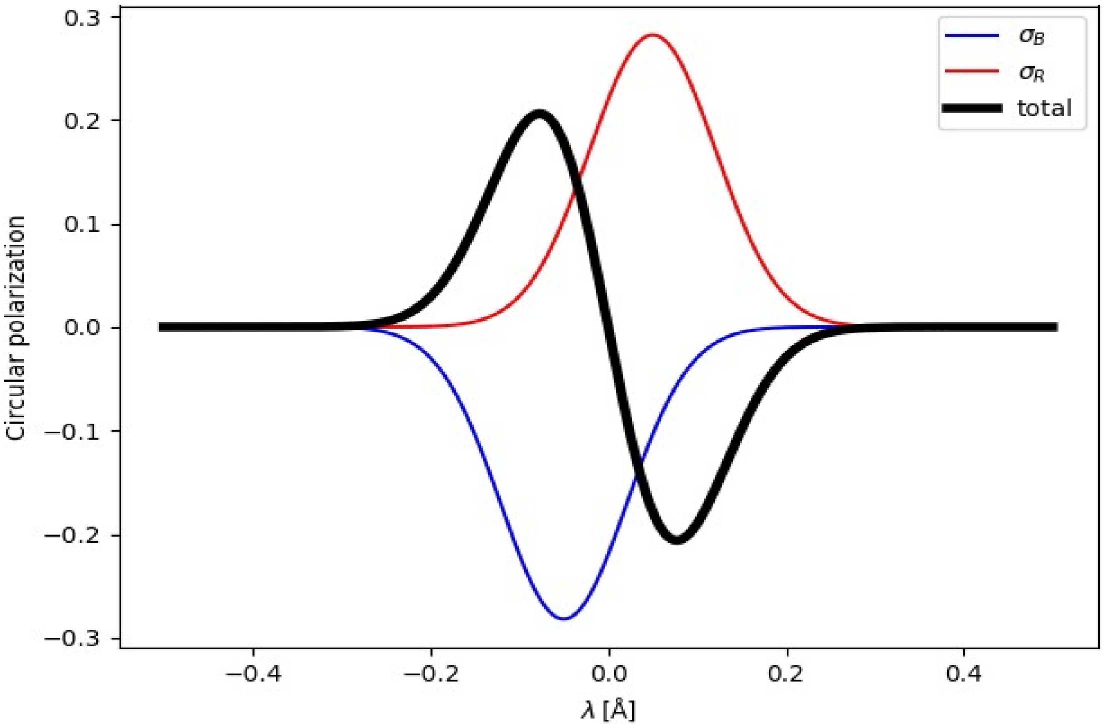
Different Δm transitions – different polarizations!

Parallel with B: only positive and negative circular polarization (σ_{blue},σ_{red})

Perpendicular to B: σ_{blue},σ_{red} seen as negative linear polarization, π as positive linear polarization

Parallel to **B**





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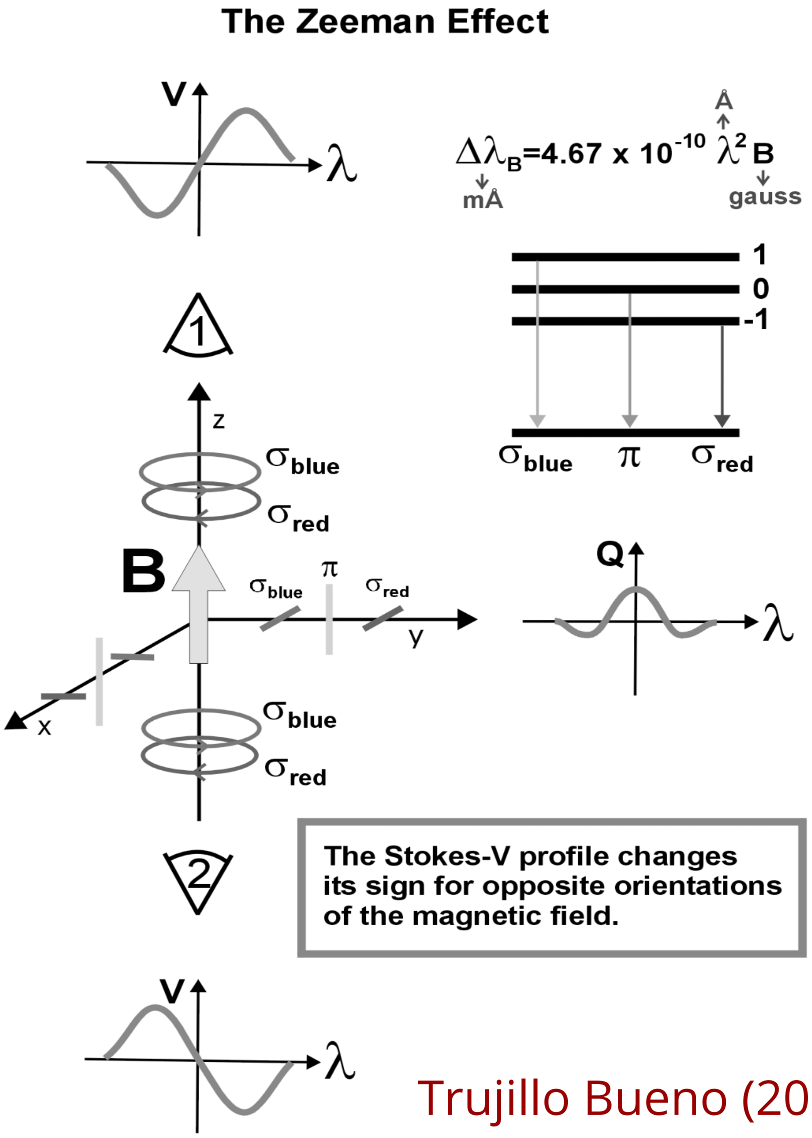
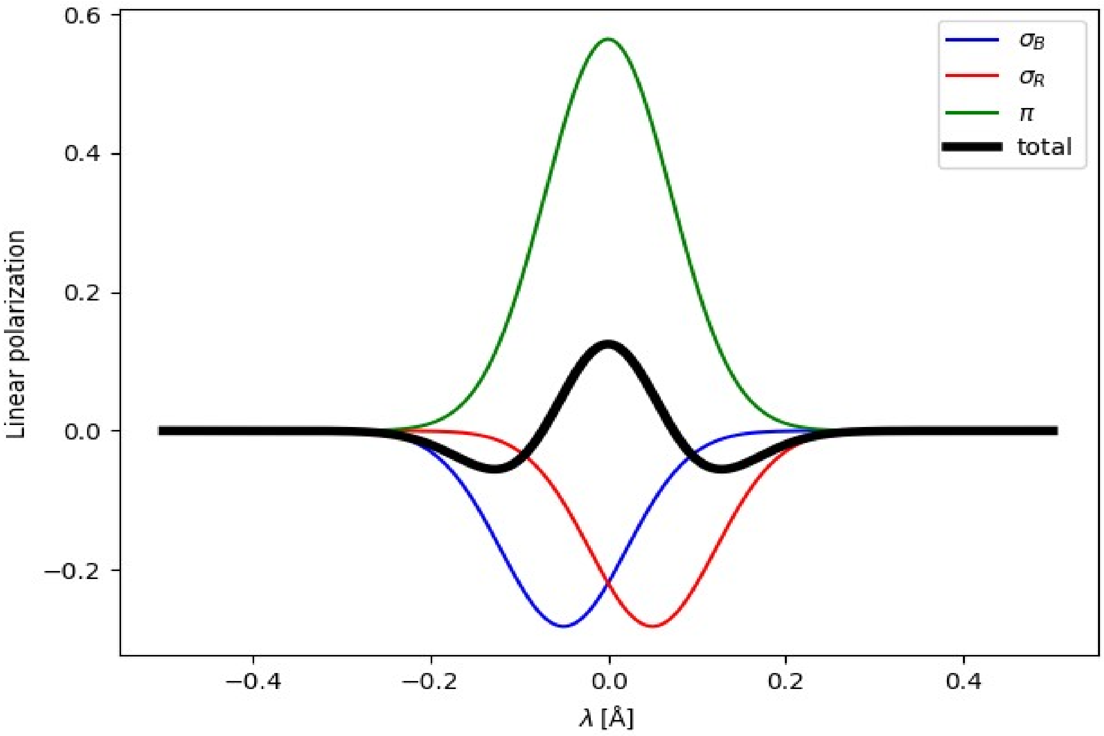
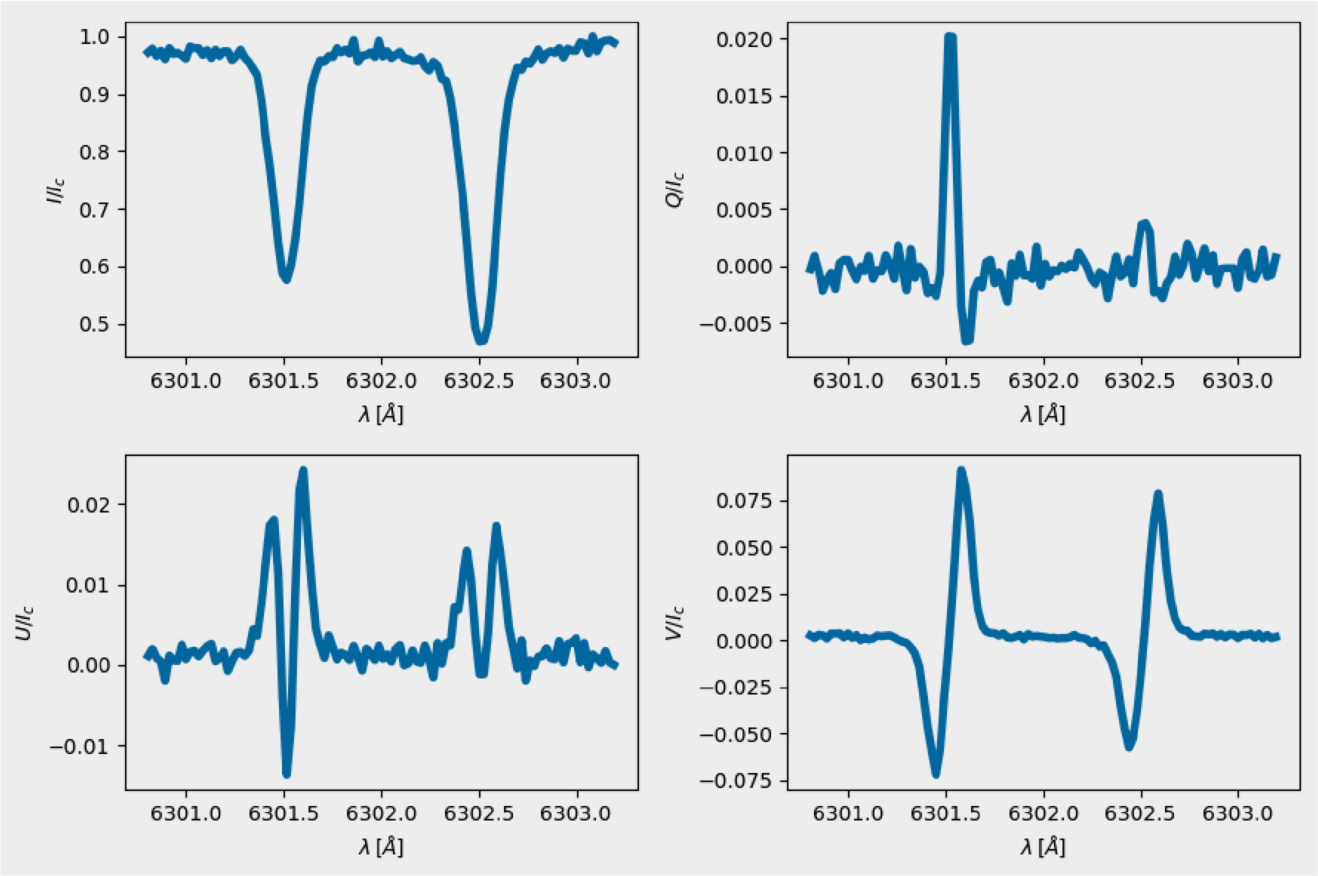
Perpendicular to **B**





Trujillo Bueno (2006)

Does it make more sense now?



"Quiet-ish" Sun observed with HINODE/SOT SP

For the end some maps of **polarization**