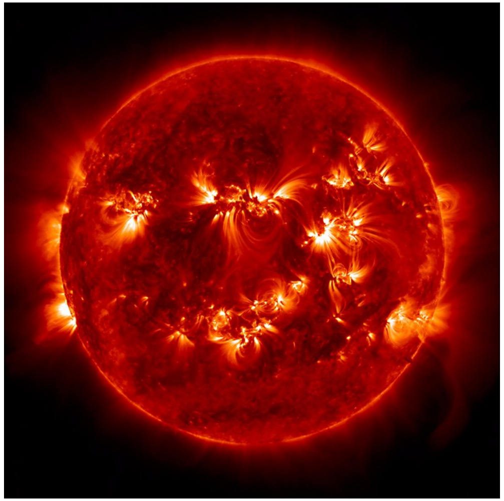
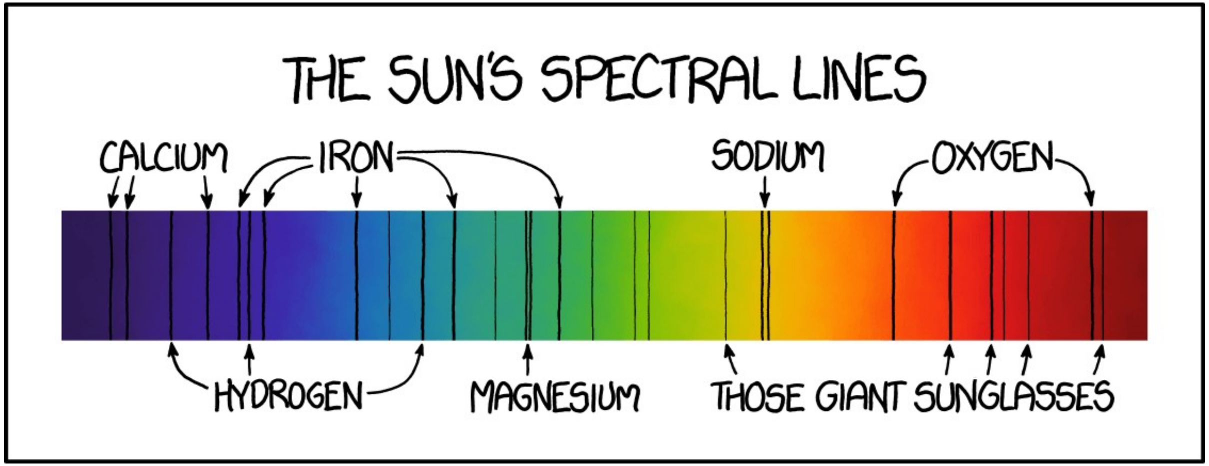
PHYS 7810 / COLLAGE2021:

Solar Spectral Line Diagnostics





Lecture 07: Sensitivies of Spectral lines to Physical Parameters

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Plan for today:

- This is, after all, a course on spectral line diagnostics
- Diagnostics: drawing conclusion about the medium from the observed spectrum
- For example, looking at the presence and strength of the spectral lines we can conclude something about the temperature of the star *(bonus points for figuring out that there is not one temperature ;))*
- We will get into this more when we get to specific solar scenarios, but for the moment we can understand some details.

Today, we will use the code we examined on Tuesday

- We solved RTE, wavelength by wavelength, numerically:

$$I_\lambda^+ = I_\lambda^0 e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S_\lambda(t) e^{-t} dt$$

- Where we had a run of temperature on the optical depth, $T(\tau)$ and we assumed the following:

Follows from LTE, and the Planck function

$$S \propto T$$

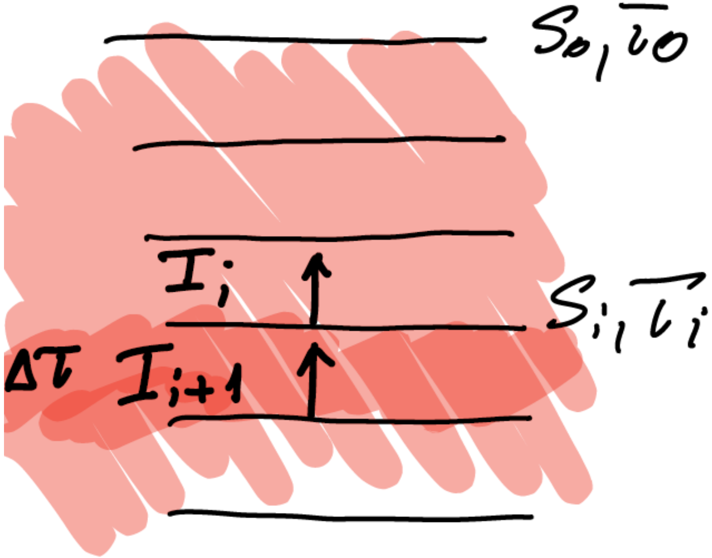
$$\tau_\lambda = \tau(1 + r\phi_\lambda)$$

$$\phi_\lambda = H(\lambda, \Delta_\lambda, a)$$

Very ad-hoc, we did it just to have some simple model and capture line formation

Damping, presumed to be constant

Also important, how did we solve the integral?



$$I_i = I_{i+1} e^{-\Delta\tau_i} + \int_0^{\Delta\tau_i} S(t) e^{-t} dt \approx$$
$$I_{i+1} e^{-\Delta\tau_i} + S(1 - e^{-\Delta\tau_i})$$

- We assumed that each little chunk is homogeneous. (Not the atmosphere, just each chunk separately)
- What would be a better way to do this?

Let’s dissect each of these approximations

$$S_\lambda = B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

- You might remember Kirchhoff’s law. Ratio of emissivity and opacity is equal to a “yet unknown” function that turns out to eventually be Planck function.
- This is actually pretty good approximation for plasma where collisional processes are more frequent than the radiative ones.
- We will discuss the departures from this *very soon.*
- For the second part (S ~ T), figure out the exponent in the above equation for solar conditions (wavelength 500 nm, T = 6000 K) – 2 mins

Wavelength dependent optical depth

$$\tau_\lambda = \tau(1 + r\phi_\lambda)$$

Given by the model atmosphere

Imposed as a free parameter by us

- What would be a proper way to do this?

$$d\tau_\lambda = -dh\chi_\lambda$$

$$\chi_\lambda = \chi_c + \chi_\lambda^L$$

$$\chi_\lambda^L = (n_l B_{lu} - n_u B_{ul})\frac{hc}{4\pi\lambda}\phi_\lambda$$

These are extremely cumbersome to calculate! (Remember all the Saha eqs.)

Wavelength dependent optical depth

- So, we simplified all that with one free parameter, r
- The goal here was not to *realistically* model 525 nm Fe I line, but to try and see what happens and how we could do it
- If you are interested in a more realistic case, you can check last year's hands on exercise that involved a bit more realistic assumptions (Lecture 19):

  https://nso.edu/students/collage/collage-2020/

- And definitely do the homework! :)

Line profile

Voigt function, not much to see here

$$\phi_\lambda = H(\lambda, \Delta_\lambda, a)$$

Except for constant turbulent velocity, we nailed this

$$\Delta_\lambda = \frac{\lambda_0}{c}\sqrt{\frac{2kT}{m} + v_t^2}$$

$$a = \frac{\Gamma_{\mathrm{damping}}}{\Delta_\nu}$$

This is just a definition basically

$$\Gamma_{\mathrm{damping}} = \Gamma_{\mathrm{natural}} + \Gamma_{\mathrm{collisional}}$$

Here we again took an ad hoc number (1E7). Presumably collisional damping depends on the density and velocity (temperature) of the particles!

Great, so let’s analyze, one-by-one importance of each physical parameter for the spectral line formation

Temperature

- Determines ionization state (higher temperature higher ionization, etc.)
- Determines excitation state (higher temperature more excitation, higher source function)
- Line broadening (especially for not heavy atoms)
- Collisional rates (higher the temperature, more collisions we have)
- **Probably the most important parameter for the line formation**

Gas Pressure (and electron pressure)

- Gas pressure = total number of particles
- So, puts a constraint on the solution of Saha equation
- In a way, determines total abundance of absorbers
- Higher pressure, more collisions

Velocities

- Turbulent velocity broadens the lines (especially for heavy atoms)
- LOS velocity (or, sometimes even horizontal velocities) important as it shifts the line profile
- Blue/red shift, or line asymmetry or both
- Can increase equivalent width of very strong lines
- Can result in some very interesting aspects in NLTE case (Doppler dimming/brightening)

Magnetic field

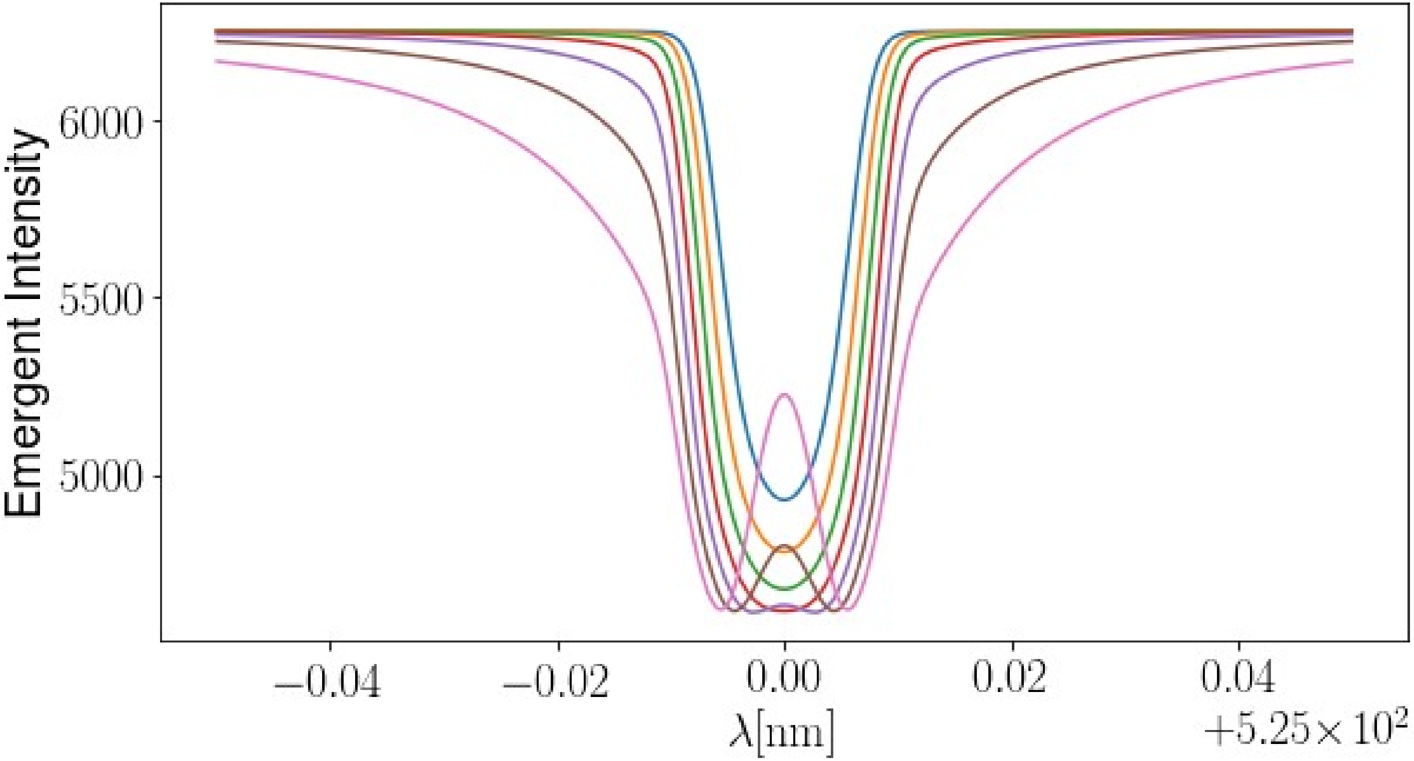
- **Will be more discussed when we get to photosphere, but:**
- We know that it can broaden / split the lines via Zeeman effect
- But more importantly: It can induce polarization via
- Zeeman effect (Paschen-Back effect)
- Hanle effect (exotic thing we will learn about when we discuss prominences)

So in the rest of the course we will:

- Investigate these in more detail
- Involve new physics
- Relate spectral line formation more closely to specific solar phenomena
- Look at some specific observations
- And who knows what else!

But first, some answers!

- Why does the line strength (r) that we used in our code, influence the spectral line shape in this specific way?



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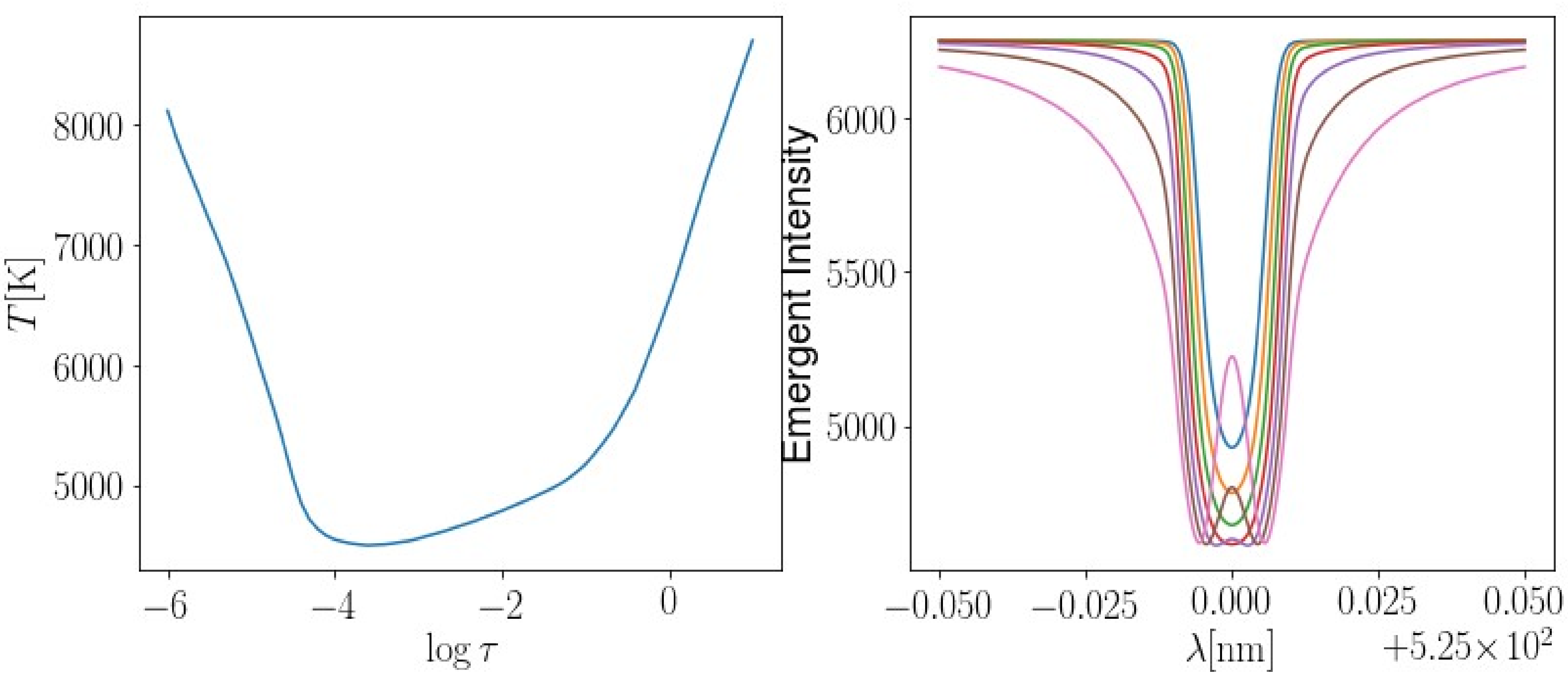
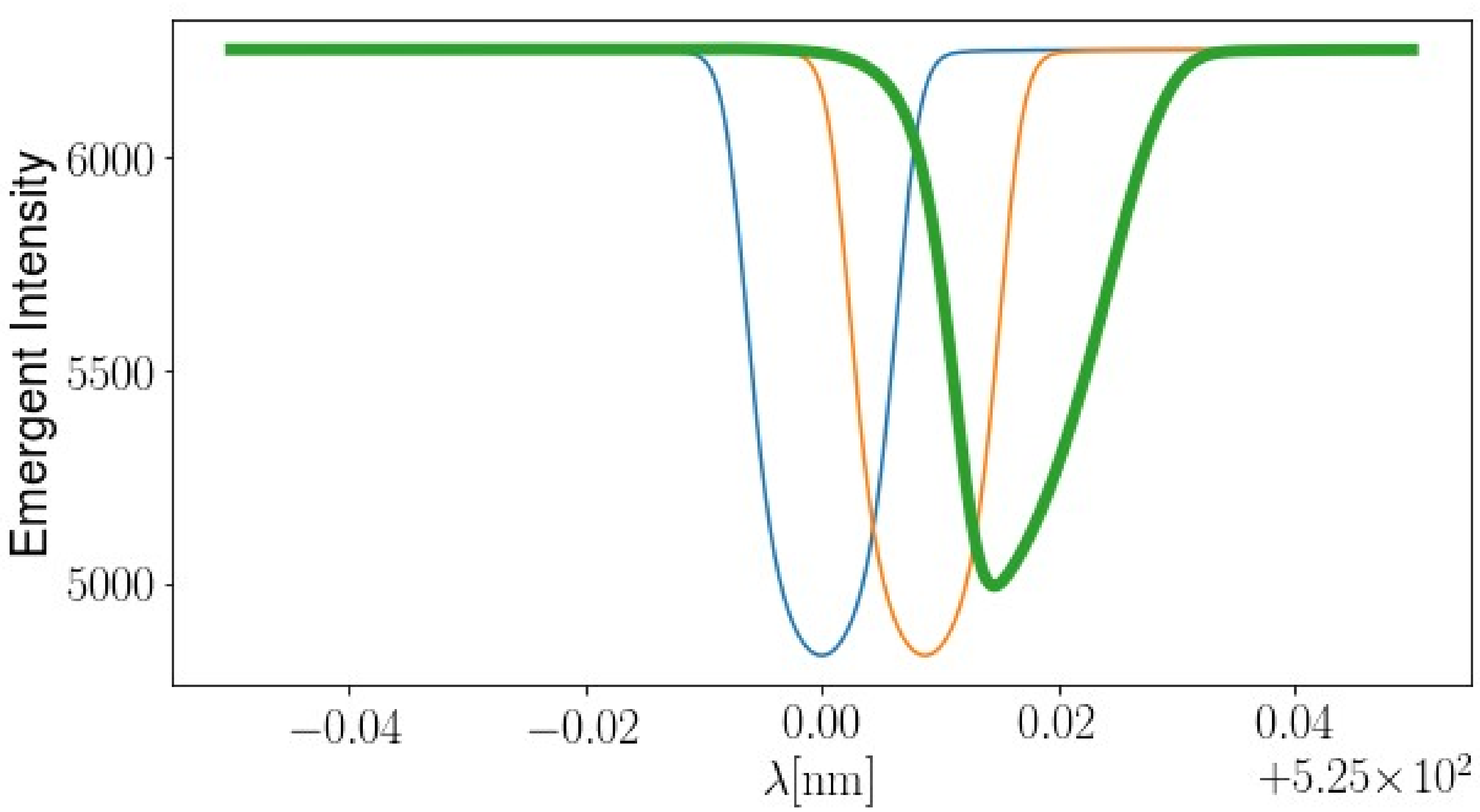


Explain the asymmetry due to velocity gradient?



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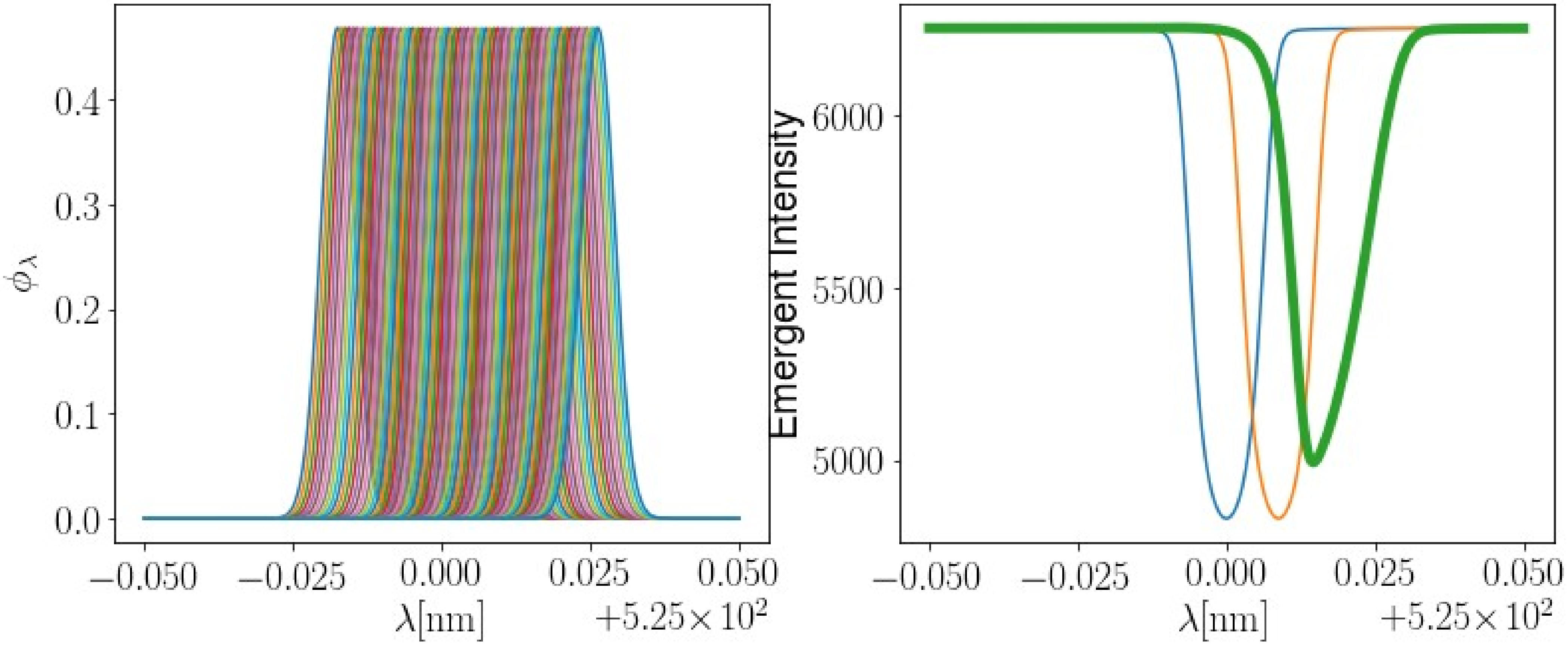
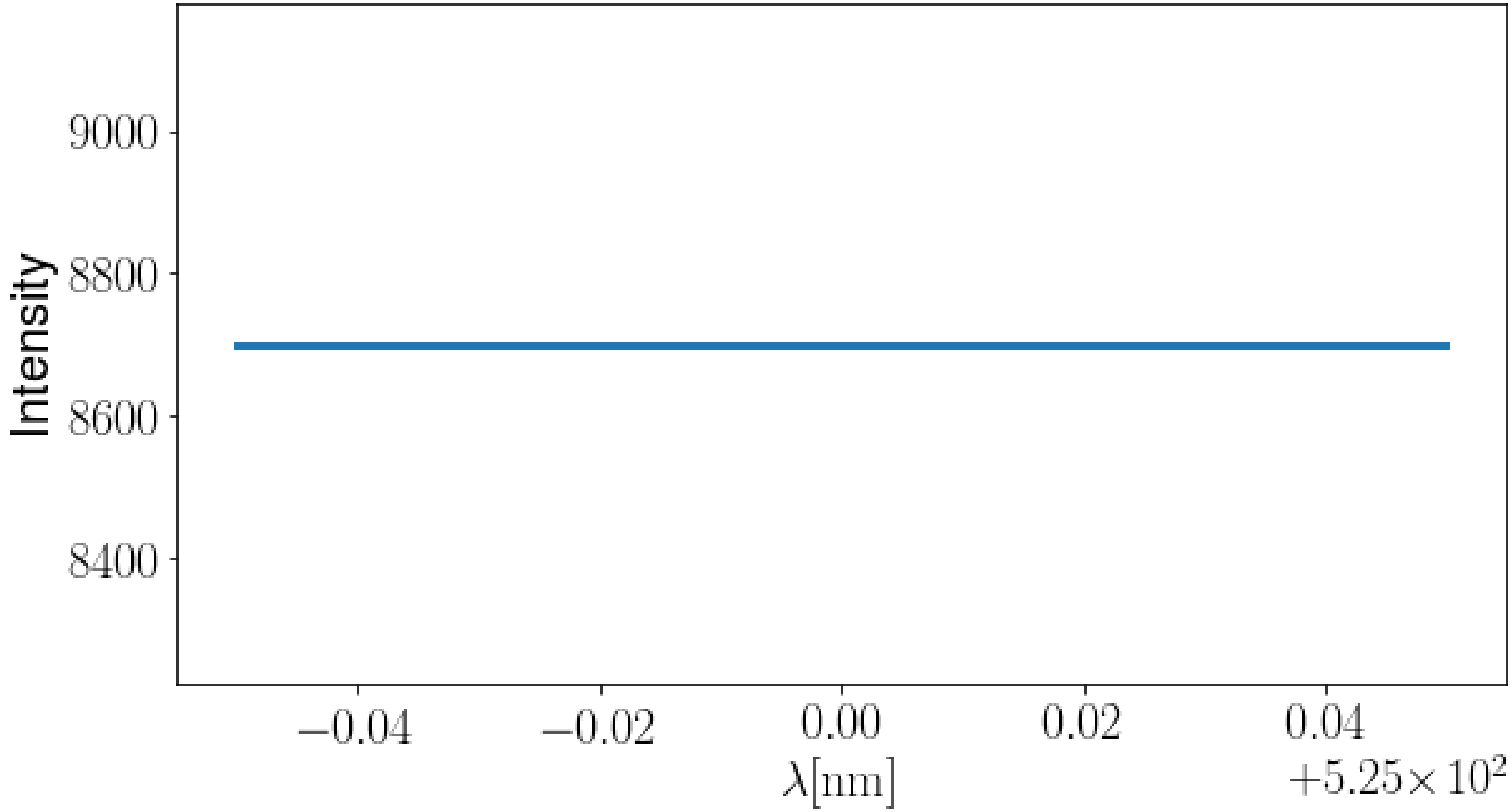


If we look at the intensity in the very deep layers…

- There is no spectral line, why?



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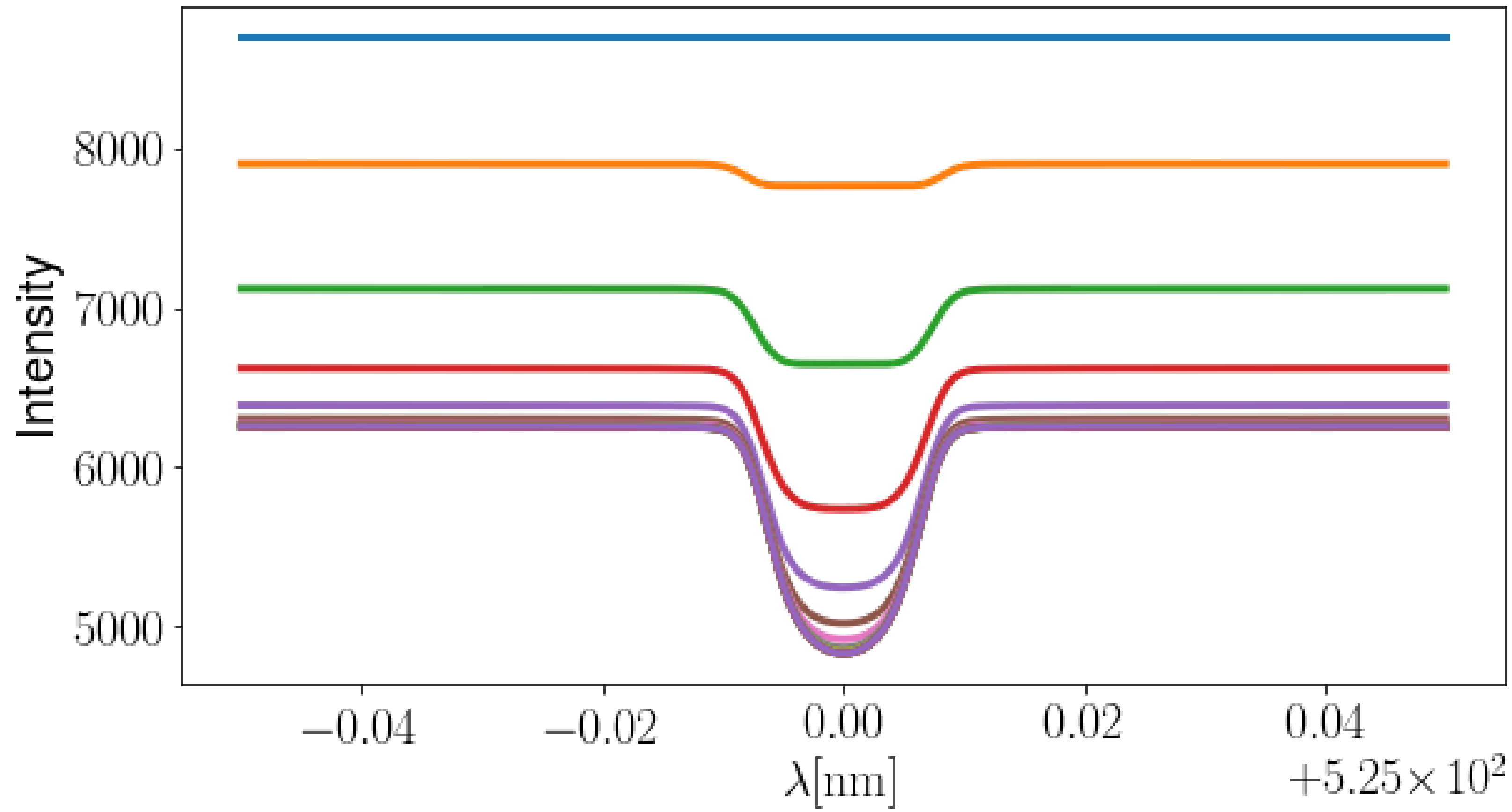


This leads us to a very important question

- **Which layers of the solar atmosphere are the most important for the formation of the given line?**
- We can answer that question with our little model, we know that:

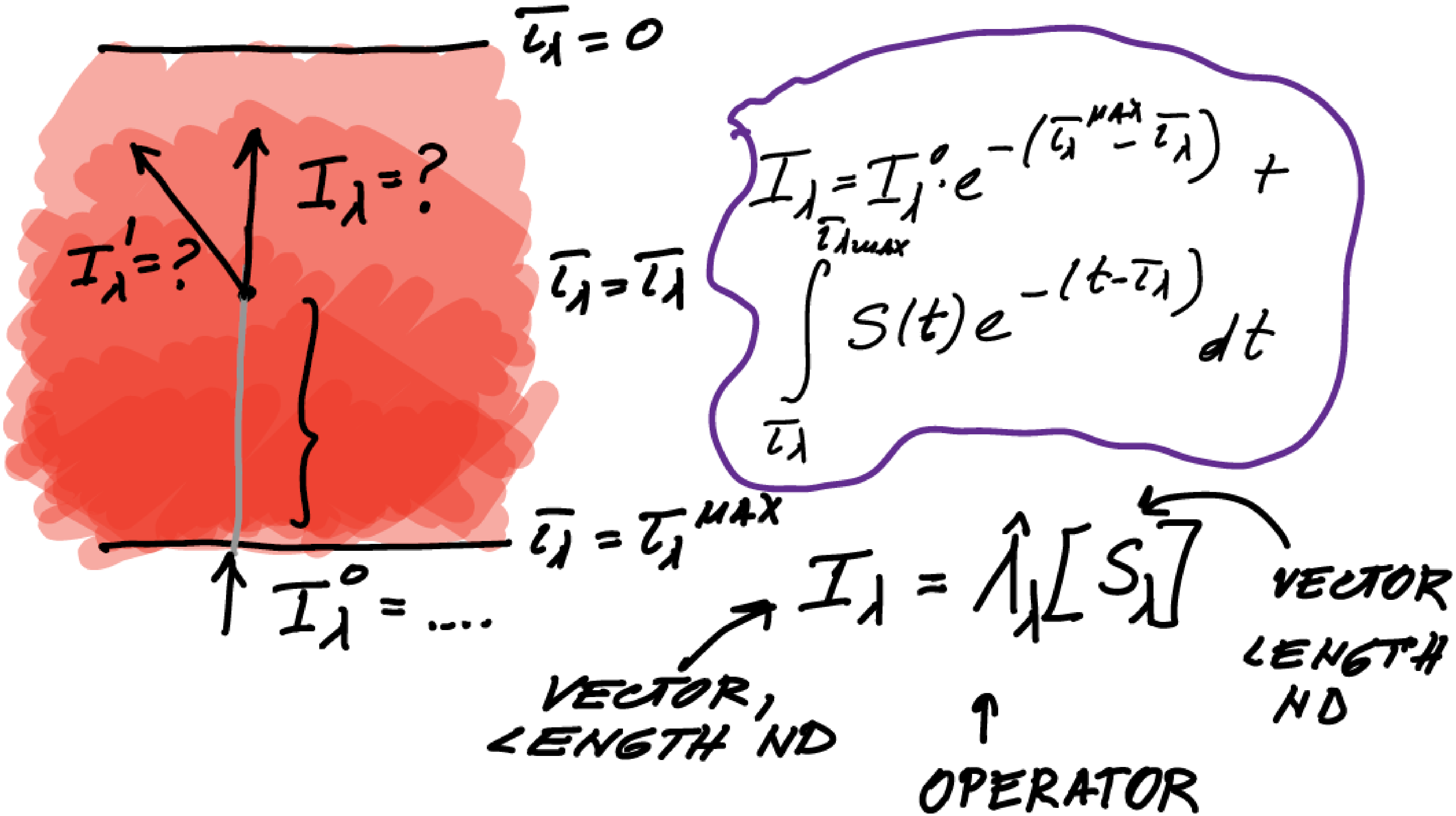
$$I_i = I_{i+1} e^{-\Delta \tau_i} + \frac{S_i + S_{i+1}}{2} (1 - e^{-\Delta \tau_i})$$

- Now, let's take a moment to agree that, technically, we can say:

$$I_i = \sum w_{i'} S_{i'}$$

(Let's move to whiteboard for this)

Lambda operator (remember this!)



This form is indeed very important!

$$I_i = \sum w_{i'} S_{i'}$$

- Formally speaking, specific monochromatic intensity at one point in the atmosphere depends on the values of the source function everywhere (*well, everywhere "upwind" from the point i).*
- This is another reason why I love to say that intensity is **non-local.** What we see (and not only what we see, intensity everywhere), is some weighted contribution of the source functions along the line of sight.
- Historically, following, Schwarzshield:

$$I_i = \Lambda[S] = \sum_{i'} \Lambda_{i,i'} S_{i'}$$

Ahem, earlier I used the word "contribution"

- Looking at how much each depth point contributes to the emergent intensity, we can construct the so called **Contribution function** (here I have explicitly noted wavelength dependence)

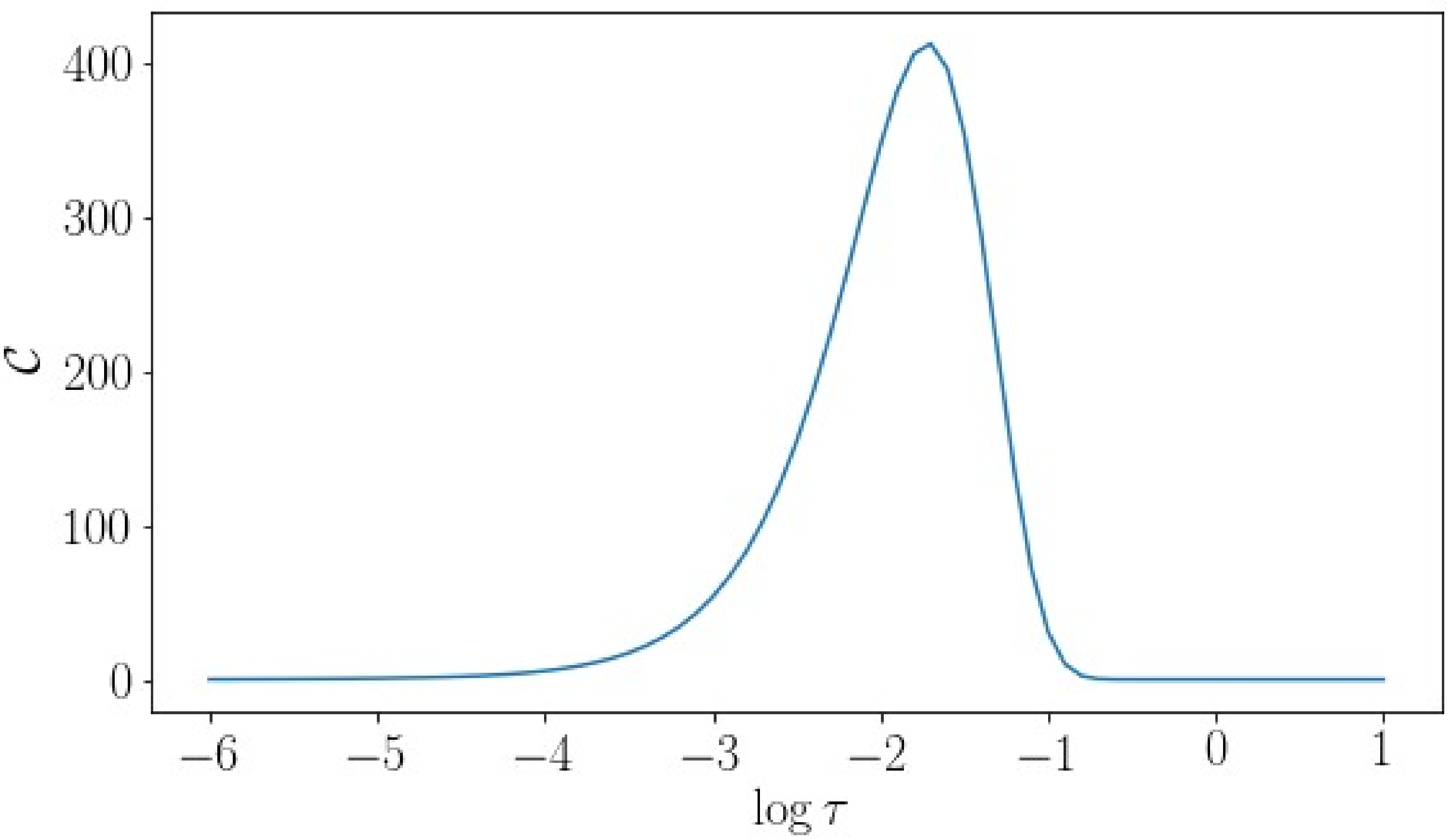
$$\mathcal{C}_{i,\lambda} = \Lambda_{0,i;\lambda} S_{i,\lambda}$$

- Analytically, we could express the contribution function from the formal solution as:

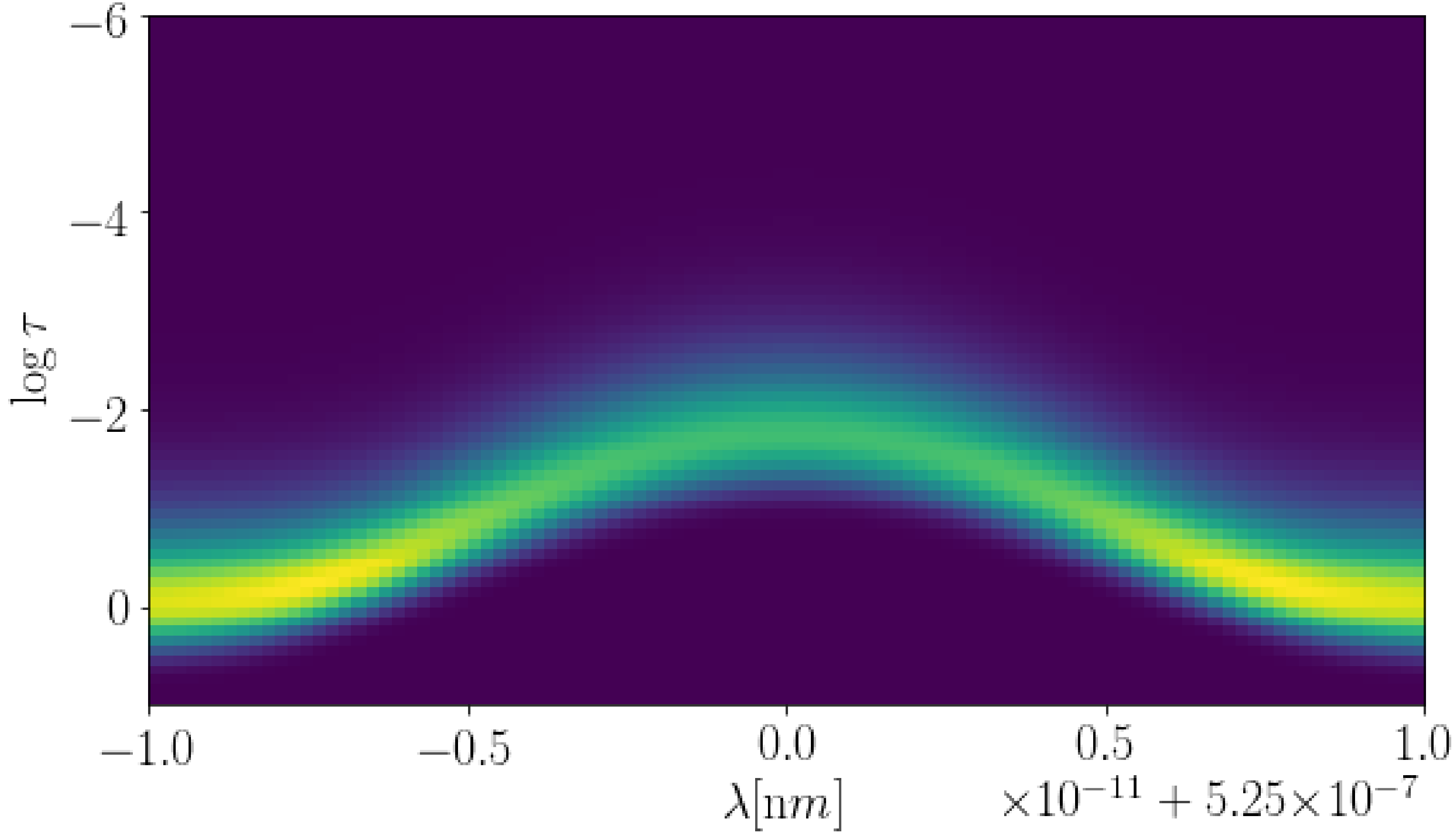
$$\mathcal{C}_{\lambda}(\tau_\lambda) = S(\tau_\lambda) e^{-\tau_\lambda} \tau_\lambda$$

This one might be a little tricky to figure out, but give it a go when you have the time

Let’s plot contribution function for one wavelength:



And then some fancy image for multiple wavelengths



Now, hanging around the people at NSO…

- You might have heard another fancy term **Response function**
- Response function tells us how the emergent intensity, ahem, **responds** to the changes at specific quantity at specific depth:

$$\mathcal{R}^{q}_{i,\lambda} = \frac{\partial I^{+}_{\lambda}}{dq_i}$$

- Where q is not an index, but a placeholder for a specific quantity (say, Temperature, velocity, magnetic field… etc.

Calculating the response functions

- Much harder. You need to take into account all the inter-dependencies
- In our case, we could find a derivative with respect to the source function easily:

$$I_\lambda^+ = \sum_i \Lambda_{0,i;\lambda} S_i$$

$$\frac{dI_\lambda^+}{dS_i} = \Lambda_{0,i;\lambda}$$

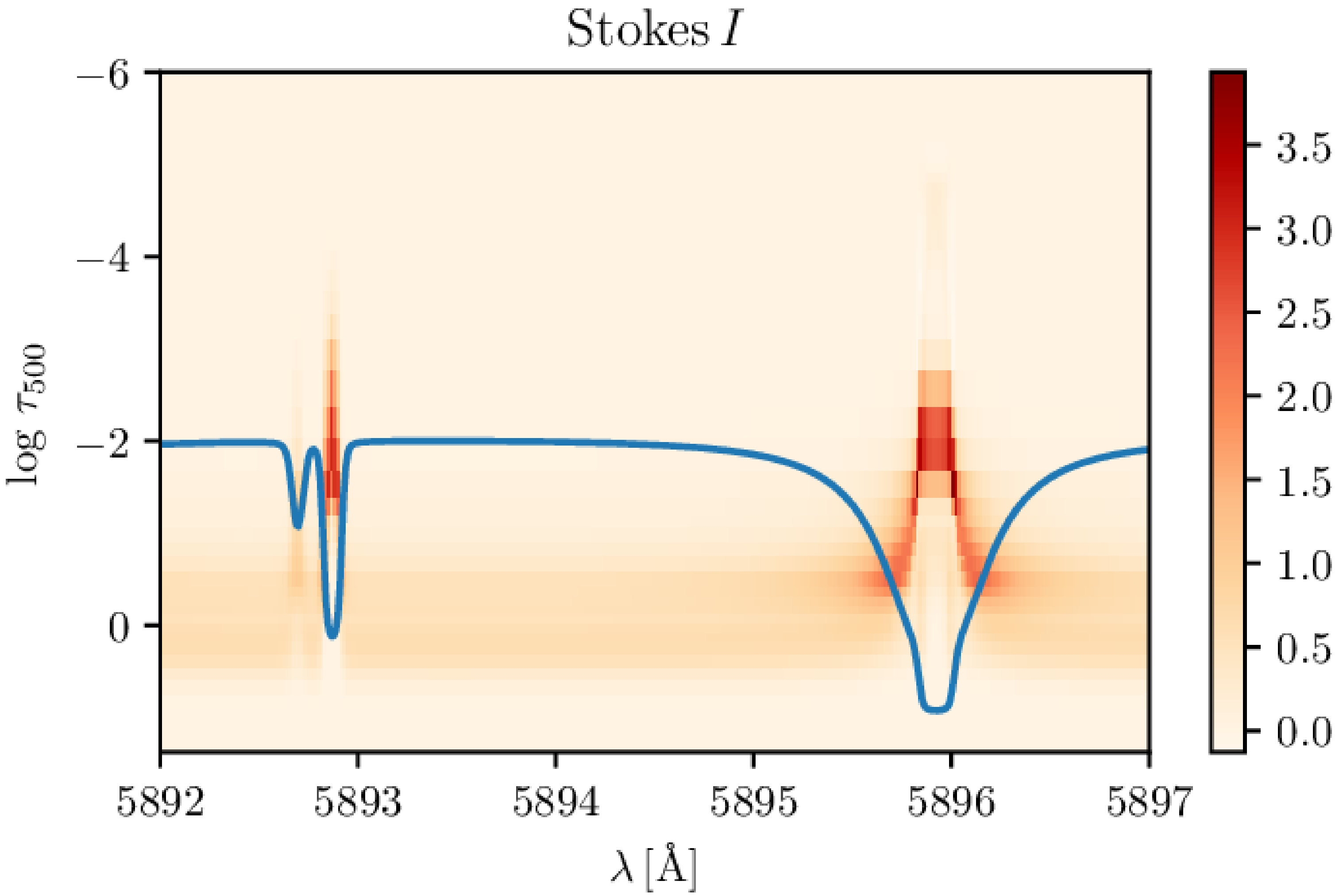
- But this is because relationship is linear, start thinking about temperatures, and it gets complicated real quick.

So to summarize that real quick:

- **Contribution function**
- Tells us how much each layer contributes to the emergent intensity

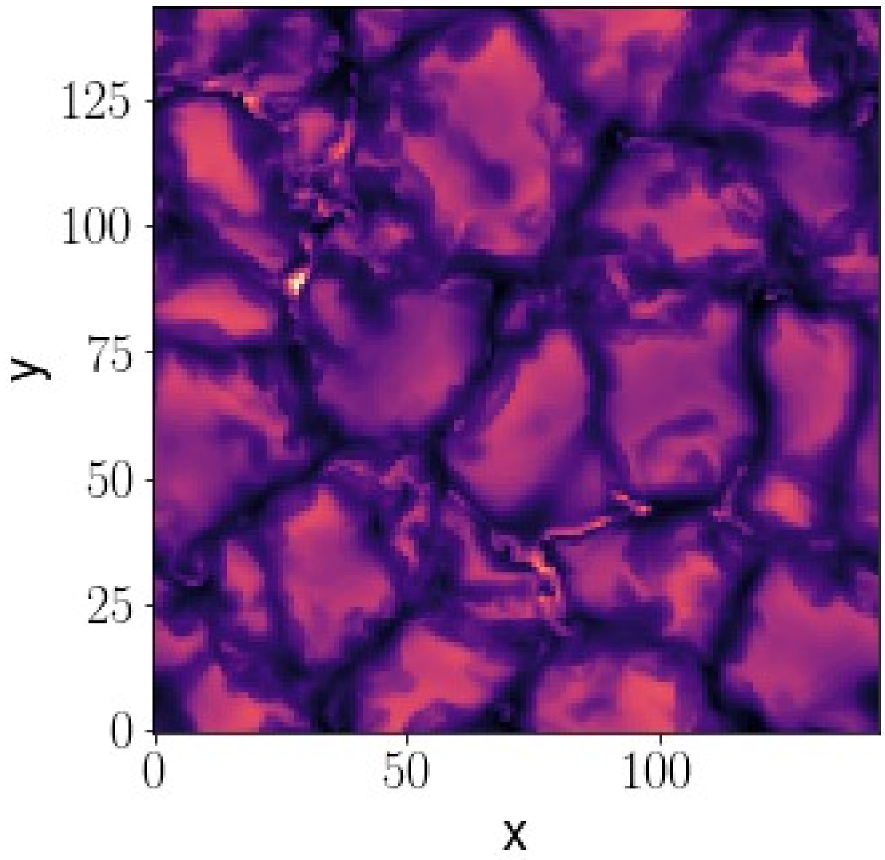
- **Response function**
- Tells us how changes at a layer reflect to emergent intensity
- Takes into account more of the coupling (will become clearer when we talk about NLTE)
- Different for different parameters
- Yields more diagnostic potential but harder to calculate

Example response function



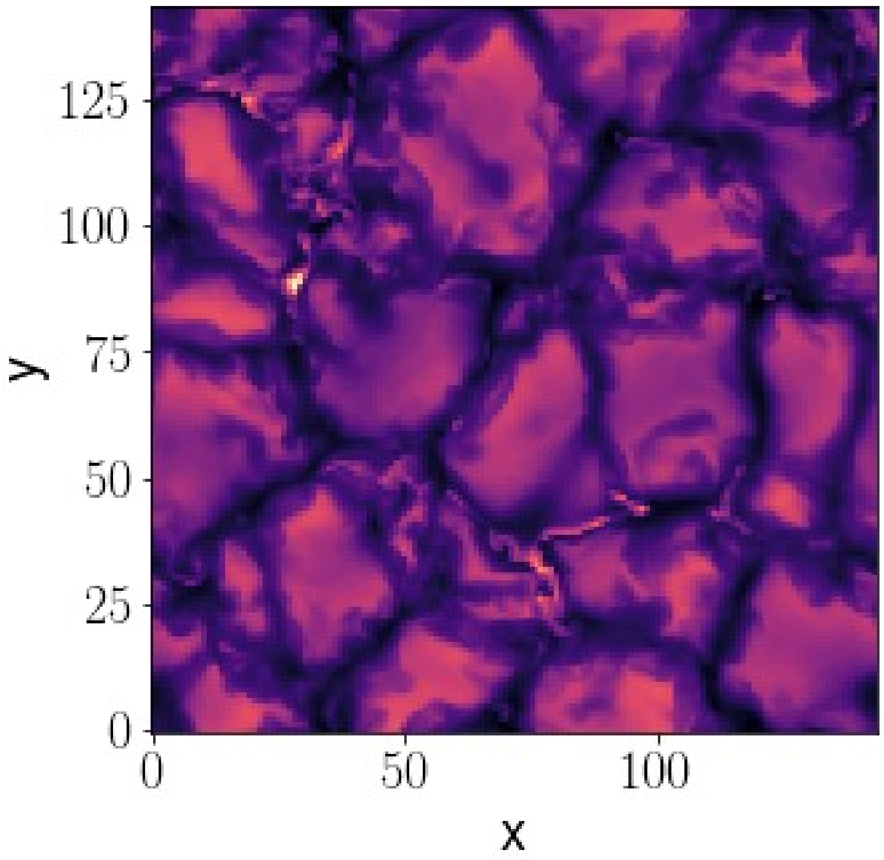
And for the end, a problem with real, well, simulated data

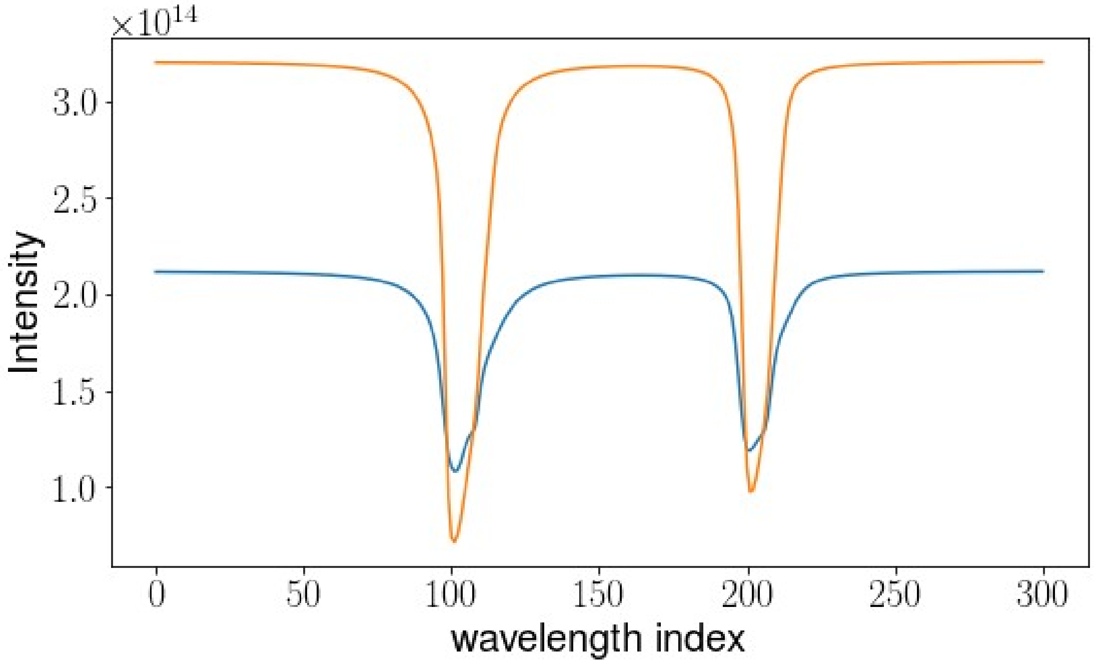
- Let’s look at this map, calculated from a MURAM simulation:



But we also have a spectrum for each point

- What does this difference between the spectra tell us?





These are spectra of a bright and a dark pixel.

Let’s sum it up

- Different levels of intensity at different wavelengths tell us about the source function stratification
- This is basically temperature stratification
- So granules are hotter deep but cooler somewhere above
- *Conveniently we call this layer “inverse granulation”*
- Now you can already imagine that there are methods how to reverse engineer the spectrum in order to recover the variations of the physical parameters – **these are called inversion methods**
- And that’s all for today!