Collage 2021 : Homework 1

February 15, 2021

To respect your other out-of-class commitments, a flexible deadline for this homework is 03/06/2021. However, the sooner you get to it, the easier it will be.

Problem 1: A slab of gas at temperature $T = 7000\,\mathrm{K}$ is kept at constant gas pressure equal to $70\,\mathrm{Pa}$. Let' assume that the slab is in the state of local thermodynamic equilibrium, so Saha and Boltzmann distributions are valid approximations.

- Calculate the total particle density assuming the ideal gas law.
- Assuming that the slab consists entirely of atomic hydrogen, find the number densities of electrons, protons and neutral hydrogen atoms.
- Make a plot that shows how the degree of ionization of this plasma changes with the gas pressure.
- Assume now that plasma consists both of atomic hydrogen and atomic helium. Ratio of He/H is known as abundance of Helium and is considered to be given. Write down a system of equations that would, in principle, allow you to solve for the number densities of all the involved particles (electrons, protons, neutral hydrogen, neutral helium, once ionized helium and twice ionized helium).

Problem 2: The assumption that the Source function at given wavelength λ depends linearly on the optical depth at that wavelength is known as Milne-Eddington approximation:

$$S_\lambda = a + b\tau_\lambda. \tag{1}$$

- Show that this yields emergent intensity in the direction of the atmospheric normal equal to:

  $$I_\lambda = a + b = S(\tau_\lambda = 1)$$

  . Assume that the lower boundary of the atmosphere is in infinity.

- Now, spend some time convincing yourself that this is not a good approximation for spectral lines formed in LTE. Namely, source function in LTE is almost constant with wavelength, while optical depth varies dramatically with wavelength. Also, the source function directly depends on the temperature. Obviously, source function is a physical quantity that cannot have same variation over different scales. So, for the spectral lines we will introduce something slightly different.

  We will assume that there is some referent optical depth scale, τ, and that:

  $$\tau_\lambda = \tau(1 + r\phi_\lambda)$$

  where r is a parameter describing how strong is the line absorption compared to the continuum and ϕ_λ describes variation of line opacity with wavelength. Assume that both of these are constant with τ and that $S_\lambda = S = a + b\tau$.

  Show that:

  $$I_\lambda = a + \frac{b}{1 + r\phi_\lambda}$$

  .

- Using the equation above show (both by reasoning as well as by plotting the results) that the depth of the line (ratio between continuum and the line "minimum") depends both on the line strength r as well as the source function gradient b.

Problem 3: This problem uses numerical calculations we did during the hands-on on Tuesday 02/02/2021, that is available at this link.

- Change the expression for the source function ($S \propto T$) to the one that assumes that the source function is equal to Planck Function and show that by changing the wavelength of the line center (and thus the dependence of the source function on the temperature) you can obtain different looking spectral lines even though all the other parameters, as well as the model atmosphere are the same.

- Set r to a moderately high value, say, 500. Your line should now be an absorption line with an emission core. Now, find value of the source function for $\tau_\lambda = 1$ at each wavelength. Plot the result as $S(\tau_\lambda = 1)$ vs λ. What can you conclude?

Note: if you want to re-write the hands-on to a programming language of choice, come and see me during virtual office hours (Monday afternoon or by appointment.)

Hint: The easiest way to visualize the spectral line is to normalize the intensity to the continuum value.