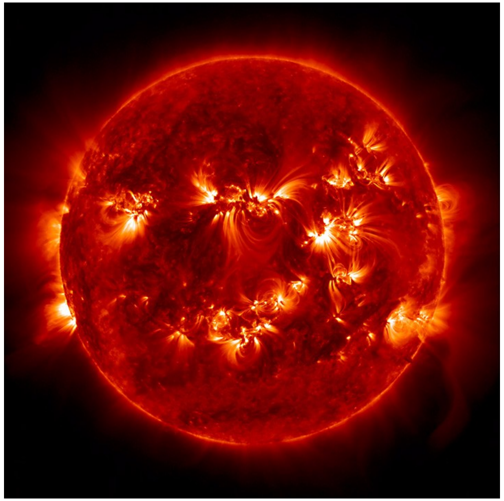
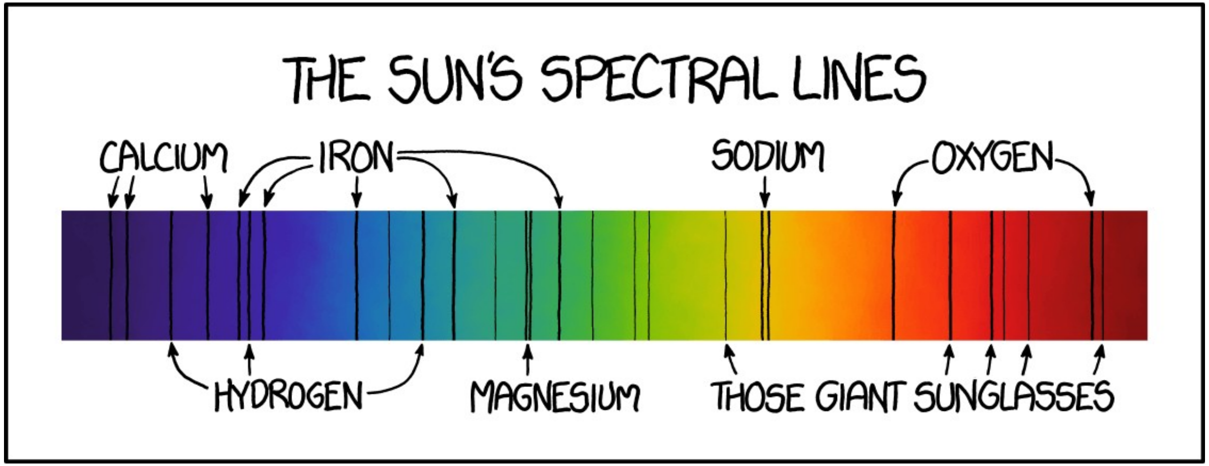
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 05: Opacity and Emissivity in Spectral Lines (cont)

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Plan for today:

- On Tuesday we entered the discussion talking about emissivity and deriving an expression
- Today we are first going to do the same for opacity (but much faster)
- Then we are going to investigate some relationships between the two
- And to analyze, through equations how each physical parameter (T, p**, v, B**), influences opacity/emissivity.

Once again, this is equation that includes it all

This whole equation tells us how the things behave on big scales. (dl is geometrical length, and light propagates over very large distances inside of astrophysical objects.

$$\frac{dI(\vec{r},\mathbf{n},\lambda)}{dl} = j(\vec{r},\mathbf{n},\lambda) - \chi(\vec{r},\mathbf{n},\lambda) I(\vec{r},\mathbf{n},\lambda)$$

These two coefficients, on the other side, depend on specific physics of emission and absorption. **We talk about them now**

Emissivity due to spectral line transitions

$$j_\lambda = \frac{hc}{4\pi\lambda} n_u A_{ul} \phi_{ul,\lambda}$$

- This is only due to **bound-bound** processes, other sources would look differently
- Finding the number density of the atoms that are in the upper energy state (population of the upper level) will be the most cumbersome task.
- Also, line emission/absorption profile (often the same) contains many dependencies

The absorption coefficient (opacity)

- We can have a very similar story here:

$$\frac{dI_\lambda^{\rm abs}}{dl} = -\chi_\lambda I_\lambda$$

- The intensity should have units of inverse length. What if we relate it somehow to number density of absorption events (absorbers?)

$$\chi_\lambda[m^{-1}] = n^{\rm absorbers}[\rm m^{-3}] \times \sigma[\rm m^2]$$

- This is more “classical” than the previous argument, but it very intuitively tells us what opacity depends on!

Let's write an equation for the opacity by analogy

$$\chi_\lambda = \frac{hc}{4\pi\lambda} n_l B_{lu} \phi_{ul,\lambda}$$

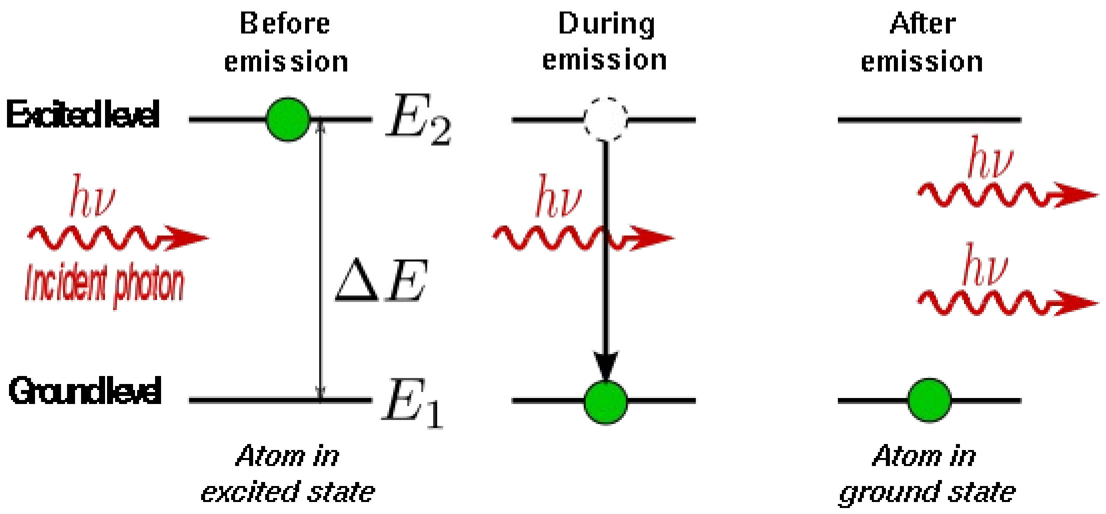
- Does this make sense?
- What changed with respect to emissivity?
- Can you figure out what units for B_lu have to be?

Let's write an equation for the opacity by analogy

$$\chi_\lambda = \frac{hc}{4\pi\lambda} n_l B_{lu} \phi_{ul,\lambda}$$

- Does this make sense?
- What changed with respect to emissivity?
- Units for B_lu have to be 1/(s x Units of Intensity) $B_{lu} \times I = R_{\rm abs}$

What about stimulated emission though?



$$E_2 - E_1 = \Delta E = h\nu$$

- Should we not include this as an emission source too?
- Still, it’s a bit awkward, it depends on the direction, it scales with incoming intensity… it’s almost as if it…

Stimulated emission is negative absorption!

$$\chi_\lambda = \frac{hc}{4\pi\lambda}(n_l B_{lu} - n_u B_{ul})\phi_\lambda$$

- Wait Ivan, does that mean that the absorption can be negative?
- Can it?
- Can it?

Yes! This is how lasers work!

- To obtain lasing we need to obtain the so called population inversion (not necessary inversion, depends on the values for B)
- We also have naturally occurring lasers, they are called MASERs (Microwave Amplification by Stimulated Emission of Radiation).
- This does not happen (at least not to my knowledge) in the atmosphere of the Sun that much

Now we know our opacity and emissivity

$$j_\lambda = \frac{hc}{4\pi\lambda} n_u A_{ul} \phi_\lambda$$

$$\chi_\lambda = \frac{hc}{4\pi\lambda} (n_l B_{lu} - n_u B_{ul}) \phi_\lambda$$

- We can now calculate optical depth and the source function, right?

Let's start with the source function

$$S = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$$

- The Source function in the line does not depend on the wavelength
- (This is generally not true as we have other op/em sources, but can be really good approximation)
- So, why don't we talk about these level populations a little bit? How to calculate them?

Boltzmann excitation distribution

Total number density of the atom (ion)

Energy with respect to the ground level

$$n_i = n \frac{g_i e^{-E_i/kT}}{Z}$$

Population of the level i

Partition function

- If we know the total number of the atom (ion), and the temperature, we can calculate the population of each of the atomic levels. Easy enough.

Boltzmann excitation distribution

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- If we know the total number of the atom (ion), and the temperature, we can calculate the population of each of the atomic levels. Easy enough.
- There is a slight problem. What we usually work with is total number of particles, say, hydrogen atoms. However… what figures in the above equation is the number of **neutral** hydrogen atoms.

So I am going to slightly re-write it

Energy of the state i in the ionization state j

Total number density of the atoms in ionization state j

$$n_{j,i} = n_j \frac{g_{j,i} e^{-E_{j,i}/kT}}{Z_j}$$

Population of the level i, in the ionization state j

- But this leads to another the question: How to find the number density of given ion.

Example: How many of the total hydrogen atoms are neutral?

Saha ionization equation

Number density of the ions in the next ionization state

Energy needed to ionize ion j

$$n_j = C_I \times n_{j+1} n_e \frac{Z_j}{Z_{j+1}} \frac{e^{E_j/kT}}{T^{3/2}}$$

Number density of the ions in the ionization state j

Electron number density

- Let’s spend a couple of minutes to see if this equation makes sense
- It of course complicates things, now there is this annoying electron density here.

Let’s try and figure out how to apply this to an example:

- Number density of atomic hydrogen atoms is 10^{22} m^{-3}. Temperature of the plasma is 6000 K. Using Saha ionization equation try to find the number density of electrons and protons. You can symbolically represent Saha equation as:

$$n_j = n_{j+1} n_e \times f(T)$$

Let’s try and figure out how to apply this to an example:

- Number density of atomic hydrogen atoms is 10^{22} m^{-3}. Temperature of the plasma is 6000 K. Using Saha ionization equation try to find the number density of neutral hydrogen and protons. You can symbolically represent Saha equation as:

$$n_j = n_{j+1} n_e \times f(T)$$

$$n_0 + n_+ = 10^{22}$$

$$n_+ = n_e$$

Does this help a bit?

Solving Saha ionization equilibrium

- However, in general, we don't have only one element, but many
- At first it might seem that Hydrogen is the most important one but….

Solving Saha ionization equilibrium

- However, in general, we don't have only one element, but many
- At first it might seem that Hydrogen is the most important one but:

  Metals, despite being much less abundant, have much smaller ionization potential, and are thus more likely ionized. So, metals are the ones who are determining the electron density of the gas.

- Technically all the species are "coupled", and for given point in the atmosphere we have to solve Saha equation for all the elements (species) simultaneously

Equation of state

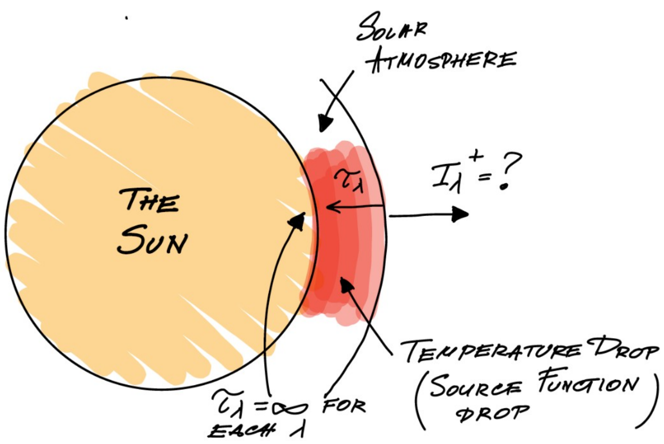
- Generally, by equation of state, I mean a relationship that connects, **pressure, mass density and temperature**
- From any two of these we should be able to calculate the third. Say we have **pressure and temperature:**
- Use $p = nkT$, to find the total number density of all the particles
- Use Saha ionization equation to calculate number densities of each constituent
- Add them with their masses to find **mass density.**

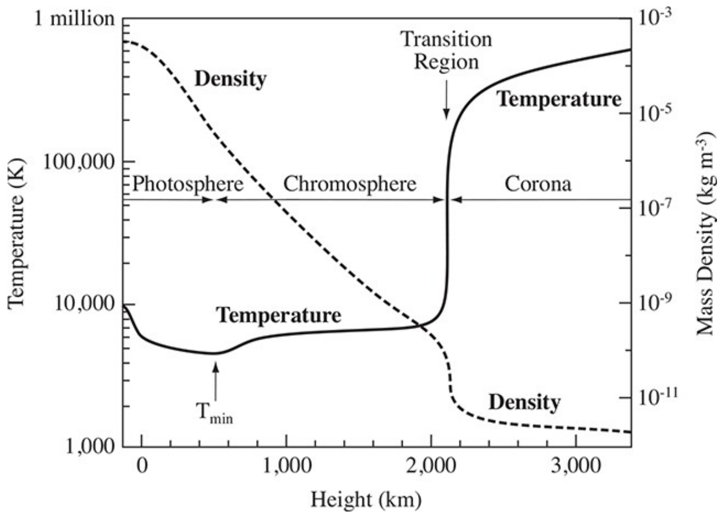
So, to recoup:

If we know the temperature and the pressure in the gas, we can solve Saha and then Boltzmann equation to find population of each relevant level.
It’s cumbersome, but doable.

Disclaimer

- Saha-Boltzmann equation only works in equilibrium.
- We need sufficient number of collisions between the particles to establish that equilibrium
- This means we need “dense” plasma (well, dense, for astrophysical contexts ;)
- In tenuous plasma, situation is even more complicated.





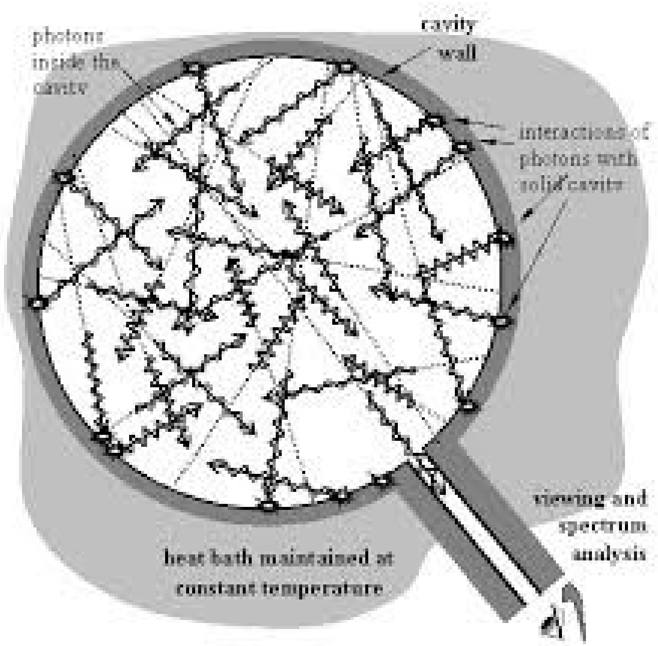
But, I told this long story because of this:

$$S = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$$

- We will now forget about the levels and everything else and go back to what Gustav Kirchhoff concluded about this
  (it seems like already he knew about RTE!)

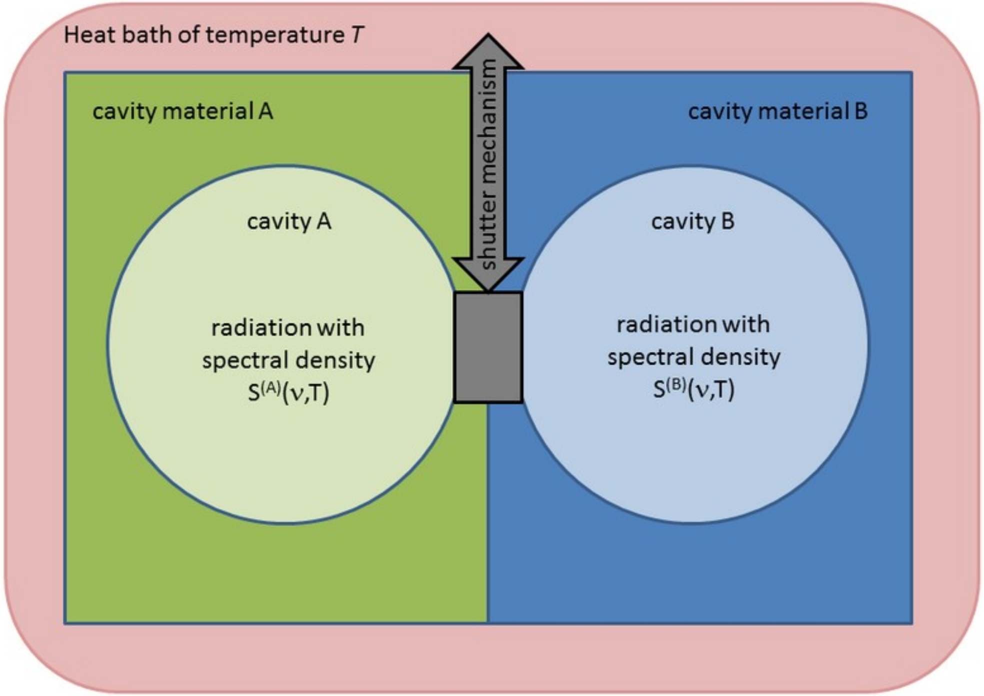
Blackbody radiation

- Usually imagined as a “cavity”, an empty hole where photons are trapped and brought to equilibrium with surrounding walls
- The distribution of the photons according to energy (wavelength), and thus the emergent specific monochromatic intensity must depend on the temperature only



Blackbody radiation

- Kirchhoff proved this by a thought experiment (how else :-))
- Imagine an obstacle that allows only a specific wavelength to pass by, T is the same in both halves
- If this "spectral density" (intensity), was different, there would be net flow of the photons, and thus energy.
- But then one part would heat up!



So Kirchhoff said:

- It seems that the intensity is universal function of wavelength and T:

$$I_\lambda = B(\lambda, T)$$

- And then he substituted this in the RTE:

$$\frac{dI_\lambda}{dl} = j_\lambda - \chi_\lambda I_\lambda$$

$$0 = \frac{dB_\lambda}{dl} = j_\lambda - \chi_\lambda B_\lambda$$

$$\frac{j_\lambda}{\chi_\lambda} = B_\lambda$$

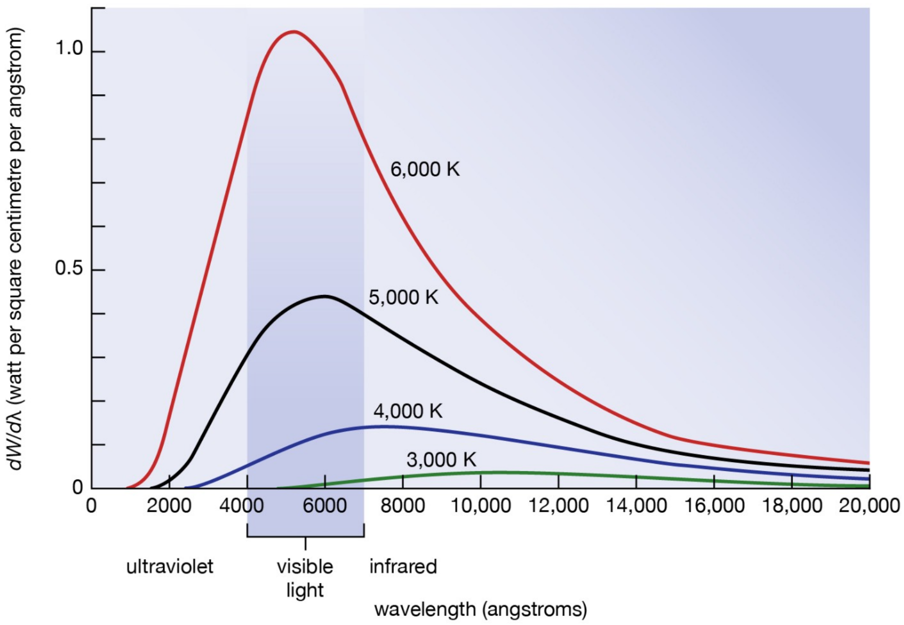
Kirchhoff's law. Does not care about where em/op come from!

So what Kirchhoff actually said:

"It is a highly important task to find this function"

Which Max Planck (and Bose, and Einstein, and ...) did

- Blackbody radiation describes distribution of photons over energies, but also the source function of the matter in the (local) equilibrium
- This might require some unpacking ;)



$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Putting together Boltzmann distribution and Planck Law:

$$n_i = n\frac{g_i e^{-E_i/kT}}{Z}$$

$$S_\lambda = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$$

$$S_\lambda = B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

- We can find relationships between the Einstein parameters
- (This will actually be your homework)
- But there is a deeper reasoning here…

Local thermodynamic equilibrium

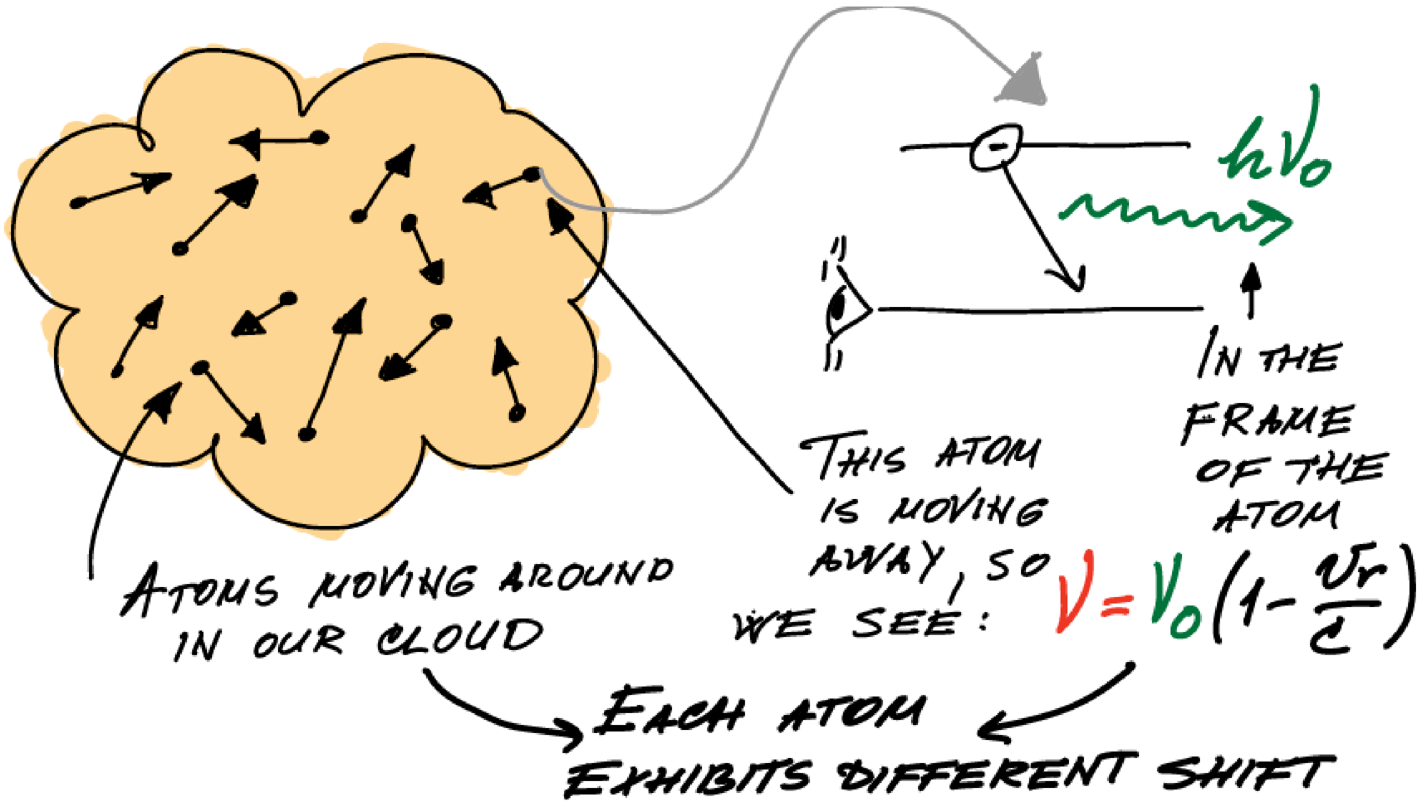
- If the matter is locally in the equilibrium (Saha-Boltzmann distribution is valid), the source function is equal to the Planck function.
- This **does not** mean that the intensity is equal to Planck function too!
- This is quite some assumption, which is valid only when collisions dominate and we can neglect influence of the radiation on the matter (so only matter influences the radiation)
- We will talk about relaxing this. That is called **non-local thermodynamic equilibrium** (weird name since non- negates the whole statement)

Line absorption (and emission) profile

- We are covered on the source function part, and it seems simple enough now.
- But we need to calculate the optical depth, for that we need opacity, and for that we need the line absorption profile:

$$\chi_\lambda = \frac{hc}{4\pi\lambda}(n_l B_{lu} - n_u B_{ul})\phi_\lambda$$

Simple spectral line broadening



The line-of-sight velocity distribution follows Gaussian distribution. Then, when we sum up emissions (absorptions) by all the particles, Doppler-shifted frequency (and wavelength) will follow Gaussian Distribution too!

The other way to write this would be:

- Each atom absorbs as a delta function (infinitely thin levels)
- But we look at large ensembles of atoms:

Delta function shifted due to Doppler effect

$$\phi_\lambda = \int_{-\infty}^{\infty} \delta(\lambda_0(1 - \frac{v}{c}))p(v)dv$$

"Average" line profile

Probability of the atom having that velocity (this is radial, or los velocity)

- This is how you would get Gaussian!

But then, the levels are **not** completely sharp

- **Natural (radiative) broadening :** Levels themselves are not perfectly sharp. The shorter is the lifetime of the level, the more smeared it is.
- **Pressure broadening (damping):** Other particle exert force on our atom and "deform" it's levels, this leads to so called pressure or collisional damping. This can even make the level be "asymmetrical" in a way
- Combination of the two yields the infamous **Voigt profile**

Lorenzian profile, due to damping, shifted due to Doppler effect

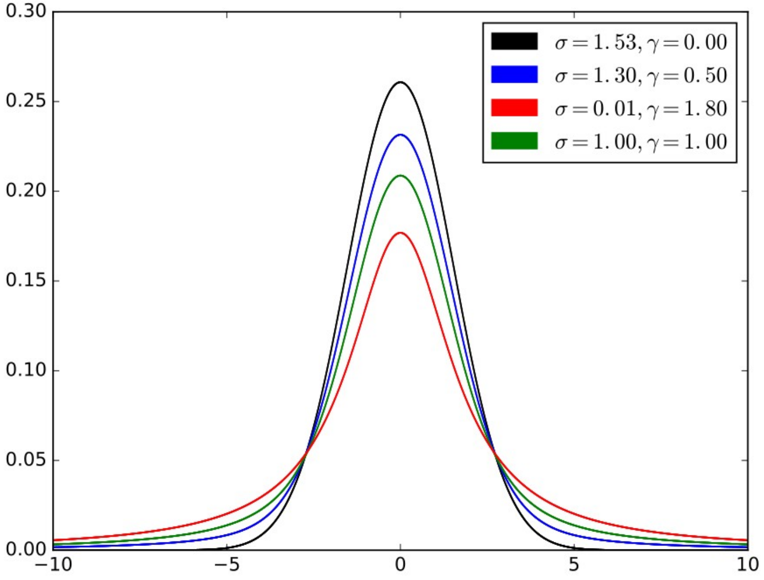
$$\phi_\lambda = \int_{-\infty}^{\infty} L(\lambda_0(1 - \frac{v}{c} - \frac{v_z}{c}))p(v)dv$$

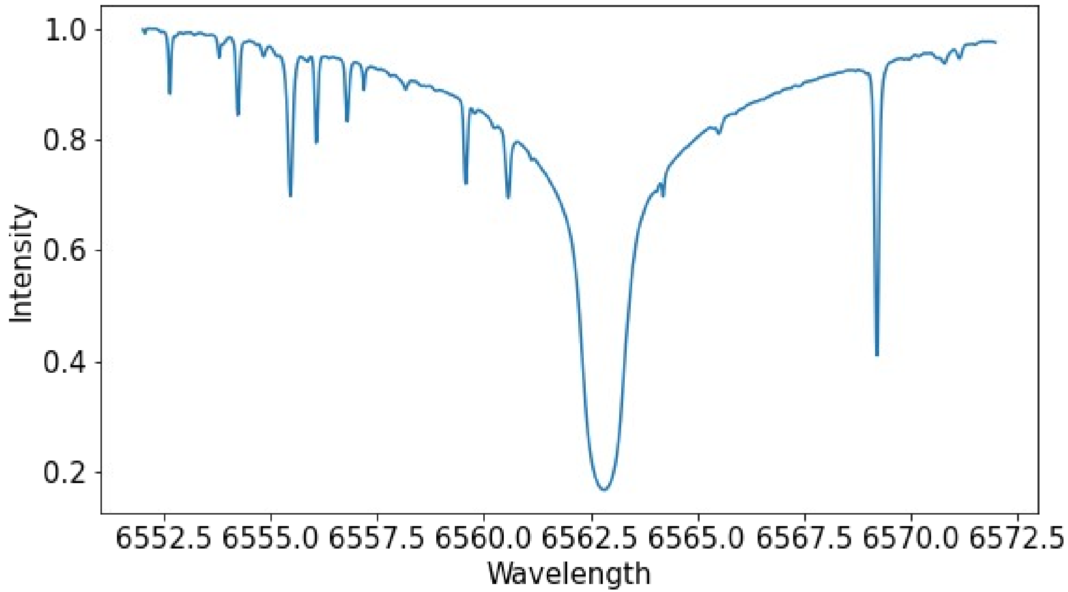
Final line profile, Voigt

Velocity distribution - Gaussian

So finally

- Voigt profile enhances the line “wings”, and it is very visible in some observed spectral lines:





Where do physical quantities come in:

- **Temperature:** ionization, excitation, thermal broadening, amount of collisions
- **Pressure:** constrains total particle density, ionization, damping
- **Large Scale velocity:** shifts the profile as a whole
- **Turbulent velocity (TBC) :** additionally broadens the line in a quasi-thermal fashion
- **Magnetic field:** a lot of things, wait for it!

The end (for now)

- We know now (in principle), how to solve for the ionization and excitation state of the gas
- We know how to find the wavelength dependence of the line absorption (and emission)
- We know how to put them together to solve the radiative transfer equation.
- We will do these things next week!