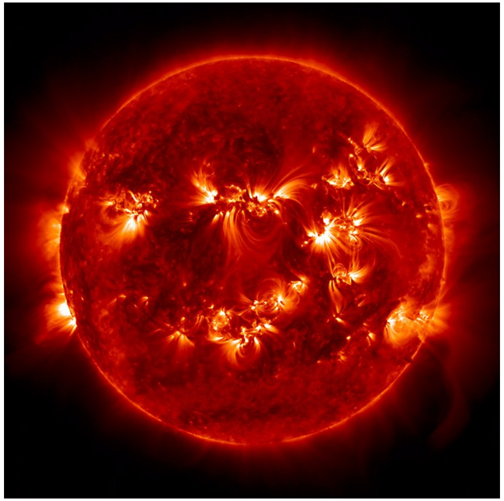
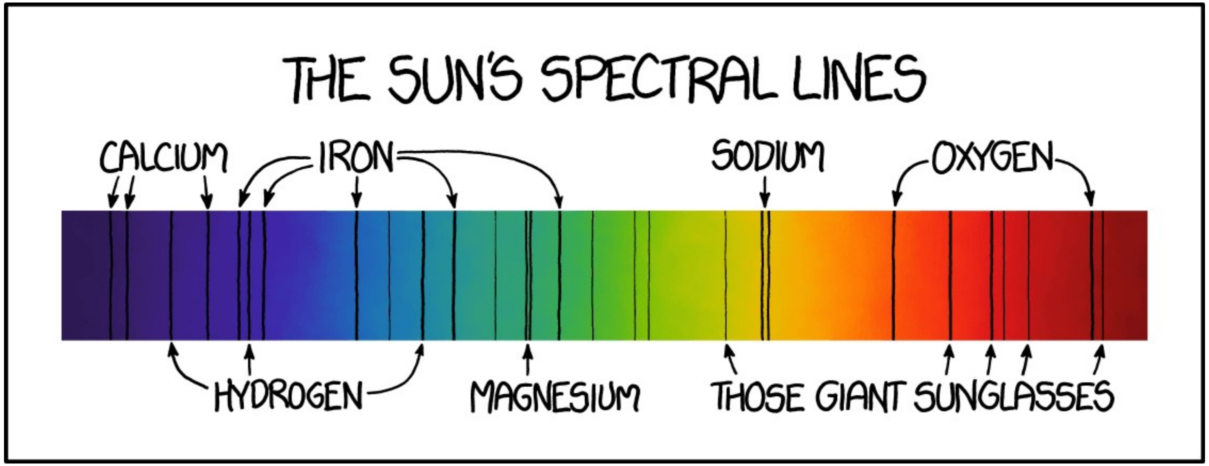
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 08: Line Formation Height

Ivan Milic (CU Boulder) ; ivan.milic (at) colorado.edu

Plan for today:

- Clarify what we mean by the spectral line formation height
- We were a little bit boggled by the shape of some spectral lines last time, let's clear that up
- We are going to talk about two useful concepts: **contribution function** and **response function**
- Both of these somehow identify layers of the atmosphere that are important for the formation of the specific spectral line, but we calculate them differently and they have somewhat different meaning

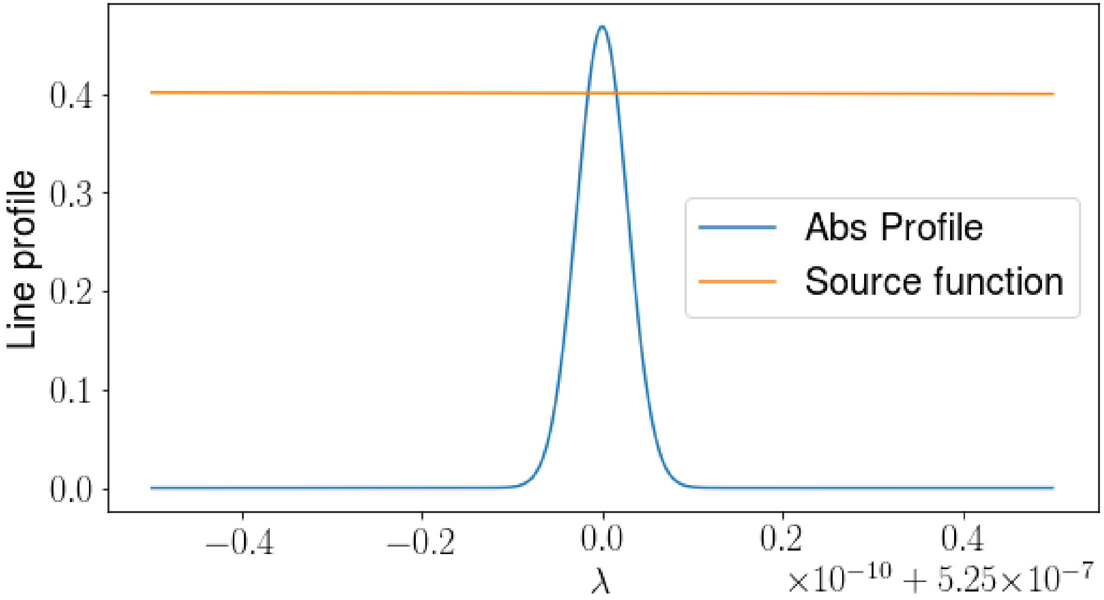
LTE line formation in the nutshell

This varies very strongly with the wavelength (Voigt profile)

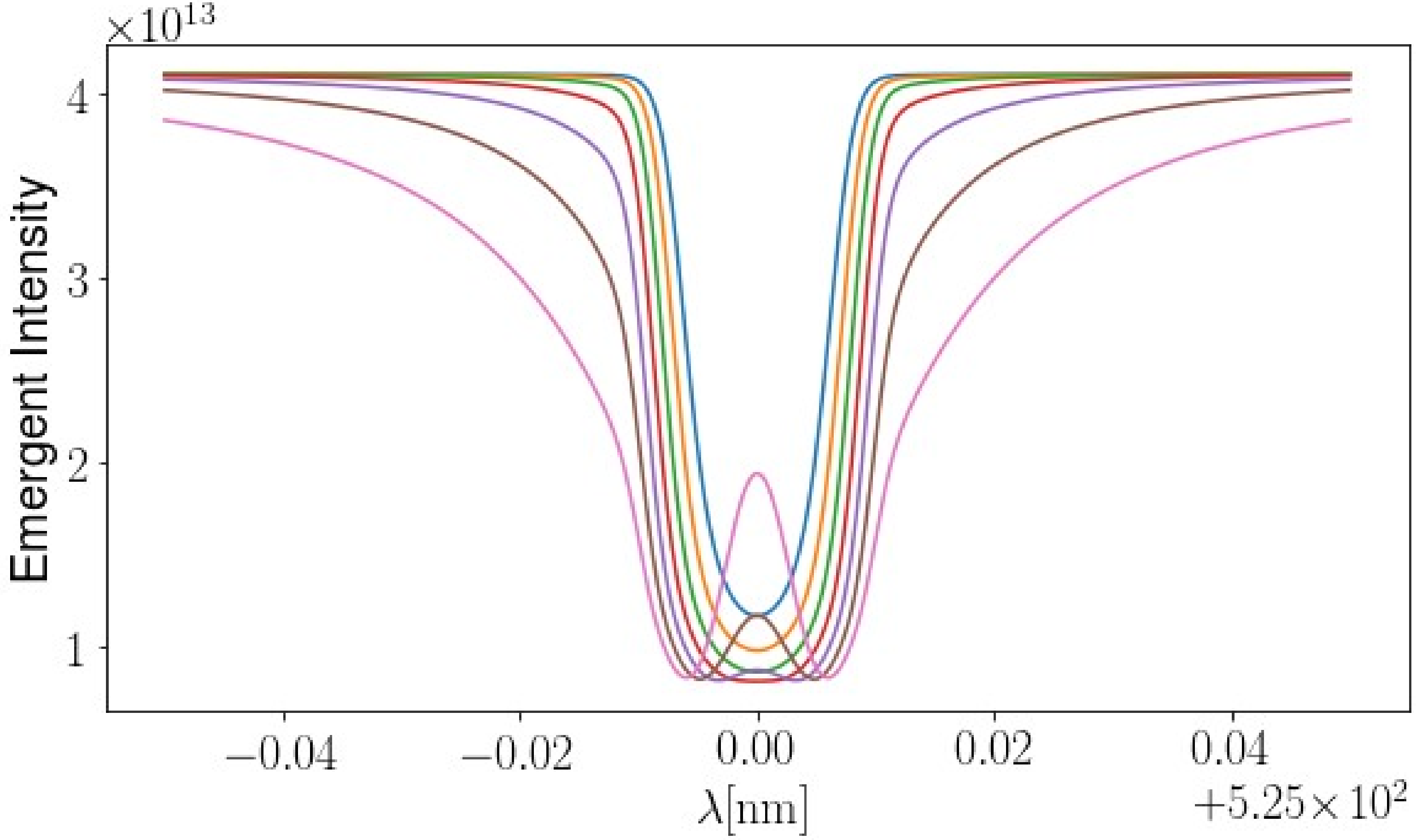
$$I_\lambda^+ = \int_0^\infty S(\tau_\lambda) e^{-\tau_\lambda} d\tau_\lambda$$

$$S = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

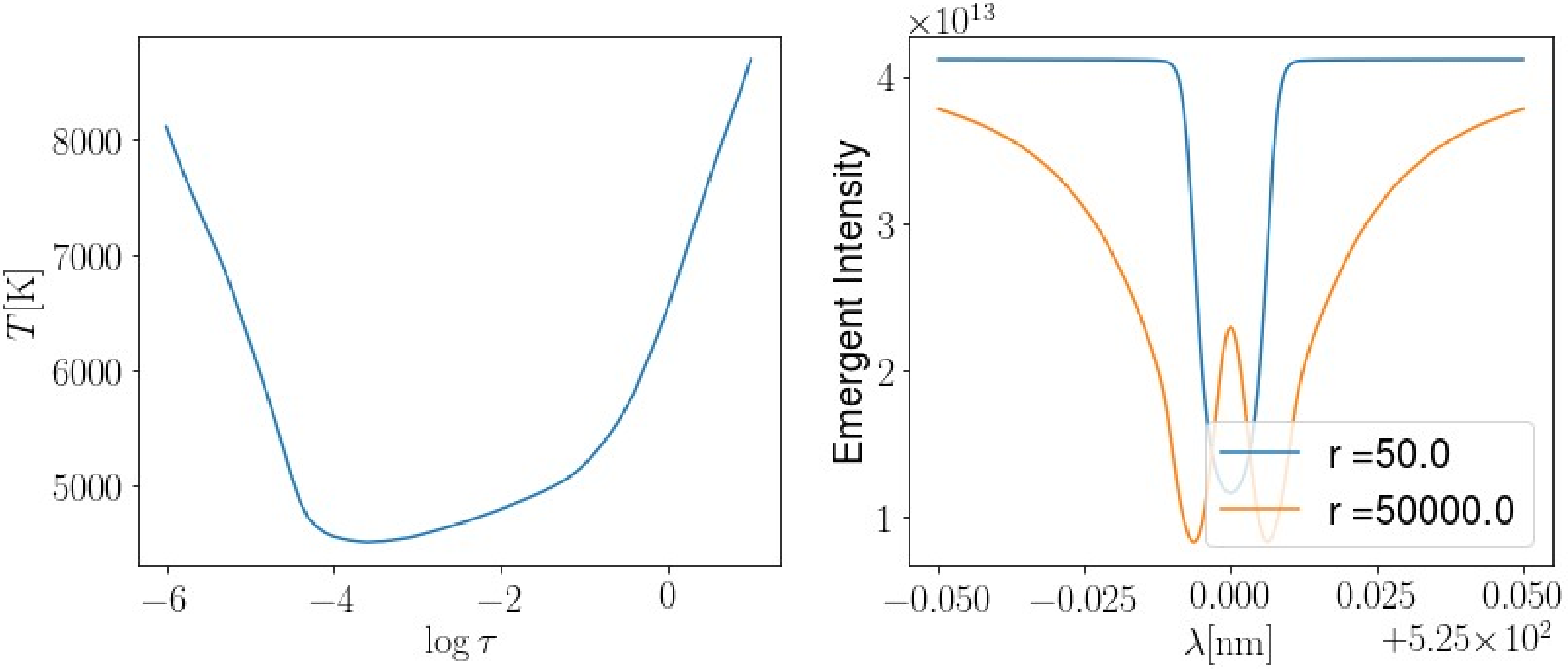
Source function actually does not vary very strongly with wavelength



So this was the plot that gave us some headache. How can the same atmosphere produce different looking lines?



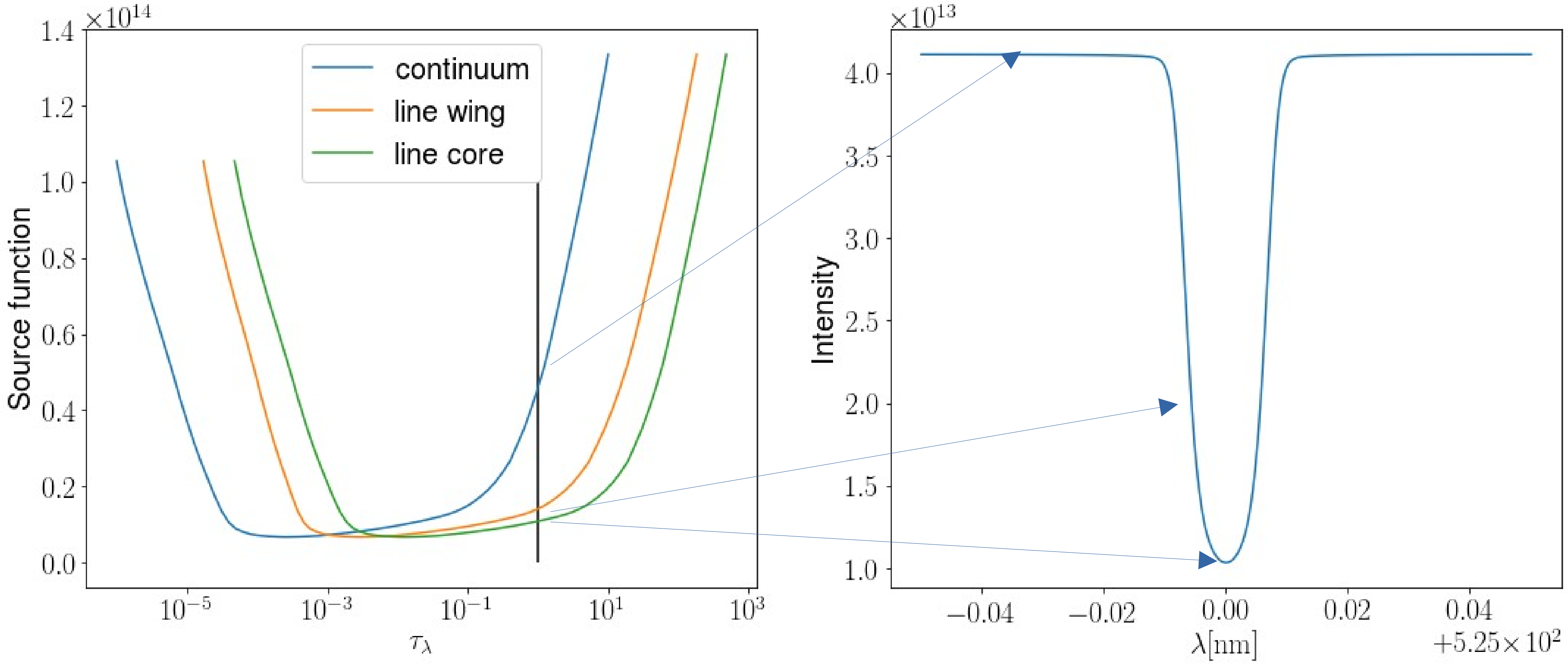
Let’s just simplify down to two different cases



Let's try to rely on this simple assumption, and explain

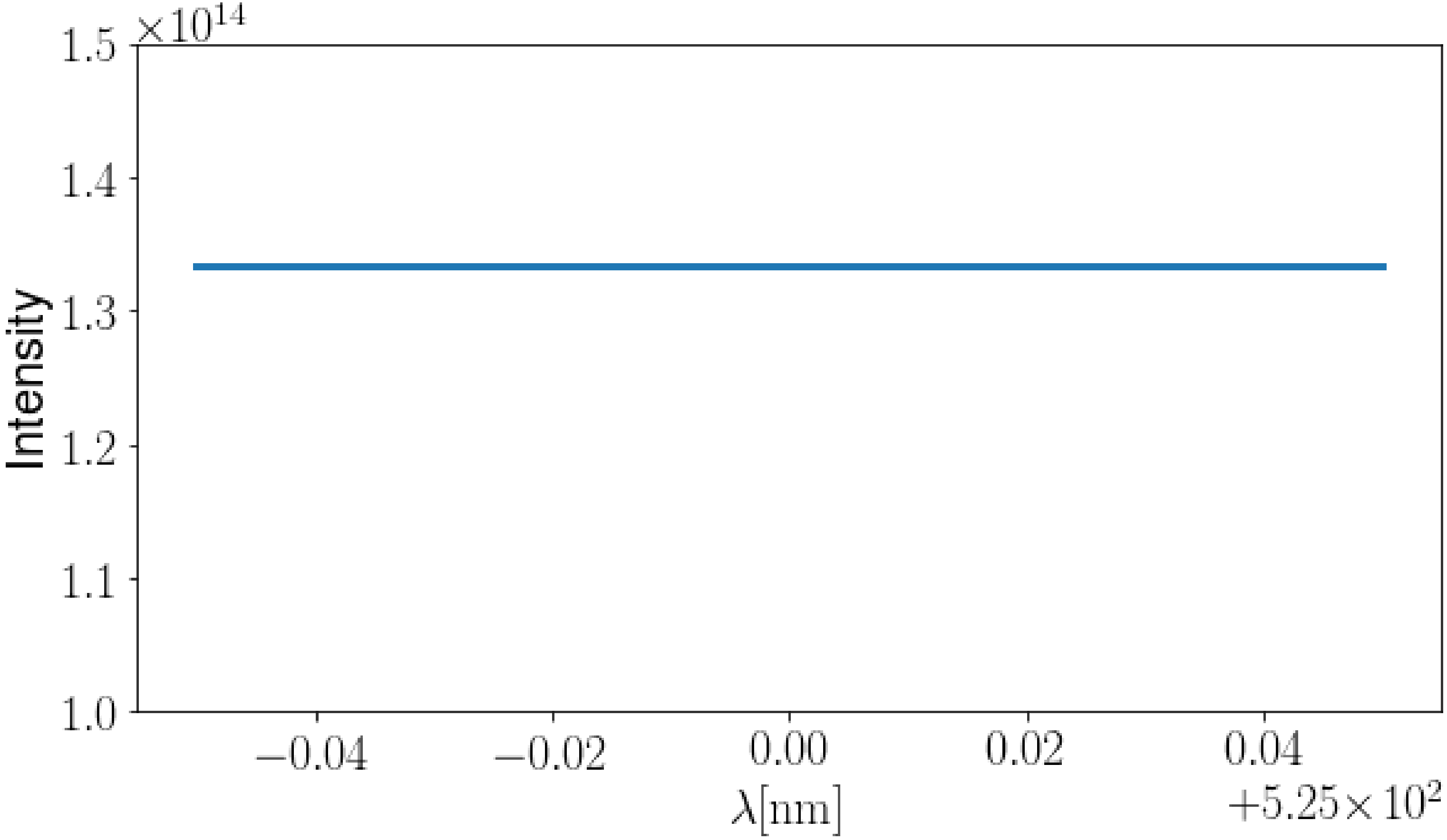
$$I_\lambda = S(\tau_\lambda = 1)$$

(But very soon we will make it more complicated, don't worry ;))



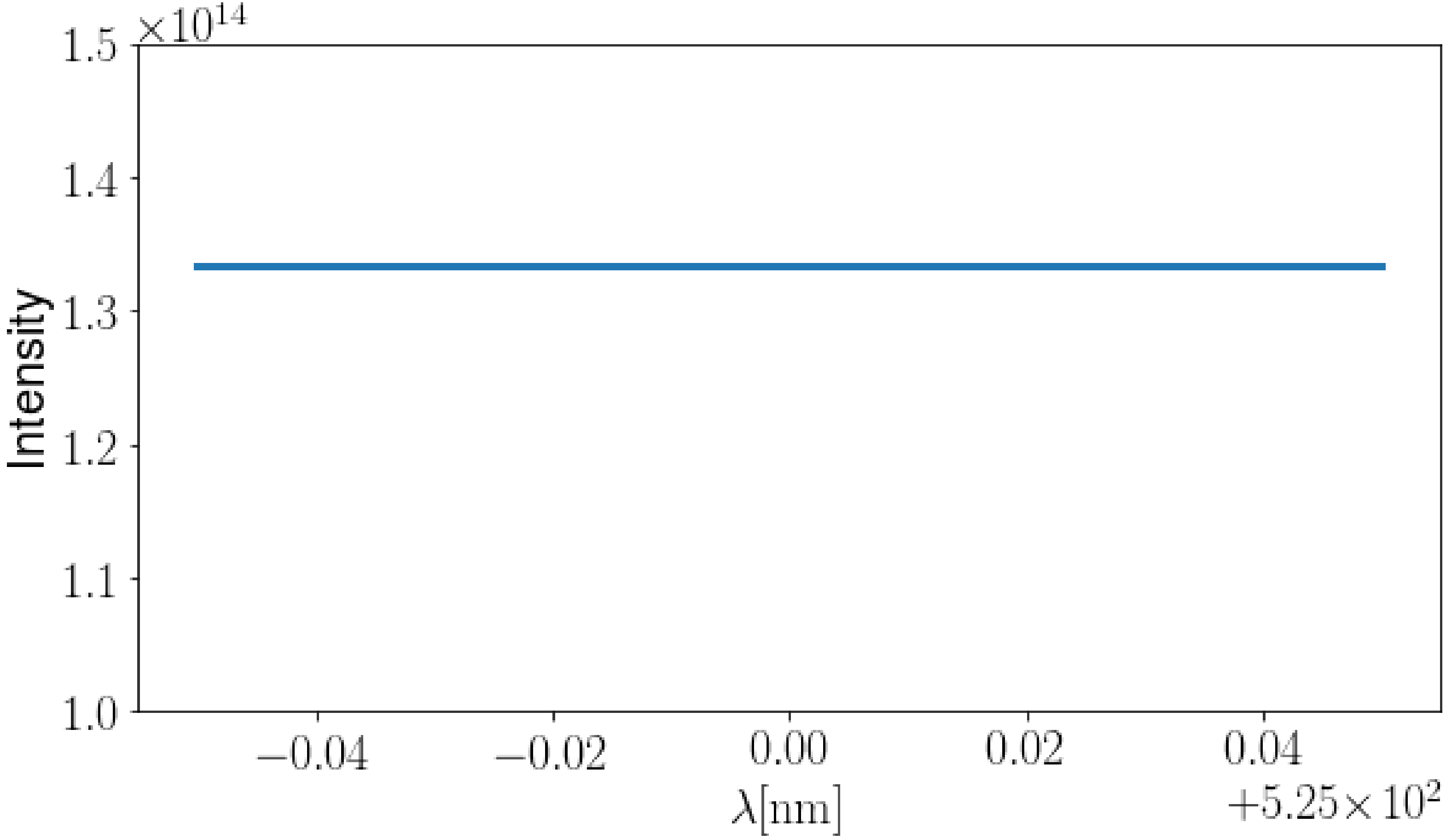
Let’s look at the problem in a different way. How does the intensity change as we move from the bottom up?

- At the bottom, there is no spectral line, why?



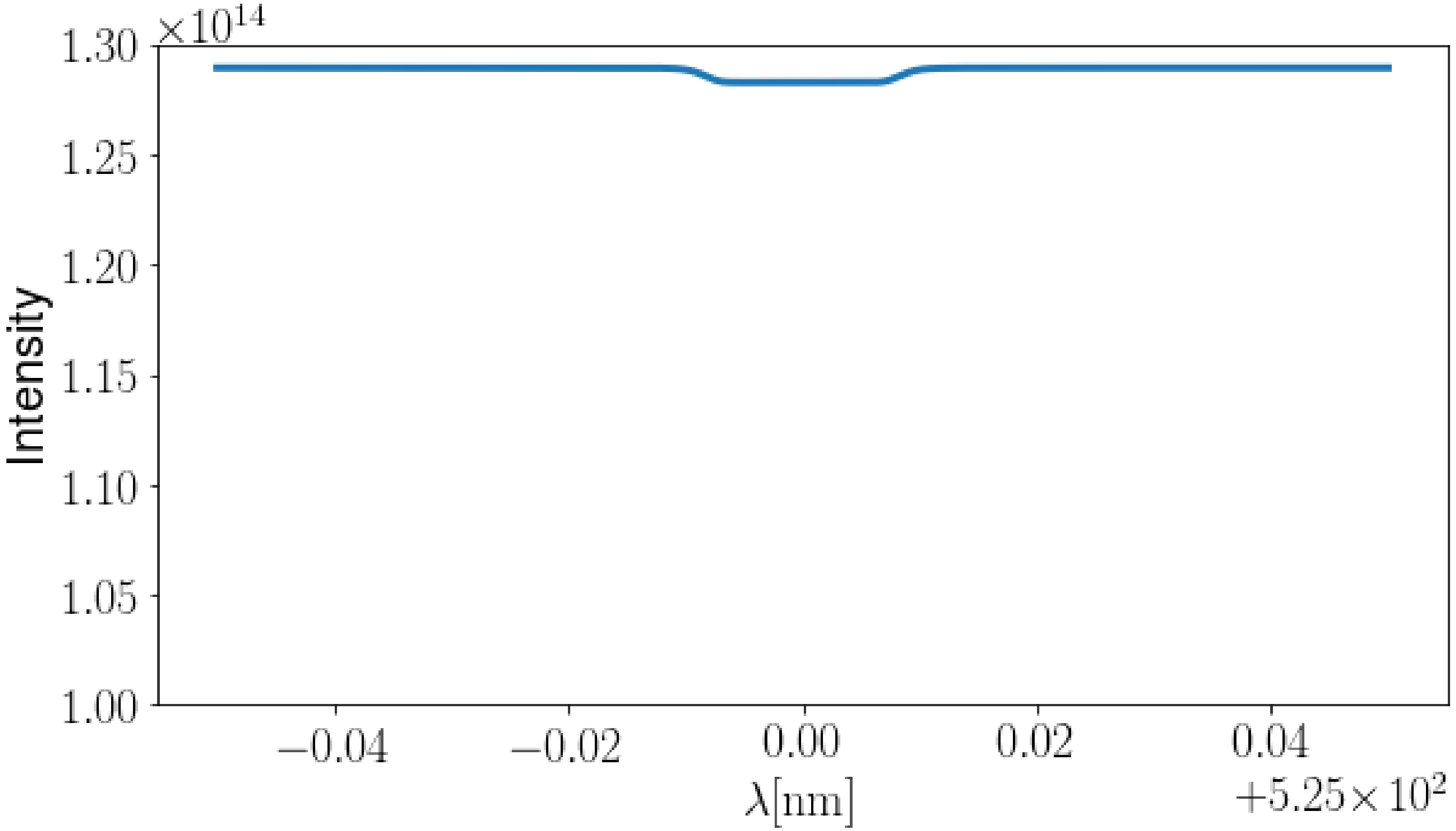
At the very bottom, i.e. the first layer…

- Ah right, because, it's the incident intensity which is Plankian (flat)



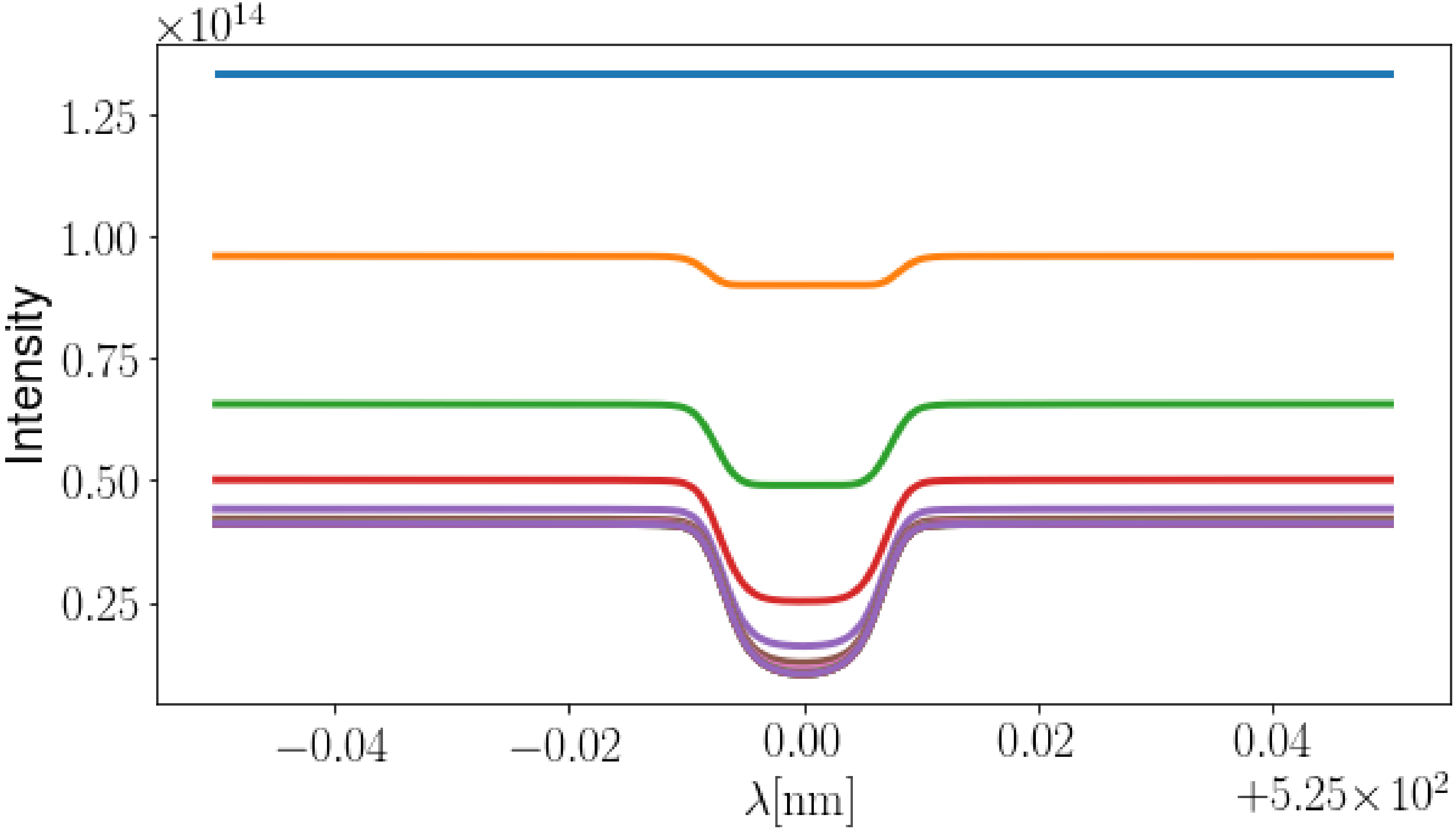
Well then the second layer:

- There is no spectral line, why?



As we go up the spectral line becomes stronger and stronger

- What is going on here? This is how the spectrum “evolves” as we go up the atmosphere (top to bottom)



This leads us to a very important question

- **Which layers of the solar atmosphere are the most important for the formation of the given line?**

This leads us to a very important question

- **Which layers of the solar atmosphere are the most important for the formation of the given line?**
- We can answer that question with our little model, we know that:

$$I_i = I_{i+1} e^{-\Delta\tau_i} + \frac{S_i + S_{i+1}}{2}(1 - e^{-\Delta\tau_i})$$

- Now, let’s take a moment to agree that (here “i” counts depth layers)

$$I_i = \sum w_{i'} S_{i'}$$

All the equations involve different weights and thus have different solutions for different wavelengths.

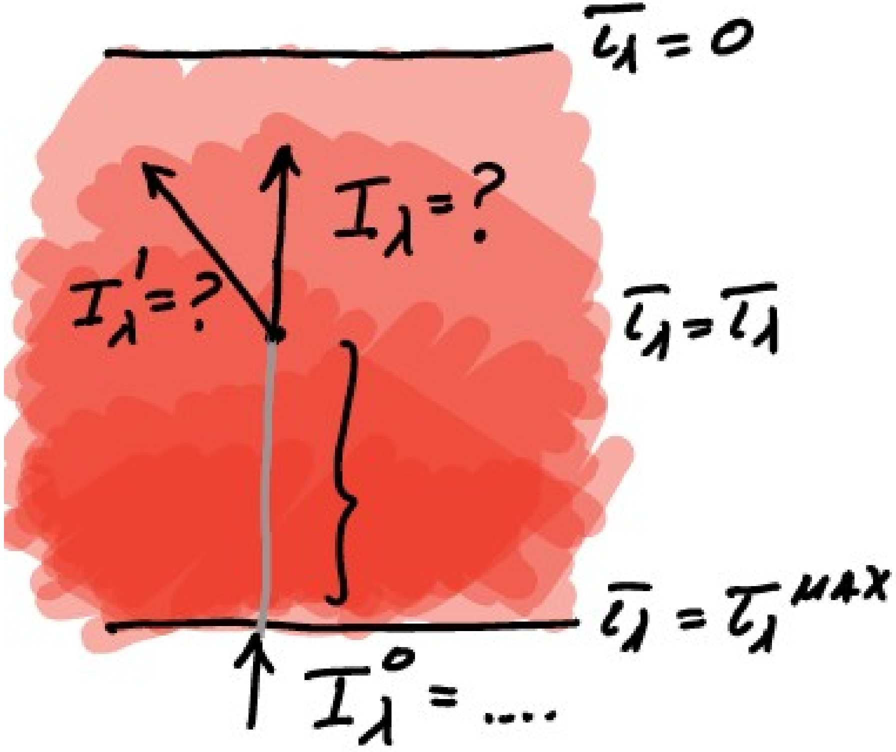
This form is indeed very important!

$$I_i = \sum w_{i'} S_{i'}$$

- Formally speaking, specific monochromatic intensity at one point in the atmosphere depends on the values of the source function everywhere (*well, everywhere "upwind" from the point i).*
- This is another reason why I love to say that intensity is **non-local.** What we see (and not only what we see, intensity everywhere), is some weighted contribution of the source functions along the line of sight.
- Historically, following, Schwarzshield:

$$I_i = \Lambda[S] = \sum_{i'} \Lambda_{i,i'} S_{i'}$$

Or, in a more analytical fashion:



- How to find the intensity in any given point?
- Extend direction toward the edge of the object (gray line), specify initial condition:
- Calculate, along that line:

$$I_\lambda(\tau_\lambda) = \int_{\tau_\lambda}^{\tau_\lambda^{\max}} S(t) e^{-(\tau_\lambda - t)} dt$$

- With little imagination. We could say, that Intensity in the given point depends on the source function everywhere, or :

$$\boxed{I_\lambda(\tau_\lambda) = \Lambda_{\lambda,\tau_\lambda}[S]}$$

Lambda operator

- Let's focus on one wavelength. We have calculated our optical depth grid and know source function on that grid.
- To calculate intensity distribution, we need source function distribution.
- So, in that context, we can maybe talk about "**lambda functional**"

$$I_\lambda(\tau_\lambda) = \Lambda_{\lambda,\tau_\lambda}[S]$$

- However, since in 99.5% of the cases we deal with arrays, we call it **lambda operator** (what are the lengths of these vectors / what is this operator)

$$\vec{I}_\lambda = \hat{\Lambda}_\lambda \vec{S}_\lambda$$

Ahem, earlier I used the word “contribution”

- Looking at how much each depth point contributes to the emergent intensity, we can construct the so called **Contribution function** (here I have explicitly noted wavelength dependence)

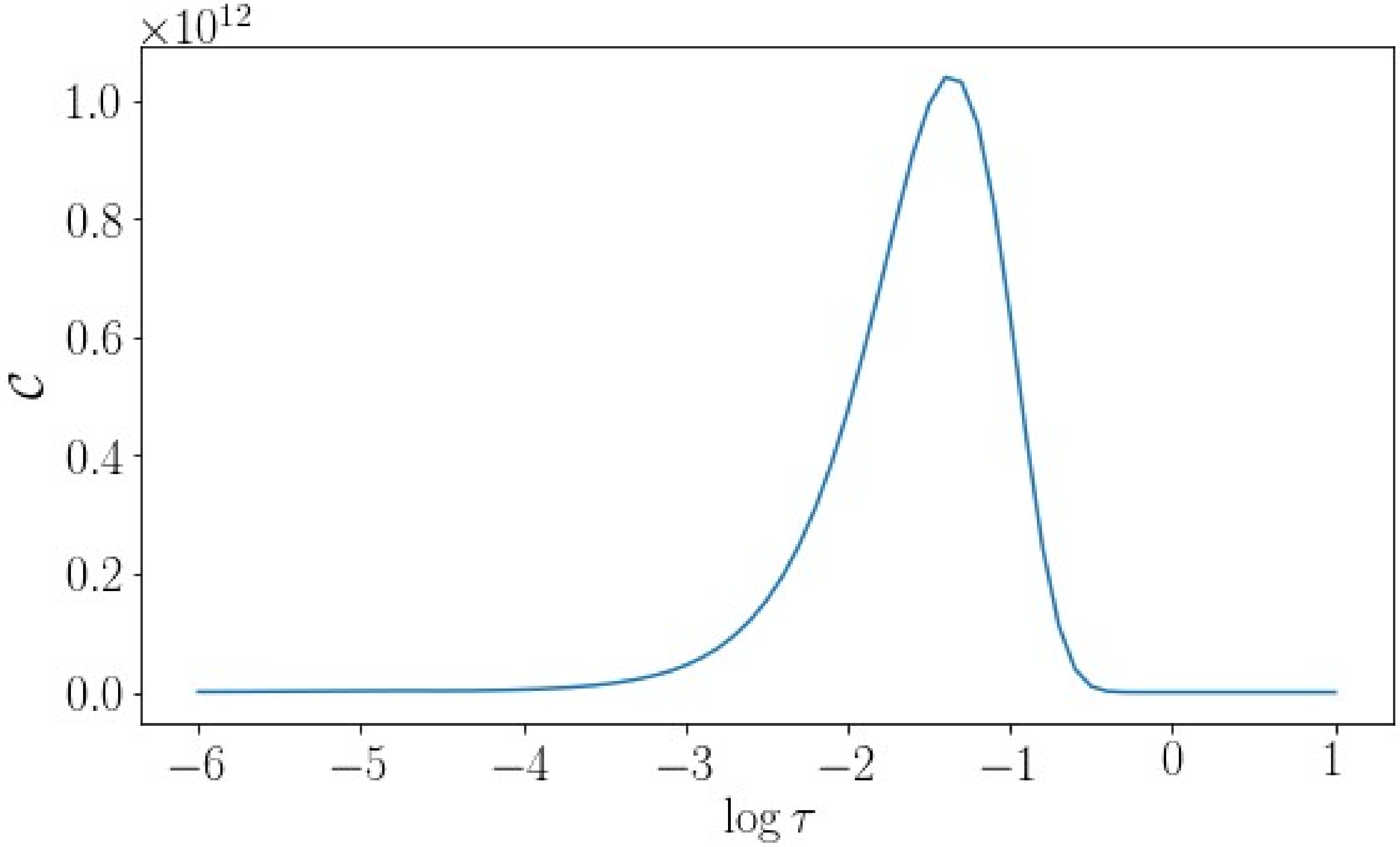
$$\mathcal{C}_{i,\lambda} = \Lambda_{0,i;\lambda} S_{i,\lambda}$$

- Analytically, we could express the contribution function from the formal solution as:

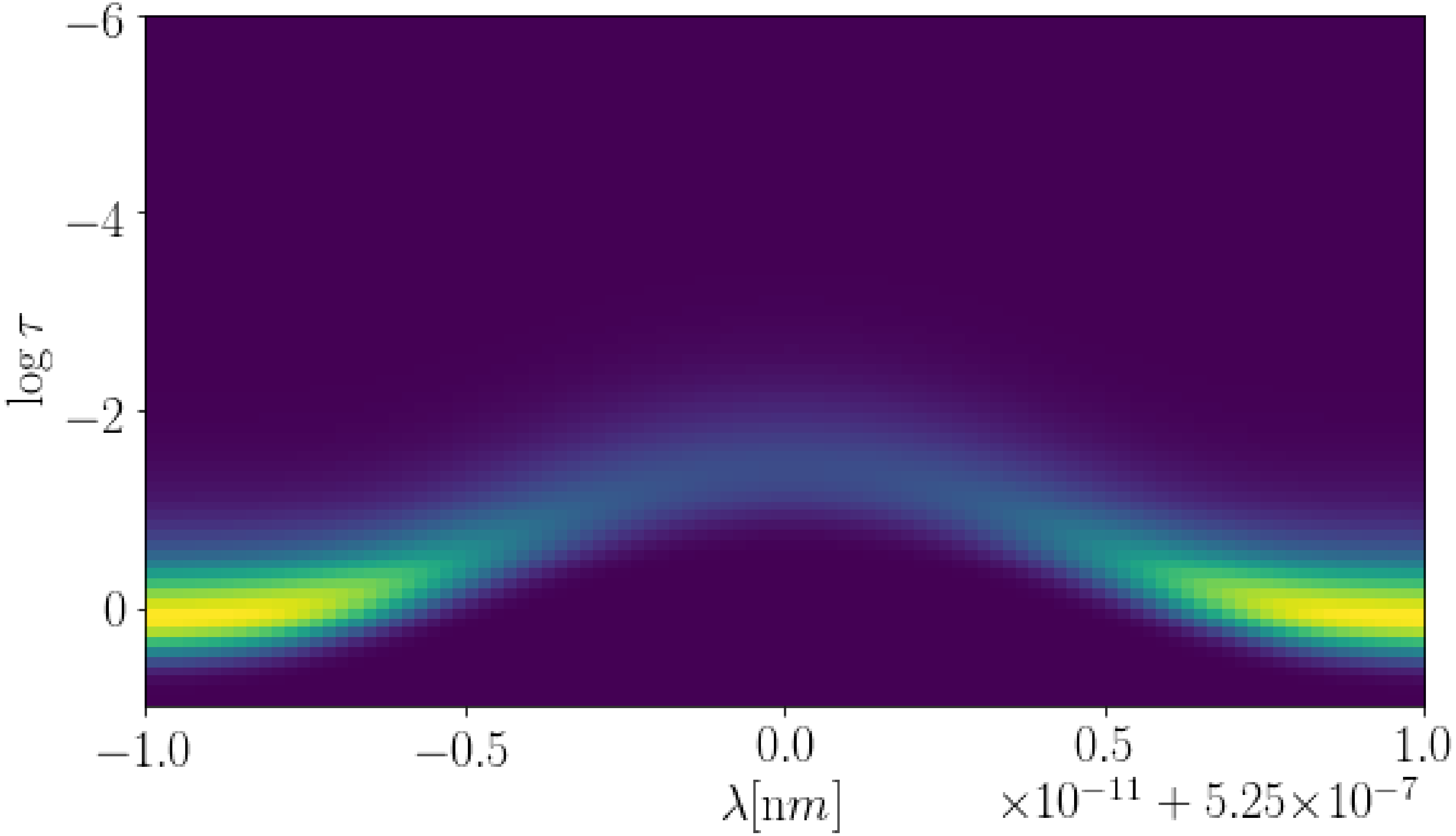
$$\mathcal{C}_\lambda(\tau_\lambda) = S(\tau_\lambda) e^{-\tau_\lambda} \tau_\lambda$$

This one might be a little tricky to figure out, but give it a go when you have the time

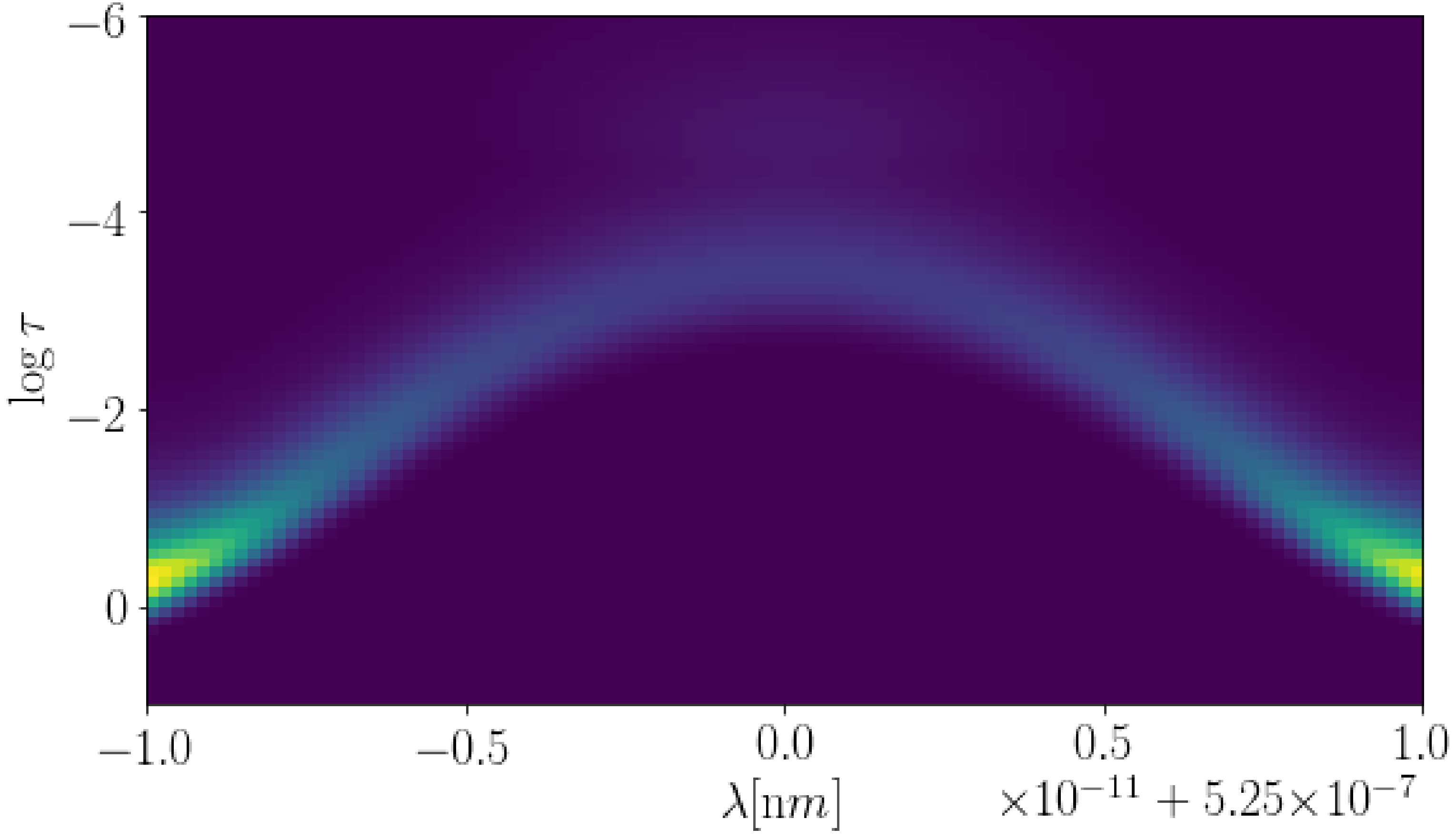
Let's plot contribution function for one wavelength:



Or in this, fancy, 2D form:



Now for a little bit stronger line:



Let’s take a little moment here to discuss what is going on

- Does the spatial distribution of the Contribution function make sense to you?

Now, hanging around the people at NSO…

- You might have heard another fancy term **Response function**
- Response function tells us how the emergent intensity, ahem, **responds** to the changes at specific quantity at specific depth:

$$\mathcal{R}^{q}_{i,\lambda} = \frac{\partial I^{+}_{\lambda}}{dq_i}$$

- Where q is not an index, but a placeholder for a specific quantity (say, Temperature, velocity, magnetic field… etc.

Calculating the response functions

- Much harder. You need to take into account all the inter-dependencies
- In our case, we could find a derivative with respect to the source function easily:

$$I_\lambda^+ = \sum_i \Lambda_{0,i;\lambda} S_i$$

$$\frac{dI_\lambda^+}{dS_i} = \Lambda_{0,i;\lambda}$$

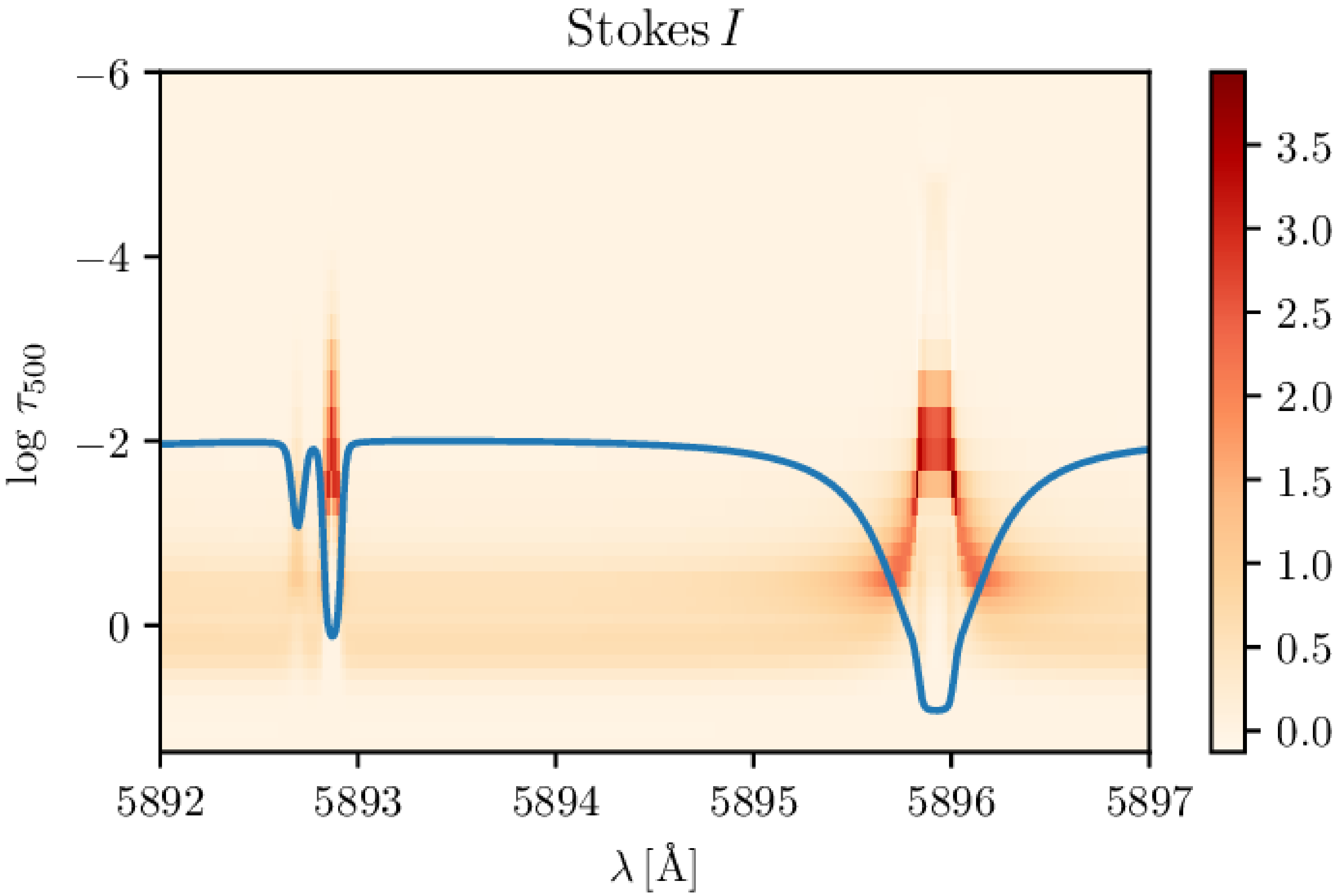
- But this is because relationship is linear, start thinking about temperatures, and it gets complicated real quick.

So to summarize that real quick:

- **Contribution function**
- Tells us how much each layer contributes to the emergent intensity

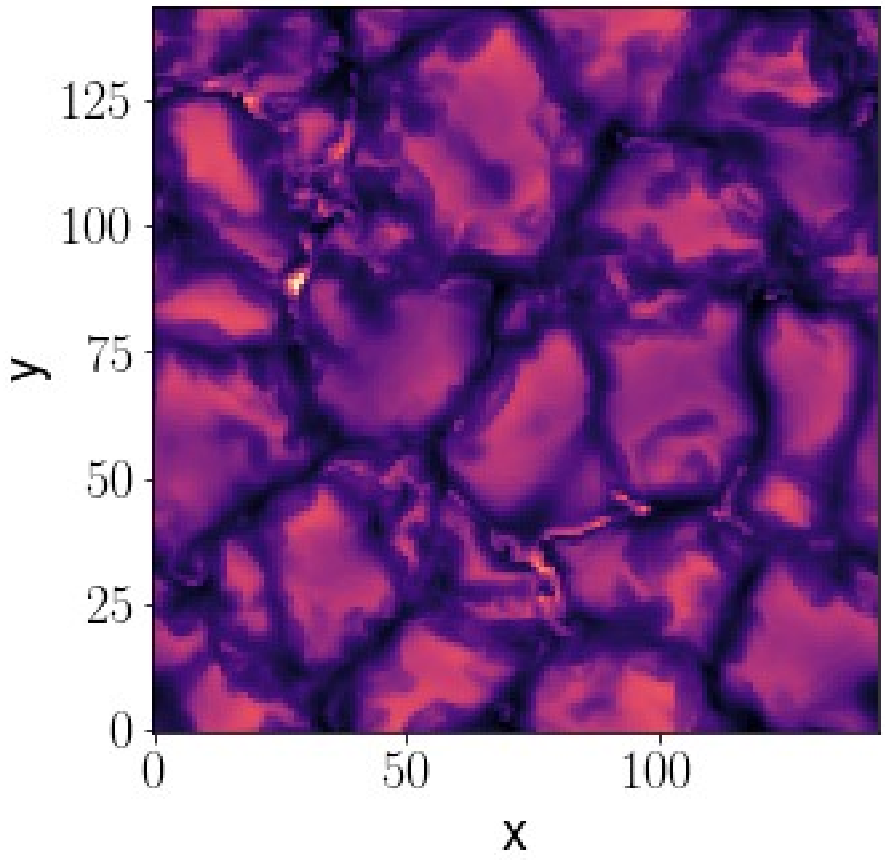
- **Response function**
- Tells us how changes at a layer reflect to emergent intensity
- Takes into account more of the coupling (will become clearer when we talk about NLTE)
- Different for different parameters
- Yields more diagnostic potential but harder to calculate

Example response function



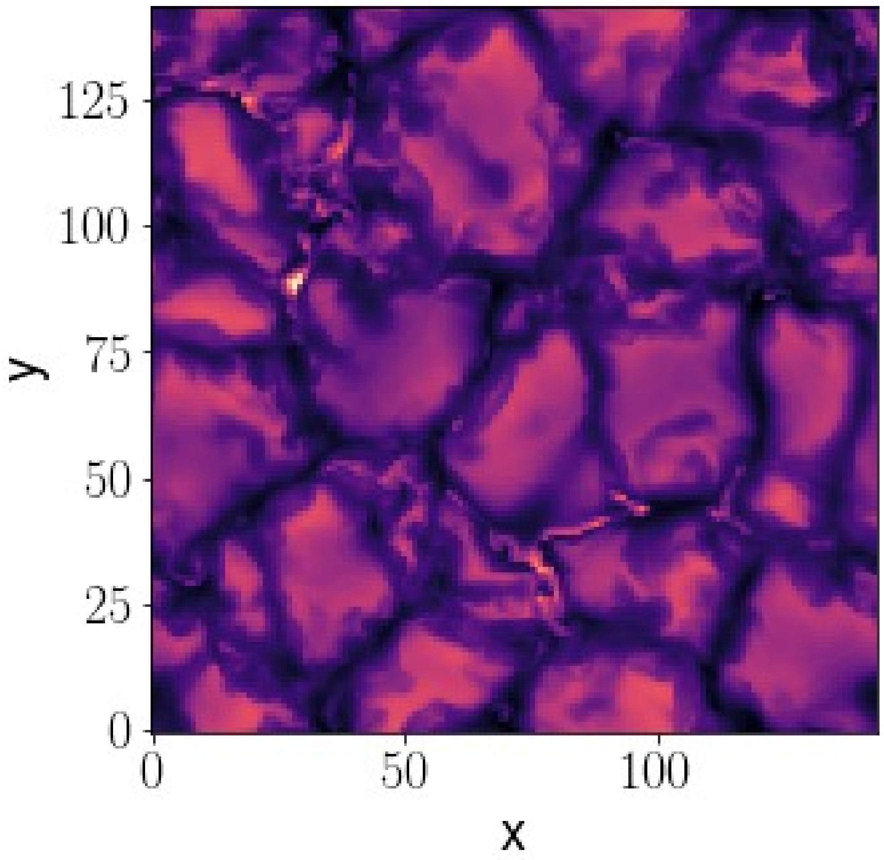
And for the end, a problem with real, well, simulated data

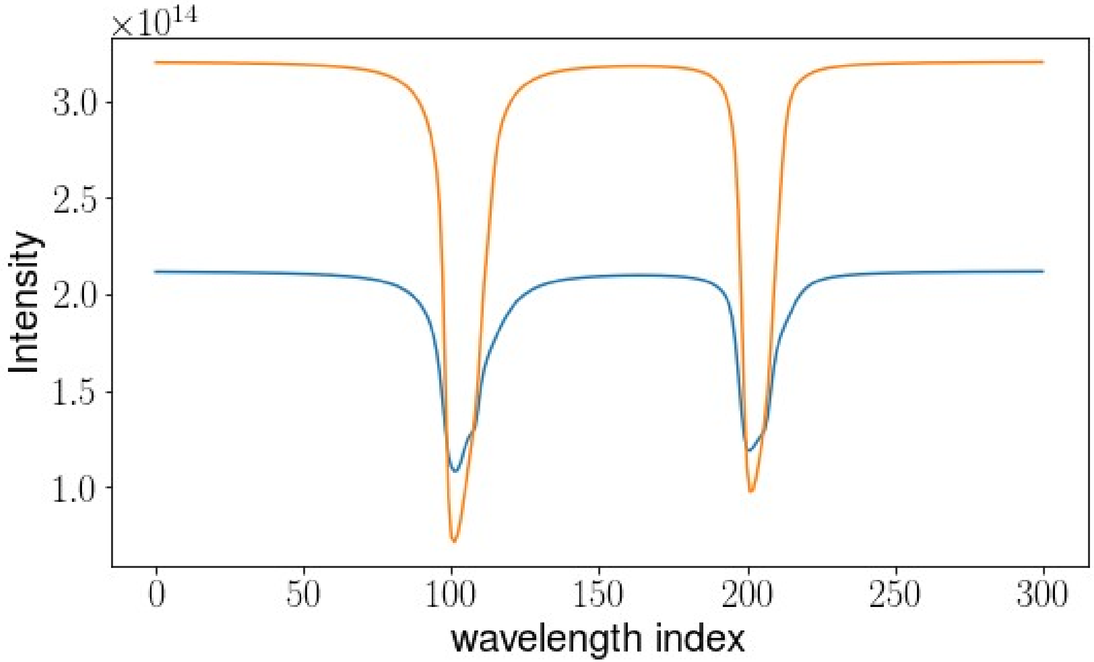
- Let's look at this map, calculated from a MURAM simulation:



But we also have a spectrum for each point

- What does this difference between the spectra tell us?





These are spectra of a bright and a dark pixel.

Let’s sum it up

- Different levels of intensity at different wavelengths tell us about the source function stratification
- This is basically temperature stratification
- So granules are hotter deep but cooler somewhere above
- *Conveniently we call this layer “inverse granulation”*
- Now you can already imagine that there are methods how to reverse engineer the spectrum in order to recover the variations of the physical parameters – **these are called inversion methods**
- And that’s all for today!