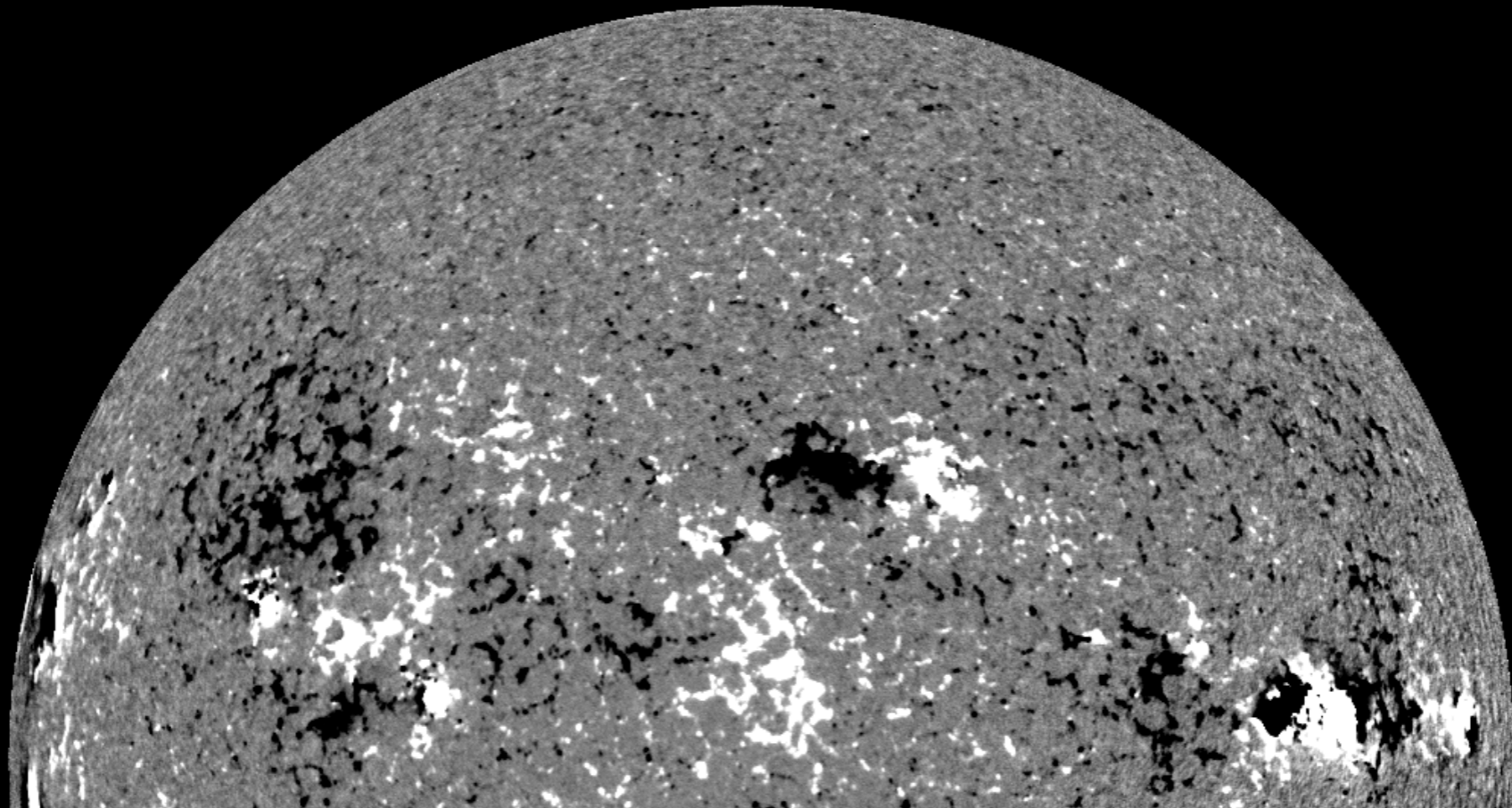


Deriving Properties of Magnetic Fields in Solar Photosphere

Maria D. Kazachenko

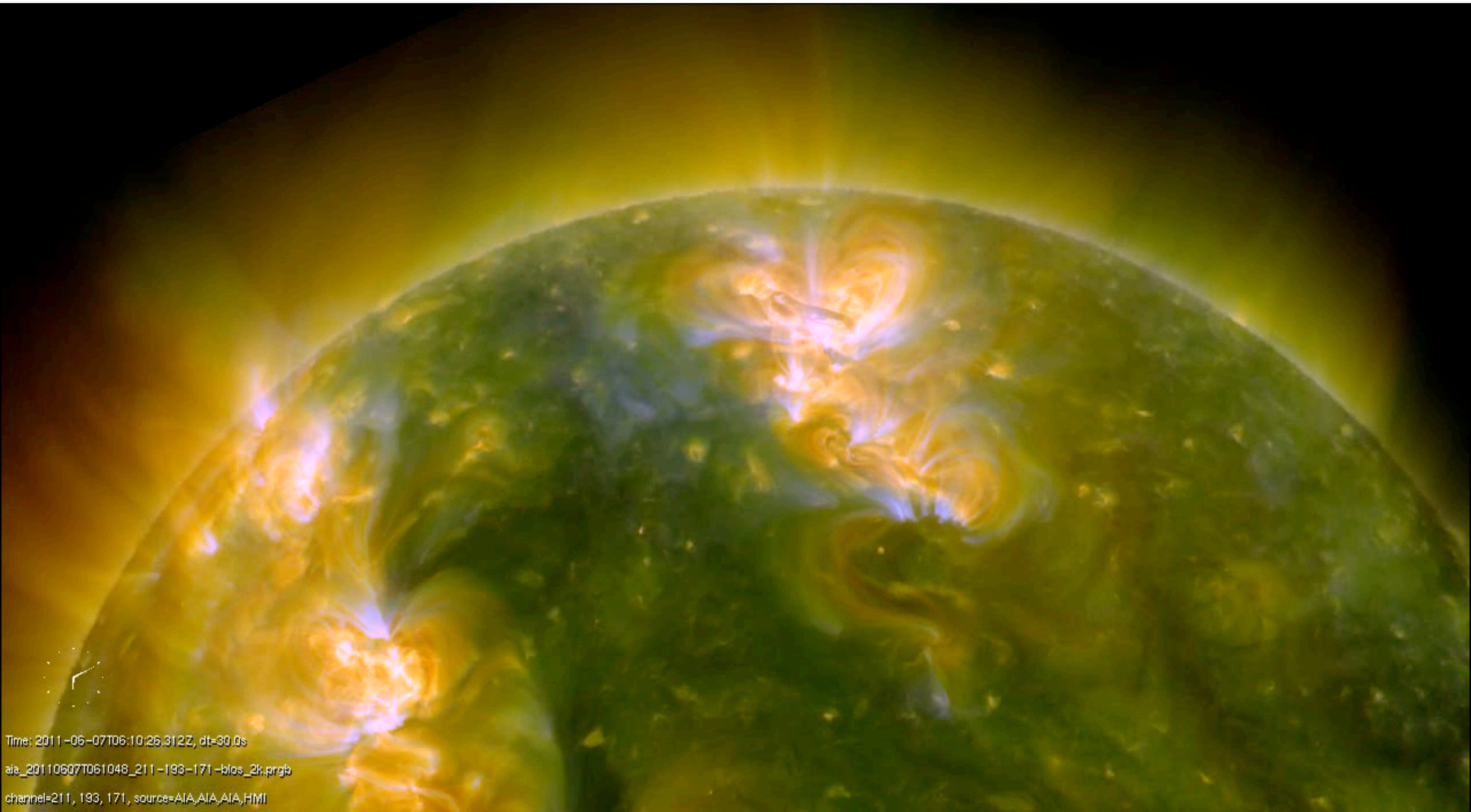
University of Colorado, Boulder — National Solar Observatory



*Photosphere
BLOS from
HMI/SDO*

**COLLAGE,
April 2 2020**

Knowledge of Coronal Magnetic Field is Vital for Understanding Long- and Short-Term Evolution of Solar Corona



Time: 2011-06-07T06:10:26.312Z, dt=30.0s

aia_20110607T061048_211-193-171-blos_2k.prgb

channel=211, 193, 171, source=AIA,AIA,AIA,HMI

**Can we use magnetic fields
measured in the photosphere to
understand magnetic properties
of solar activity?**

**During next three classes we will go over
methods used to derive some useful
properties of evolving magnetic fields:
velocity and electric fields, energy and
helicity fluxes.**

Outline for next three classes

- Today: magnetic fields in the photosphere; early methods to find magnetic field flows (velocity, electric fields) from these measurements.
- Next Tuesday: deriving velocity fields in the solar photosphere using more recent methods: DAVE4VM, PDFI, some ML methods; show examples of their application to solar data.
- Next Thursday: hands-on activity: applying FLCT to a sequence of HMI/SDO magnetograms to derive horizontal velocities, magnetic fluxes, helicity and energy fluxes.

Outline For Today

- Why Magnetic Fields (**B**) on the Sun Are Important?
- Where can we presently routinely measure magnetic fields on the Sun?
- What can we get from **B**?
- Tracking evolution of **B**: early methods for tracking **B**: tracking and inductive methods

Why Magnetic Fields Are Important?

Plasma β vs. height in solar atmosphere

Upper Corona:
Gas flows drag
magnetic fields

Solar atmosphere:
Magnetic fields
drag gas flows

Below photosphere:
Gas flows drag
magnetic fields

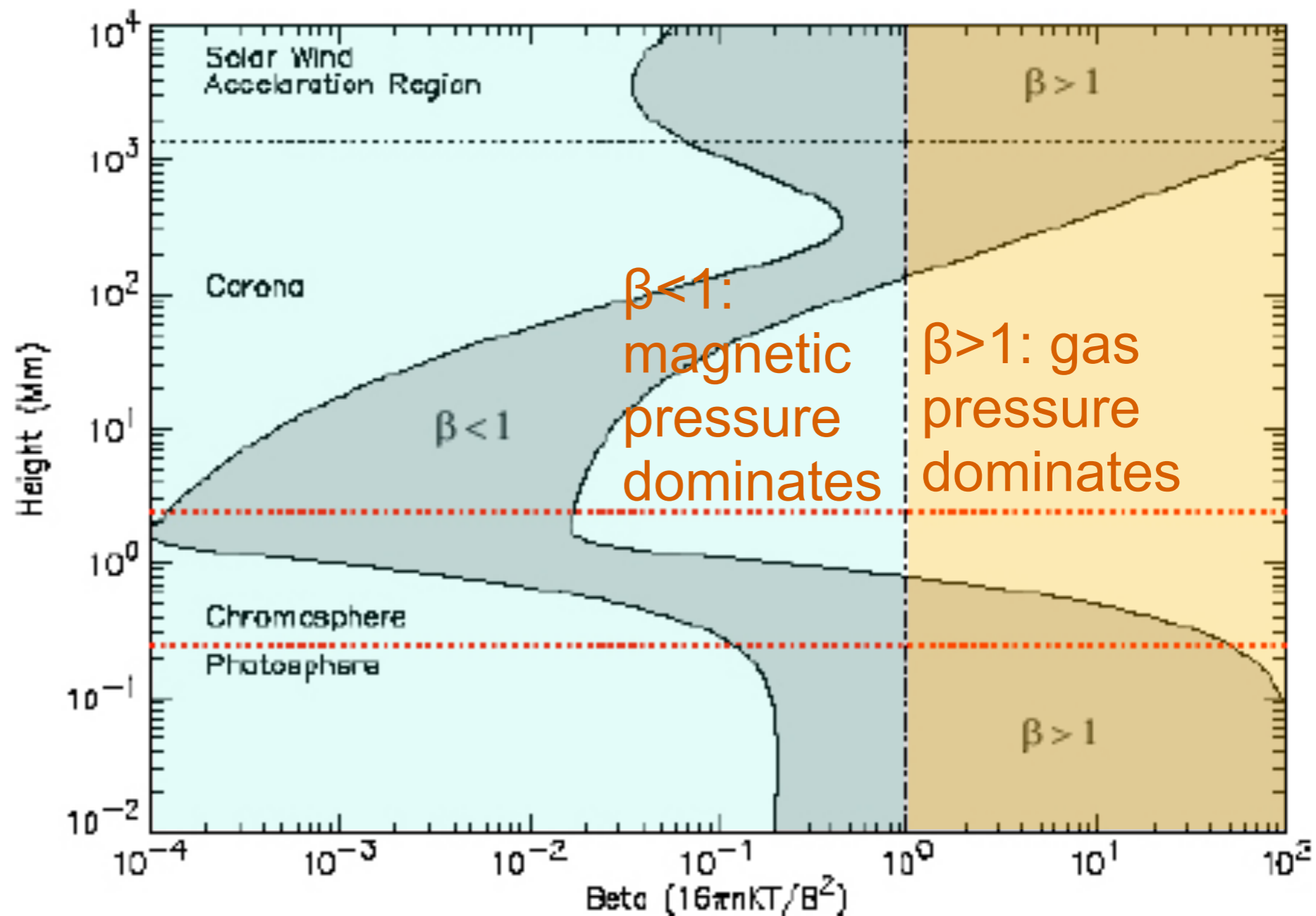


Figure 1. Ratio of gas pressure to magnetic pressure (β) as a function of height¹.

Why Magnetic Fields Are Important?

Plasma β vs. height in solar atmosphere

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Gas flows drag
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Solar atmosphere:
Magnetic fields
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Below photosphere:
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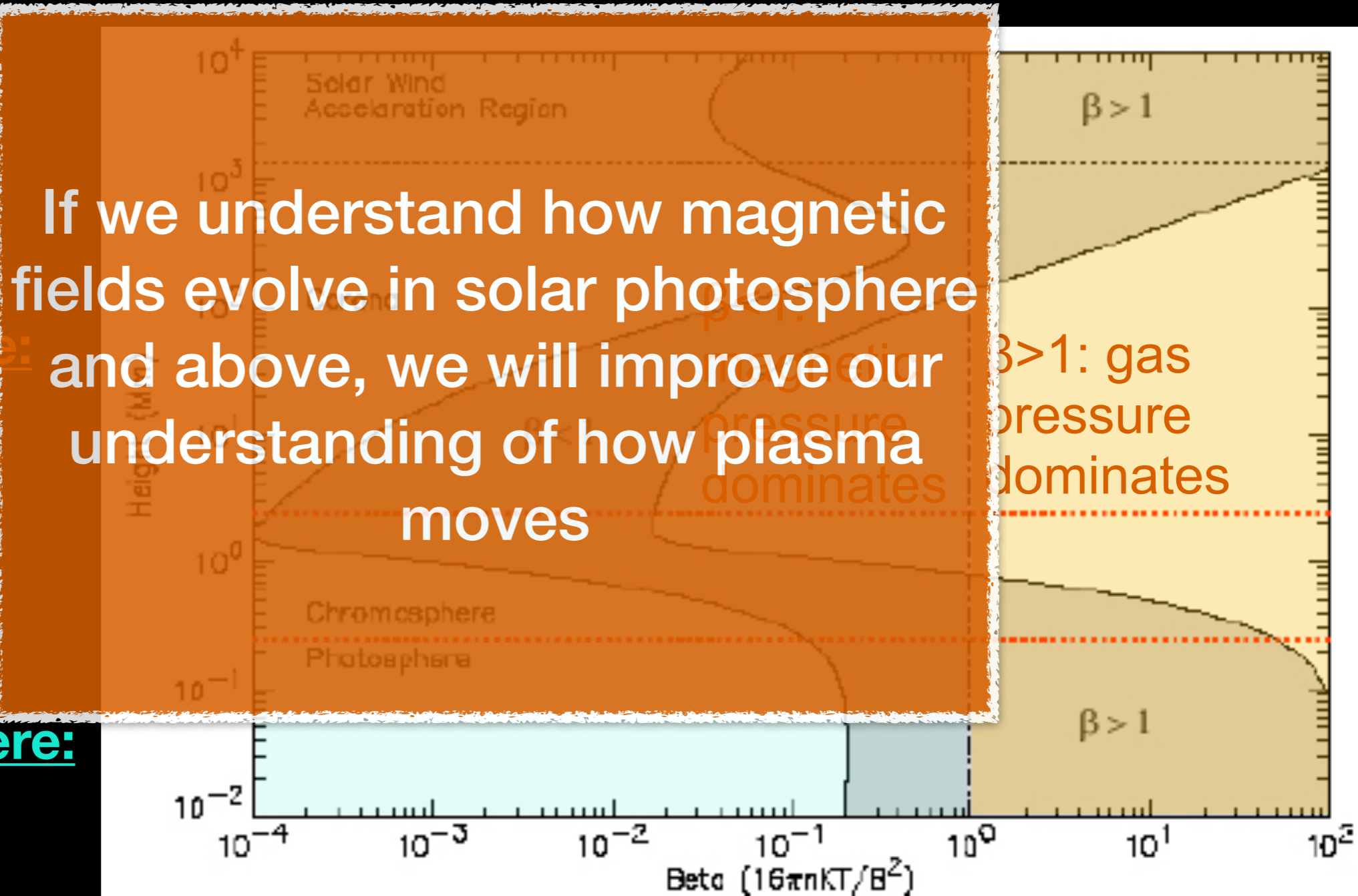


Figure 1. Ratio of gas pressure to magnetic pressure (β) as a function of height¹.

Where Does CME & Flare Energy Come From?

From the low-corona magnetic field!

Table 1. Energy Requirements for a Moderately Large CME

Parameter	Value
Kinetic energy (CME, prominence, and shock)	10^{32} ergs
Heating and radiation	10^{32} ergs
Work done against gravity	10^{31} ergs
Volume involved	10^{30} cm ³
Energy density	100 ergs cm ⁻³

Table 2. Estimates of Coronal Energy Sources

Form of Energy	Observed Average Values	Energy Density ergs cm ⁻³
Kinetic ($(m_p n V^2)/2$)	$n = 10^9$ cm ⁻³ , $V = 1$ km s ⁻¹	10^{-5}
Thermal (nkT)	$T = 10^6$ K	0.1
Gravitational ($m_p n g h$)	$h = 10^5$ km	0.5
Magnetic ($B^2/8\pi$)	$B = 100$ G	400

Where Does CME & Flare Energy Come From?

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Magnetic ($B^2 / 8\pi$)	$B = 100$ G	400

If we understand how magnetic fields evolve in solar photosphere and above, we will improve our understanding of flares and CMEs

Why Magnetic Fields Are Important?

- All solar activity, active regions and the quiet Sun, reflect evolution of magnetic fields;
- Magnetic fields observed in the photosphere are generated in the Sun's interior, extending into the photosphere and the atmosphere => Measurements in the photosphere provide insight of **B** in the interior;
- Measurements in the photosphere provide insight of **B** in the corona; how magnetic energy is stored and released in solar atmosphere.

Where Can We Measure Magnetic Fields on the Sun?

- Photosphere: both line-of-sight (LOS) and vector magnetic fields

Where Can We Measure Magnetic Fields on the Sun?

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 - SDO: Routinely since 2010, full-disk, $dt=135s$, $ds=360$ km;

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The only routine measurements

- SDO: Routinely since 2010, full-disk, $dt=135s$, $ds=360$ km;

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Where Can We Measure Magnetic Fields on the Sun?

- Photosphere: both LOS and vector magnetic fields

The only routine measurements

- SDO: Routinely since 2010, full-disk, $dt=135s$, $ds=360$ km;
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- Sunrise/IMaX: One-time measurements: $dt=(10-40)s$; $ds=40$ km;

- Chromosphere: extremely hard, very few LOS measurements

- Solar Corona: extremely hard, very few LOS measurements

Summary: We have decent measurements of **B** in the photosphere; but not in the chromosphere/corona;

DKIST instruments

HMI: 0.5 arcsec = 360 Km; DKIST: 0.025" = 20 km, 20 times better resolution

	Visible Broadband Imager, VBI	Visible Tunable Filter, VTF	Visible SP, ViSP	Diffraction-Lim. Near IR SP DL-NIRSP	Cryogenic Near IR SP, Cryo-NIRSP
ds, "	0.011	0.028	-	0.03" - 0.464	0.5 (cor) 0.15 (disk)
dt, s	3.2	0.8-13	10	0.001 (disk) 1 (cor)	>0.1
Band, nm	380-850	520-870	380-900, 3 bands	500-2500 1 vis, 2 NIR	1000-5000
FOV, '	2 x 2 square	1' - circle	2 x 2 square	2 x 2 & 2.4 - 27,	4 x 3 (cor) 1.5 x 1.5 (disk)
Location	l, phot-chrom	IQUV phot-chrom	IQUV B, phot-chrom	B, phot.chrom-cor.	B, corona, hel/sph <1.5 R _{sun}
PI	F. Vogler	O. von der Luhe	R. Cassini	H. Lin	J. Kuhn
Science	Hires imaging in lines, 20 km	High ds/dt narrow band imaging, doppler, vh, B	Simult. B at different heights, no spatial info	B in phot/cor. imaging	B in high corona

Could be used simultaneously, mosaicing

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ds, "	0.011	0.008	0.008	0.001 (disk) 0.15 (cor)	0.5 (cor) 0.15 (disk)
dt, s	3.2	0.8-13	10	0.001 (disk) 1 (cor)	>0.1
Band, nm	580-850	520-870	380-900, 3 bands	500-2500 1 vis, 2 NIR	1000-5000
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B from DKIST:

- B on disk: phot. & chromosphere; ds ~ 20km;
- B on limb: corona; ds ~ 300km;

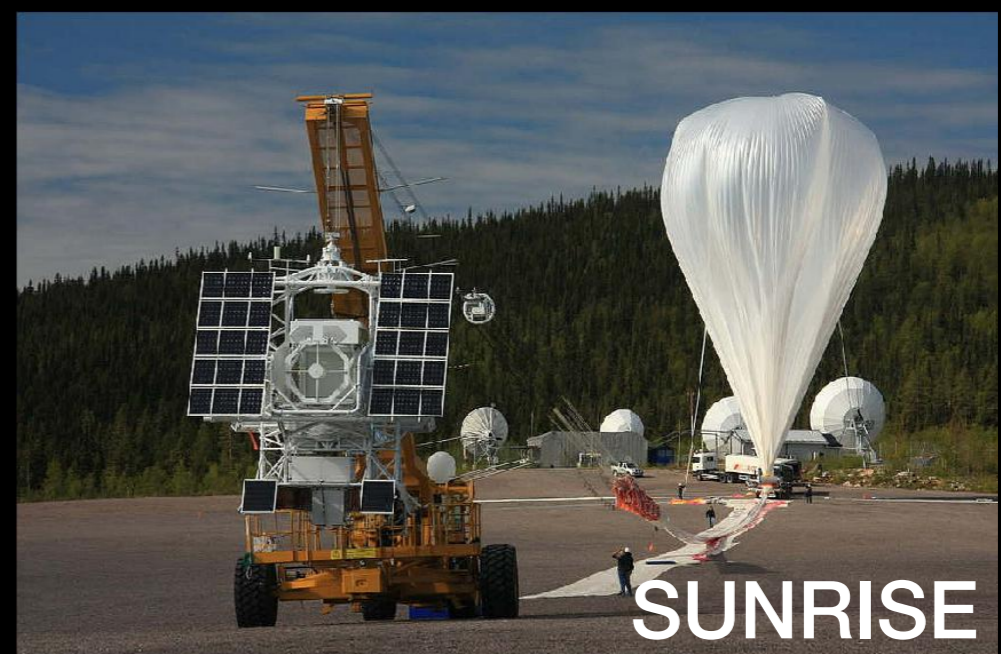
- + super high dt and ds
- + B in phot., chrom., corona;
- + revolutionary new science!
- limited FOV
- limited duration
- complex inversion;

Could be used simultaneously, mosaicing

Let's look at some examples of present day magnetic field measurements:

1) Most routine: HMI/SDO

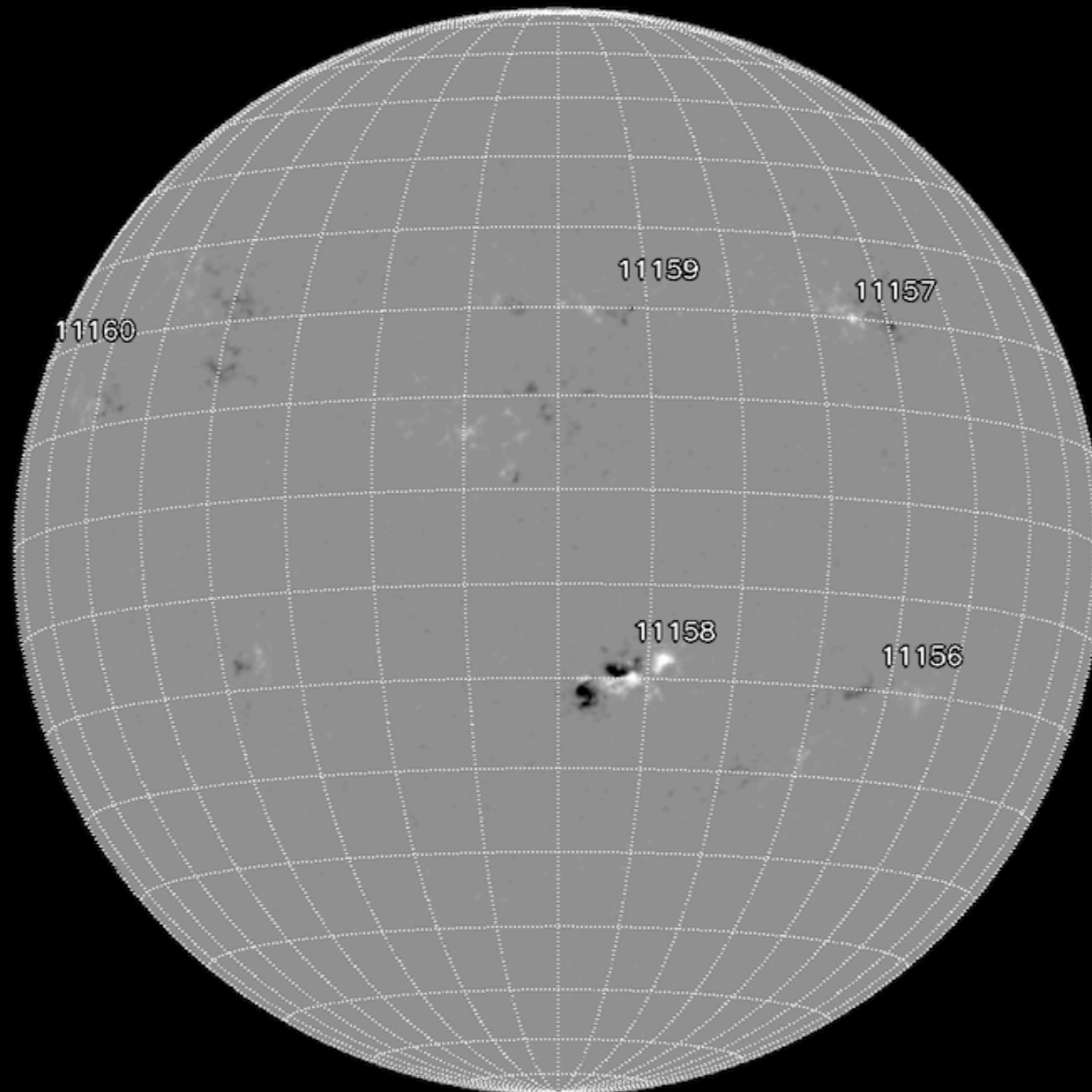
2) Highest resolution: IMaX/SUNRISE



Photosphere: Magnetic Fields in an Active region: Ground-based LOS magnetogram from GONG+ on Feb. 14 2011

Magnetic field, B_{Los}

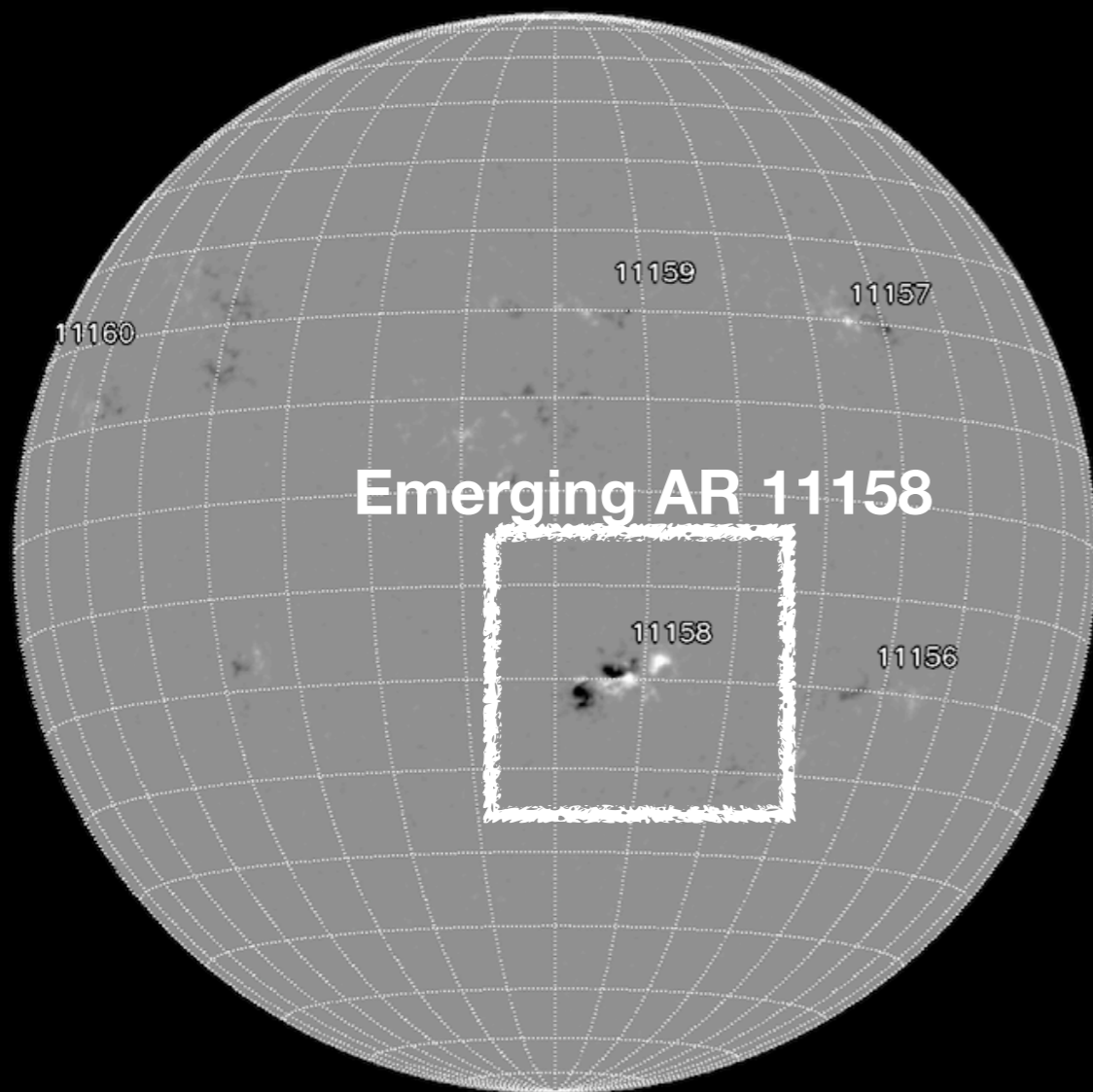
GONG+ (Teide Canary Is Sp) Magnetogram 14-Feb-2011 15:23:51.000



Photosphere: Magnetic Fields in an Active region: Ground-based LOS magnetogram from GONG+ (Canary Islands) on Feb. 11 and Feb 14 2011

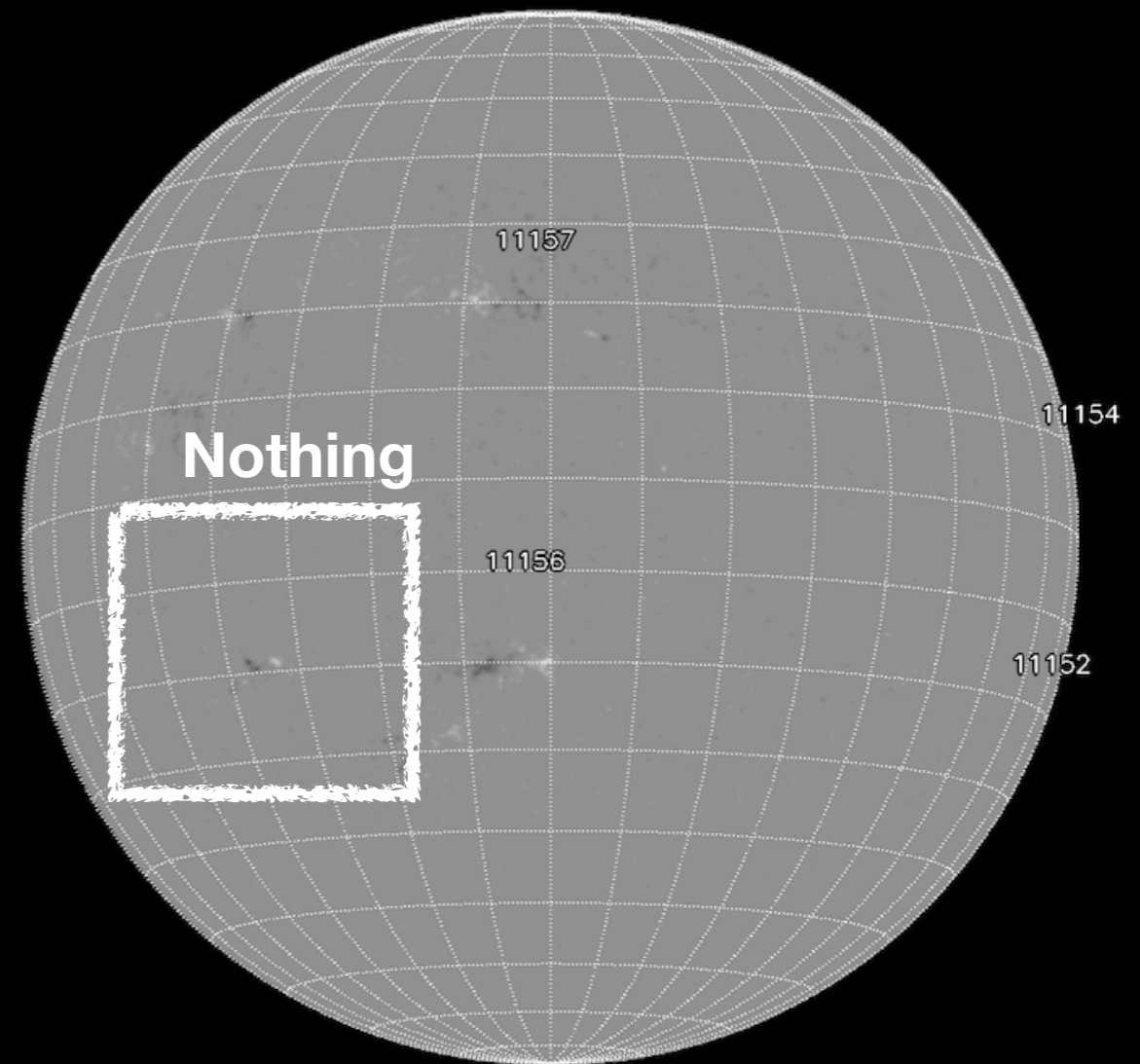
Same AR1158 we have seen

GONG+ (Teide Canary Is Sp) Magnetogram 14-Feb-2011 15:23:51.000



Three days earlier....

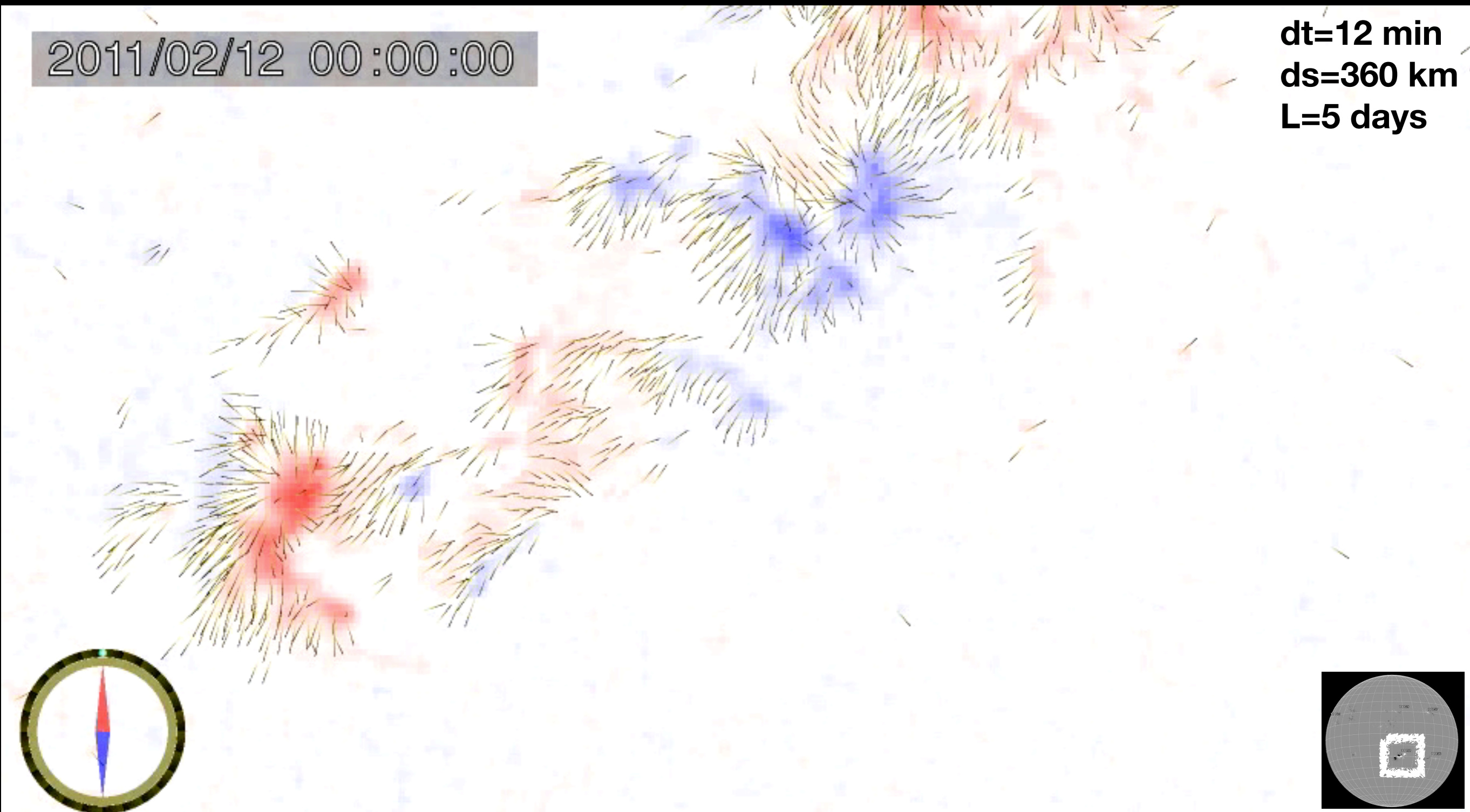
GONG+ (Learmonth WA Aus) Magnetogram 11-Feb-2011 08:09:26.000



Vector B_{phot} in this Active region: from HMI/SDO (Helioseismic and Magnetic Imager / Solar Dynamics Observatory, 2010)

2011/02/12 00:00:00

dt=12 min
ds=360 km
L=5 days

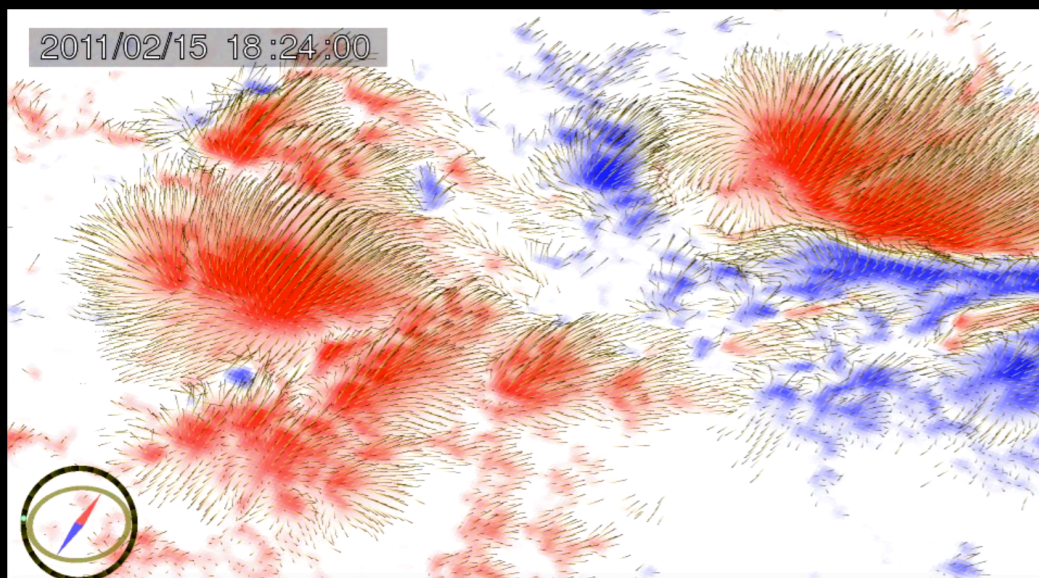
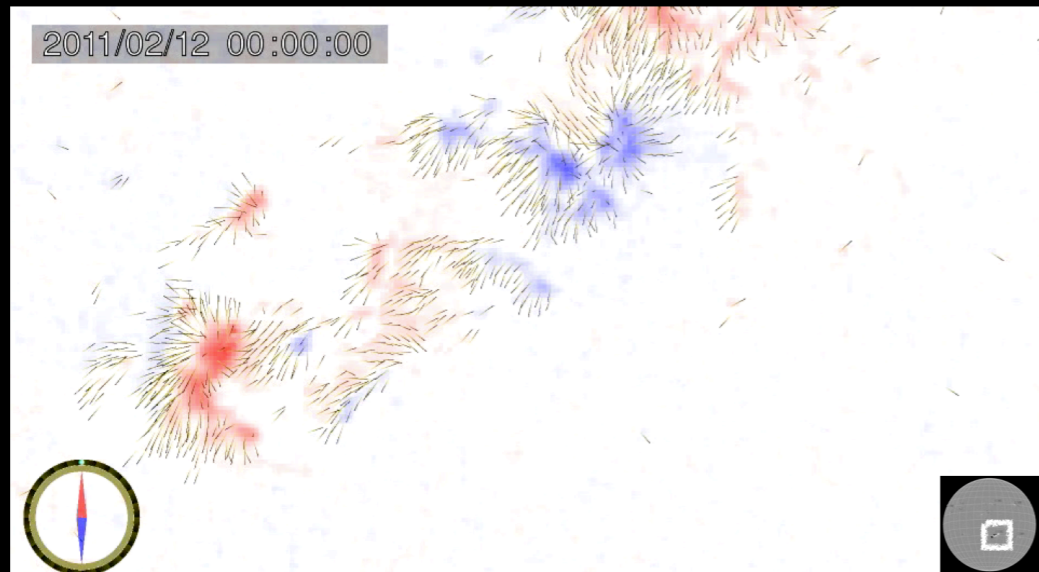


NOAA AR 11158 evolution

Red - $B_z > 0$; blue - $B_z < 0$, arrows - B_h

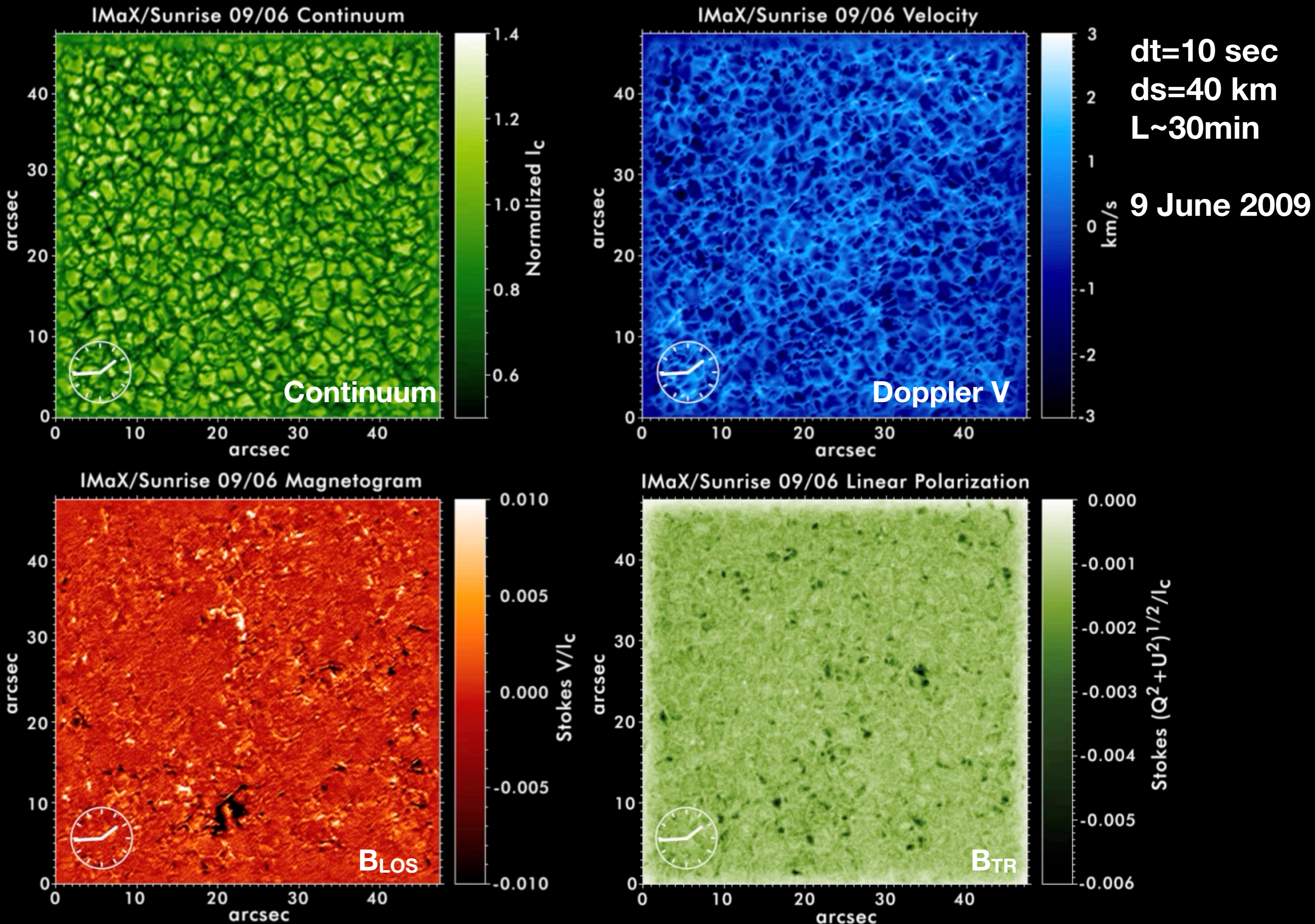
Magnetic fields at AR scale

- Highly dynamical
- Emerge, cancel
- Rotate, shear
- All this dynamics contributes into storage of magnetic energy in the coronal magnetic field; solar eruptions
- We will learn more about AR magnetic fields during the hands-on exercise



Red - $B_z > 0$; **blue** - $B_z < 0$, arrows - B_h

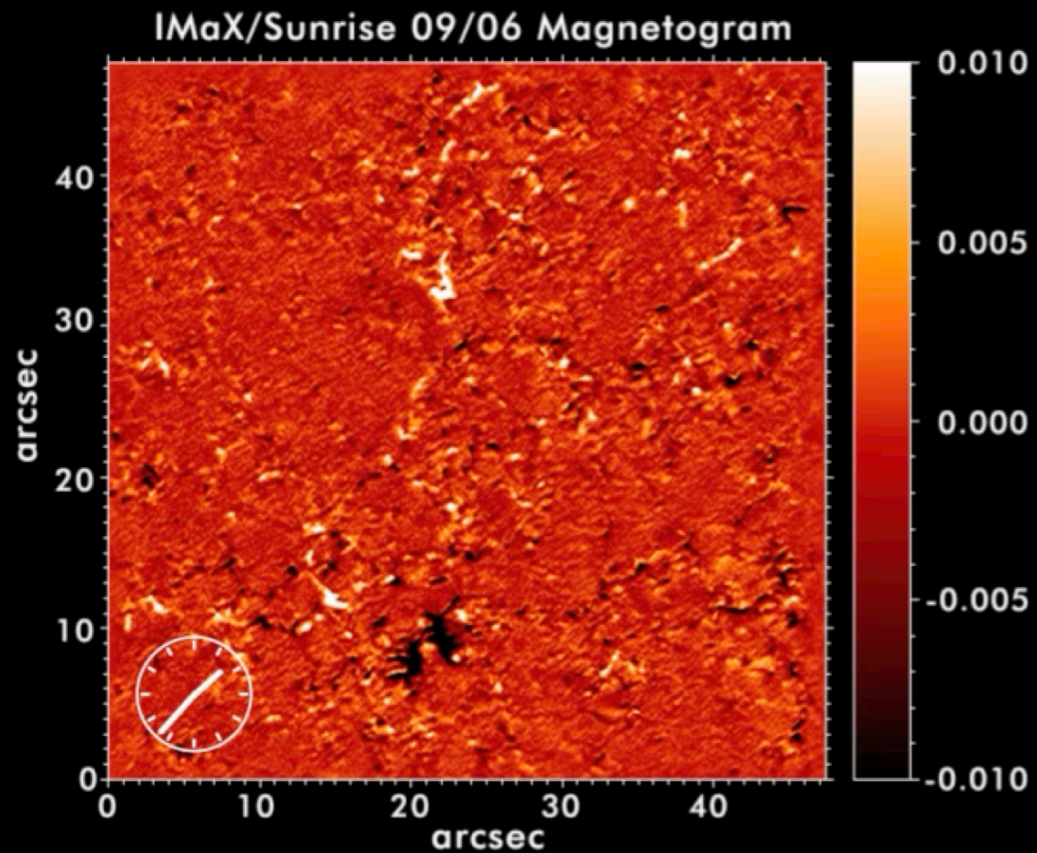
Photosphere: Magnetic Fields in the Quiet Sun (Sunrise/IMaX)



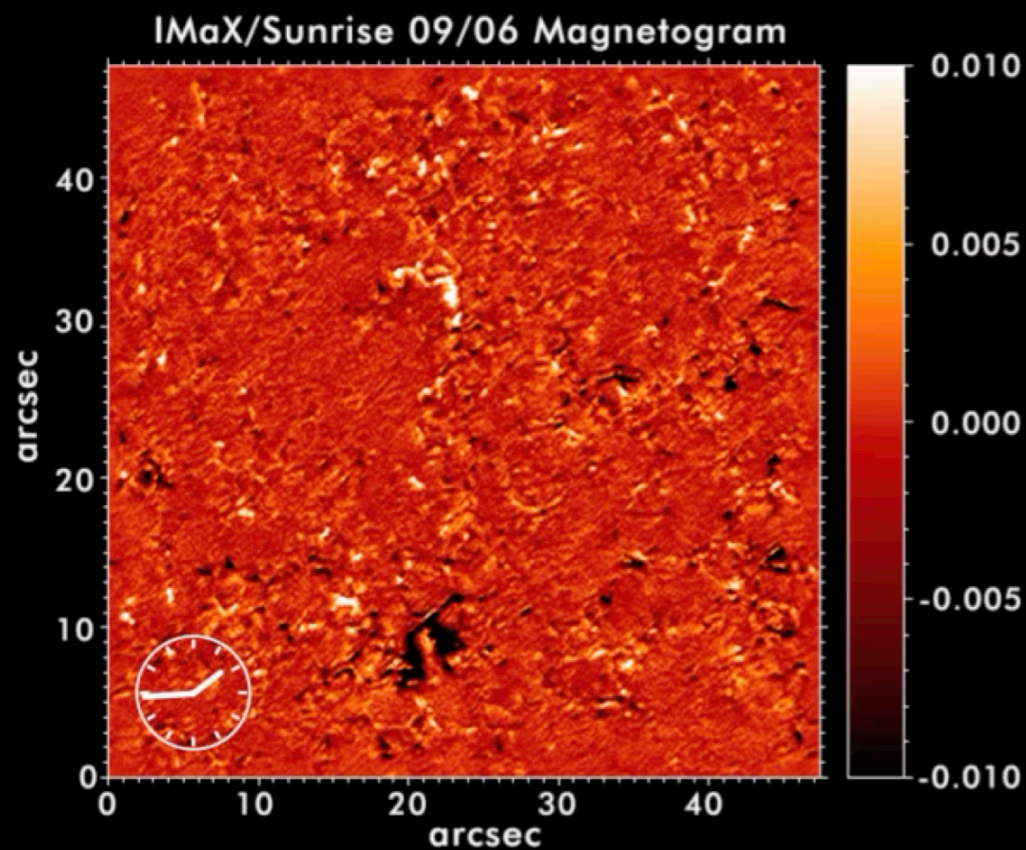
White - $B_z > 0$; Black - $B_z < 0$,

Imaging Magnetograph eXperiment (IMaX)

Magnetic fields at quiet sun scale



- Highly dynamical
- Emerge, cancel
- Rotate, shear



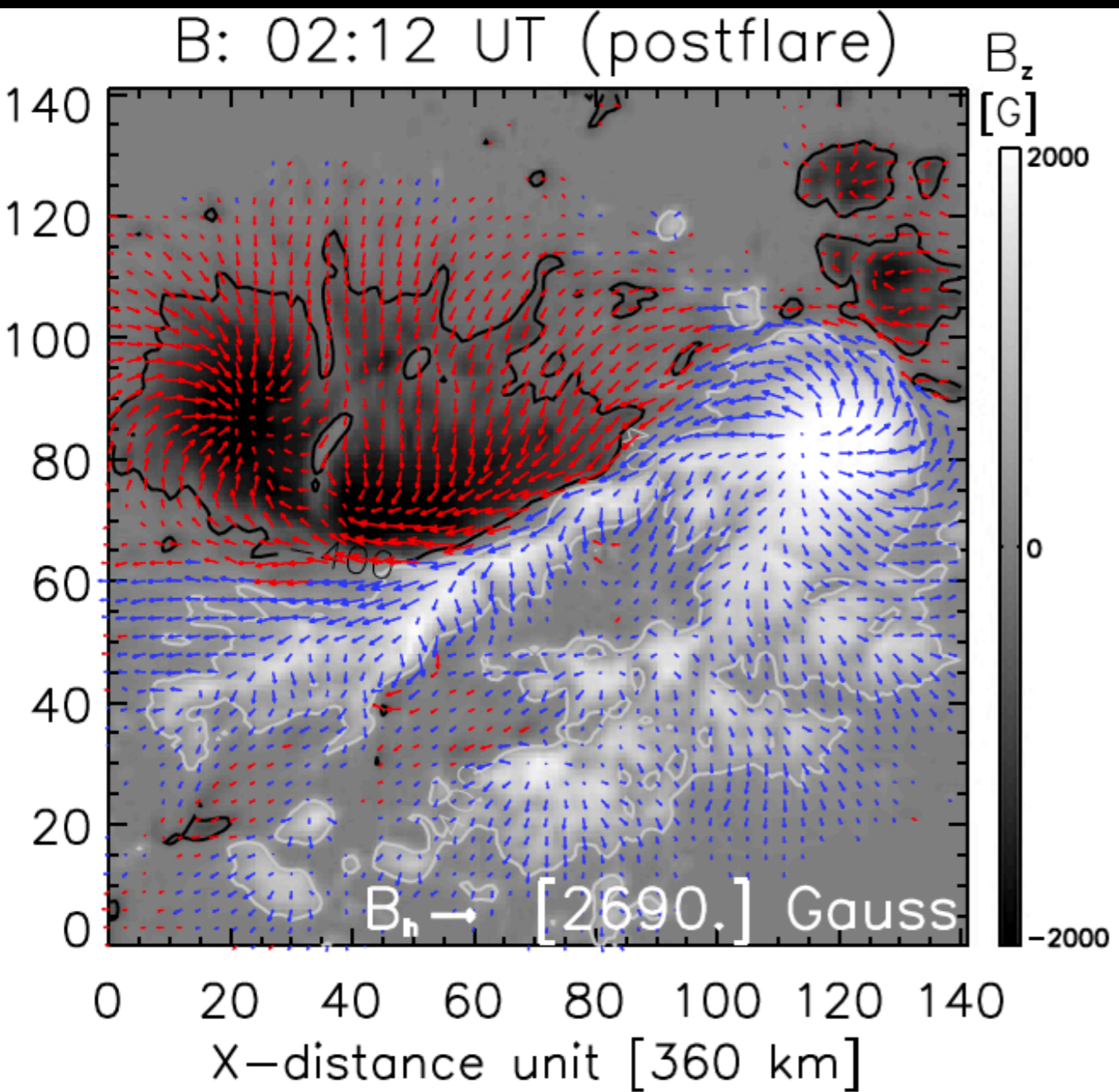
- All this dynamics contributes into storage of magnetic energy above on the smallest scales.
- Related to coronal heating?

Ok, imagine we measured magnetic field $B(x,y)$ at the photosphere.

What physical properties (variables) can we derive from $B=(B_x, B_y, B_z)$?

I will give you a minute to let leave your answers in the zoom chat.

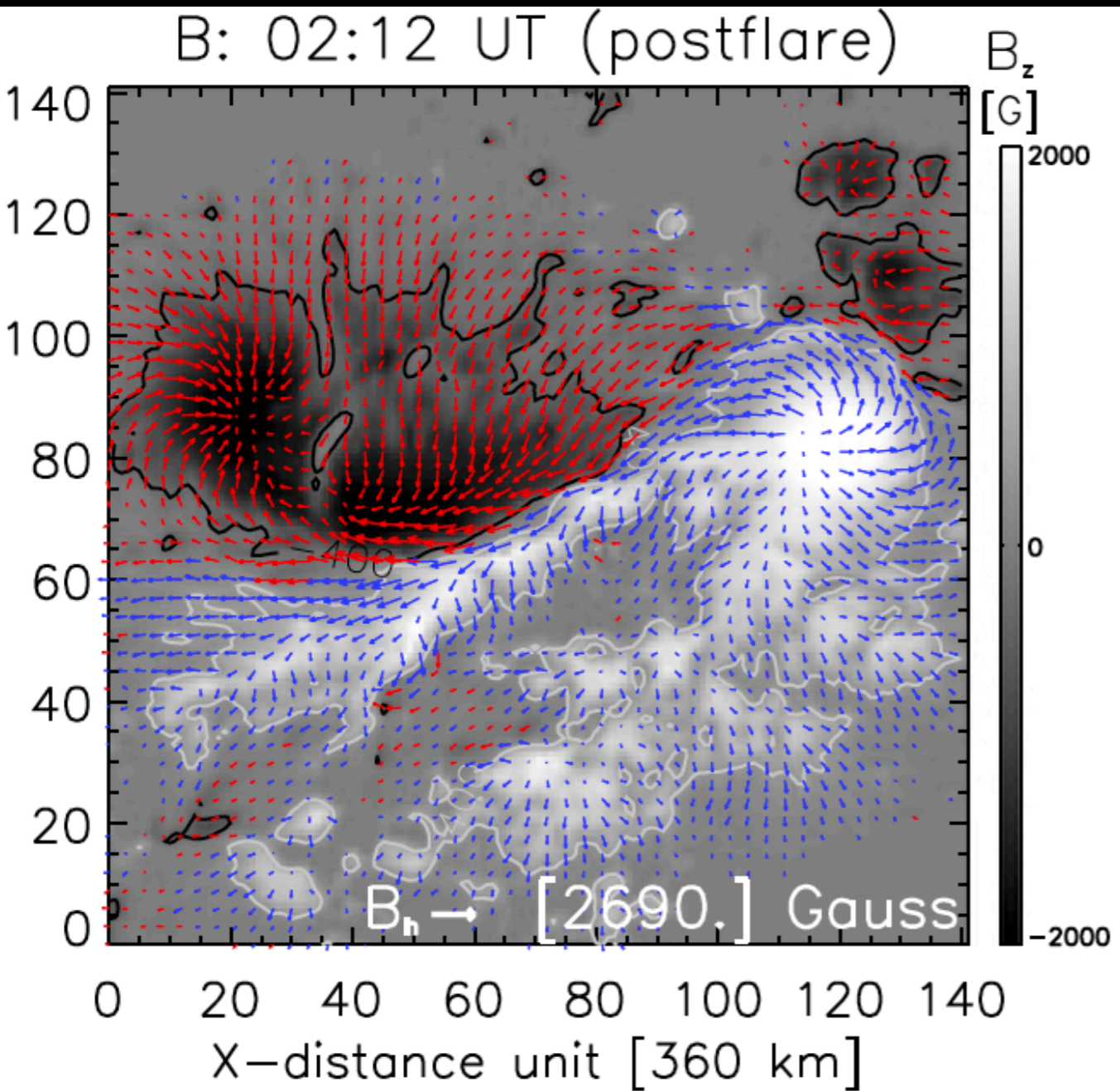
What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



**Magnetic
flux**

$$\Phi = \int |B_n| dS,$$

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



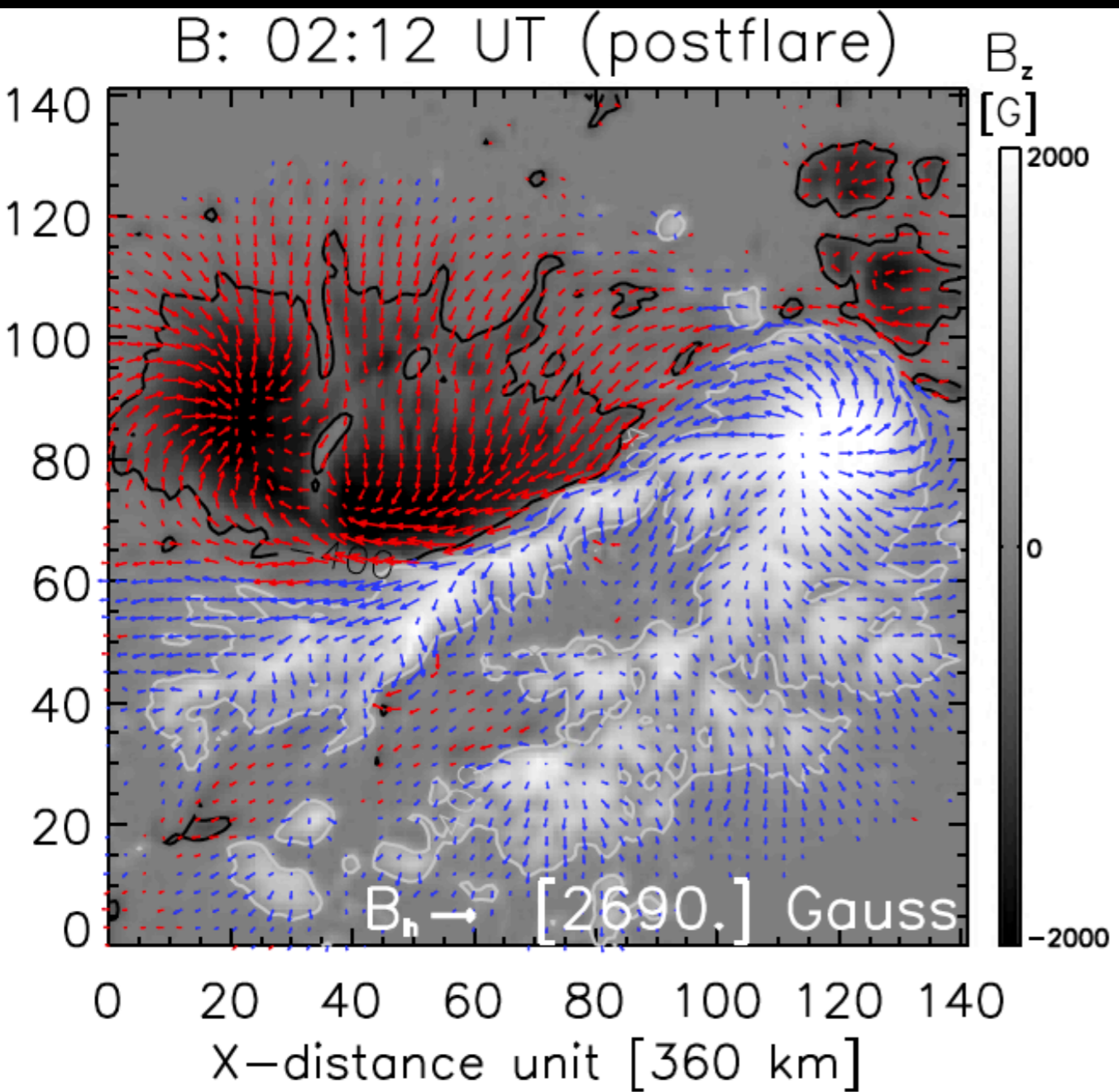
Magnetic flux

$$\Phi = \int |B_n| dS,$$

Current density

$$J_z(x, y, t) = \frac{1}{\mu_0} \nabla \times B_h$$

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



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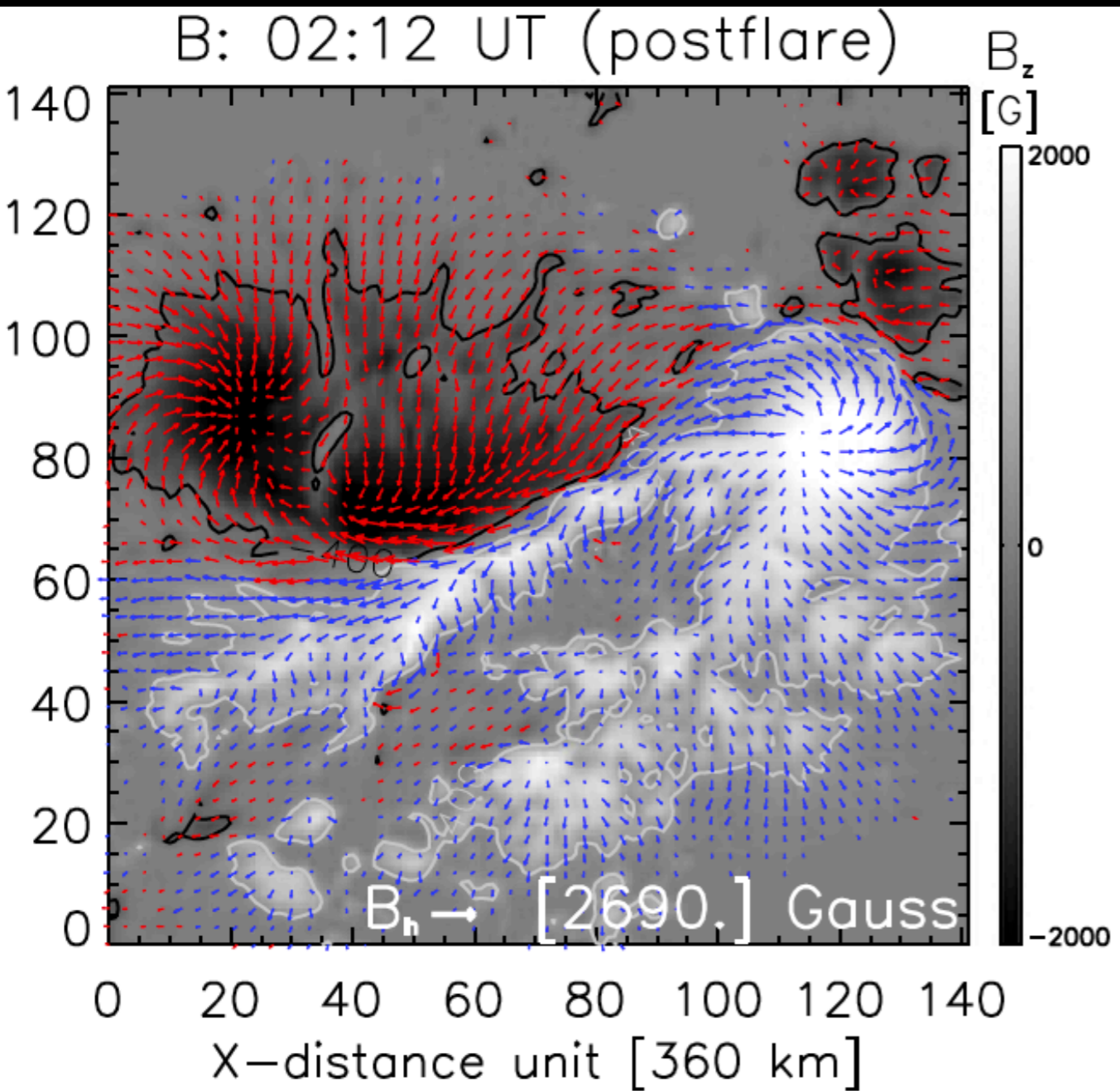
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Vector potential

$$\mathbf{A}^P = \nabla \times \mathbf{B}^P \hat{z}$$

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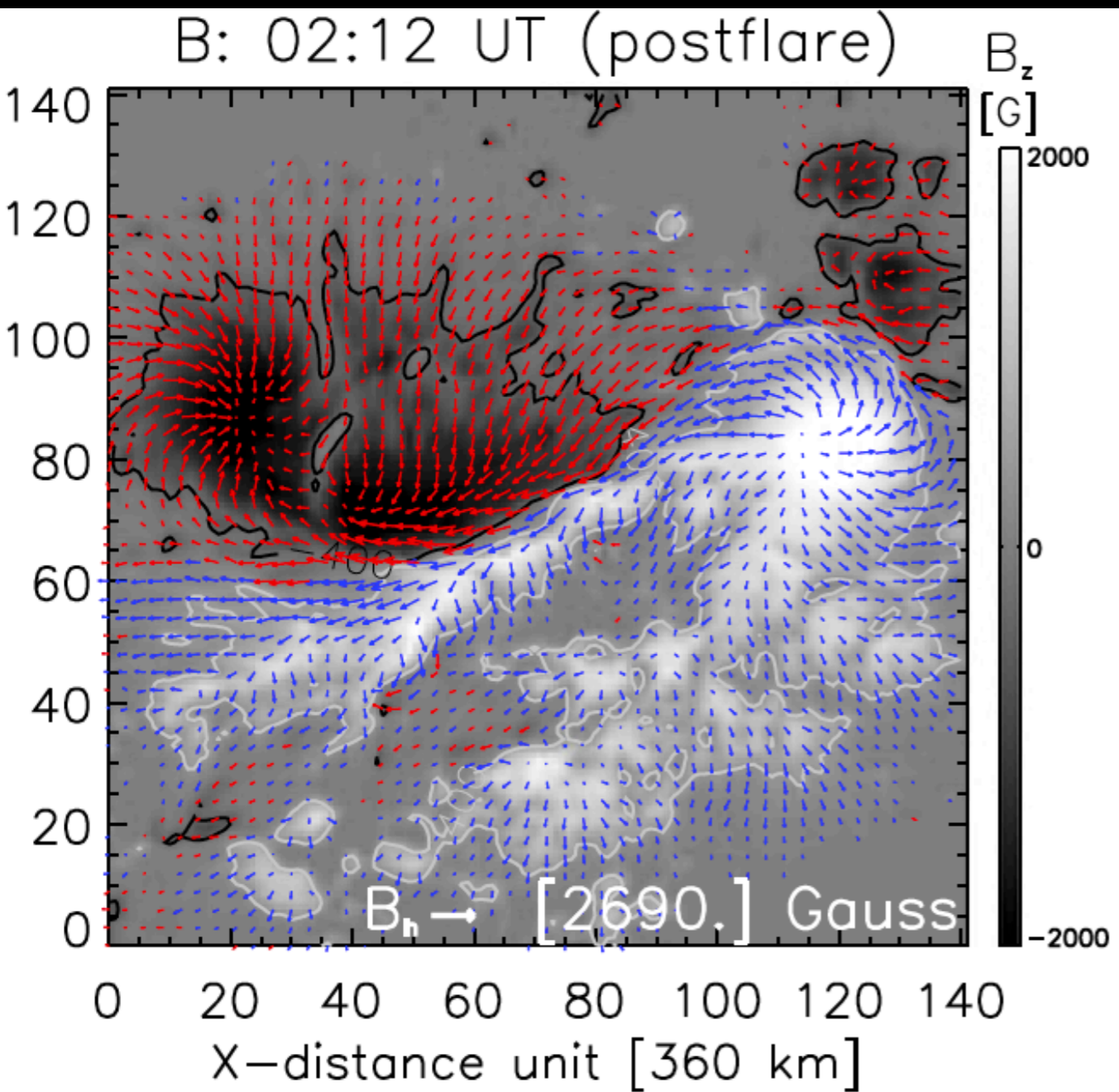
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Electric field

$$\frac{\partial \mathbf{B}}{\partial t} = -(\nabla \times c\mathbf{E}),$$

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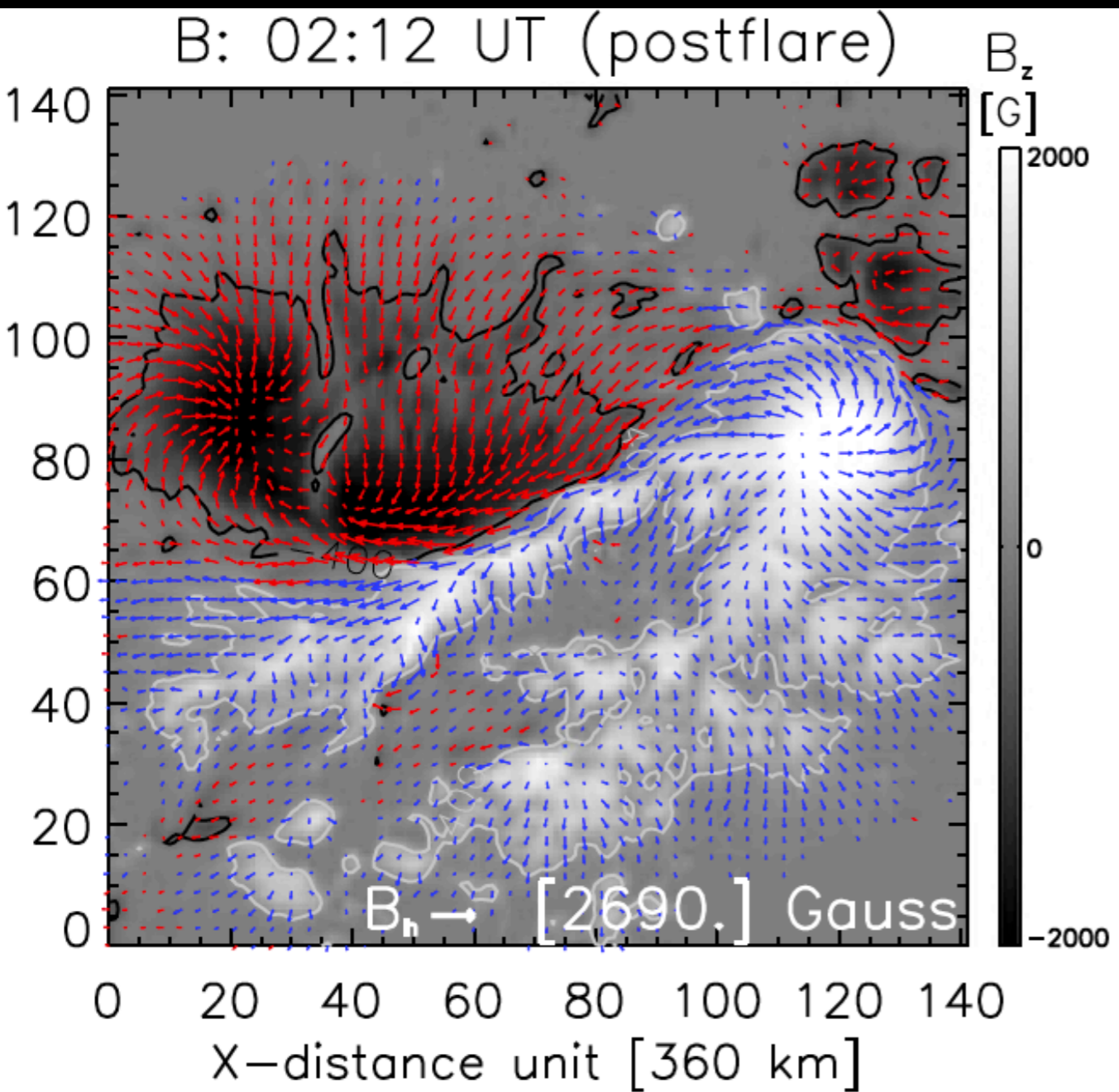
Electric field

$$\frac{\partial \mathbf{B}}{\partial t} = -(\nabla \times c\mathbf{E}),$$

Mag. Energy flux

$$\mathbf{S} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B}),$$

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



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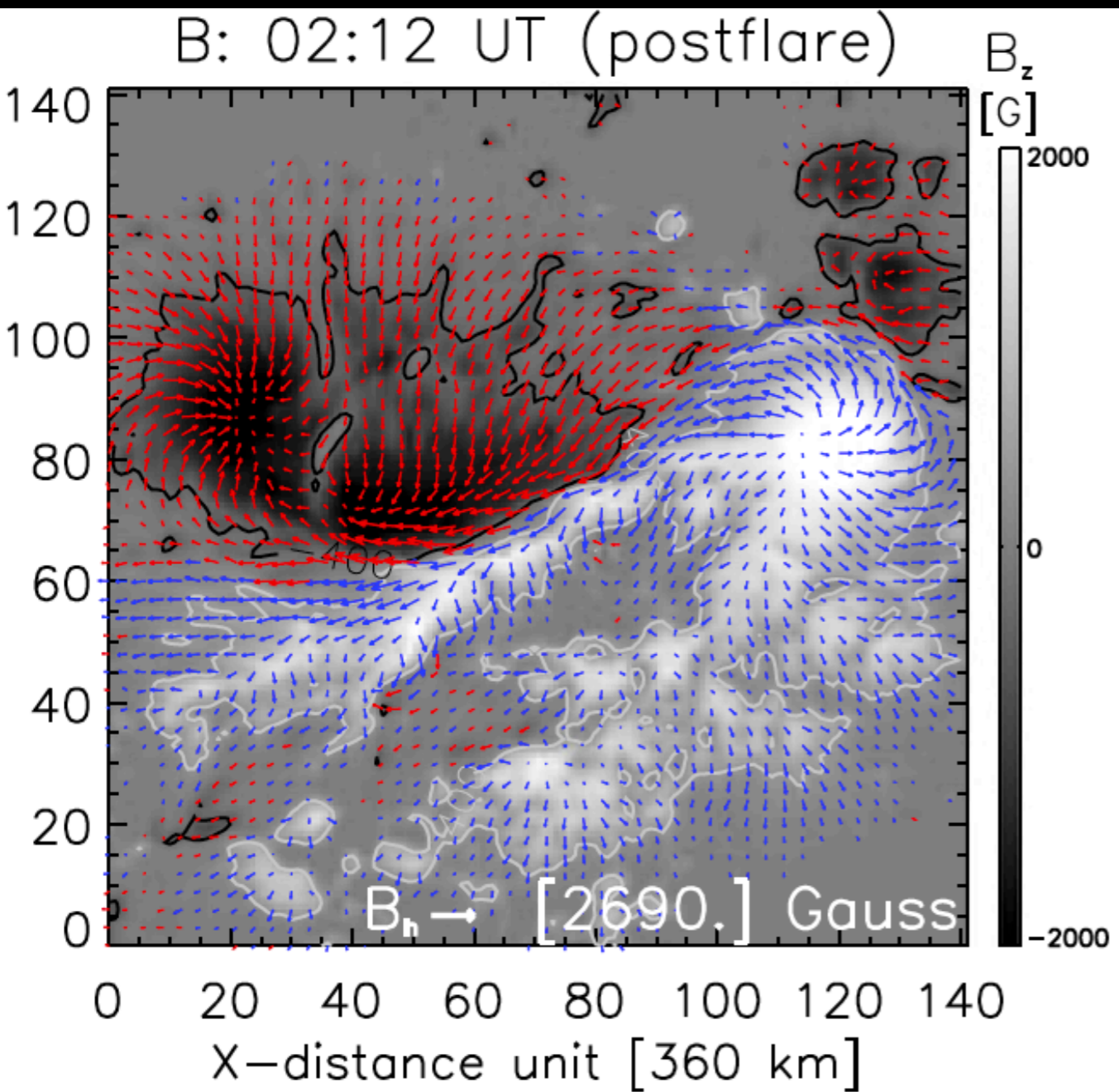
$$\mathbf{S} = \frac{c}{4\pi} (\mathbf{E} \times \mathbf{B}),$$

Helicity flux

$$\left(\frac{dH_R}{dt} \right) = -2 \int (\mathbf{A}^P \times \mathbf{E}) \cdot \hat{z} da$$

Magnetic shear

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



Magnetic flux

$$\Phi = \int |B_n| dS,$$

Current density

$$J_z(x, y, t) = \frac{1}{\mu_0} \nabla \times B_h$$

Vector potential

$$\mathbf{A}^P = \nabla \times B^P \hat{z}$$

Electric field

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Mag. Energy flux

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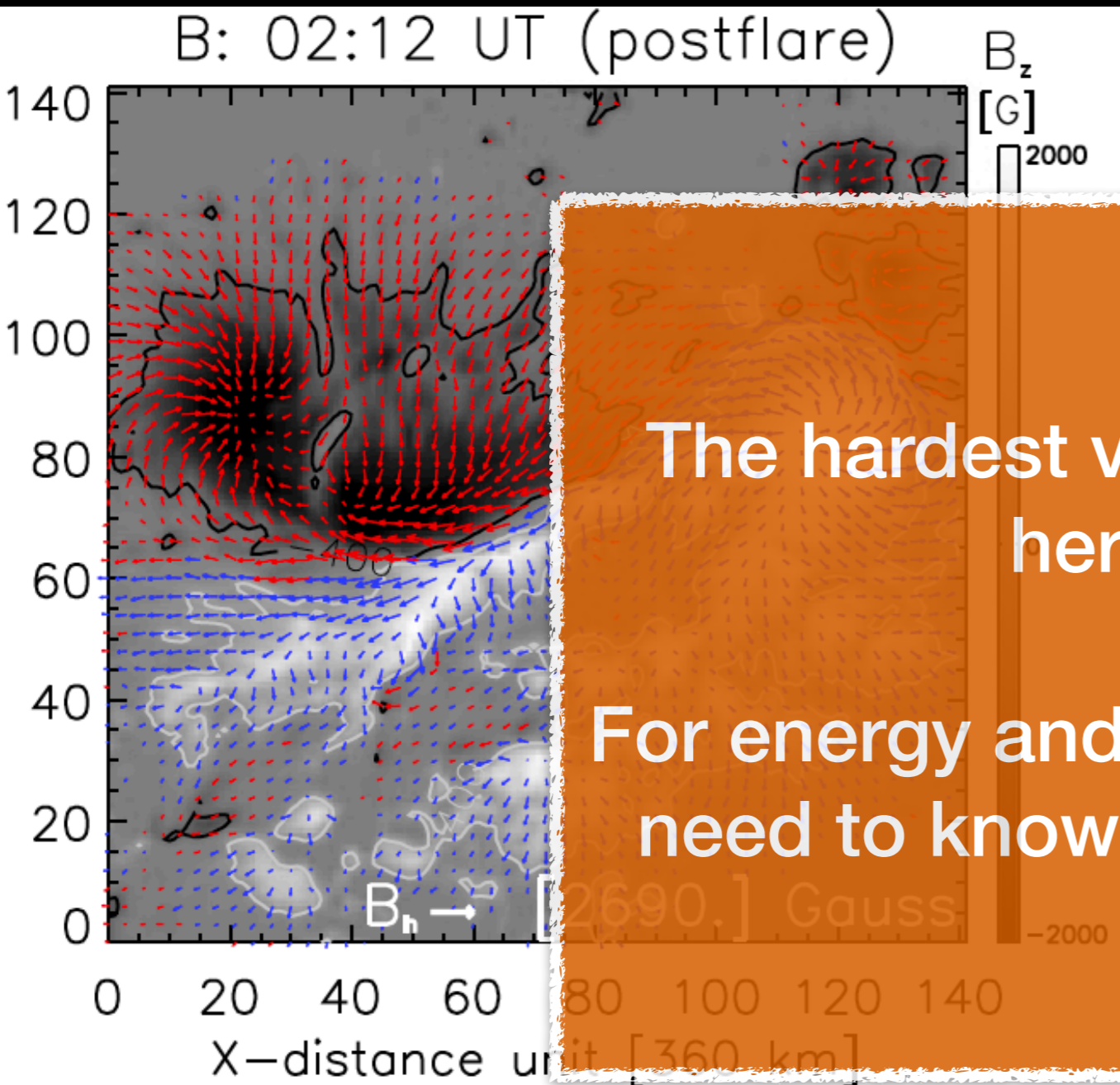
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Magnetic shear

Lorentz force

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



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$$\Phi = \int |B_n| dS,$$

Current density

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The hardest variable to derive here is \mathbf{E} .

Vector potential

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For energy and helicity fluxes we need to know electric fields \mathbf{E} ;

Electric field

$$\frac{\partial \mathbf{B}}{\partial t} = -(\nabla \times c\mathbf{E}),$$

Mag. Energy flux

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Helicity flux

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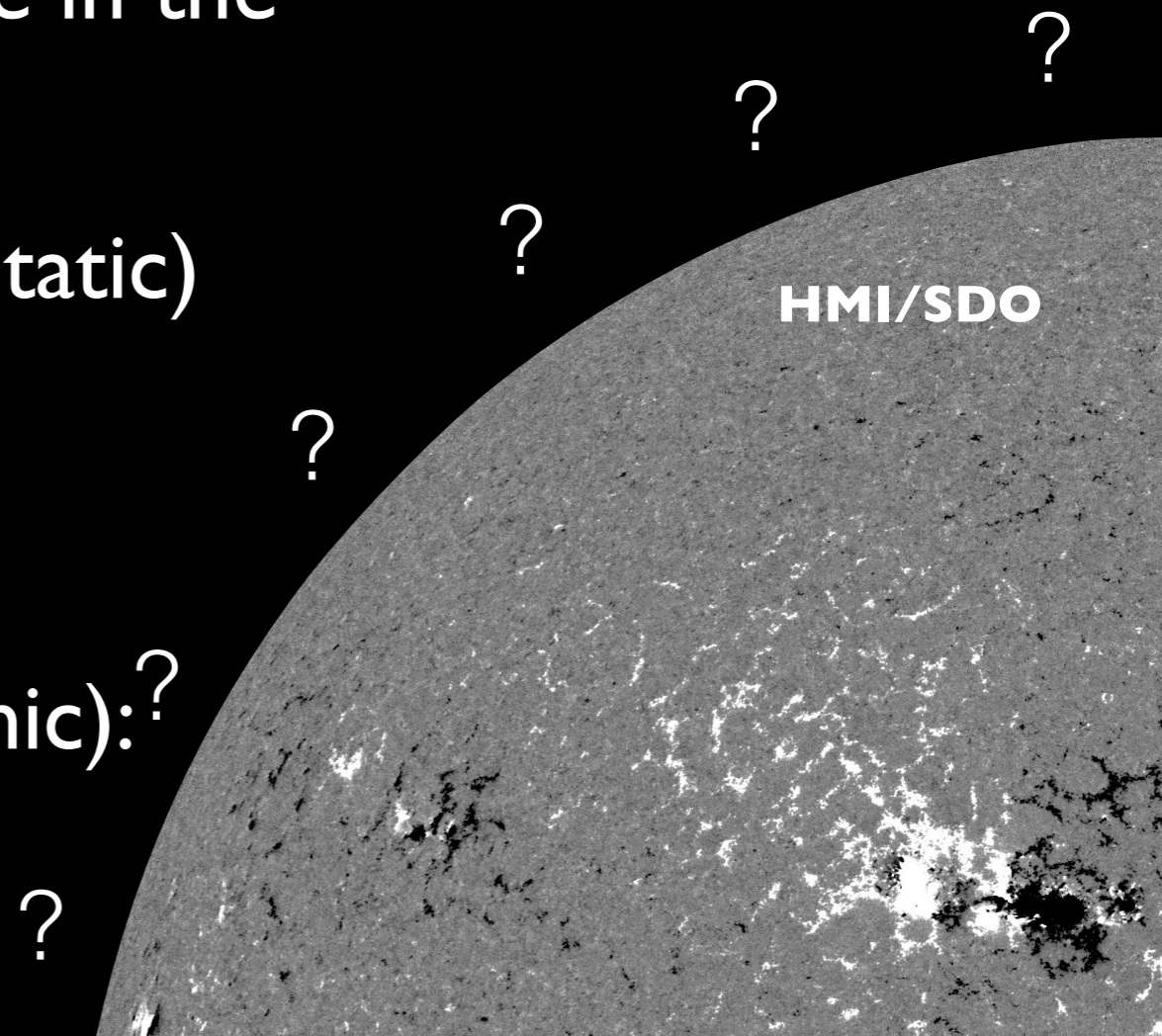
Magnetic shear

Lorentz force

What else can we get from $\mathbf{B}=(B_x, B_y, B_z)$?

- ❖ Magnetic fields at the photosphere could be used to improve forecasting of solar flares
- ❖ Magnetic fields at the photosphere could be used to find magnetic fields above in the corona:

- ❖ Potential field extrapolations (static)
- ❖ Non-linear force-free fields extrapolations (static)
- ❖ Data-driven simulations (dynamic):
force-free or full MHD



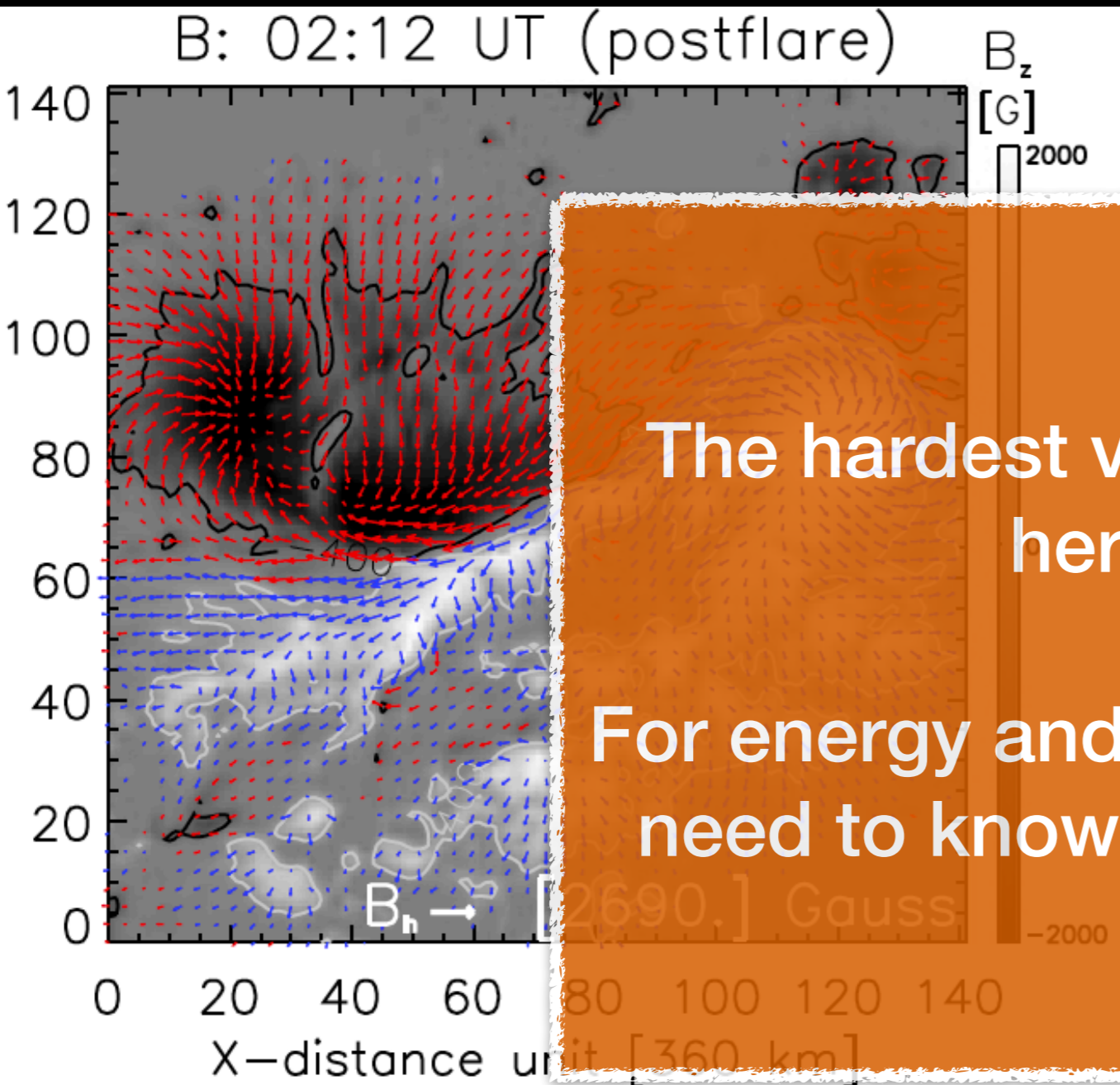
Example Coronal Field Model of AR 11158 Driven by Photospheric **B** and derived **E**

Green & purple show positive and negative photospheric flux, resp.

Coronal brightness is log of LOS-integration of field-line averaged J^2

2011-02-15T03:18

What can we get from $\mathbf{B}=(B_x, B_y, B_z)$?



Magnetic flux

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Current density

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$$\frac{\partial \mathbf{B}}{\partial t} = -(\nabla \times c\mathbf{E}),$$

Mag. Energy flux

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Helicity flux

$$\left(\frac{dH_R}{dt} \right) = -2 \int (\mathbf{A}^P \times \mathbf{E}) \cdot \hat{z} da$$

Magnetic shear

Lorentz force

Finding Electric Fields is Hard. Why?

- ❖ Directly measure from Stark Effect? Hard!

Wien(1916) ... Foukal & Behr (1995). Some planned efforts with DKIST.

- ❖ Find E from V (horizontal velocity). Possible!

- ❖ From Ideal MHD: $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$.

- ❖ But then, how to find V?

- ❖ From Faraday's Law

$$\frac{\partial \mathbf{B}}{\partial t} = -(\nabla \times c\mathbf{E}),$$

Today we will review earliest electric field inversion methods

- ★ Non-inductive or **Tracking approaches** to find V: Local Correlation Tracking (LCT, November & Simon 1988), Fourier Local Correlation Tracking (FLCT, Fisher & Welsch 2008).
- ★ First methods that implements some form of induction equation (**inductive approach**): Minimum energy fit (MEF, Longcope 2004), DAVE (Schuck 2006),
$$\frac{\partial B}{\partial t} = -(\nabla \times cE),$$
- ★ Comparison of early methods before we had routine vector magnetogram measurements: Welsch et al. 2007.

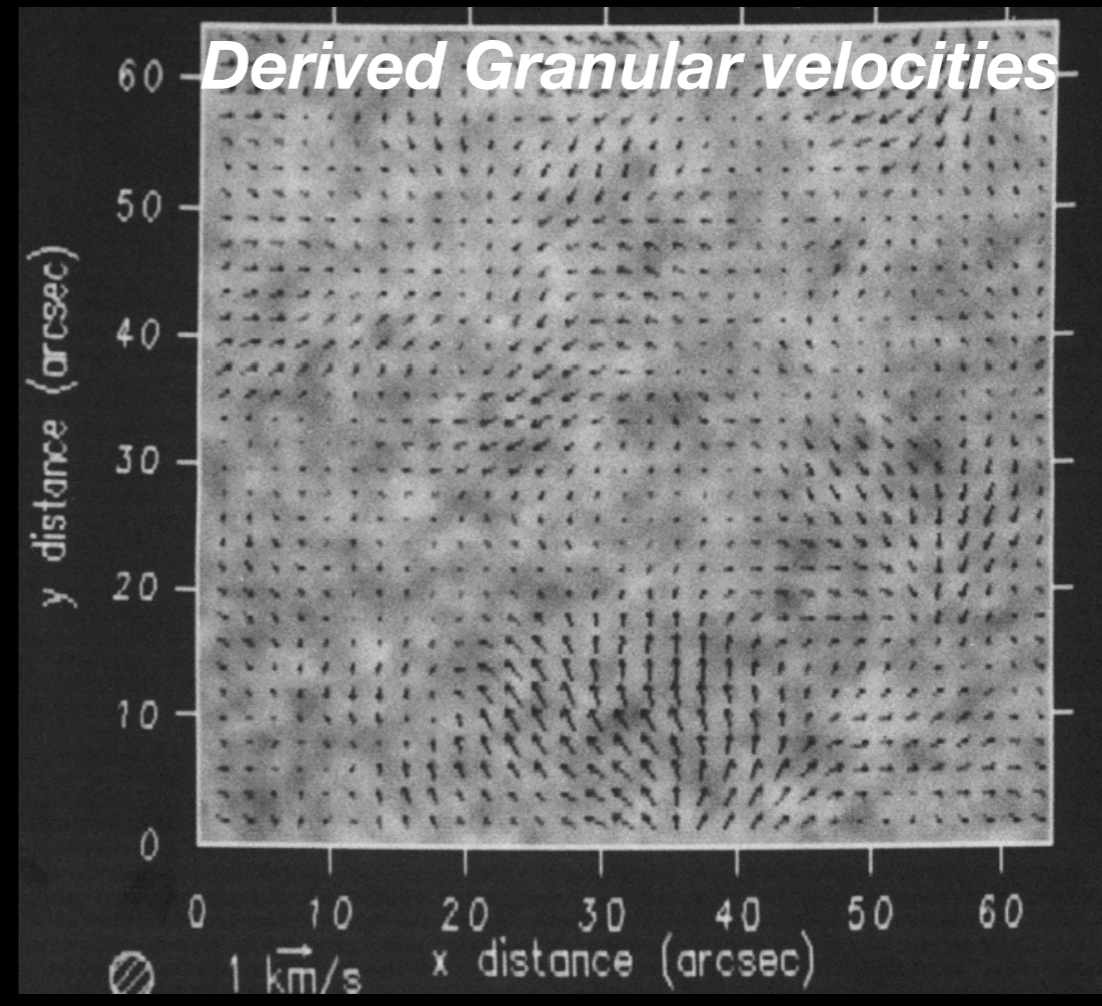
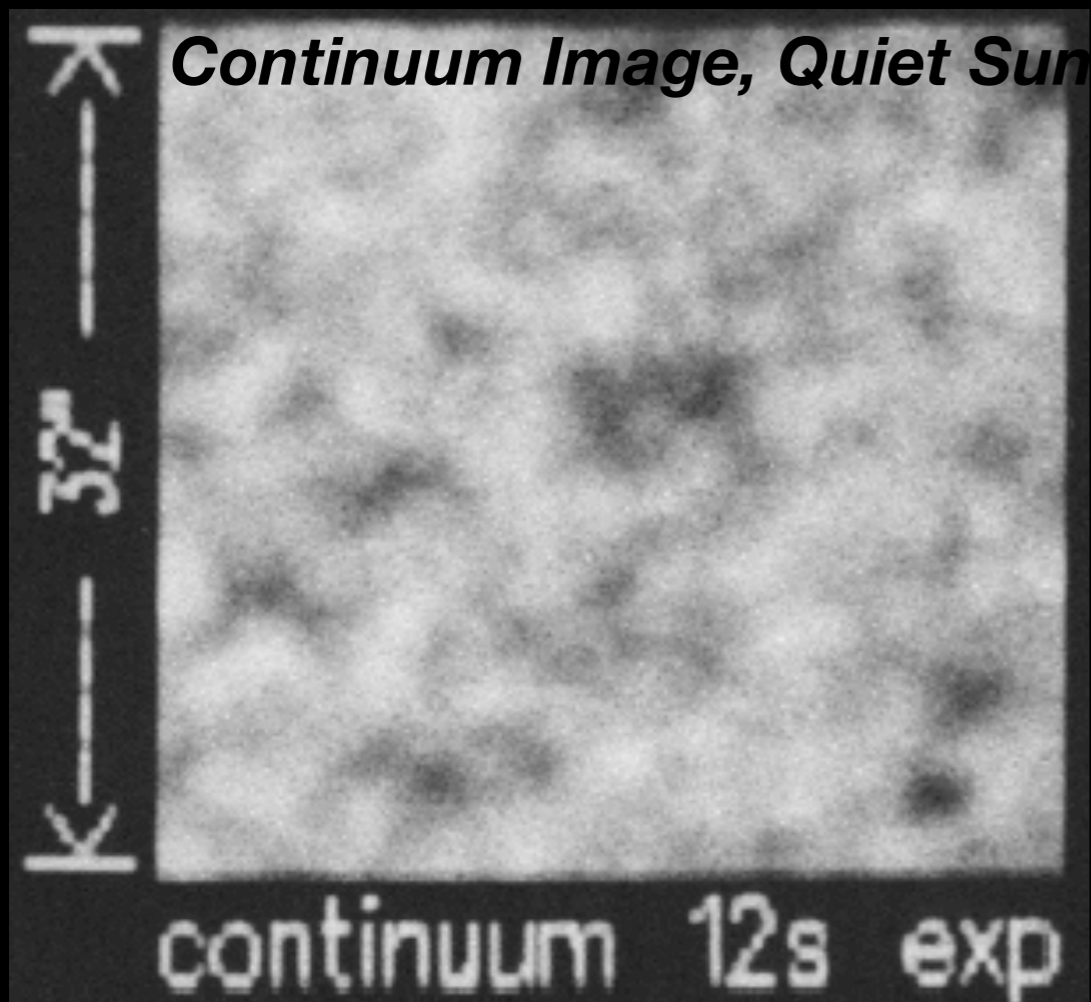
Local Correlation Tracking (LCT)

- November & Simon, 1988: introduced Local Correlation Tracking in Solar Physics; minimizes the merit function M to find relative displacement δx b/w I_1 and I_2 images:

$$M(\delta x_i) = \sum_{p=-s}^{+s} \sum_{q=-s}^{+s} \left\{ W(p, q) \left[I_1 \left(x_i + p + \frac{\delta x_i}{2}, y_i + q + \frac{\delta y_i}{2} \right) - I_2 \left(x_i + p - \frac{\delta x_i}{2}, y_i + q - \frac{\delta y_i}{2} \right) \right]^2 \right\},$$

where W - apodizing function (gaussian) which weights pixels closer to x_i more heavily in the sum.

- Used white light observations of solar granulation, $L=80\text{min}$: Sacramento Peak Vacuum Tower Telescope to derive granular velocities;
- **Results**: Solar mesogranulation and super-granulation flows of $10''$ - $40''$ spatial scales



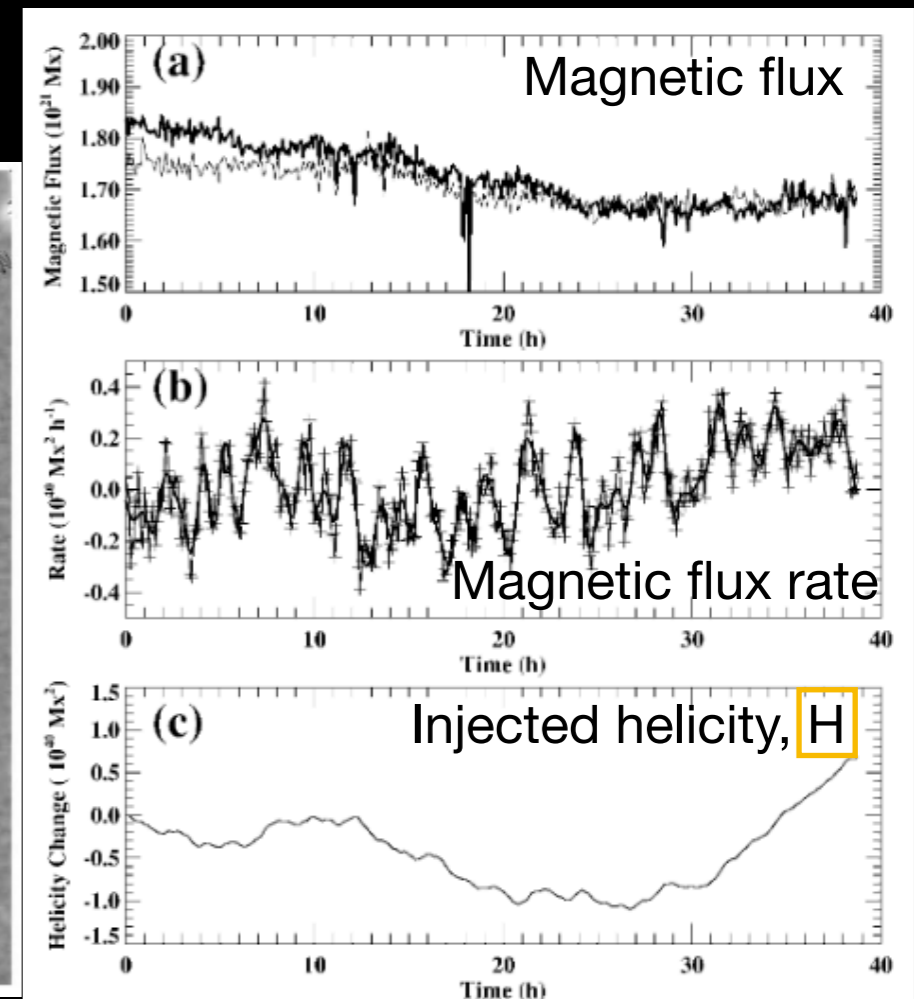
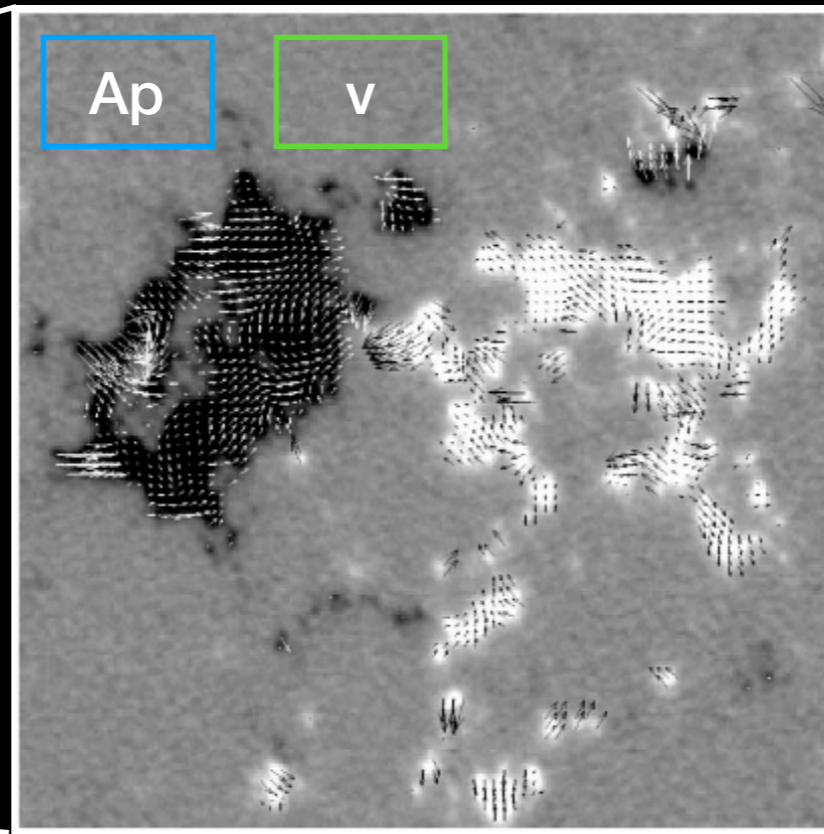
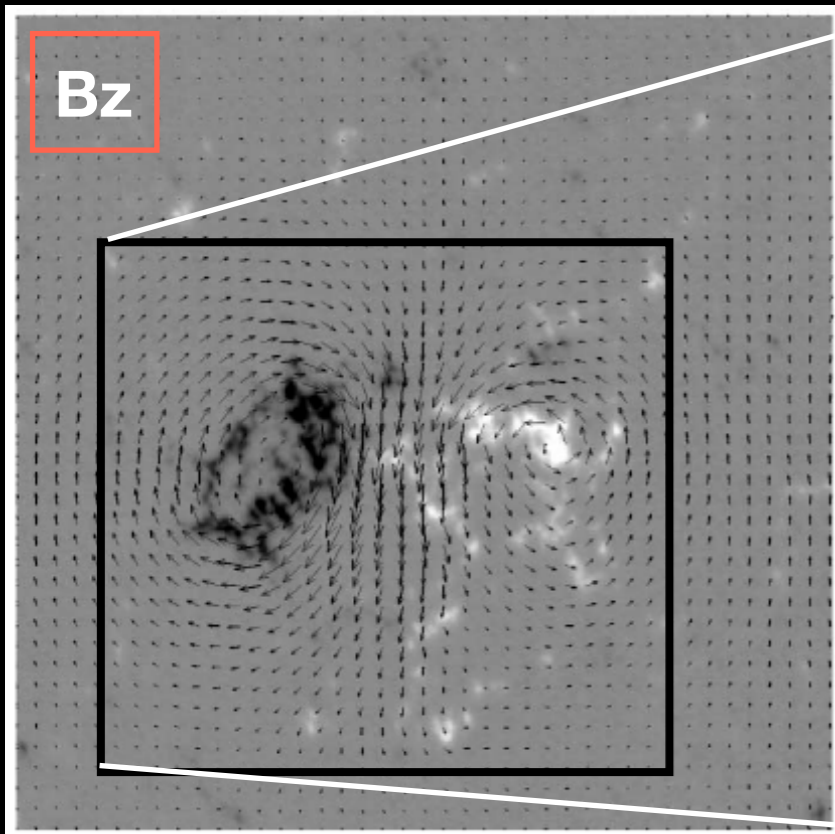
LCT: first application to a large dataset

- Chae 2001: First application of LCT to MDI/SOHO LOS magnetograms;
- L=40 hrs, AR 8011, dt=96 min; ds=720km;
- Results: found rate of magnetic helicity injection to better understand role of photospheric flows in the evolution of coronal magnetic fields in solar ARs:

$$\frac{dH}{dt} = \oint 2(\mathbf{B} \cdot \mathbf{A}_p) v_z dS + \oint -2(\mathbf{v} \cdot \mathbf{A}_p) B_z dS,$$

emergence term

shear term



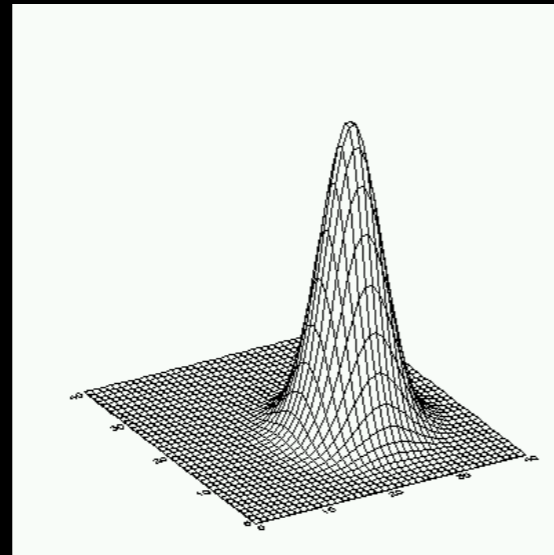
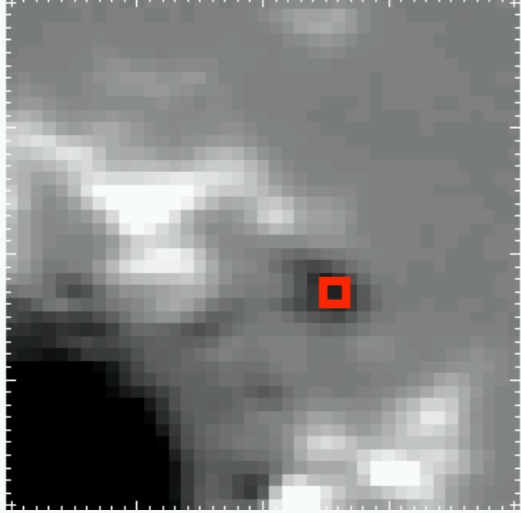
LOS B and vector potential Ap

Derived horizontal velocities

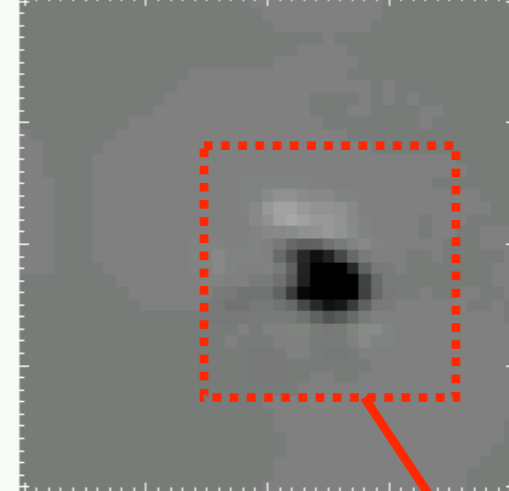
Fourier Local Correlation Tracking (FLCT)

Welsch et al. 2004: FLCT method finds horizontal velocity $V(x,y)$ by correlating subregions, to find local shifts:

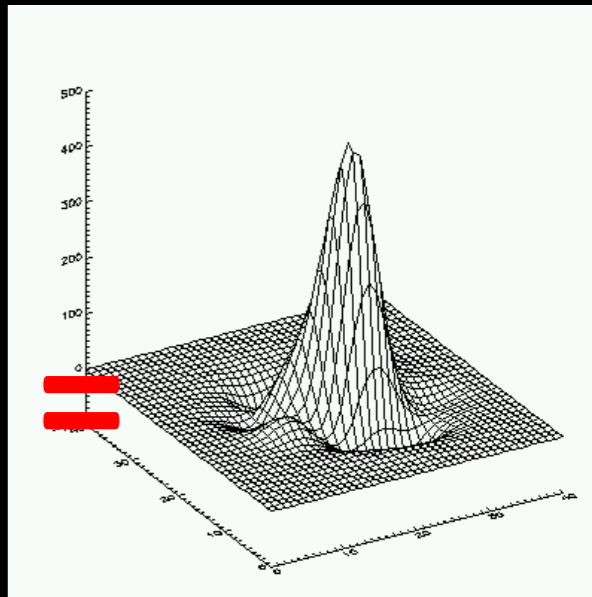
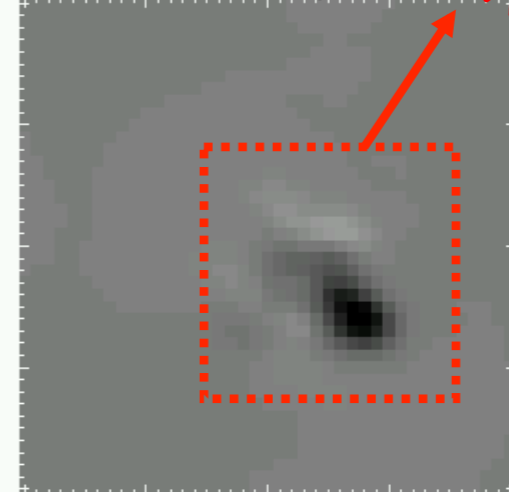
AR 8210 (IVM),1998/05/01, 17:13



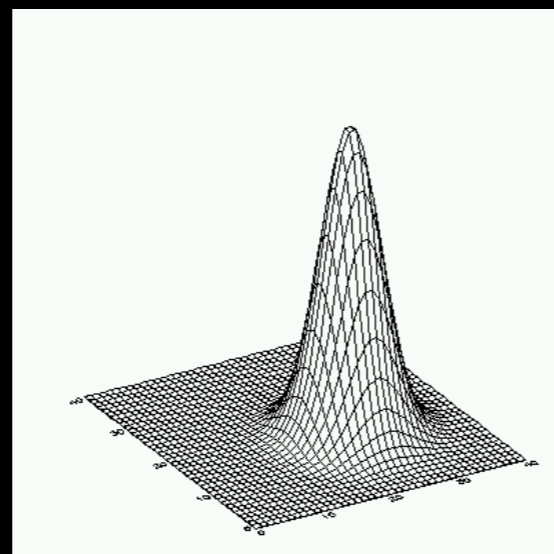
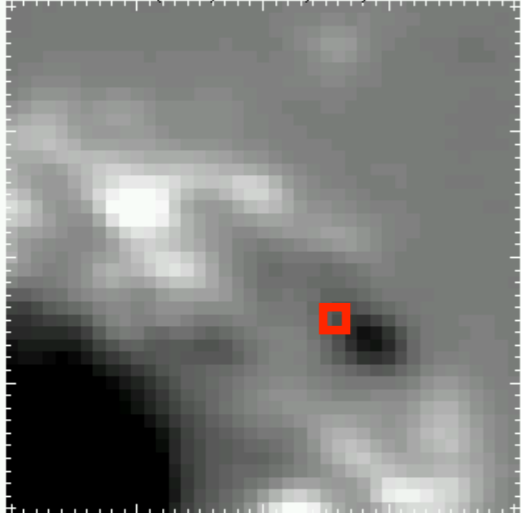
Masked Image 1



Masked Image 2



AR 8210 (IVM),1998/05/01, 21:29



1) For each (x_i, y_i) above $|B|_{\text{threshold}}$

2) Apply Gaussian mask at (x_i, y_i)

3) Truncate and cross-correlate in Fourier space

4) $\Delta x(x_i, y_i)$ is interpolated max. of correlation function

Today we will review earliest electric field inversion methods

Main Problem: derived horizontal velocity, V , is not necessarily physical

- ★ Non-inductive or **Tracking approaches** to find V : Local Correlation Tracking (LCT, November & Simon 1988), Fourier Local Correlation Tracking (FLCT, Fisher & Welsch 2008).
- ★ First methods that implements some form of induction equation (**inductive approach**): Minimum energy fit (MEF, Longcope 2004), DAVE (Schuck 2006),
$$\frac{\partial B}{\partial t} = -(\nabla \times cE),$$
- ★ Comparison of early methods before we had routine vector magnetogram measurements: Welsch et al. 2007.

First inductive method (IM)

Kusano et al. 2002: assume ideal induction eq. $\frac{\partial \mathbf{B}}{\partial t} = -c(\nabla \times \mathbf{E}) = \nabla \times (\mathbf{v} \times \mathbf{B}),$

1) Use B_z observations and LCT method $\Rightarrow V_h$;

2) Use B_z, B_h, V_h & $\frac{\partial B_z}{\partial t} = \hat{\mathbf{z}} \cdot \nabla \times (\mathbf{v} \times \mathbf{B}) = -\nabla \cdot (\mathbf{v}_h B_z - v_z \mathbf{B}_h), \Rightarrow V_z$

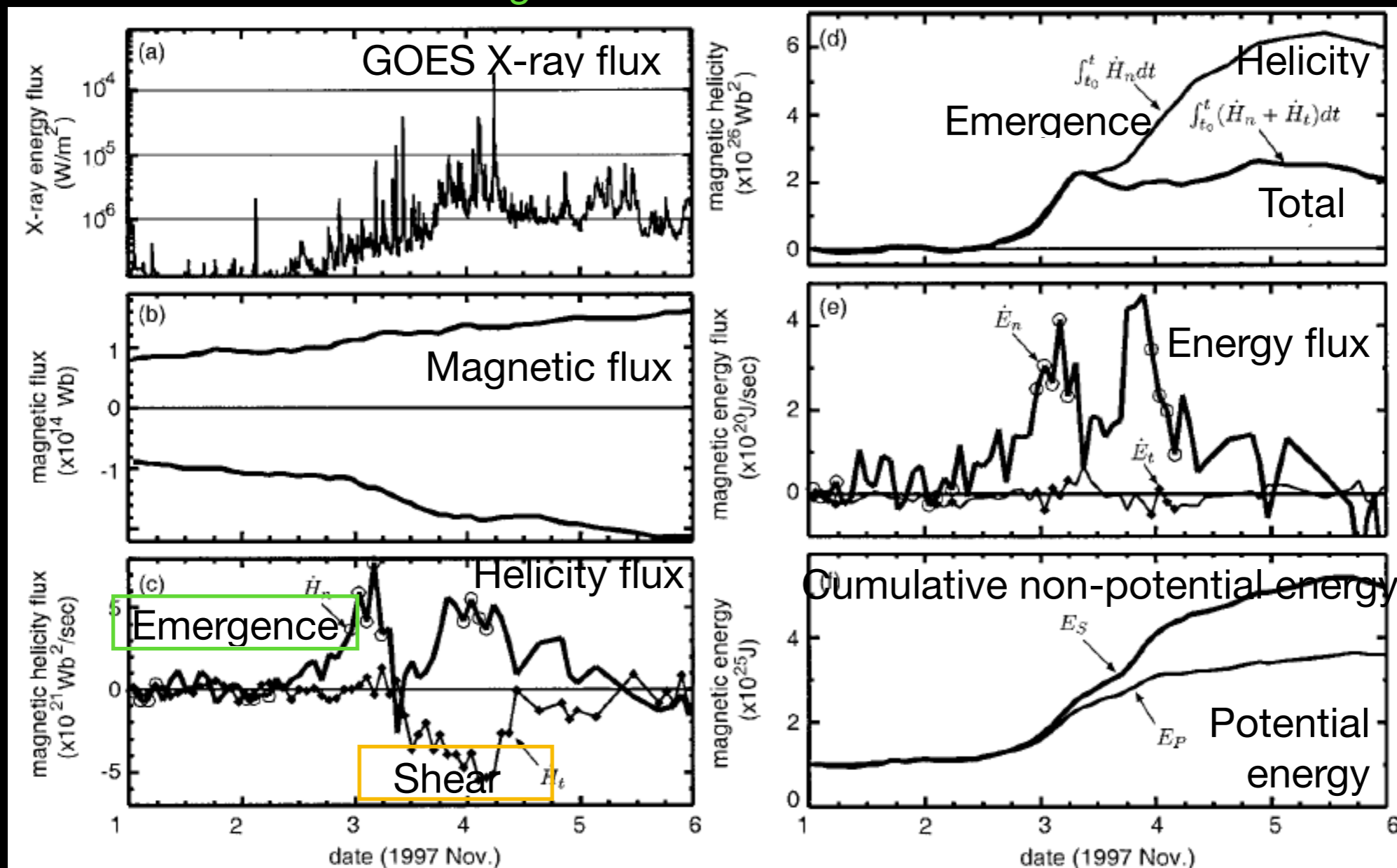
- Problems: Not defined in areas where $B_h = 0$.

First inductive method (IM): application

- Kusano et al. 2002: Applied to AR 8100, 1997 Nov 1, B_z from MDI/SOHO and \mathbf{B} from vector magnetograph, NAOJ;
- Results: shear and emergence terms are equally important;

$$\frac{dH}{dt} = \oint 2(\mathbf{B} \cdot \mathbf{A}_p)v_z dS + \oint -2(\mathbf{v} \cdot \mathbf{A}_p)B_z dS,$$

emergence term
shear term



Inductive LCT Method (ILCT)

- ❖ Welsch et al. (2004) showed that FLCT flows only approximately reproduce observed dB_z/dt .
- ❖ As an alternative they suggested to use decomposition

$$\begin{aligned}\mathbf{u}B_z &= -\nabla_h \chi + \nabla_h \times \psi \mathbf{z} \\ \frac{dB_z}{dt} &= -\nabla \cdot (\mathbf{V}_h B_z - V_z \mathbf{B}_h) = -\nabla \cdot (\mathbf{u}B_z) \\ &= -\nabla \cdot (-\nabla_h \chi + \nabla_h \times \psi \mathbf{z})\end{aligned}$$

Then χ is defined by solving Poisson equation and ψ is defined by LCT speed assuming $\mathbf{u}=\mathbf{u}(\text{LCT})$.

Assuming $\mathbf{V}_{\text{perp}} \cdot \mathbf{B} = 0$ we can find \mathbf{V}_{perp} .

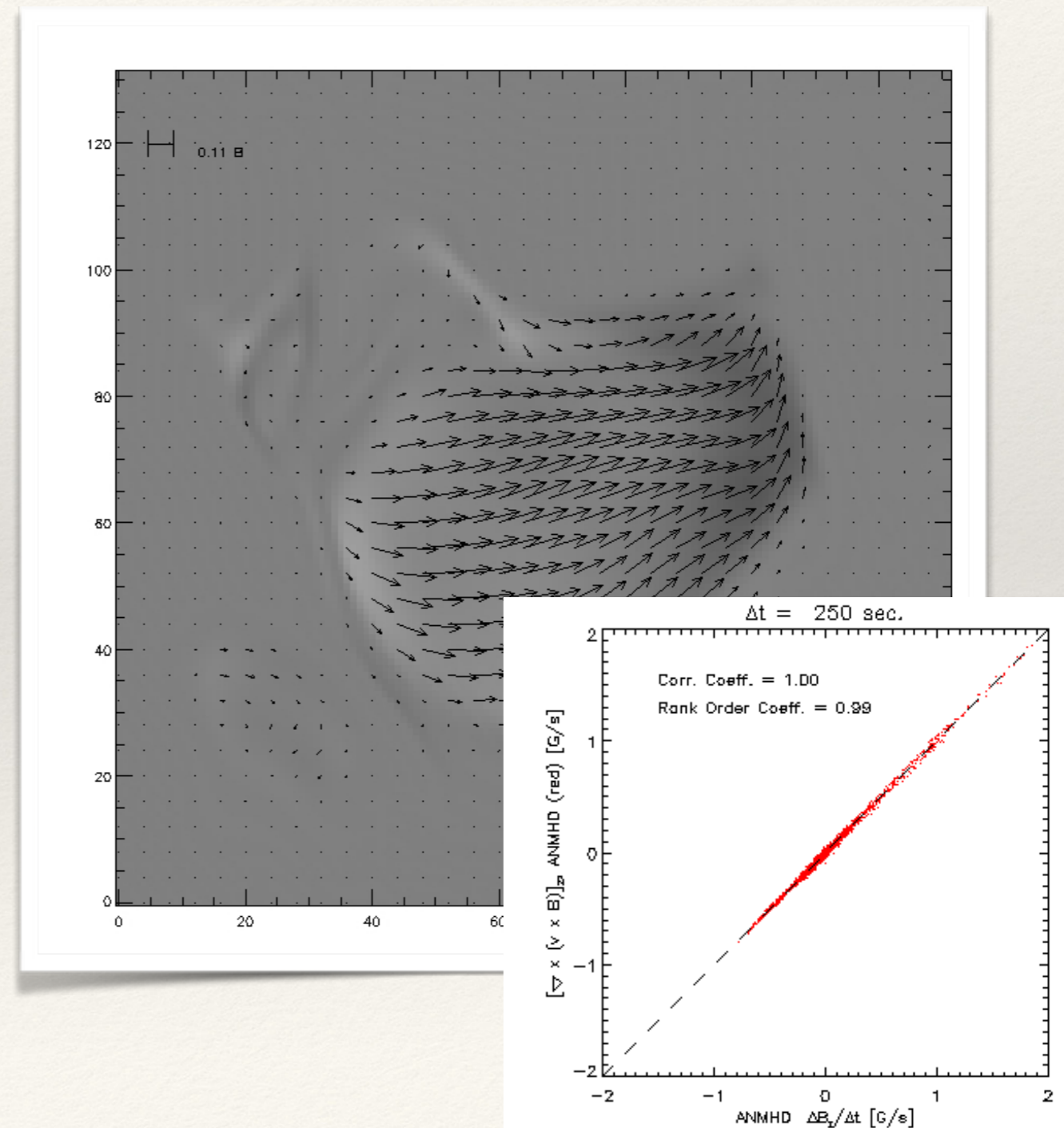
Other inductive methods

- Longcope (2004): Minimum energy fit (MEF): finds smallest plasma velocities consistent with magnetograms and magnetic induction equation
- Schuck (2006): Building on differential LCT introduced differential affine velocity estimator (DAVE); finds optical flow field that is statistically consistent with the magnetic induction equation and the affine velocity profile within the window aperture (varying window size);
- Georgoulis & LaBonte (2006): minimum structure reconstruction method (MSR); Uses inductive and electrostatic potential decomposition;
- For excellent reviews of different methods see Welsch et al. (2007), Schuck (2006);

Accuracy of Velocity Estimates

- ❖ Welsch et al. (2007) conducted quantitative tests of accuracy using several available methods.
- ❖ They created “synthetic magnetograms” from ANMHD simulations of a bipolar magnetic region rising through a convecting medium.
- ❖ In these data, both \mathbf{V} & \mathbf{B} are known exactly.
- ❖ They verified that the ANMHD data were consistent with

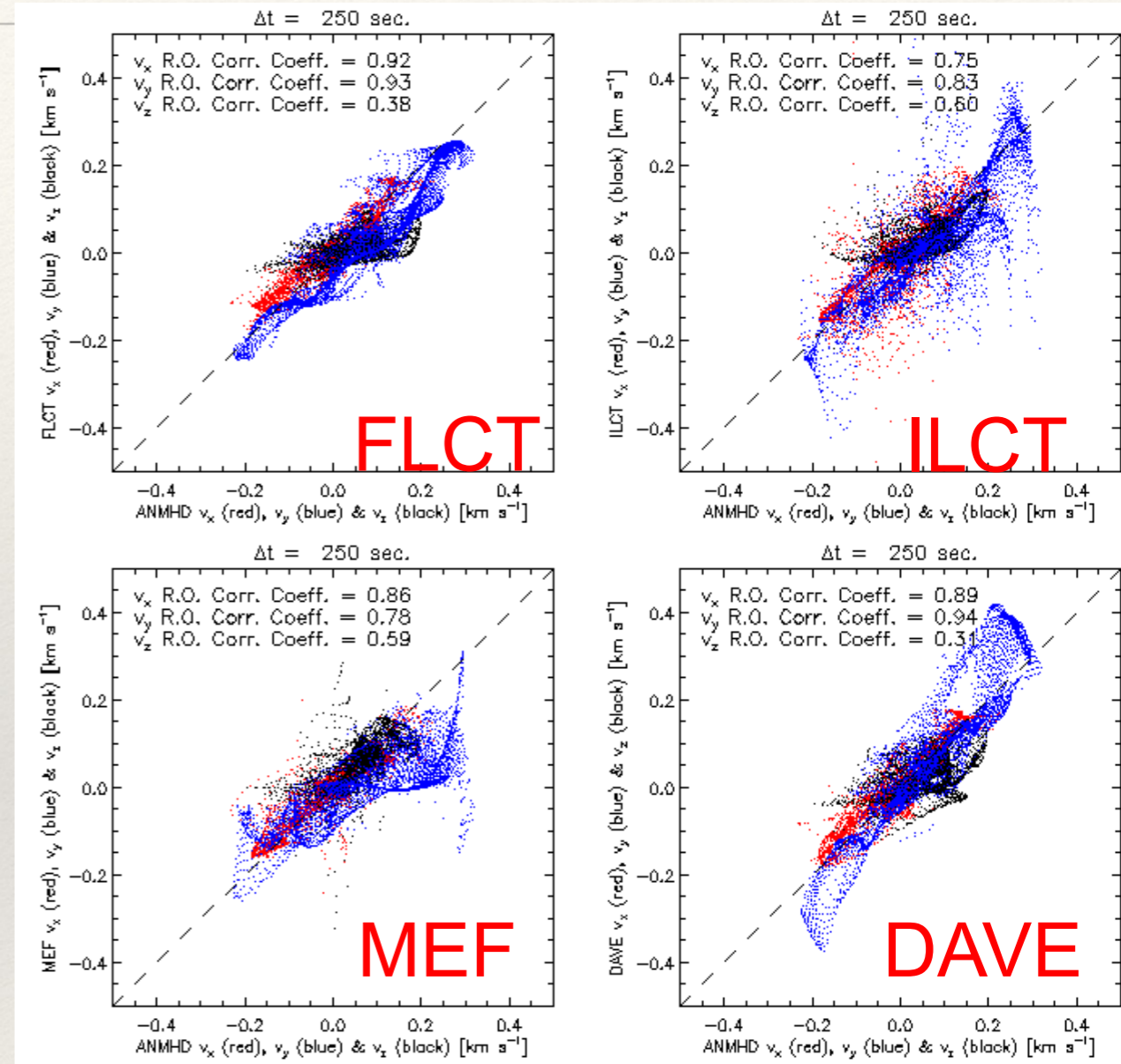
$$\partial_t B_n = \nabla \cdot (V_n \mathbf{B}_{\text{hor}} - \mathbf{V}_{\text{hor}} B_n)$$



Accuracy of Velocity Estimates

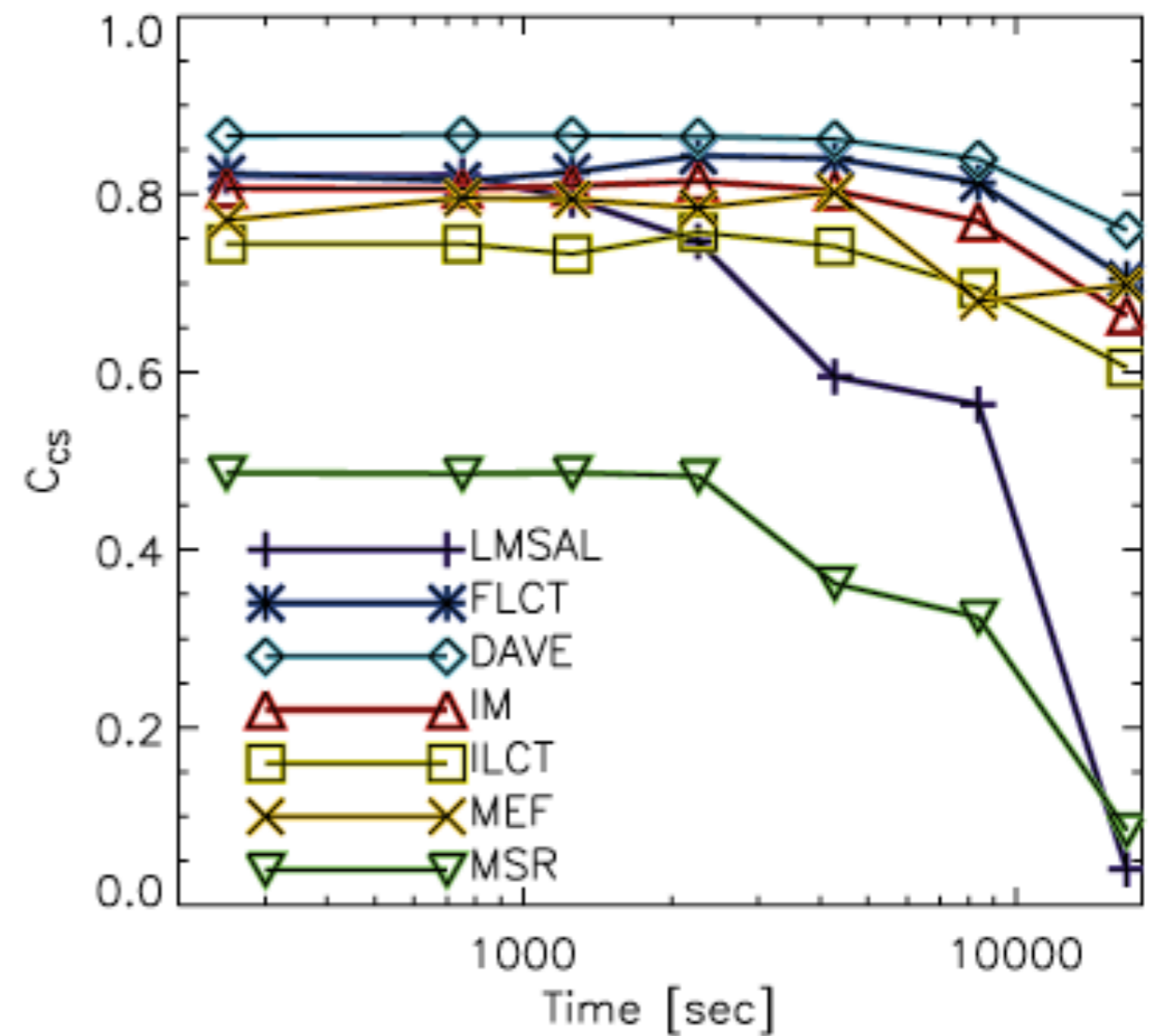
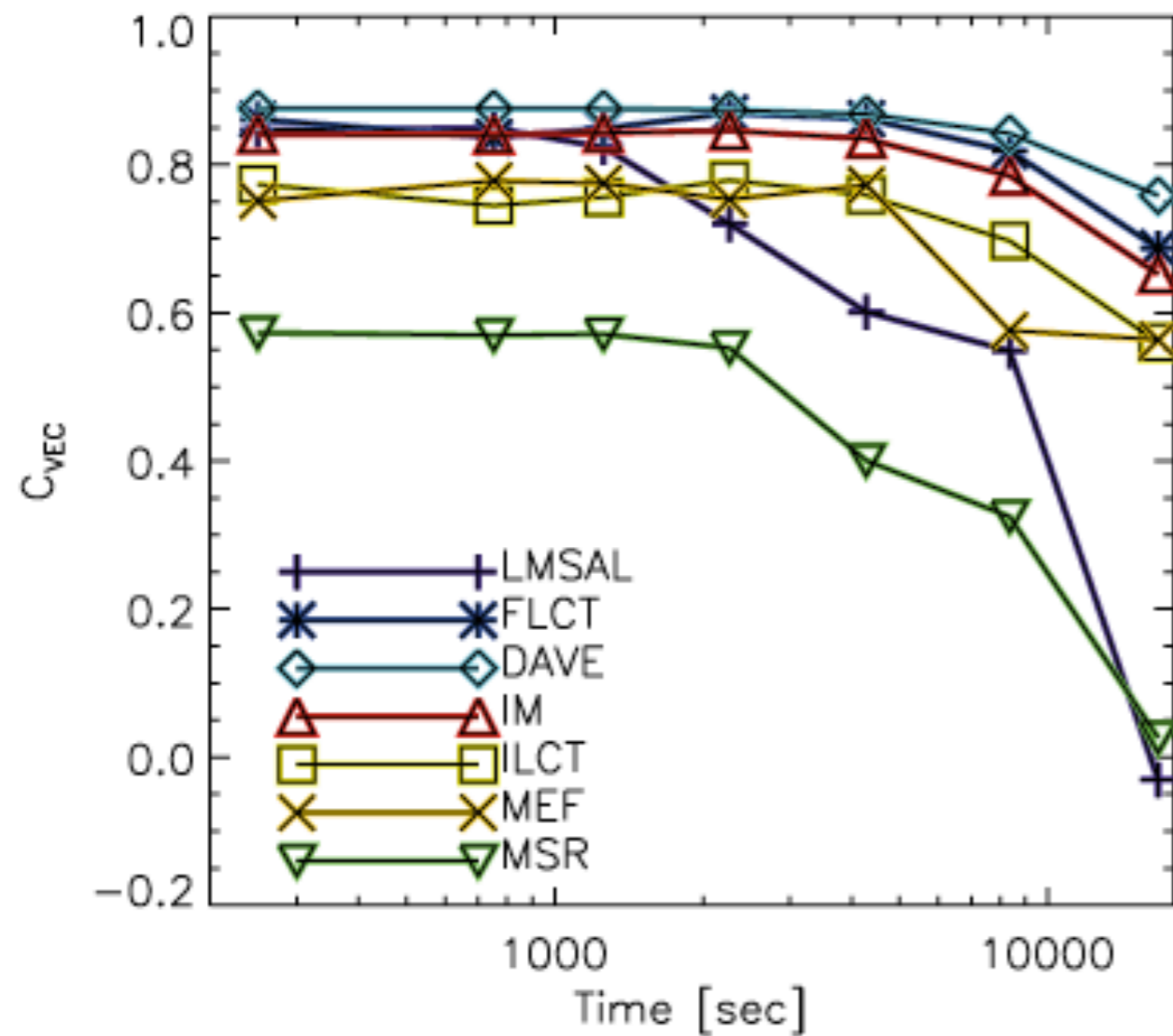
Here, I show representative results from just a few of the methods tested:

- ❖ Fourier LCT (FLCT, Welsch et al. 2004)
- ❖ Inductive LCT (ILCT, Welsch et al. 2004)
- ❖ Minimum Energy Fit (MEF, Longcope 2004)
- ❖ Differential Affine Velocity Estimator (DAVE, Schuck 2006)



Comparison of actual ANMHD and derived velocity estimated using FLCT, ILCT, MEF, DAVE.

Comparison of different methods using ANMHD test case: velocity comparison

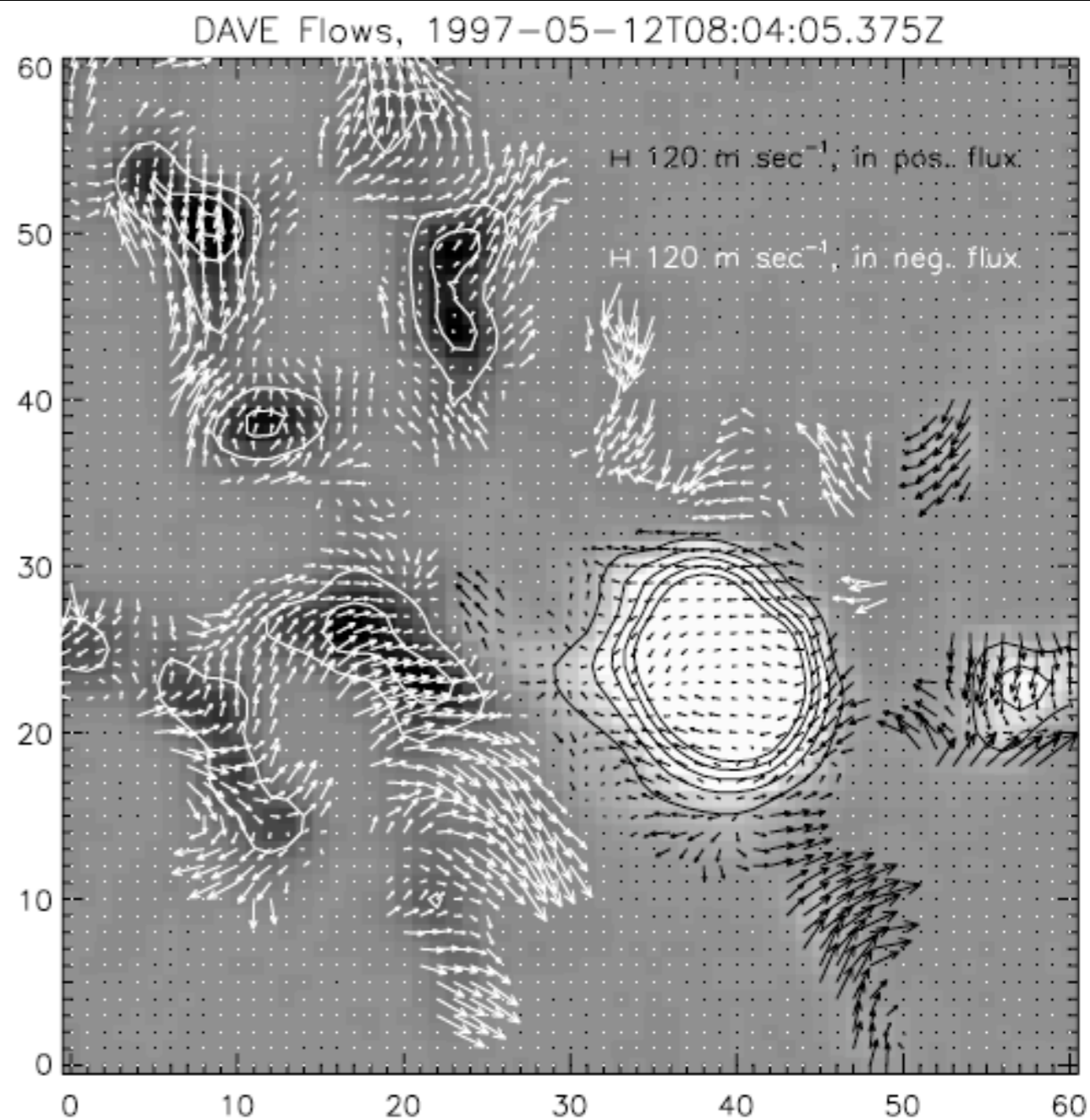
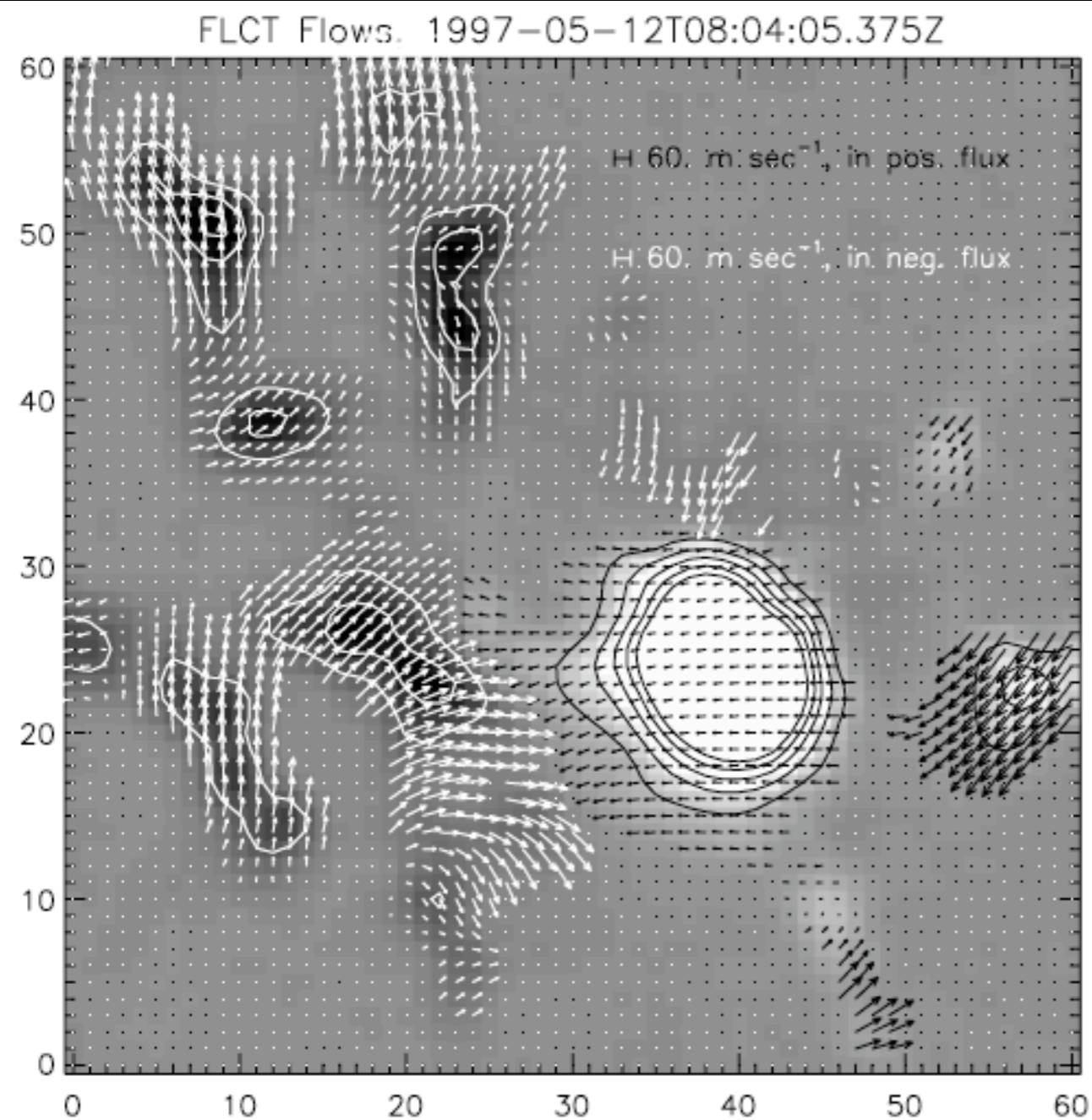


Vector correlation for varying cadence

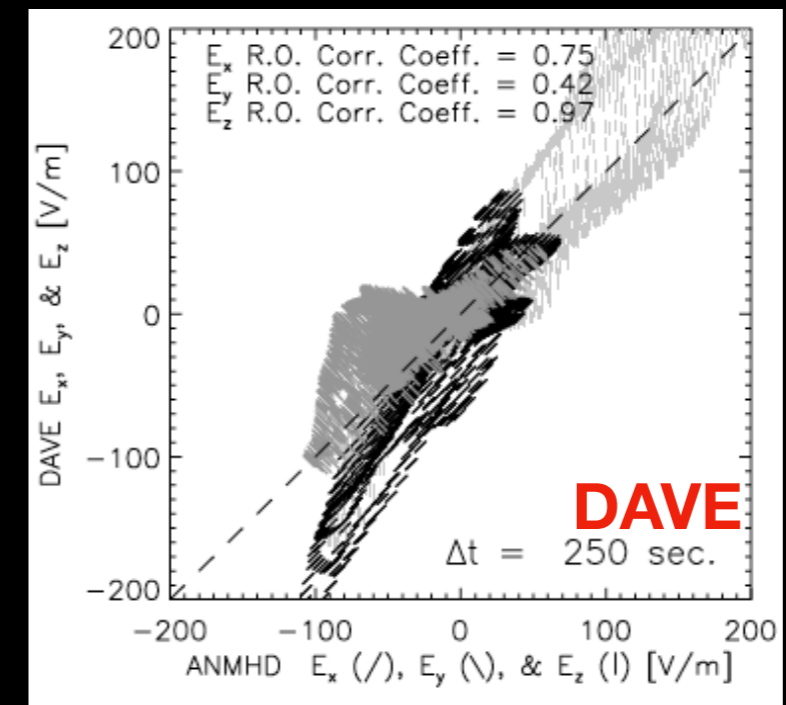
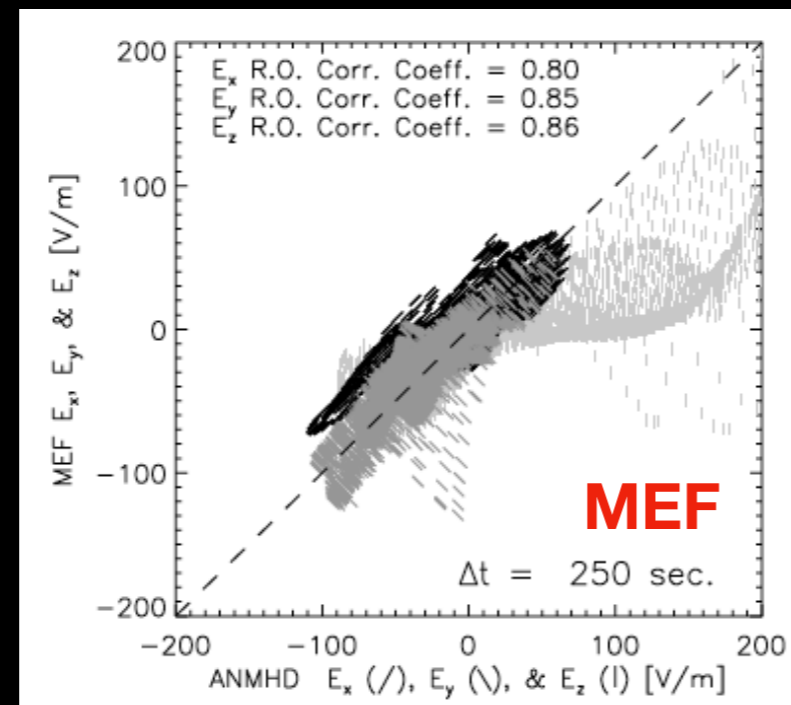
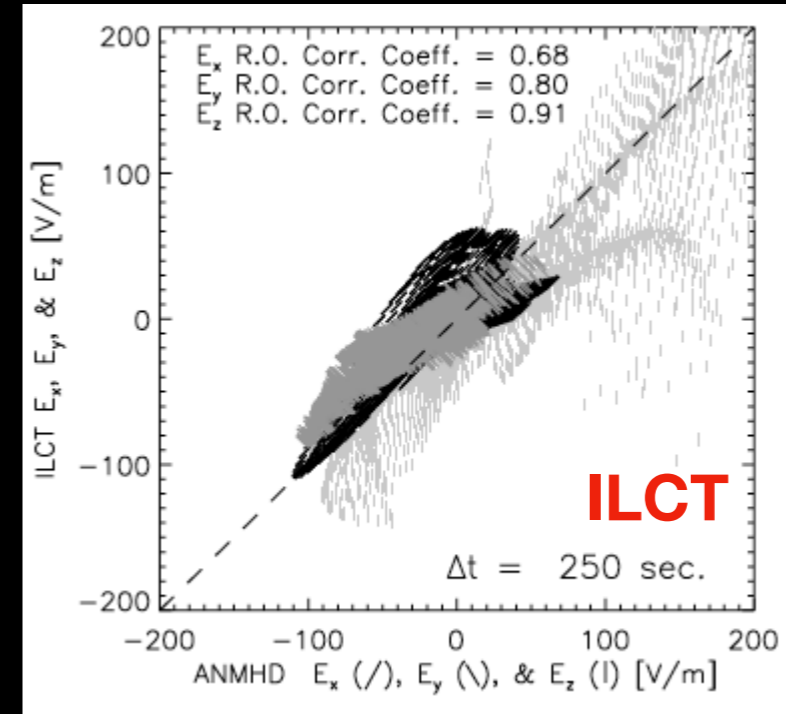
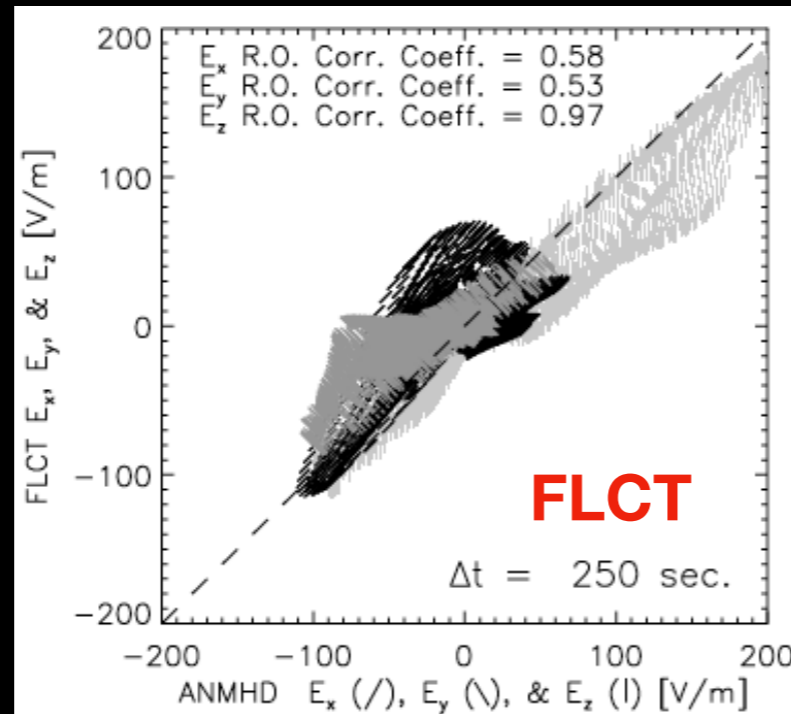
Direction correlation for varying cadence

Velocity reconstruction: all methods did poorly, IM, FLCT, and MEF performed similarly; DAVE did slightly better;

Real magnetogram: Sample maps of FLCT and DAVE flows show them to be strongly correlated, but far from identical.

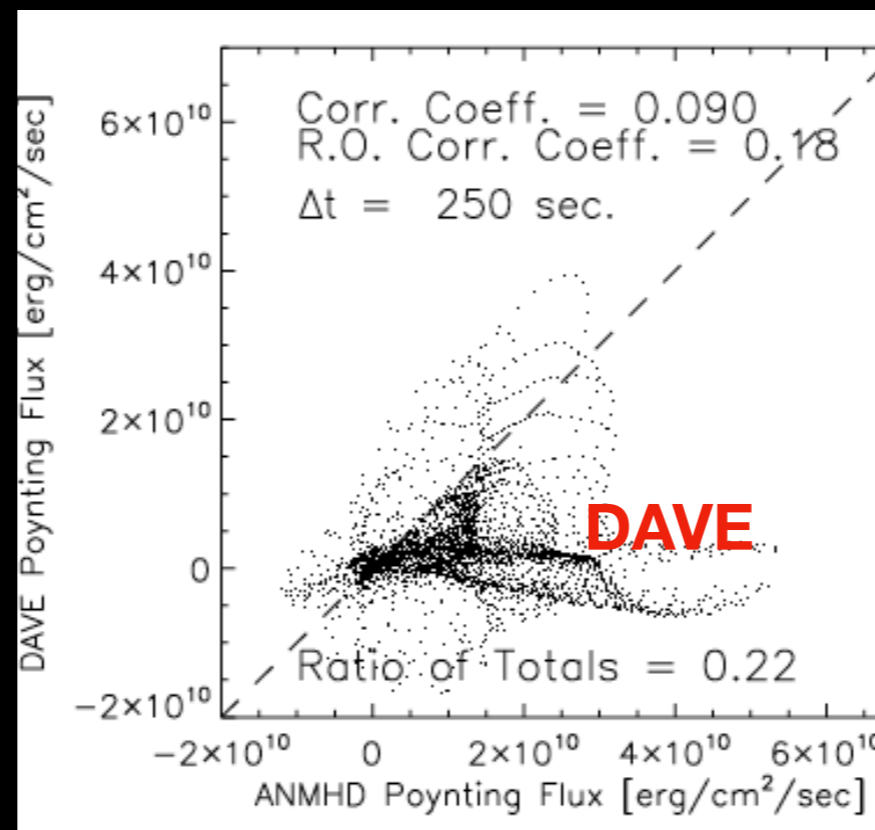
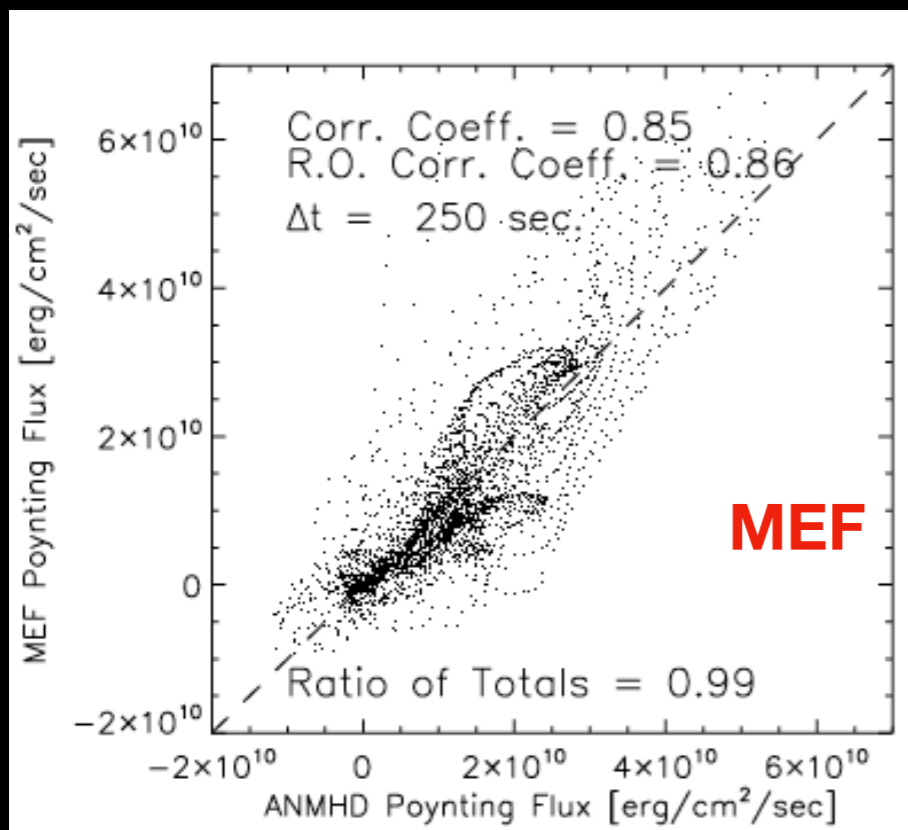
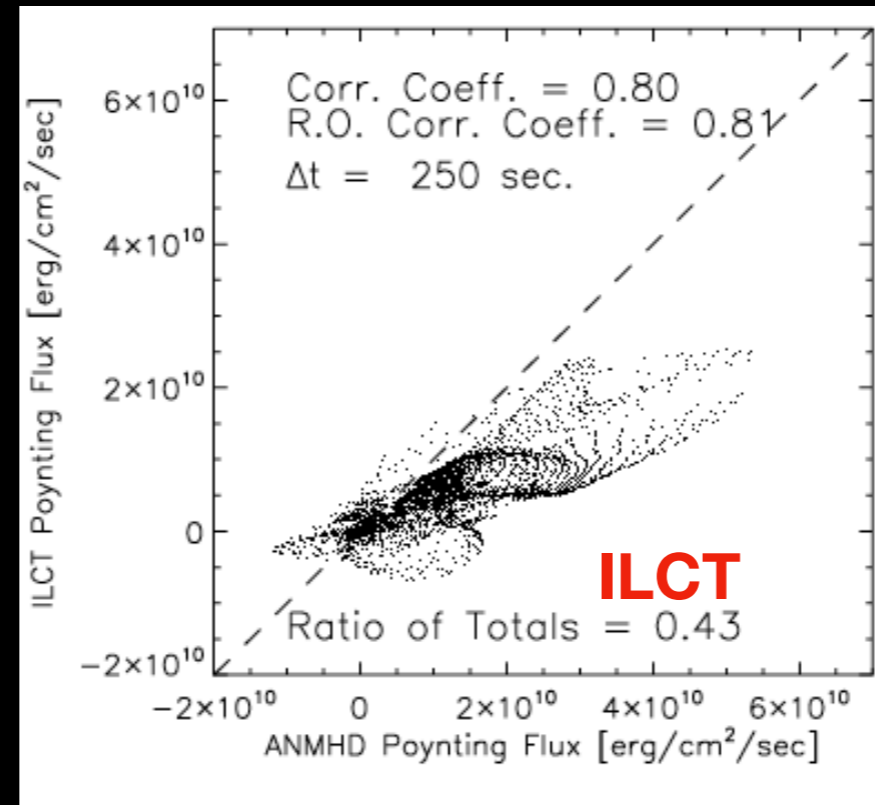
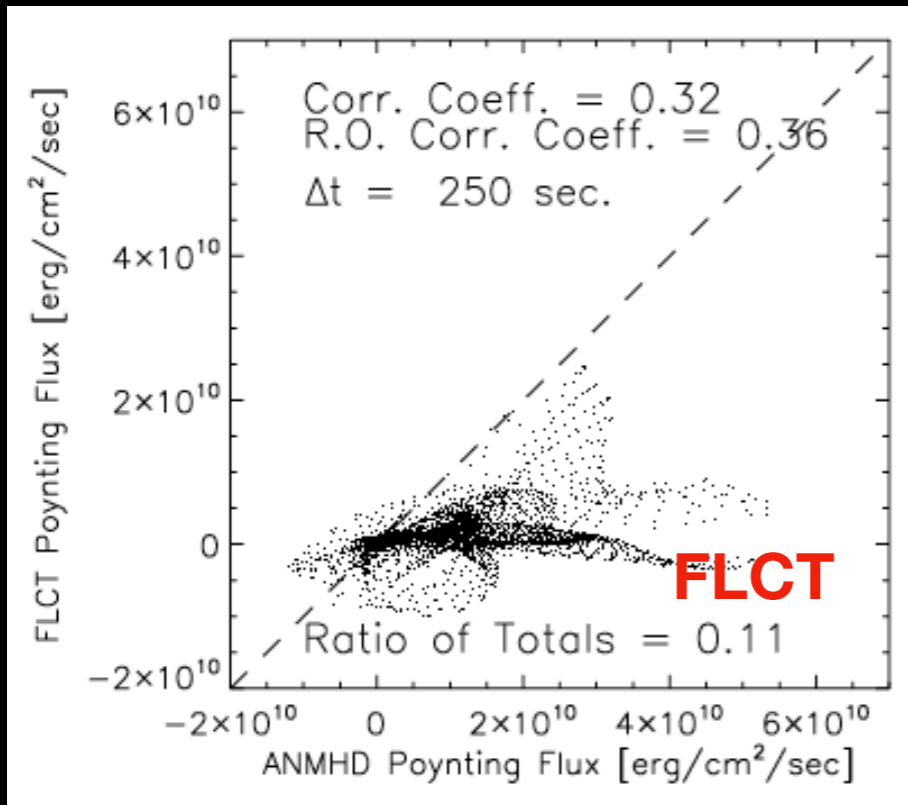


Comparison of different methods using ANMHD test case: electric fields comparison



- Electric fields: DAVE, FLCT, ILCT, MEF did OK.

Comparison of different methods using ANMHD test case: Poynting flux comparison



- Energy and helicity fluxes: MEF was the best; could be because it is most suitable physically to ANMHD test case.

Comparing performance of FLCT, DAVE, ILCT, MEF, IM etc (from Welsch et al. 2007): Summary

- Velocity reconstruction: all methods did poorly, IM, FLCT, and MEF performed similarly; DAVE did slightly better;
- Electric fields: DAVE, FLCT, ILCT, MEF did OK.
- Energy and helicity fluxes: MEF was the best; could be because it is most suitable physically to ANMHD test case.

So which inversion should we use to get horizontal velocity V or E ?

Most of the described methods only used **one component of B , B_R** and the normal component of induction equation to find horizontal velocity:

$$dB_z/dt = -c[\nabla_h \times E_h]_z = [\nabla \times (v \times B)]_z$$

Now with routine vector and Doppler velocity field measurements we could solve **vector B** and induction equation:

$$dB/dt = -c[\nabla \times E] = [\nabla \times (v \times B)]$$

- Examples of methods that use vector magnetic fields:
 - DAVE4VM (Schuck et al. 2008)
 - PDFI (Fisher et al. 2020, Kazachenko et. al. 2014).

Summary

- Today we learnt: Why Magnetic Fields (**B**) on the Sun Are Important? Where can we presently routinely measure magnetic fields on the Sun? What can we get from **B**? Finding E (or V) from **B**: Tracking & Inductive methods.
- Next Tuesday: we will learn how one could derive velocity fields in the solar photosphere using vector **B** and more recent methods: DAVE4VM, PDFI and also some ML methods. I will show examples of their application to HMI data.
- Next Thursday: hands-on activity: applying FLCT to a sequence of HMI/SDO magnetograms to derive horizontal velocities, magnetic fluxes, helicity and energy fluxes.