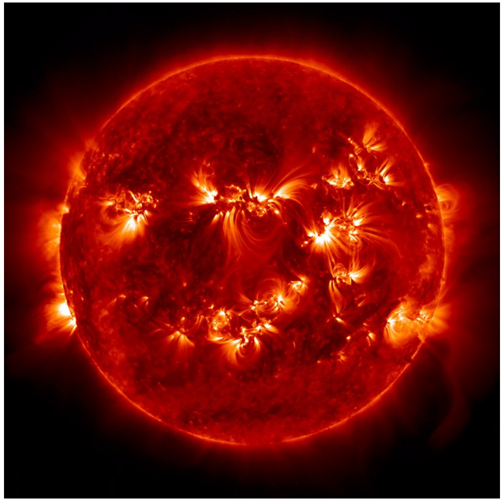
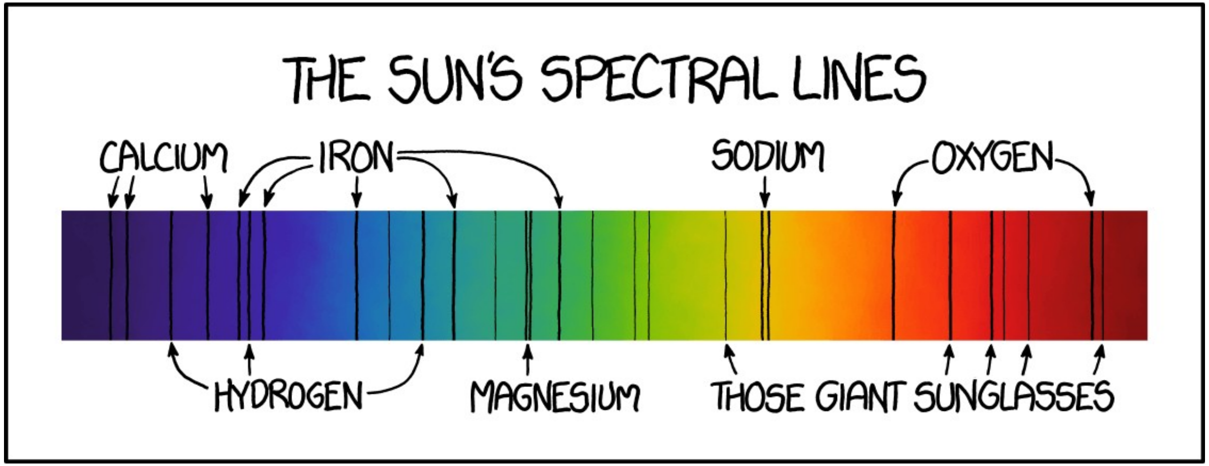
PHYS 7810 / COLLAGE2021:
Solar Spectral Line Diagnostics





Lecture 04: Opacity and Emissivity in Spectral Lines

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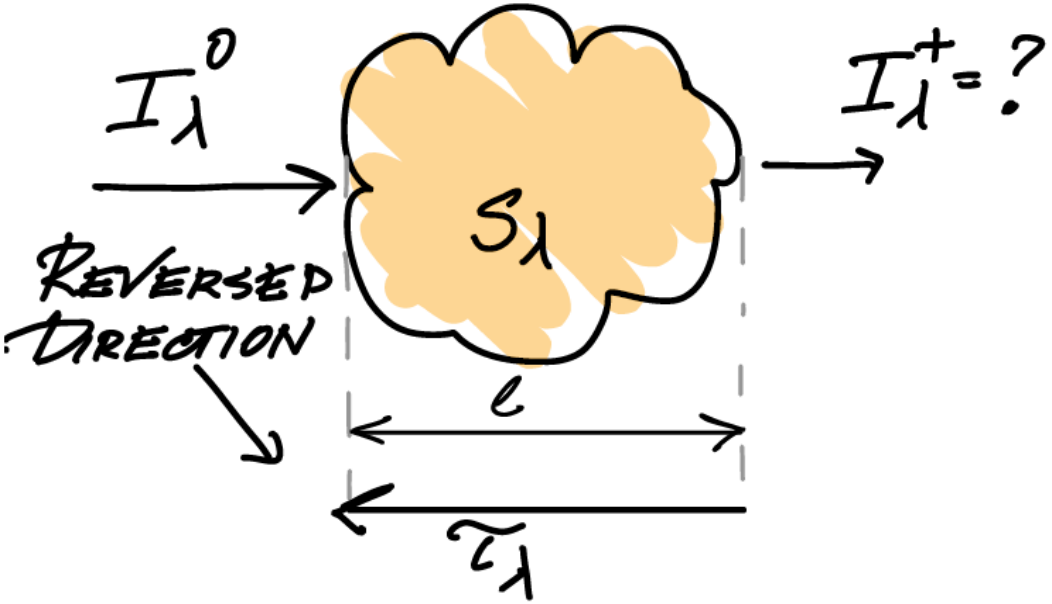
Plan for today:

- Last week we tackled things from mathematical / geometrical point of view (refresher incoming)
- The question remains: **Where do the opacity and emissivity (coefficients of absorption and emission) come from?**
- We will focus on spectral line processes, so the title could have been **Opacity and Emissivity due to spectral line processes**

Formal solution - reminder

- We will often want to reverse the direction of optical depth so that 0 is at the top. This leads to the form you are maybe most familiar with:



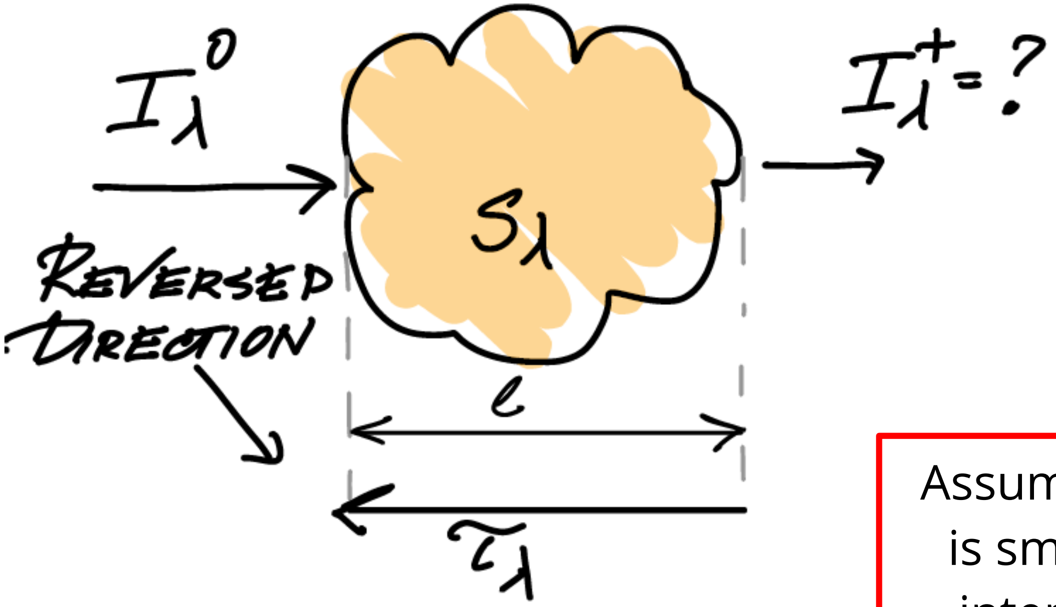
$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

With a solution:

$$I_\lambda^+ = I_\lambda^0 e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t) e^{-t} dt$$

Let's first solve a problem related to this

- We had a look at the solution of this when optical depth depends on wavelenght as: $\tau_\lambda = \tau\,\phi_\lambda$ where the line profile has some shape (say, a Gaussian)



$$I_\lambda^+ = I_\lambda^0 e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t) e^{-t} dt$$

For zero incident intensity and constant source function results in:

$$I_\lambda^+ = S(1 - e^{-\tau_\lambda})$$

Assuming that optical depth at each wavelength is small, how does the shape of the emergent intensity depend on the line profile (5 mins)?

That's right, it would look like this: $I_\lambda^+ = S\,\tau\,\phi_\lambda$

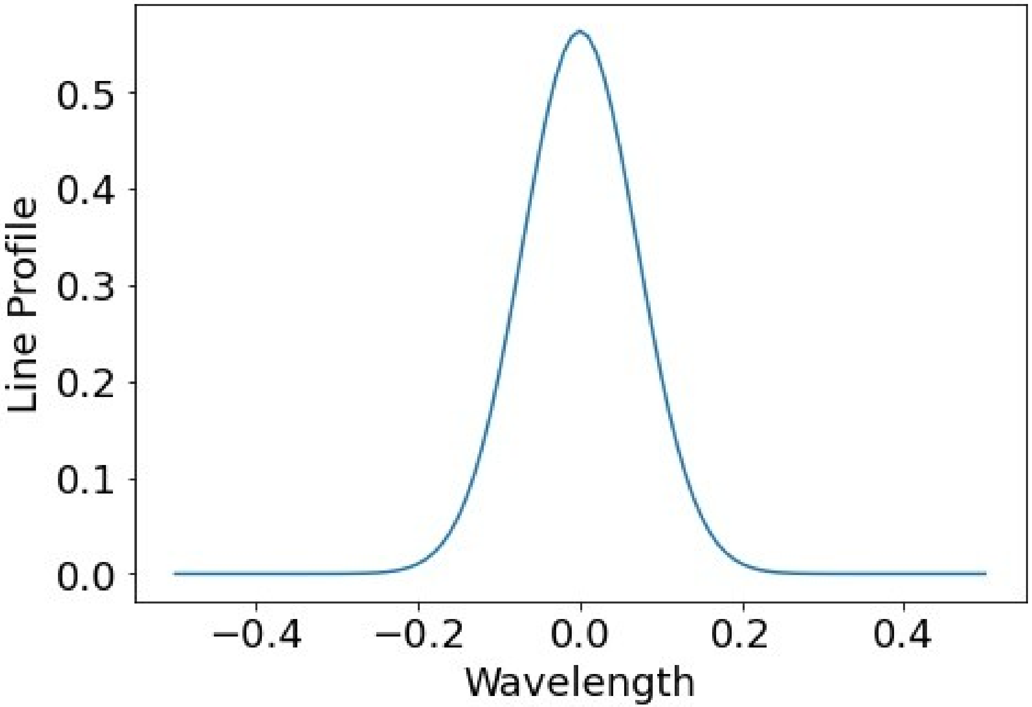
- The shape of the emergent intensity is the same as the shape of the line (absorption) profile
- This is not trivial! Line profile is a **local quantity,** intensity is a **non-local quantity**
- So if line profile is Gaussian, so is the emergent intensity
- Would this line be emission or absorption line?
- What does local/non-local mean to you? (2 mins)

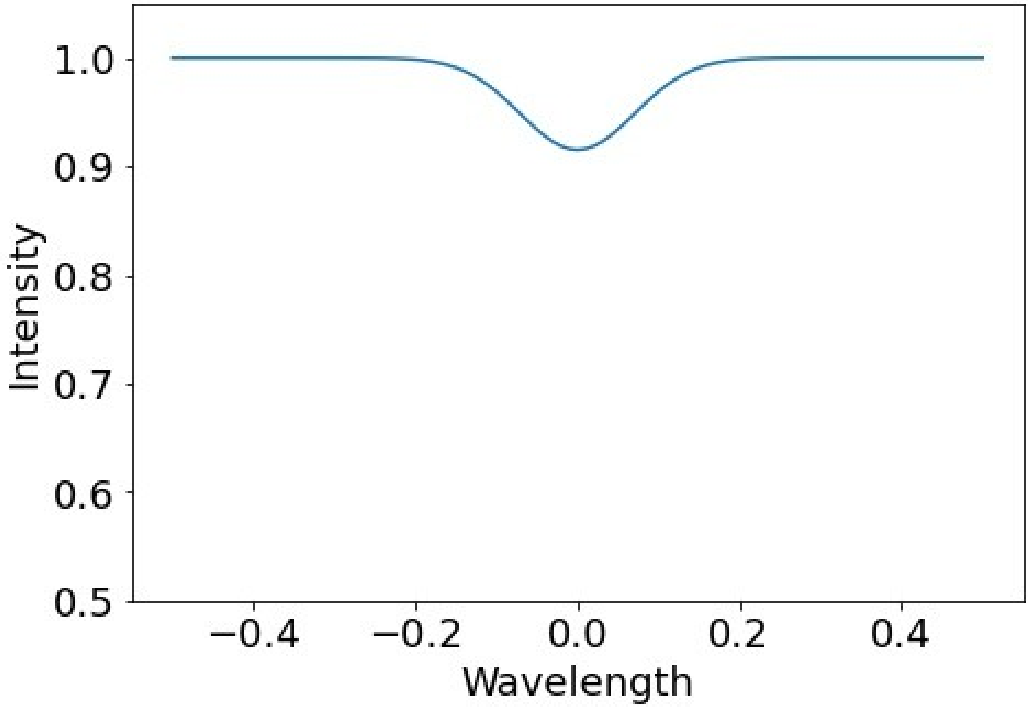
Now, purely by intuition, answer the follow up question:

How would the result change if there was an incident, wavelength-independent radiation, greater than the source function in the slab? (30 s answers in the chat)

That's correct:

$$I_\lambda^+ = I + (S - I)\,\tau\,\phi_\lambda$$

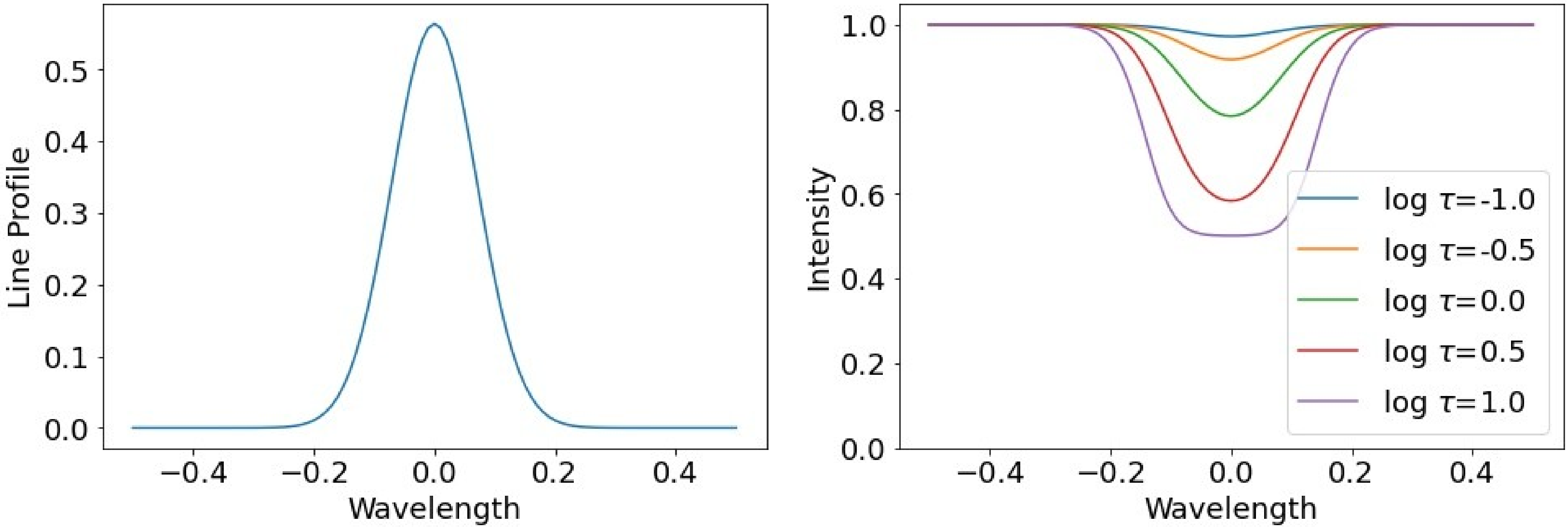




Spectral lines have Gaussian shapes only when they are formed in a constant, optically thin medium!

Otherwise….

- Well, look at this, and tell me what you conclude (2mins)



The main point of this “prelude” was to show you radiative transfer is important!

Ok back to RT, it's a combination of small and big scales

This whole equation tells us how the things behave on big scales. (ds is geometrical length, and light propagates over very large distances inside of astrophysical objects.

$$\frac{dI(\vec{r}, \mathbf{n}, \lambda)}{dl} = j(\vec{r}, \mathbf{n}, \lambda) - \chi(\vec{r}, \mathbf{n}, \lambda) I(\vec{r}, \mathbf{n}, \lambda)$$

These two coefficients, on the other side, depend on specific physics of emission and absorption. **We talk about them now**

A simple radiative transfer flowchart

- We usually have a spatial grid (x,y,z), and then values of relevant quantities (T, p, v, B…) on it
- We calculate emissivity and opacity on this grid (we will see now how), at each of the **wavelengths of interest.**
- Thus, we also know the **source function**
- We integrate opacity on the spatial grid to get **optical depth at each wavelength.**
- We integrate the source function on the optical depth to get **emergent intensity** at each wavelength (and everywhere else, btw)
- Note directional dependence, will be important

Understanding coefficient of emission

If we focus on the “emitted” part of the intensity, we would get:

$$\frac{dI_\lambda^{\text{emitted}}}{dl} = j_\lambda$$

So, the emissivity quantifies amount of intensity added per unit length.

- Now, we can unpack it further:

$$j_\lambda = \frac{d^5 E}{dS\, dl\, dt\, d\Omega\, d\lambda}$$

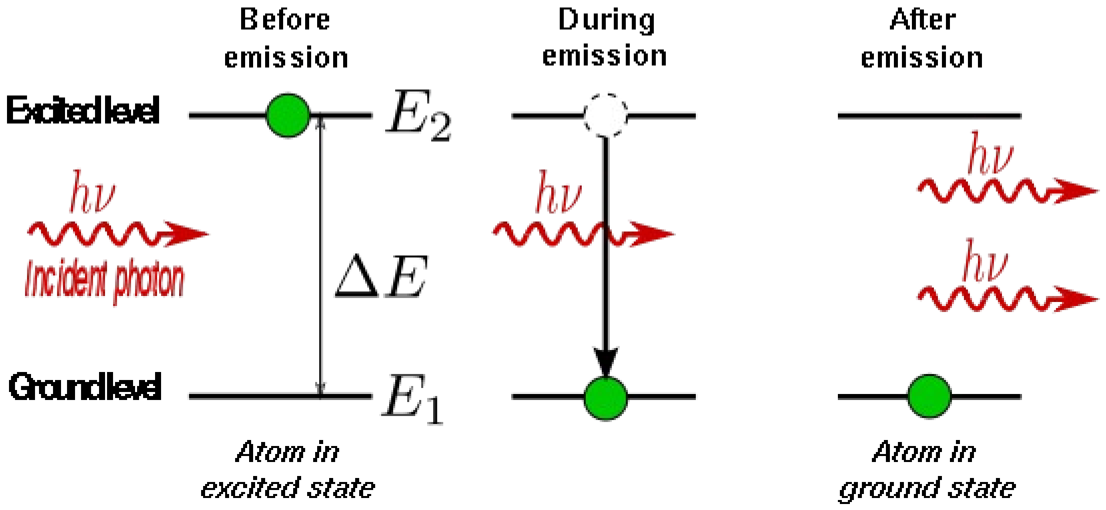
But let's also assume we are talking about spectral lines

- We are talking about spectral line processes, that are discrete and happen between two bound states. We focus only on these two levels
- To emit in this specific **transition** atom needs to be excited. We don't care how (speaking of which, how?)
- If you remember correctly, there are two emission processes:

Spontaneous emission

Stimulated emission

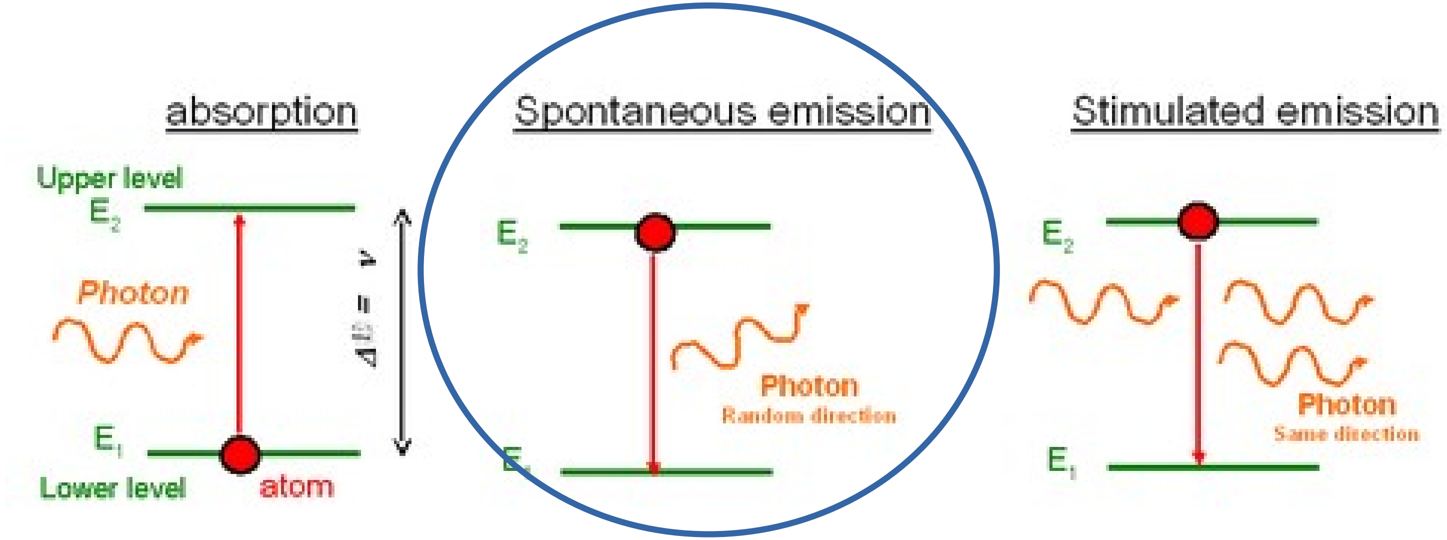
Since we are here, let's talk about these two processes



$$E_2 - E_1 = \Delta E = h\nu$$

- What are notable differences between the two?

Since we are here, let's talk about these two processes



- Spontaneous is **isotropic**, stimulated follows the **distribution of incoming radiation.**
- Spontaneous depends on the state of the gas, stimulated depends on the state of the gas **and on incoming intensity.**

Ok step by step, let's deal first with the volume

$$\frac{d^3}{dt\, d\Omega\, d\lambda} \frac{dN}{dV} \frac{hc}{\lambda} = \frac{d^3 n}{dt\, d\Omega\, d\lambda} \frac{hc}{\lambda}$$

Ok step by step, let's deal first with the volume

$$\frac{d^3}{dt\, d\Omega\, d\lambda} \frac{dN}{dV} \frac{hc}{\lambda} = \frac{d^3 n}{dt\, d\Omega\, d\lambda} \frac{hc}{\lambda}$$

- This is number density of what?
- Correct, this is number density of the atoms that are capable of emitting the atoms. For us, this means atoms that are in the upper state of the transition we are considering.

Then let's deal with the angle

- I guess it is reasonable to assume that the atoms emit isotropically.
- What would you say, when is this reasonable and when not? (2 mins)
- Anyway, with the assumption of isotropy, do we all agree that:

$$\frac{d^2}{dt\,d\lambda}\frac{dn}{d\Omega}\frac{hc}{\lambda} = \frac{d^2}{dt\,d\lambda}\frac{n}{4\pi}\frac{hc}{\lambda}$$

And then with the time…

- We can, again, assume that we somehow magically keep number of excited atoms constant (*why? How? What is the prerequisite for that? - 3 mins*)
- And that the **rate** of radiative de-excitations is constant (reasonable)
- Can we all then agree that:

$$\frac{d}{d\lambda}\frac{dn}{dt}\frac{hc}{4\pi\lambda} = \frac{d}{d\lambda} n A_{\mathrm{ul}} \frac{hc}{4\pi\lambda}$$

Emission rate. Actually what we call it is: Einstein Coefficient of Spontaneous Emission. Units are 1/s or, if you prefer, Hz. What would you say is the typical magnitude of this parameter?

And finally wavelength!

- There has to be some distribution of the emission over the velocity, we call it **line profile**

$$\frac{dn}{d\lambda} A_{\rm ul} \frac{hc}{4\pi\lambda} = n\phi_\lambda A_{\rm ul} \frac{hc}{4\pi\lambda}$$

- If the lines were the way we imagined them (remember first class), this profile would be a delta function, right?
- However we already saw during the last class that it has a thermal broadening component (+ many other broadenings, wait a bit).
- For the moment just remember: $\int_{-\infty}^{\infty} \phi_\lambda d\lambda = 1$

So finally:

$$j_\lambda = \frac{hc}{4\pi\lambda} n_u A_{ul} \phi_{ul,\lambda}$$

- Does it make sense overall?
- Can you spend a few minutes checking the units here?
- Finding the number density of the atoms that are in the upper energy state will be the most cumbersome task.
- I will often call this number density - **population**

The absorption coefficient (opacity)

- We can have a very similar story here:

$$\frac{dI_\lambda^{\mathrm{abs}}}{dl} = -\chi_\lambda I_\lambda$$

- The intensity should have units of inverse length. What if we relate it somehow to number density of absorption events (absorbers?)

$$\chi_\lambda[m^{-1}] = n^{\mathrm{absorbers}}[\mathrm{m}^{-3}] \times \sigma[\mathrm{m}^2]$$

- This is more “classical” than the previous argument, but it very intuitively tells us what opacity depends on!

Let’s write an equation for the opacity by analogy

$$\chi_\lambda = \frac{hc}{4\pi\lambda} n_l B_{lu} \phi_{ul,\lambda}$$

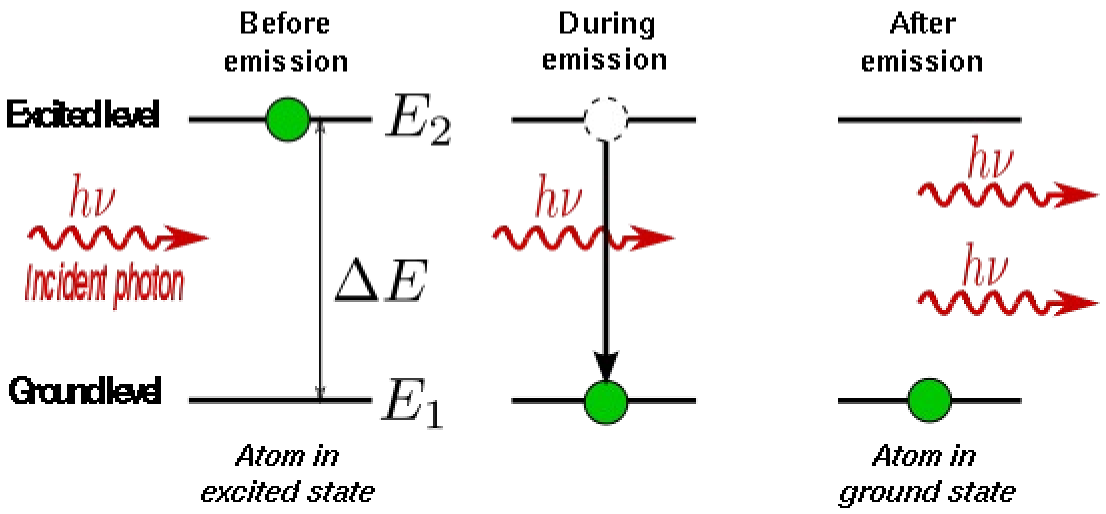
- Does this make sense?
- What changed with respect to emissivity?
- Can you figure out what units for B_lu have to be?

Let's write an equation for the opacity by analogy

$$\chi_\lambda = \frac{hc}{4\pi\lambda} n_l B_{lu} \phi_{ul,\lambda}$$

- Does this make sense?
- What changed with respect to emissivity?
- Units for B_lu have to be 1/(s x Units of Intensity) $B_{lu} \times I = R_{\rm abs}$

What about stimulated emission though?



$$E_2 - E_1 = \Delta E = h\nu$$

- Should we not include this as an emission source too?
- Still, it’s a bit awkward, it depends on the direction, it scales with incoming intensity… it’s almost as if it…

Stimulated emission is negative absorption!

$$\chi_\lambda = \frac{hc}{4\pi\lambda}(n_l B_{lu} - n_u B_{ul})\phi_\lambda$$

- Wait Ivan, does that mean that the absorption can be negative?
- Can it?
- Can it?

Yes! This is how lasers work!

- To obtain lasing we need to obtain the so called population inversion (not necessary inversion, depends on the values for B)
- We also have naturally occurring lasers, they are called MASERs (Microwave Amplification by Stimulated Emission of Radiation).
- And with this I let you go!
  (Hopefully 10 mins earlier, to make up for the last class)