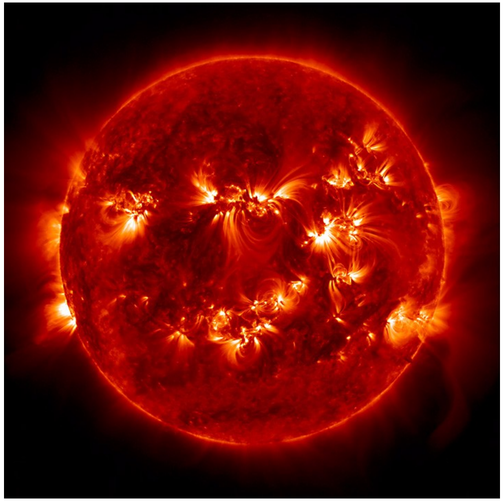
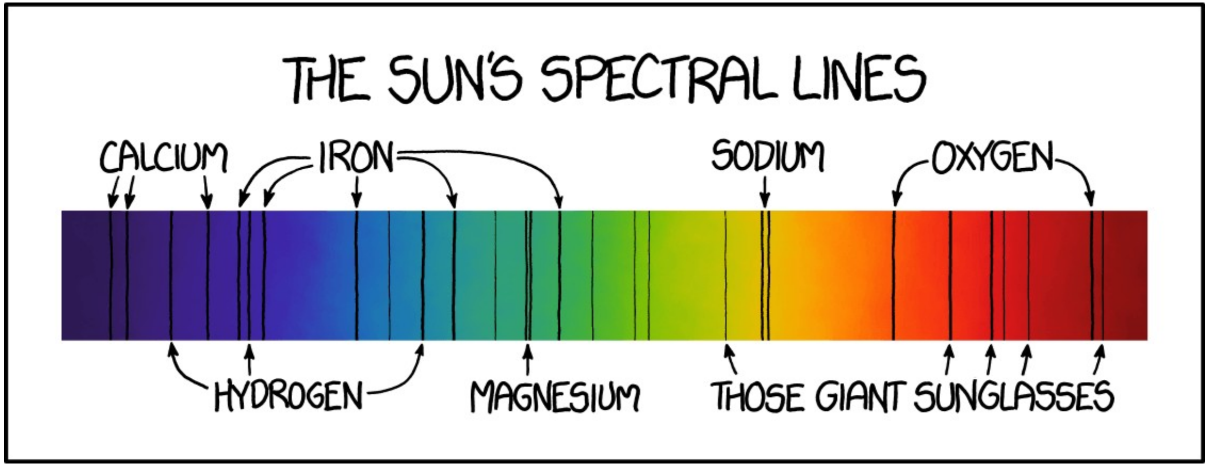
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 10: Radiative Transfer with Scattering

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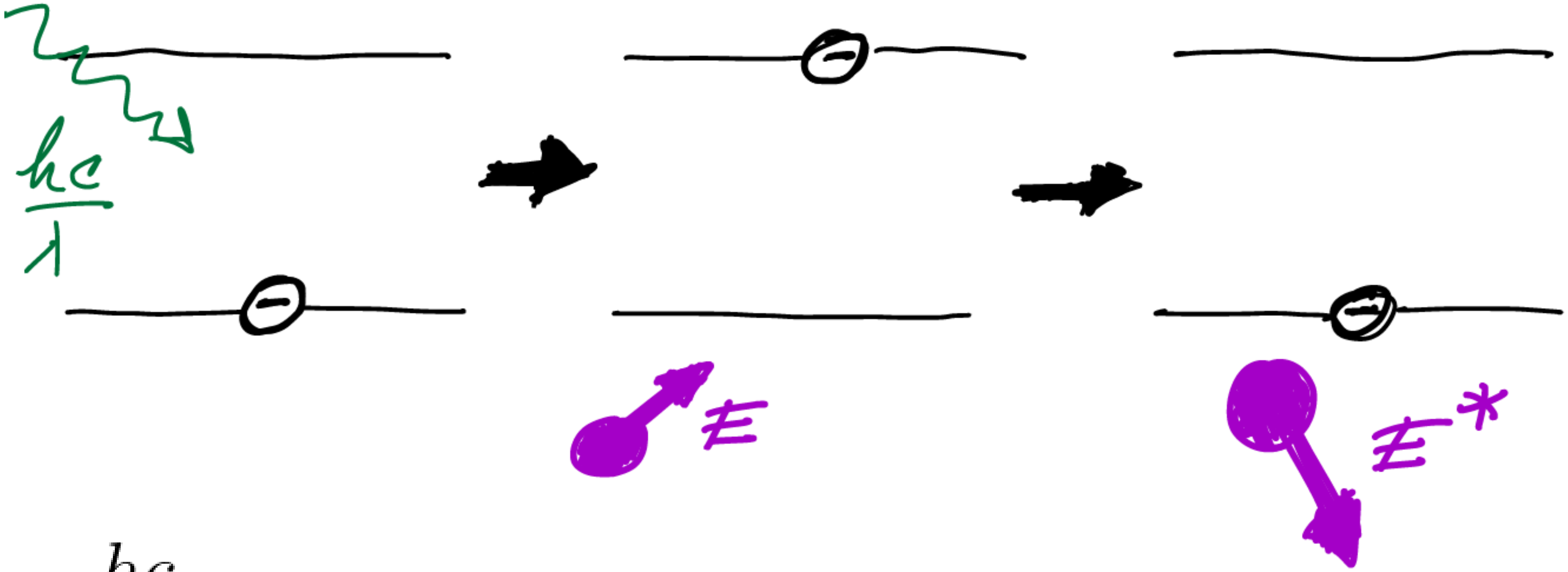
What did we do so far

- We solved radiative transfer equation for multiple wavelengths. That gave us emergent spectrum emerging from an atmosphere.
- We either assumed very simple parametrized models (see the homework).
- Or we worked with a model atmosphere, assuming LTE (Source function = Planck function; see the hands on).
- We also derived expressions for line opacity / emissivity and saw that *level populations* are the most important factor
- Saha and Boltzmann distributions allow us to calculate the level populations
- Problem is long, but conceptually straightforward.

LTE and what it implies

- We mentioned LTE (Local Thermodynamic Equilibrium), quite a few times
- For LTE to work we need to have ionization, excitation (as well as velocity distribution) governed by one and the same temperature
- Wait, what does this mean?
- Well in principle, you could define **excitation temperature, ionization temperature, etc, and they would not have to be the same**
- Even kinetic temperature would be something weird if velocity distribution was not Maxwellian

Role of collisions



$$A + \frac{hc}{\lambda} \to A^* \to A + \Delta E_{\text{kinetic}}$$

- Collisions help us establish this equilibrium
- Collisions redistribute excess energy (actually excess energy is negligible compared to the kinetic energy of the particles)

Mini detour – Spectral line heating and cooling

- In tenuous media, this extra energy can be important and contribute to **heating** of the medium (we know that photons can heat up medium)
- There is also opposite process:

  Collisions cause the excitation.

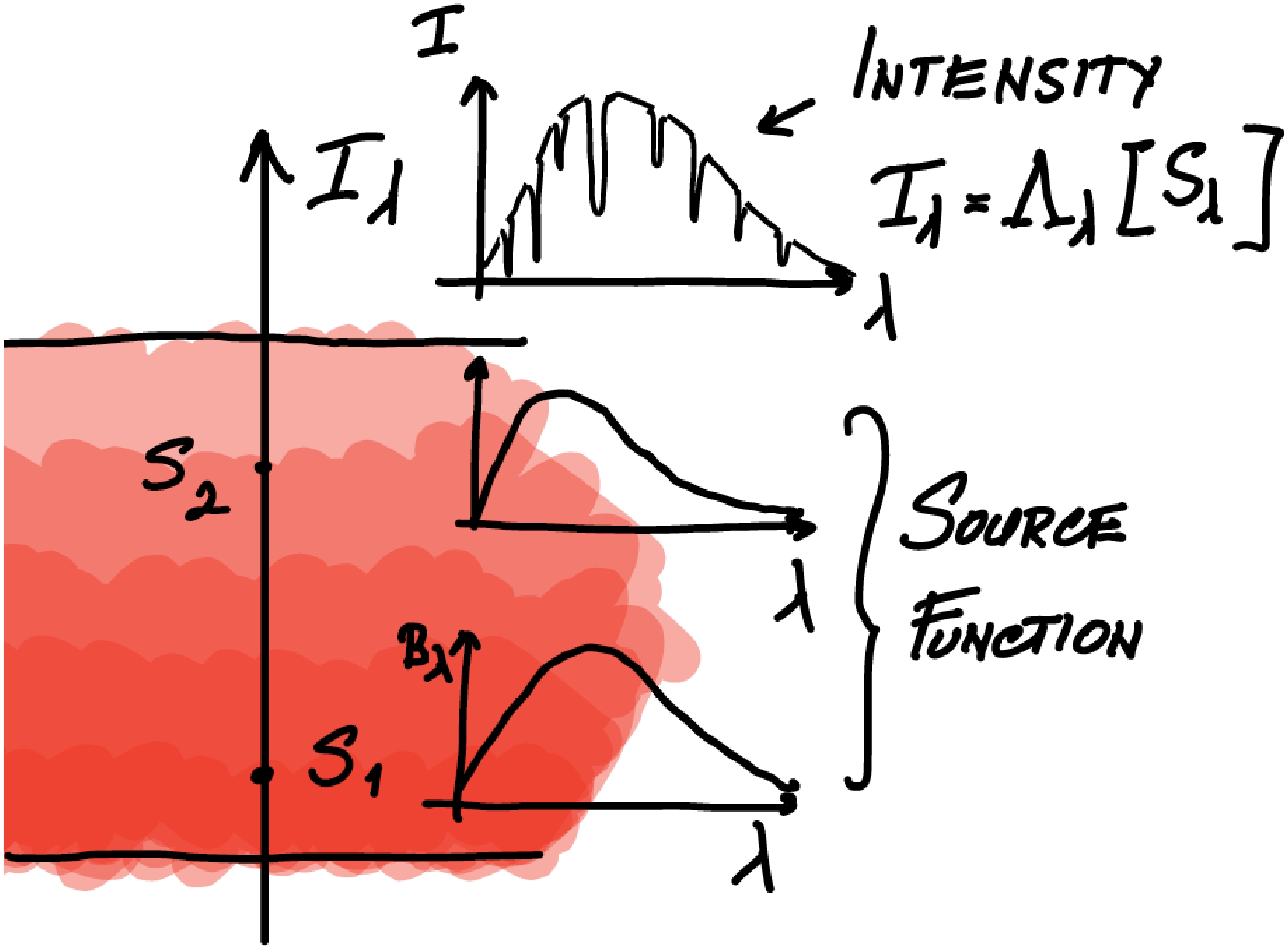
  Emitted photons escape the medium (not enough absorption), thus taking the kinetic temperature away and **cooling** the gas
- Important in interstellar matter and in transition region and corona

But wait, does that mean

- That the absence of collisions would break LTE?
- Actually, yes, but take it easy ;)

LTE and radiation

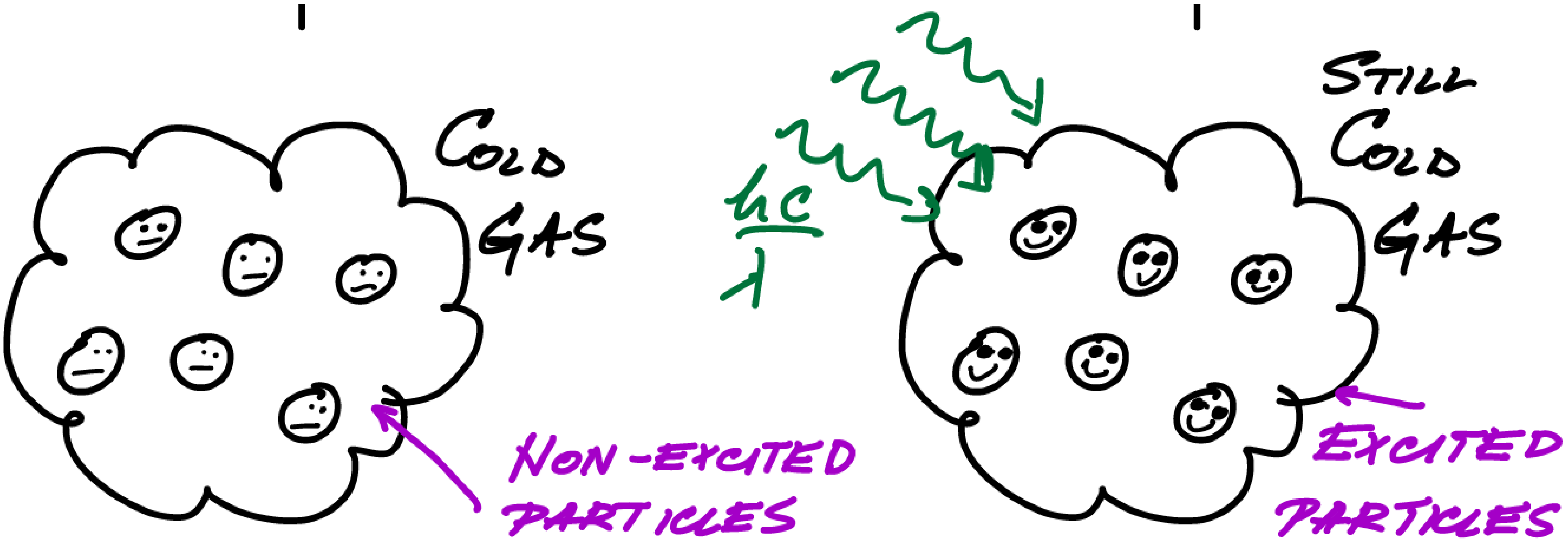
- **Contrary to what we might think, LTE does not imply that the intensity looks like blackbody**
- Opacity and emissivity obey equilibrium distributions, and the **source function will be Planck function**, but the intensity is still solution of RTE



LTE and radiation

- In LTE, radiation does not influence the state of the matter
- Matter influences the radiation (i.e. excitation state determines absorption and emission), but not vice versa
- That actually makes LTE much much more simple than the more general case
- We will see very soon that abandoning LTE brings many complications

What is the more general case? Well something like this:



$$n_u = \frac{g_u e^{-E_u/kT}}{Z} \qquad\qquad n_u \neq \frac{g_u e^{-E_u/kT}}{Z}$$

The general point is

- **Radiation can influence the** state of matter, or as Mihalas and Hubeny put it nicely:

  "… the material's absorption and emission coefficients are set by the local radiation field. But in turn, local radiation field is the result of not only local photon emission and absorption, but also photons that have penetrated from other (perhaps remote) points in the atmosphere where the physical conditions might be quite different. In short, the radiation field determines the non-equilibrium properties of the material, but those properties in turn determine the radiation field. The two are inextricably coupled. This is the central problem in computing a theoretical stellar spectrum"

Might not be obvious but it is related to this question:

r/askscience · Posted by u/Forestlight_ 1 month ago 2

2.5k

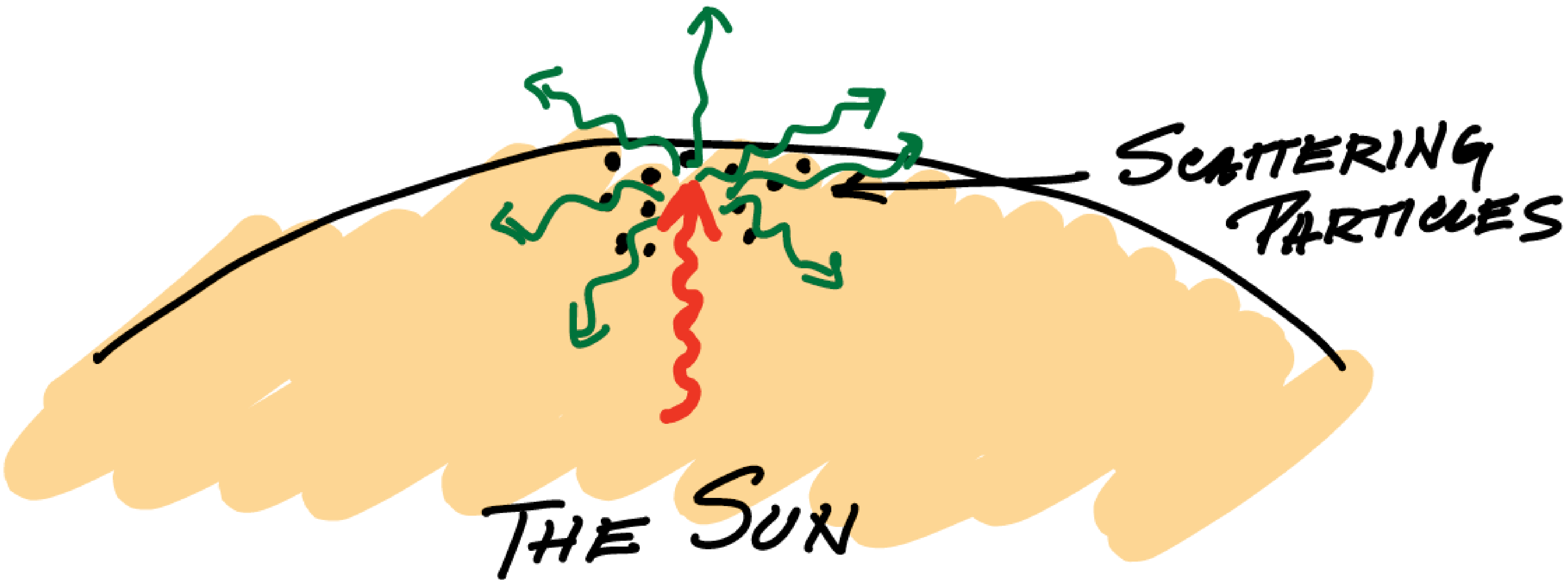
Why is the absorbtion spectra from stars not cancelled out by re-emission?

Astronomy

I have done some googling on this topic but I haven't found a definite answer. I know that atoms in stars can absorb photons of certain wavelengths and that these atoms can re-emit these photons. So the answer I usually read on this particular question is that while the light from the sun has to travel in a certain direction to reach the earth, the photons of a particular wavelength can be absorbed and re-emitted in every possible direction so that the amount of photons from that particular wavelength that reaches the earth is less than the other wavelengths. So here's my question: While the above is true for light traveling through a small region of a star, there have to be other regions in the sun that would send their light to a different part of the universe than our earth but their absorbed and re-emitted light can also be scattered in all directions so some of that light could be re-emitted in our direction when it otherwise wouldn't (without absorption and emission). So wouldn't all these regions cancel each other out? We receive less light from a particular wavelength from a certain region from the sun, but we also receive some light of that wavelength from other regions of the sun that don't send their light of "normal wavelengths" to us? If this wasn't true, where would these photons go? They can't just dissapear right? What am I missing here?

To put it in the form of a sketch, this is the question:

- Green is scattering (emission), red is absorption



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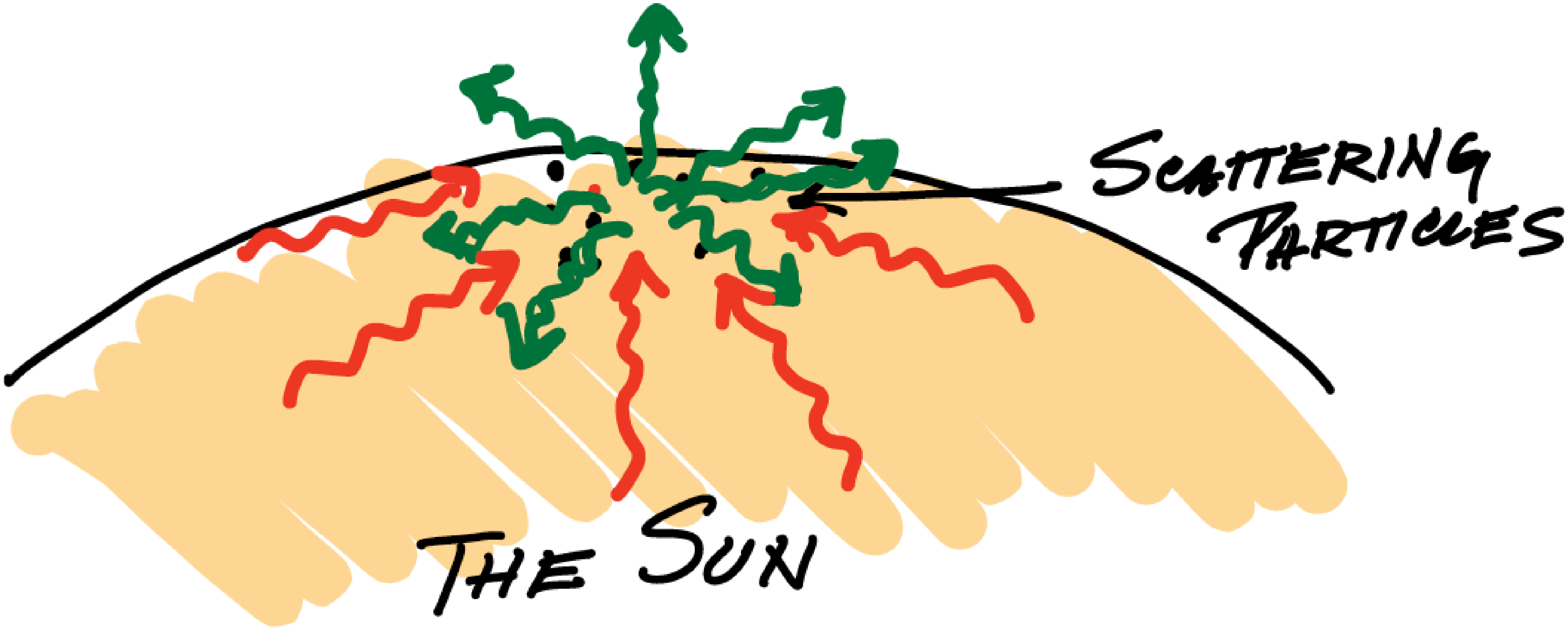
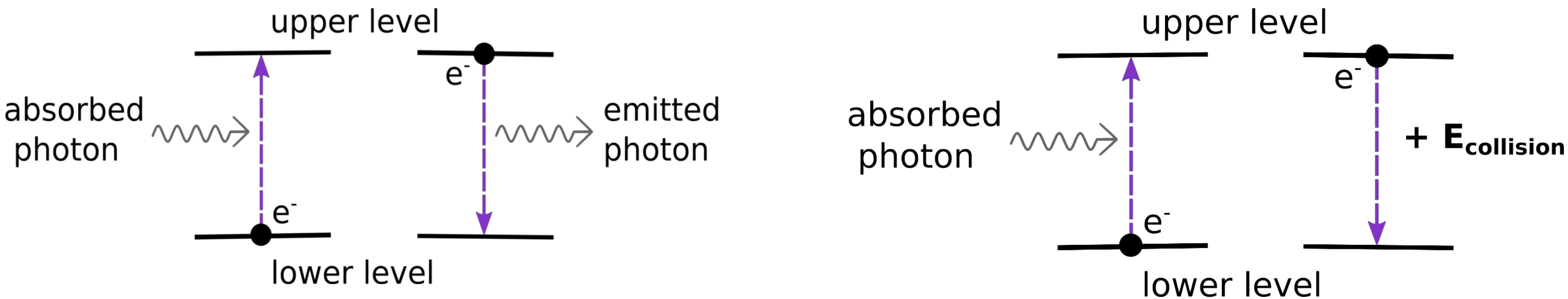


I will argue the following:

- **Simple to see:** There is less added emission due to the scattering than the overall loss due to scattering
- **Less simple to see:** Inclusion of scattering complicates the emission and makes is non-local
- **Even less simple to see:** Treating this properly turns RTE from a differential into an ***integro-differential*** equation, that cannot be solved directly
- We have moved from LTE to the so called NLTE (non-Local Thermodynamic Equilibrium) approximation

But first, let's see what is spectral line scattering



- Absorption **immediately** followed by excitation.
- I will argue that the probability of getting "true" absorption in which the photon is destroyed is:

$$\boxed{\epsilon = \frac{C_{ul}}{A_{ul} + C_{ul}} = \frac{\chi_{\rm abs}}{\chi_{\rm abs} + \chi_{\rm scatt}}}$$

So, to be clear:

- Spectral line scattering is **the absorption of the photon** followed immediately by **radiative de-excitation and thus the emission of a very similar photon**
- In continuum problems one could imagine this as having some “virtual excited state”
- In spectral line processes it’s easier to imagine, as there is an actual excited state
- There is, however, “shuffling” in terms of wavelengths, that we call **redistribution**

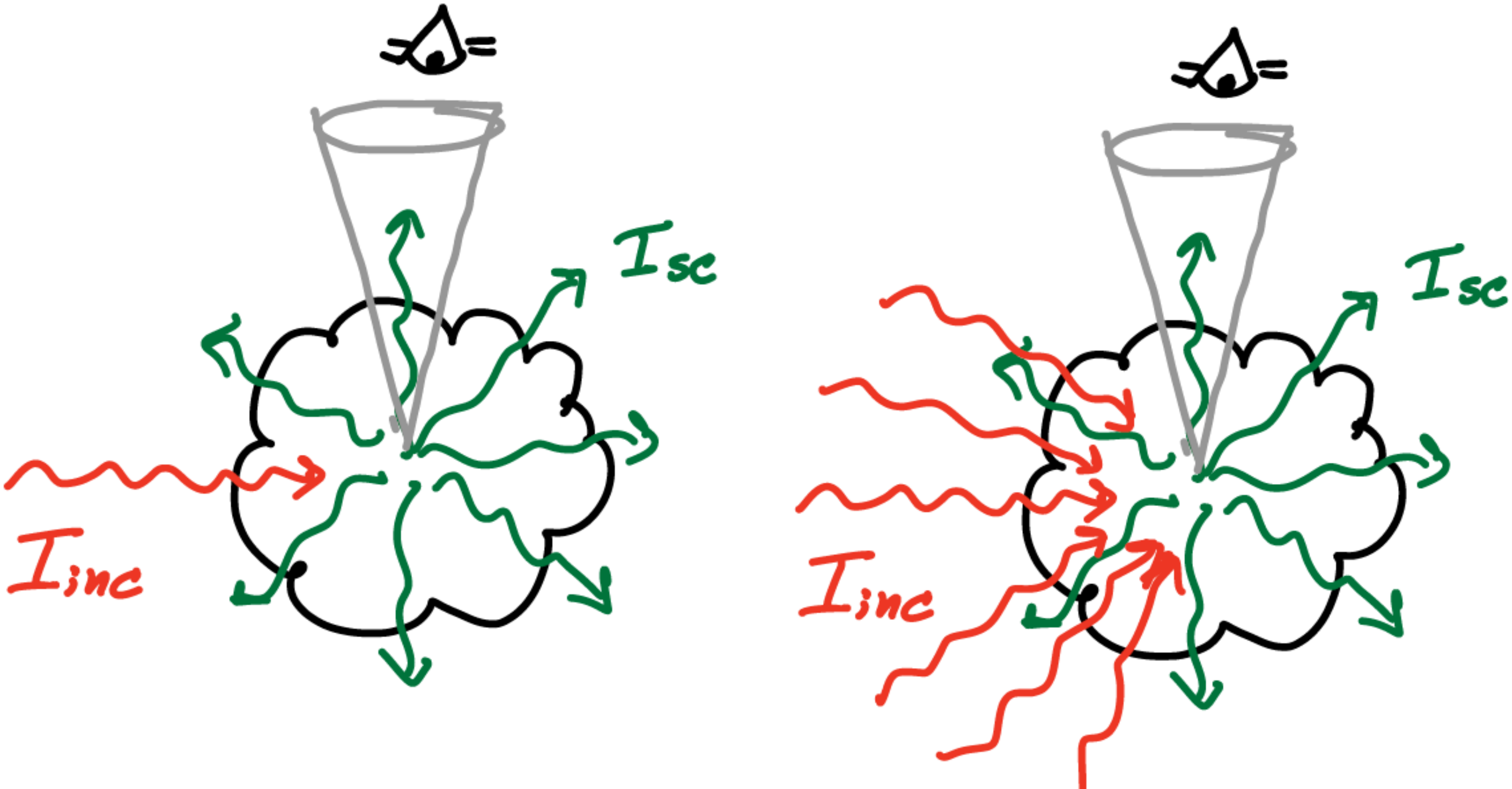
But, step by step

- Let’s assume the following:
- We are considering one wavelength only (for now)
- The scattering is ***coherent,*** meaning no change in wavelength (good for continuum problems)
- The scattering is **isotropic,** meaning: no matter where they come from, photons scatter equally in all the directions
- Let’s derive something useful

$$\epsilon = \frac{\chi_{\rm abs}}{\chi_{\rm abs} + \chi_{\rm scatt}}$$

In which case will there be more scattered radiation?

- Or: “In which case will scattering emissivity be higher?”



Right, more “total” incoming radiation, more emission

- This begs for introduction of intensity integrated over directions:

This is “scattered” flux. It belongs to absorption.

$$\frac{dF_{\rm sc}}{dl} = \chi_{\rm sc} \oint I(\lambda, \hat{\Omega}) d\Omega$$

$$j_{\rm scattered} = \frac{\chi_{\rm sc}}{4\pi} \oint I(\lambda, \hat{\Omega}) d\Omega$$

But it also belongs to emission, in all the directions.

To keep the formalism, we treat scattered photons as being removed from our ray, and then we add them to the emission coefficient.

More generally

- Emission is going to be a combination of thermal (LTE), and scatttered (NLTE) emission, so:

J is the so called mean intensity. It is usually introduced much sooner, but I wanted us to actually need it first.

$$S = \frac{j}{\chi} = \frac{j_{\mathrm{Planck}} + \chi_{\mathrm{sc}} J}{\chi_{\mathrm{abs}} + \chi_{\mathrm{sc}}}$$

$$J = \frac{1}{4\pi} \oint I(\Omega) d\Omega$$

$$S = \epsilon B + (1 - \epsilon) J$$

The source function is combination of thermal emission and scattered radiation.

If I write this now as a single equation:

$$\boxed{\frac{dI}{dl} = I - \epsilon B - (1-\epsilon)\frac{1}{4\pi}\oint I(\Omega)d\Omega}$$

- The spatial change of the intensity depends on the very same intensity (integrated over the wavelengths).

This is, in a way, similar to writing: $\boxed{x = \ln x}$

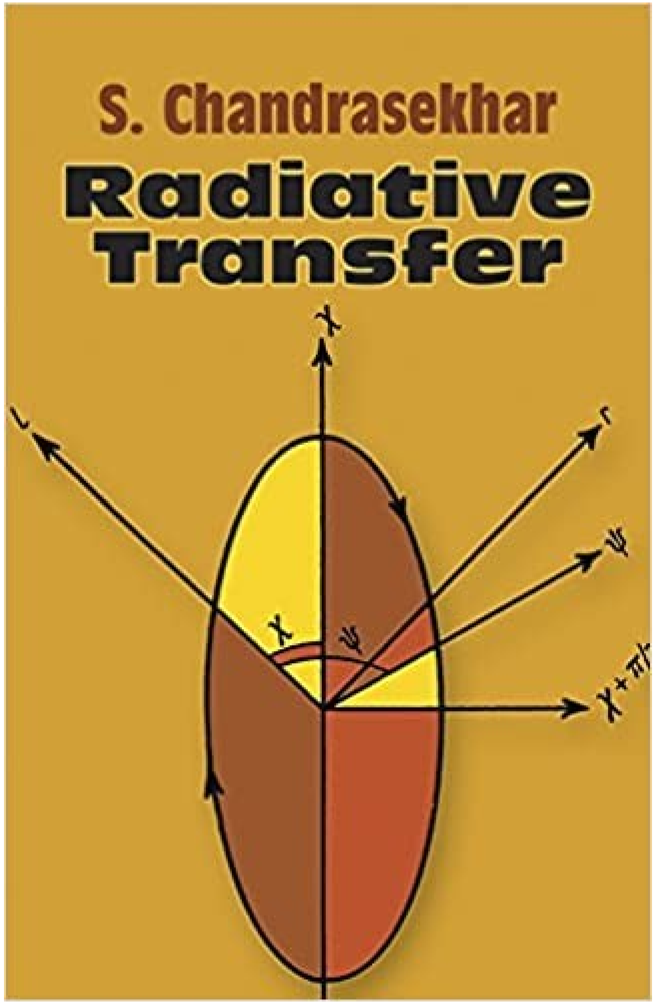
- It’s similar in the way that it is unsolvable directly. Knowing B and epsilon does not help us simplify this in any way

$$\boxed{\frac{dI}{dl} = I - \epsilon B - (1-\epsilon)\frac{1}{4\pi}\oint I(\Omega)d\Omega}$$

- This is known as an integro-differential equation. Note that we are integrating over direction and differentiating with respect to path (spatial coordinate)

- Many books and papers are written about solving this.

And one of the first to solve it was, of course:



We will soon see the solution, but first:

- Assume an isothermal atmosphere (B = const with depth)
- Assume epsilon = 0 (or a very small number)
- Convince yourself that the source function will drop toward the top of the atmosphere

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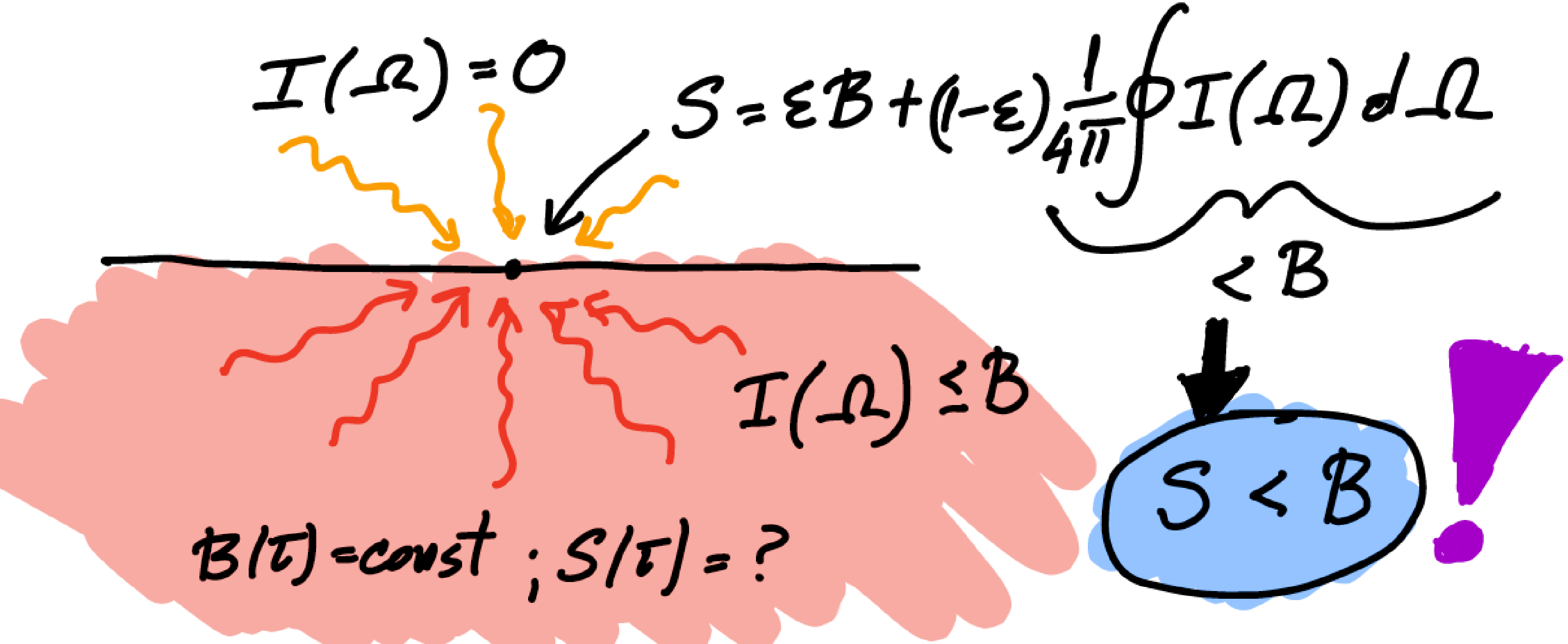
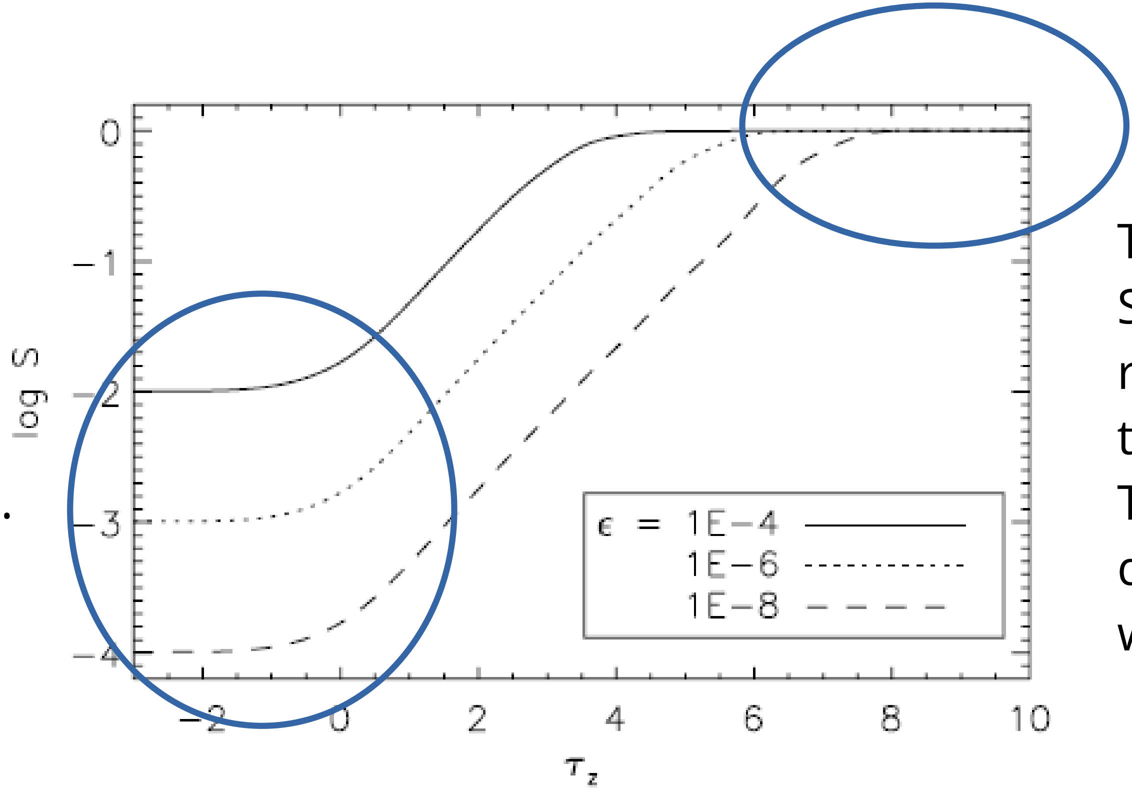


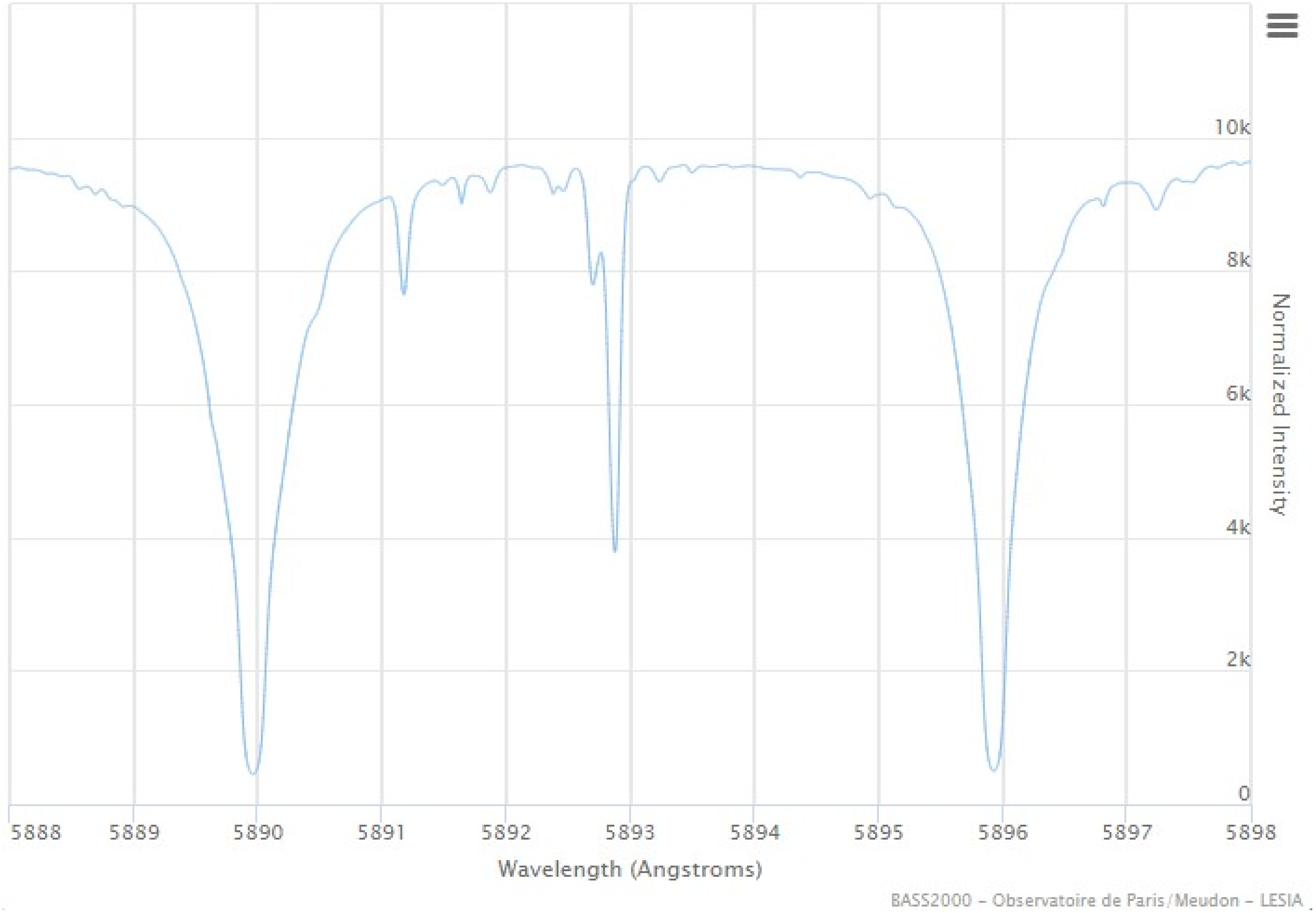
The source function actually drops quite a bit



Top of the atmosphere, very low source function. This is because the photons escape easily and the value of the mean intensity drops.

Thermalization. Source function reaches the value of the Planck Function. This has nothing to do with density, only with opacity.

This causes some very deep spectral lines:



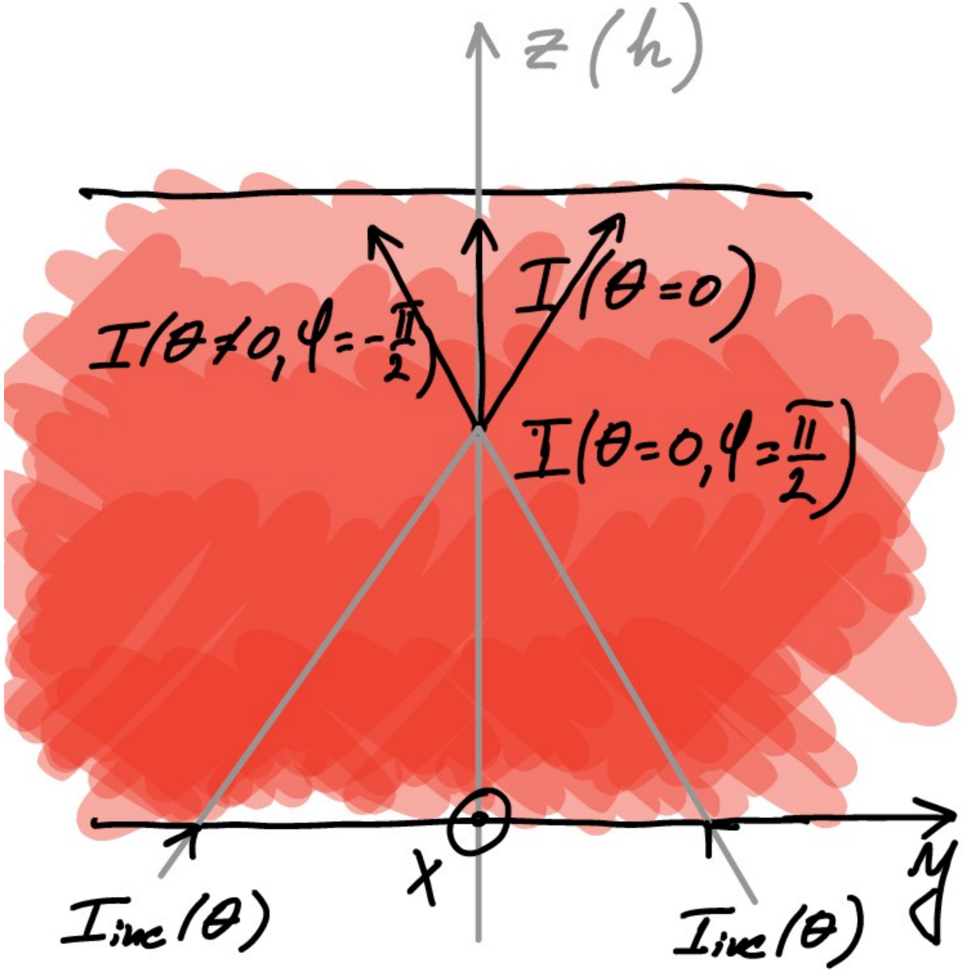
Final remark, calculating Intensity in other directions

- Necessary part of scattering is treating the mean intensity

$$J = \frac{1}{4\pi} \oint I(\Omega) d\Omega$$

- If we want to calculate it, we need intensity in all the directions.
- How to obtain that?

1D plane-parallel atmosphere



- Introduction of the famous "mu", the cosine of the heliocentric angle
- If nothing depends on x and y, phi is obsolete!

$$\frac{dI}{dl/\cos\theta} = I - S$$

$$\mu\frac{dI}{dl} = I - S$$

$$J = \frac{1}{2}\int_{-1}^{1} I(\mu)d\mu$$

Summary

- Scattering couples directions: To know what scatters towards us we need to know whole radiation field (in all directions)
- RTE becomes much more complicated
- Solution is non-trivial, it is actually only numerically doable, even in idealized cases.
- We will see this next week on Thursday in a hands-on exercise
- Next class, all this but for atoms!