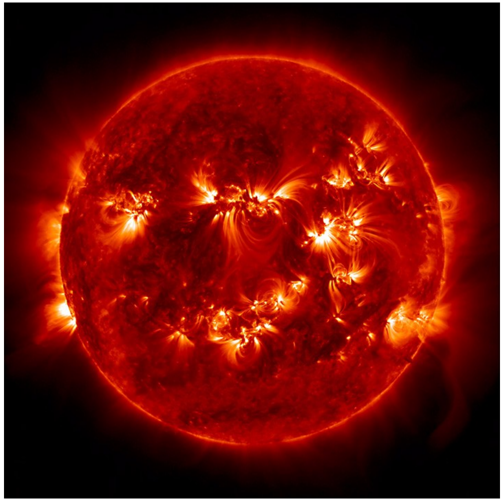
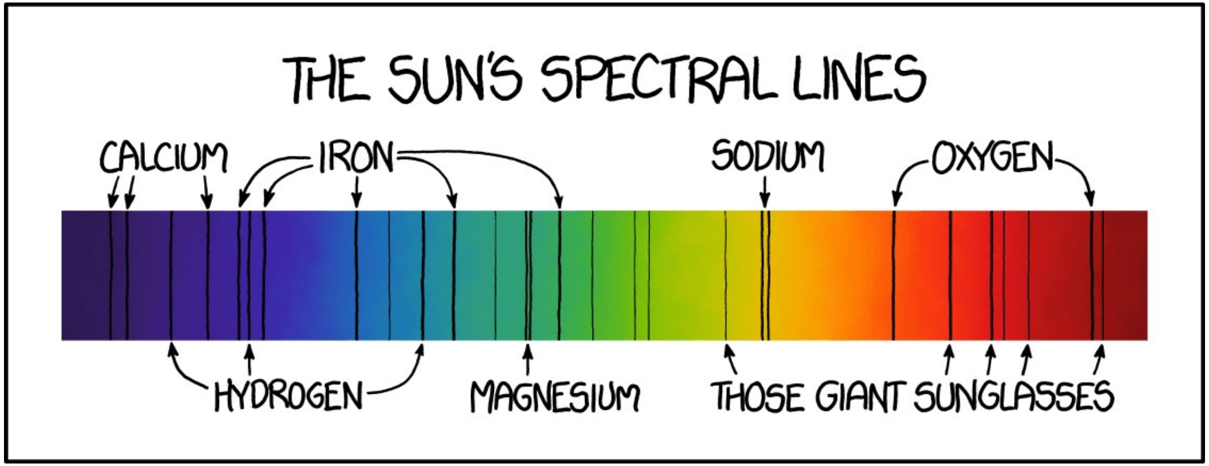
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 13: Scattering Hands-On

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What is problem with scattering again?

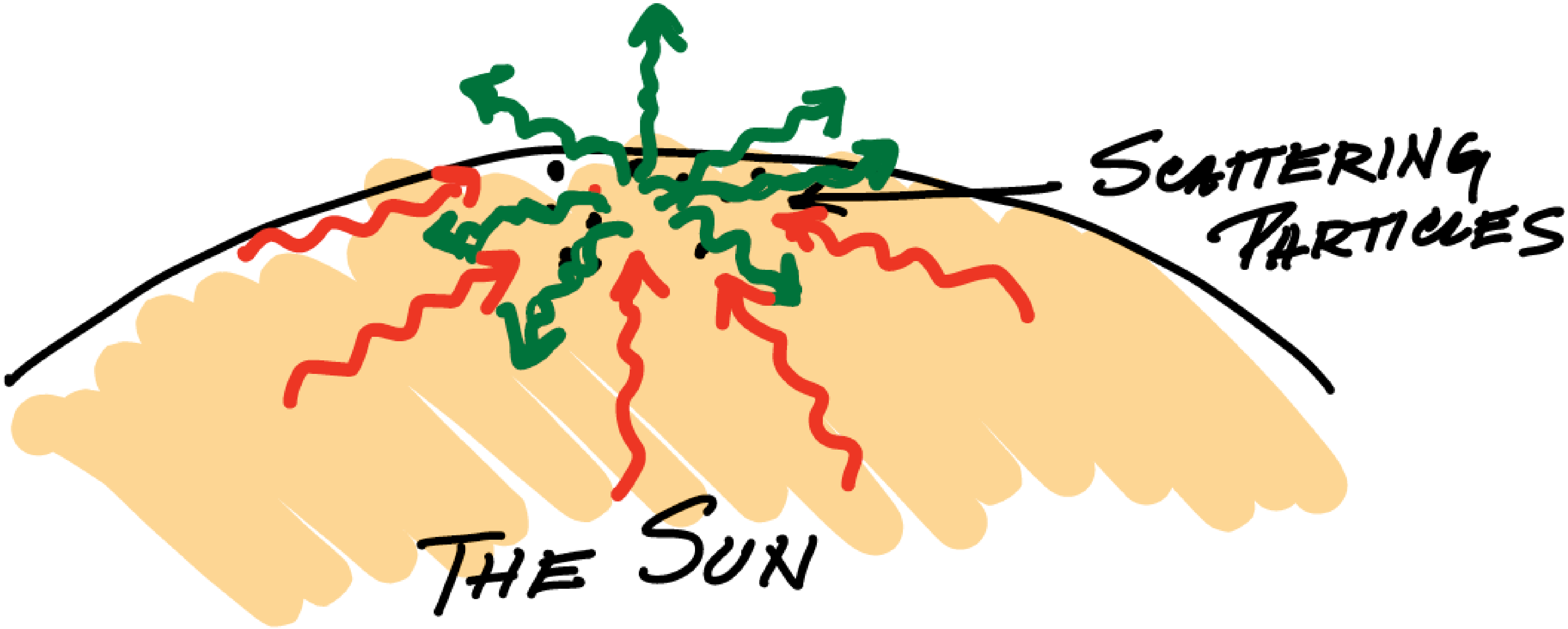
- In LTE, emission and absorption depends on the local quantities
- Source function is equal to the Planck function, depends only on temperature and (weakly) on wavelength
- If there is scattering, situation is more complicated, even in the continuum, and especially so in spectral lines.
- *Remember, cold gas can emit a lot if we illuminate it with strong radiation and it scatters it around.*

Example: Prominences!

- Right: prominence seen in H alpha (Copyright: Peter Desypris)
- This is a bunch of plasma somehow suspended above the Sun that scatters radiation coming from below. Remember this as we will model it soon!

Remember this?

- Red is incoming radiation, that is getting absorbed, but actually scattered as green lines (colors have nothing to do with wavelength here)



Remember the formalism

- Source function is a combination of thermal (LTE), and scatttered (NLTE) source function so something like this:

J is the so called mean intensity. It is the intensity averaged over angle.

$$S = \frac{j}{\chi} = \frac{j_{\mathrm{Planck}} + \chi_{\mathrm{sc}} J}{\chi_{\mathrm{abs}} + \chi_{\mathrm{sc}}}$$

$$J = \frac{1}{4\pi} \oint I(\Omega) d\Omega$$

$$S = \epsilon B + (1 - \epsilon) J$$

Famous “epsilon” tells you what is the probability of “truly” absorbing (destroying) the photon. In chromosphere epsilon drops to 10^{-8} or even less!

If I write this now as a single equation:

$$\frac{dI}{dl} = I - \epsilon B - (1 - \epsilon)\frac{1}{4\pi}\oint I(\Omega)d\Omega$$

- The spatial change of the intensity depends on the very same intensity (integrated over the wavelengths).

This is, in a way, similar to writing: $x = \ln x$

- It's similar in the way that it is unsolvable directly. Knowing B and epsilon does not help us simplify this in any way

$$\frac{dI}{dl} = I - \epsilon B - (1 - \epsilon)\frac{1}{4\pi}\oint I(\Omega)d\Omega$$

- This is known as an integro-differential equation. Note that we are integrating over direction and differentiating with respect to path (spatial coordinate)
- Many books and papers are written about solving this.

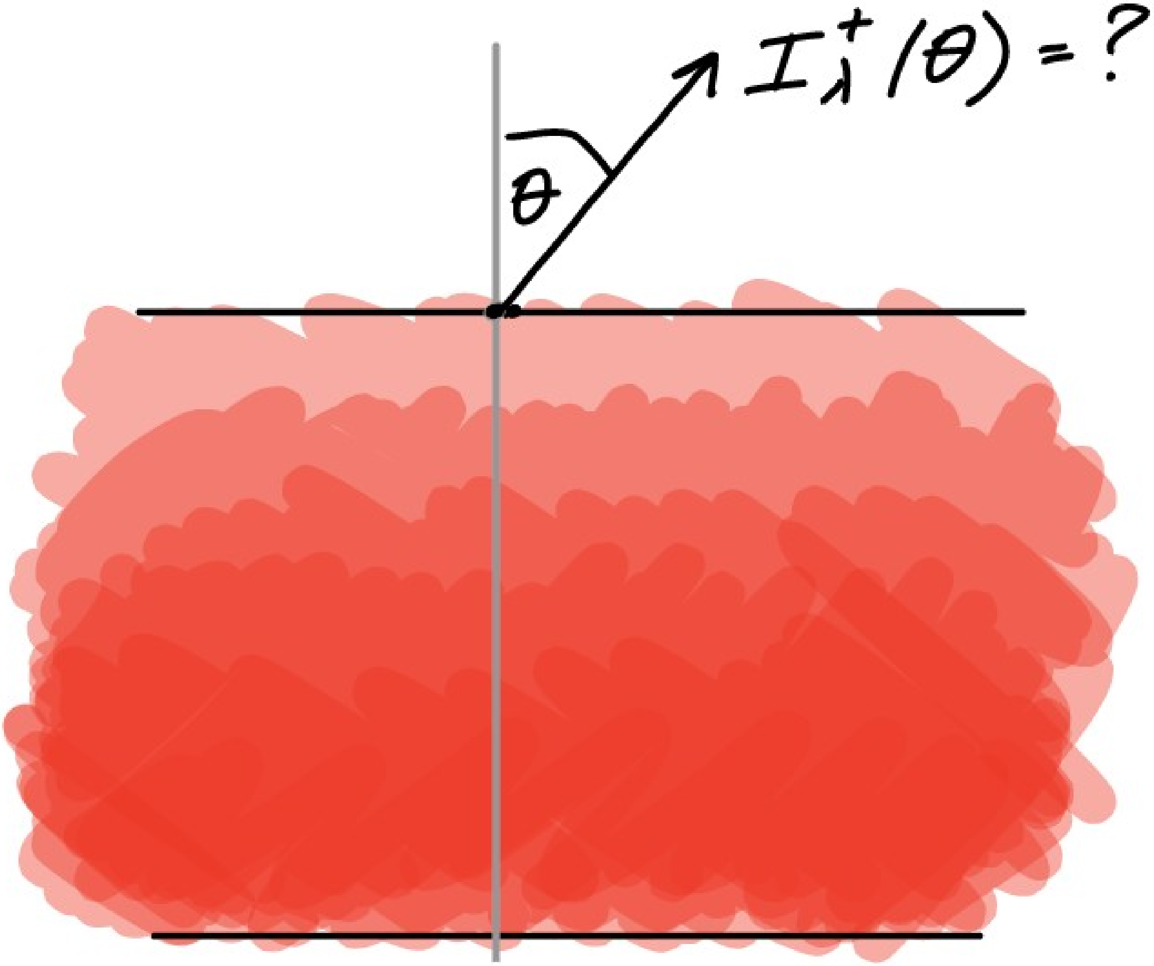
Let's say we want to try to do this.

- Necessary part of scattering is treating the mean intensity

$$J = \frac{1}{4\pi} \oint I(\Omega) d\Omega$$

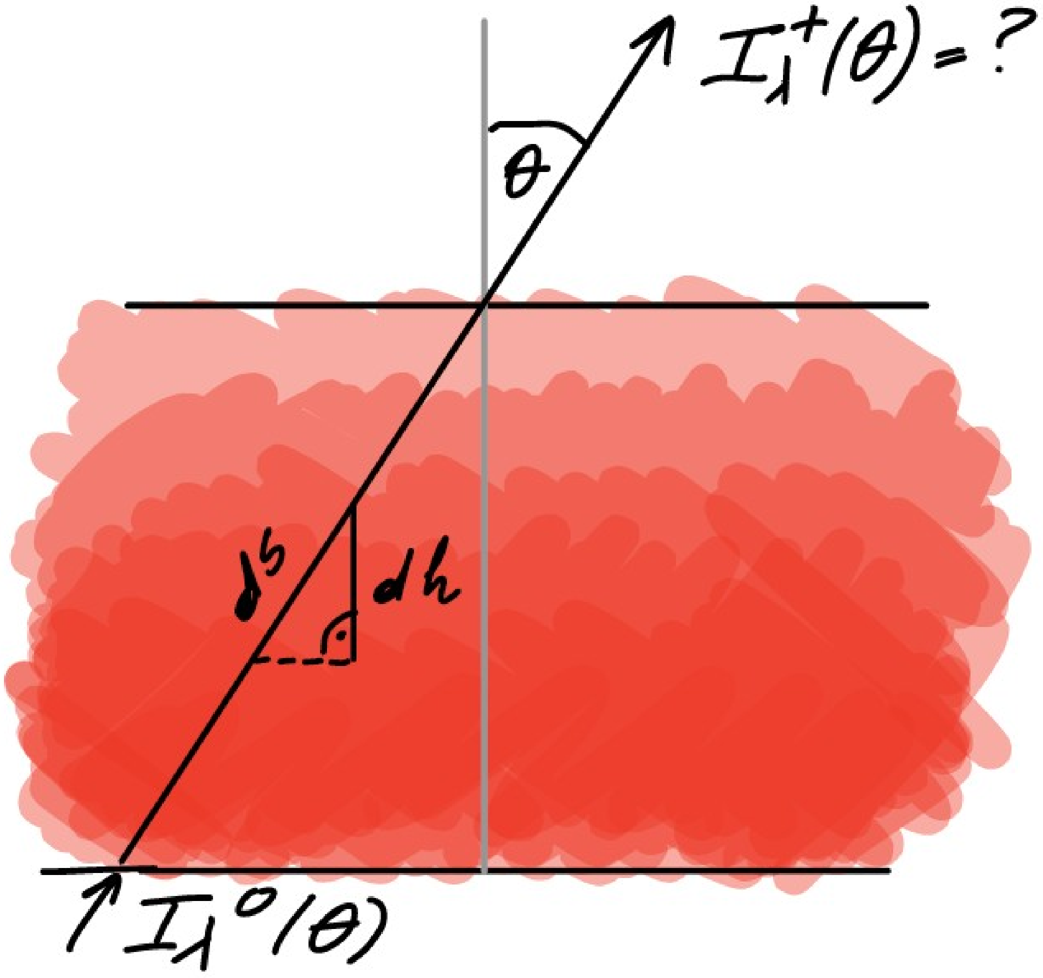
- If we want to calculate it, we need intensity in all the directions.
- How to obtain that?

Calculating intensity in arbitrary direction



- Assume that this atmosphere varies only along one coordinate, height.
- How would you go on about finding the emergent intensity in an arbitrary direction?

Calculating intensity in arbitrary direction

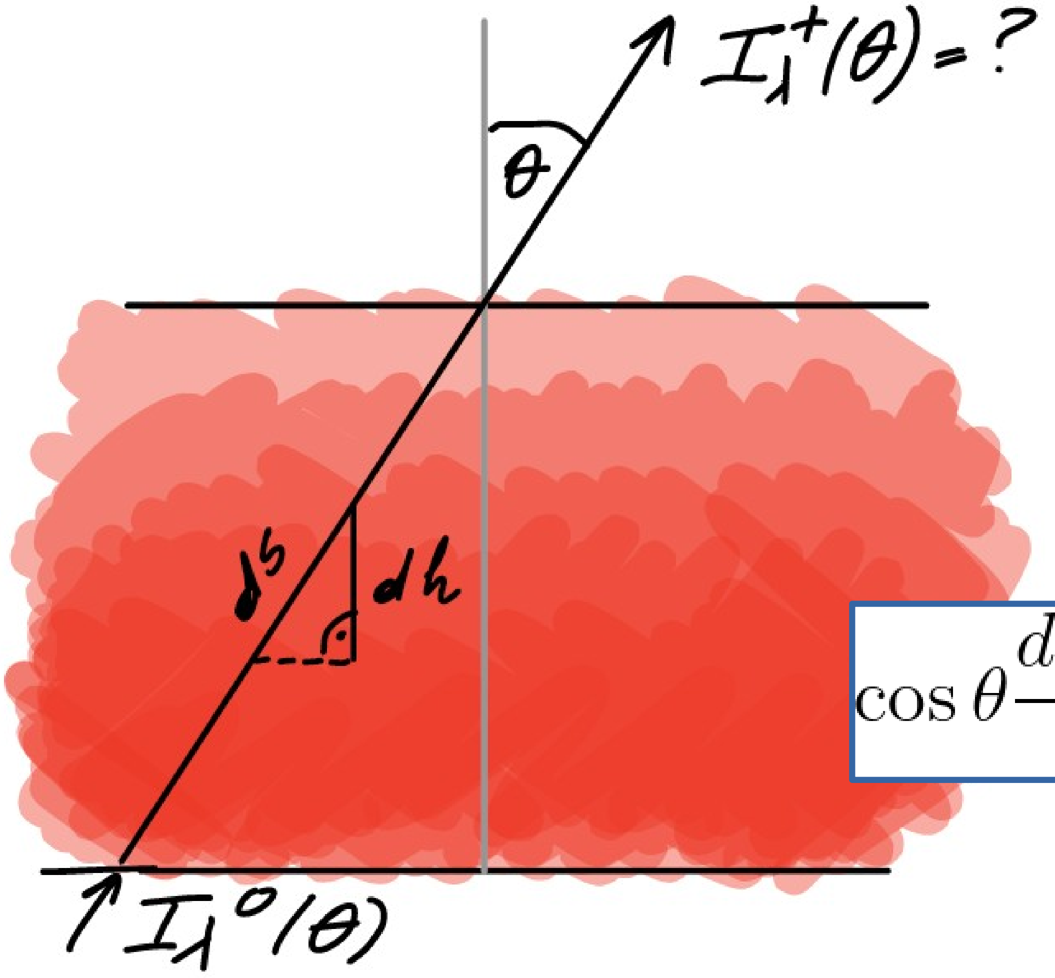


- We need to solve the RTE along this inclined line:

$$\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$

Comfortable with this? We will use this on multiple occasions.

Calculating intensity in arbitrary direction



- We need to solve the RTE along this inclined line:

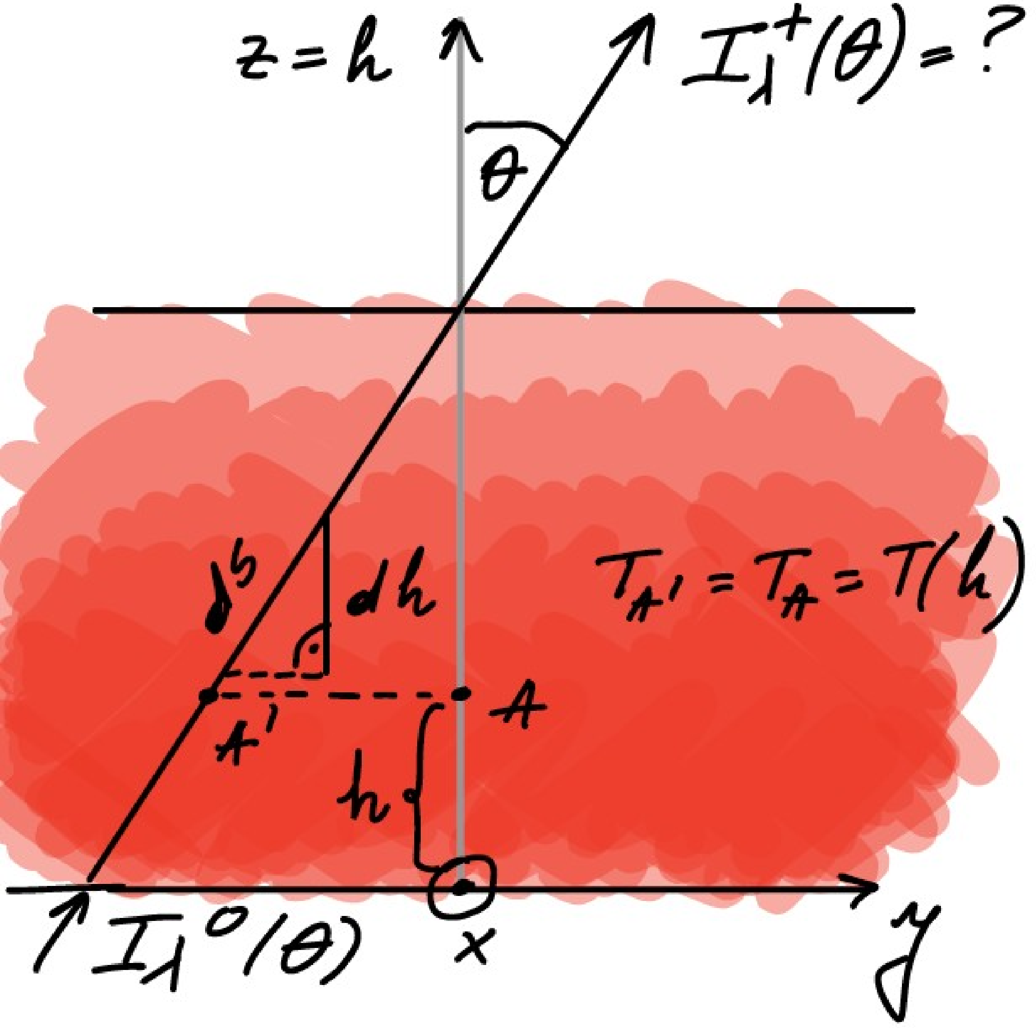
$$\frac{dI_\lambda(\theta)}{ds} = -\chi_\lambda I_\lambda + j_\lambda$$

Which is, in essence:

$$\cos\theta \frac{dI_\lambda(\theta)}{dh} = \mu \frac{dI_\lambda(\mu)}{dh} = -\chi_\lambda I_\lambda + j_\lambda$$

Famous “mu value”!

Don't forget that this is only true in 1D atmosphere!



- So in 1D atmosphere, everything depends only one coordinate (height, optical depth, whatever you like)
- Convince yourself that azimuth angle is not important. So:

$$\mu \frac{dI_\lambda(\mu)}{dh} = -\chi_\lambda I_\lambda + j_\lambda$$

$$\mu \frac{dI_\lambda(\mu)}{d\tau_\lambda} = I_\lambda - S_\lambda$$

So to summarize

- In 1D atmospheres intensity in given direction is a solution of RTE where spatial coordinate is tilted (divided with cosine of so called heliocentric angle)
- This is true for any point in the atmosphere, not only for the outgoing intensity
- We can choose any number of directions and solve RTE along these directions, to get the dependence of the intensity on the direction (or, more precisely “mu”)

So, what do we do now:

- First we are going to solve RTE for some “inclined lines of sight”
- Then we are going to solve this system of equations:

$$\mu \frac{dI_\lambda(\mu)}{d\tau_\lambda} = I_\lambda(\mu) - S_\lambda$$

$$S_\lambda = \epsilon B_\lambda + (1-\epsilon)\frac{1}{2}\int_{-1}^{1} I_\lambda(\mu) d\mu$$