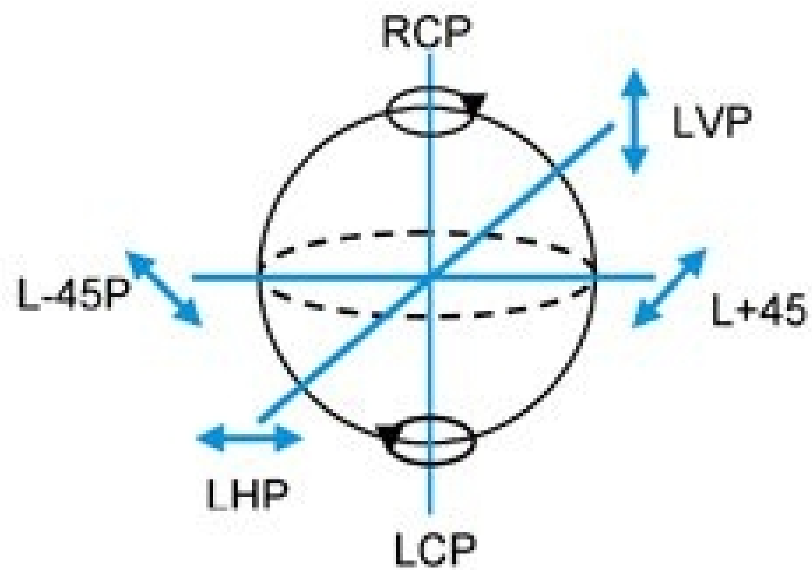
PHYS 7810: Solar Physics with DKIST

Lecture 9: Stokes Formalism

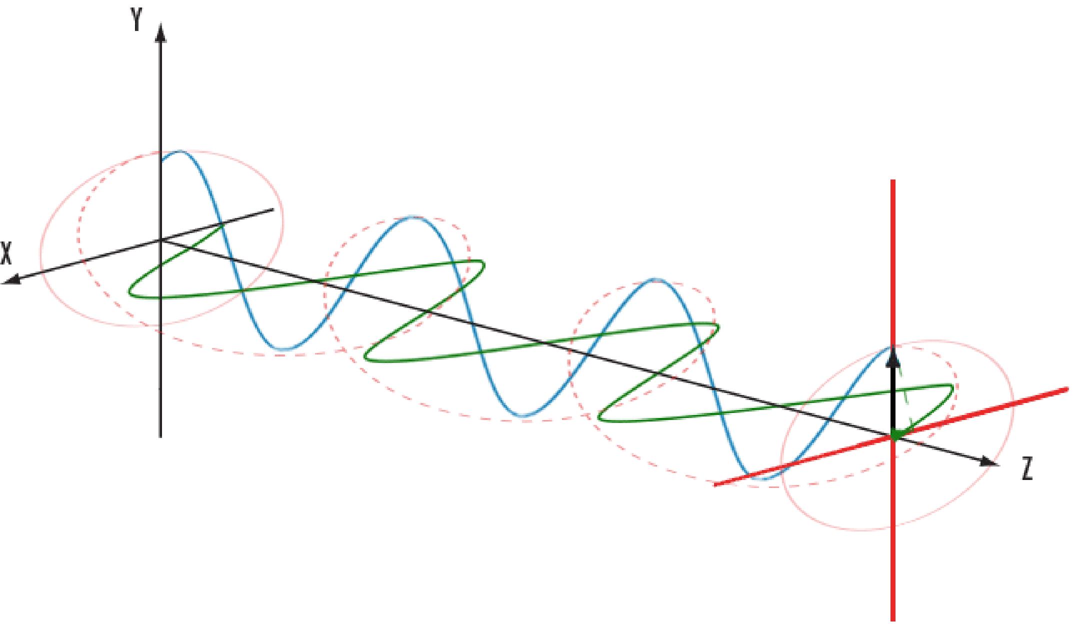
Ivan Milic *ivan.milic@colorado.edu*



Previous classes

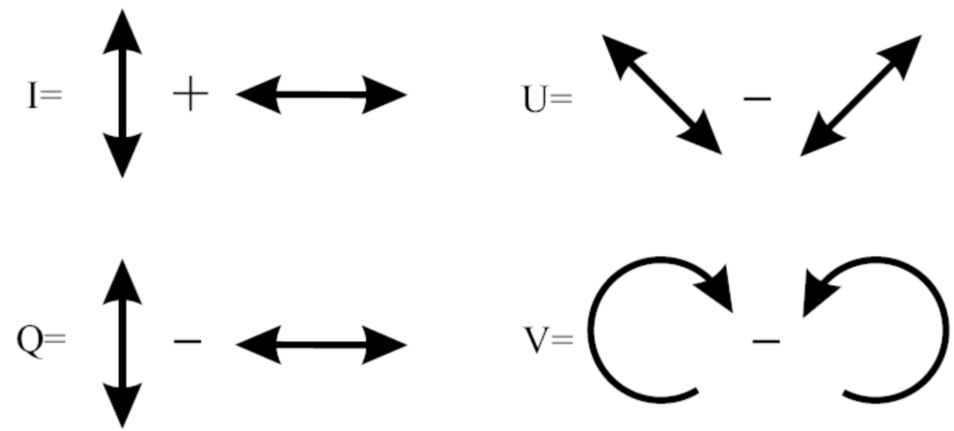
- We learned what specific intensity is
- We learned how to probe the **spatial dependency** (high spatial resolution observations)
- We learned how to probe the **wavelength (freq) dependency** (high spectral resolution observations)
- But, we only counted the number of photons.
- Because light is a transversal wave, we can also probe the polarization of the light.

Usually I just show this slide – let’s dissect this a bit!



Credits: www.edmundoptics.com

Stokes parameters:



Let’s dissect this a bit

- E always perpendicular to B
- We can have two perpendicular electric field propagating along the same direction without impeding each other!

$$E_x = A_x e^{i(kz-\omega t+\delta_x)}$$

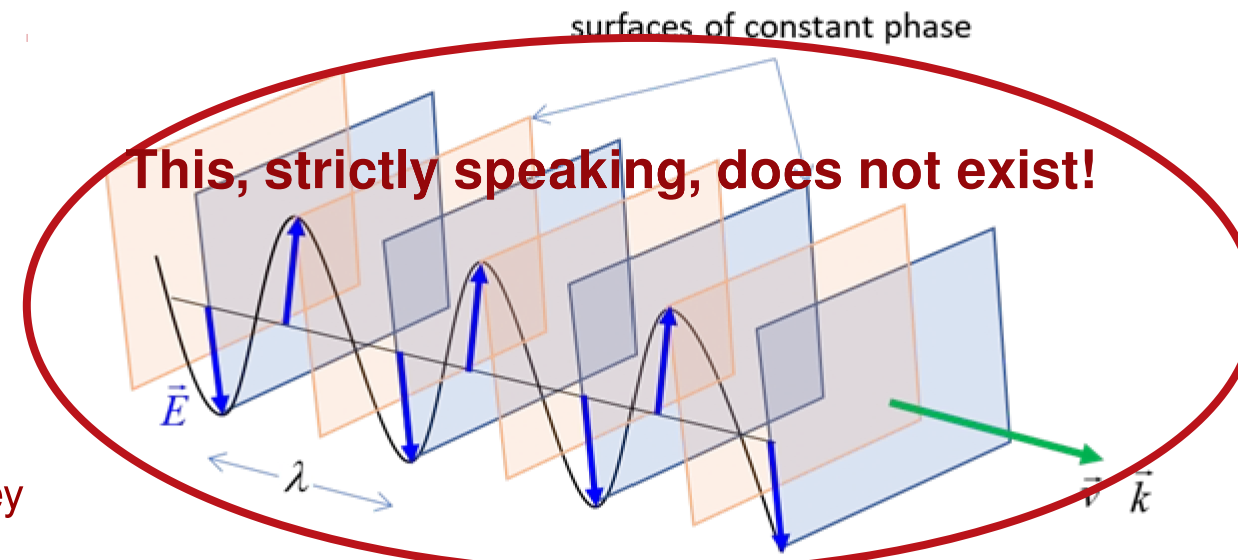
$$E_y = A_y e^{i(kz-\omega t+\delta_y)}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$



Credits: University of Sydney

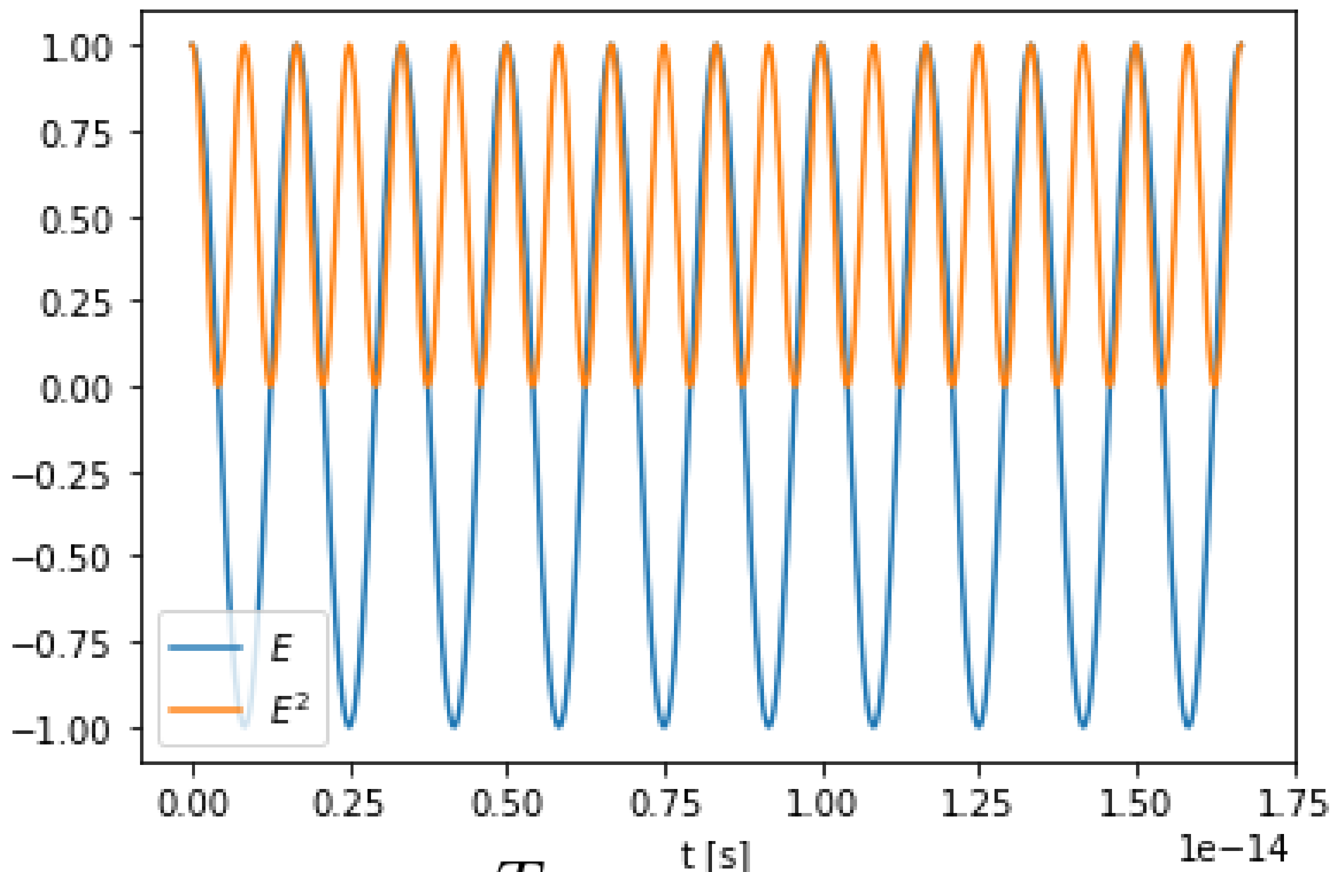
So, we can always separate our EM wave into two components

$$E_x = A_x e^{i(kz-\omega t+\delta_x)} \qquad E_x = A_x e^{i(kz-\omega t)}$$

$$E_y = A_y e^{i(kz-\omega t+\delta_y)} \qquad E_y = A_y e^{i(kz-\omega t+\Delta)}$$

- So total of ~~four~~ three quantities that completely describe the vectorial state of the light, two amplitudes, and the difference between the phases.
- Why does the absolute phase not matter?
- Strictly speaking, we neglected the vectorial nature so far when we talked about imaging and spectroscopy
- For full treatment, see Born & Wolf

Reminder : intensity of an EM wave



$$I = \lim_{T \to \infty} \frac{1}{T} \int_0^T E(t) E^*(t) dt = \frac{1}{2} E_0 E_0^*$$

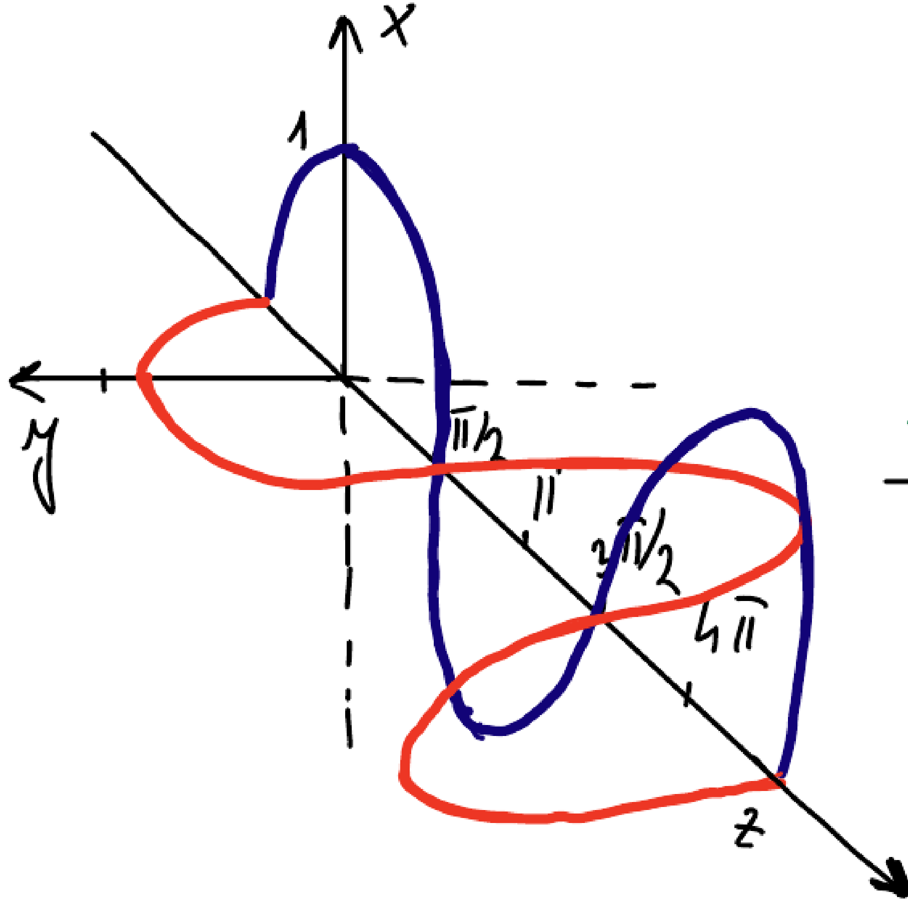
Convince yourself that all these waves have the same intensity

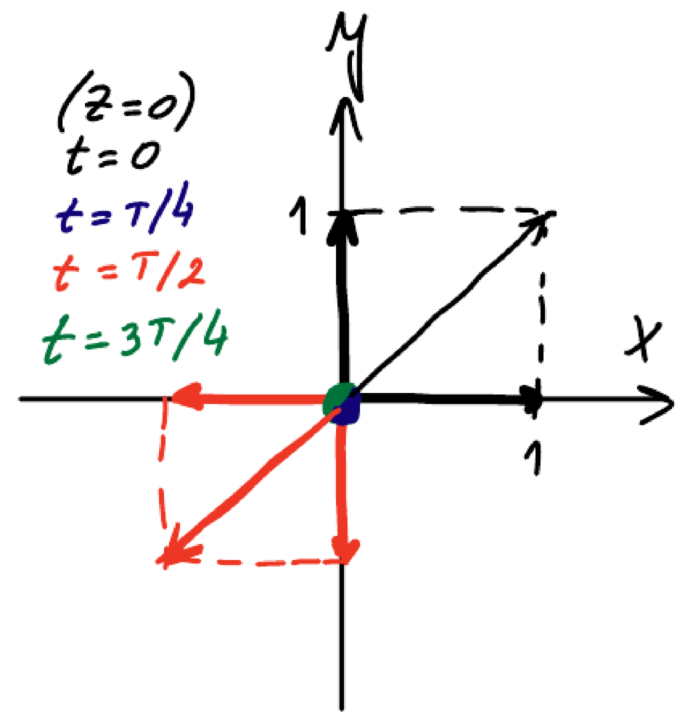
$$\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t)}\vec{e}_y$$

$$\vec{E} = \sqrt{2}e^{i(kz-\omega t)}\vec{e}_x$$

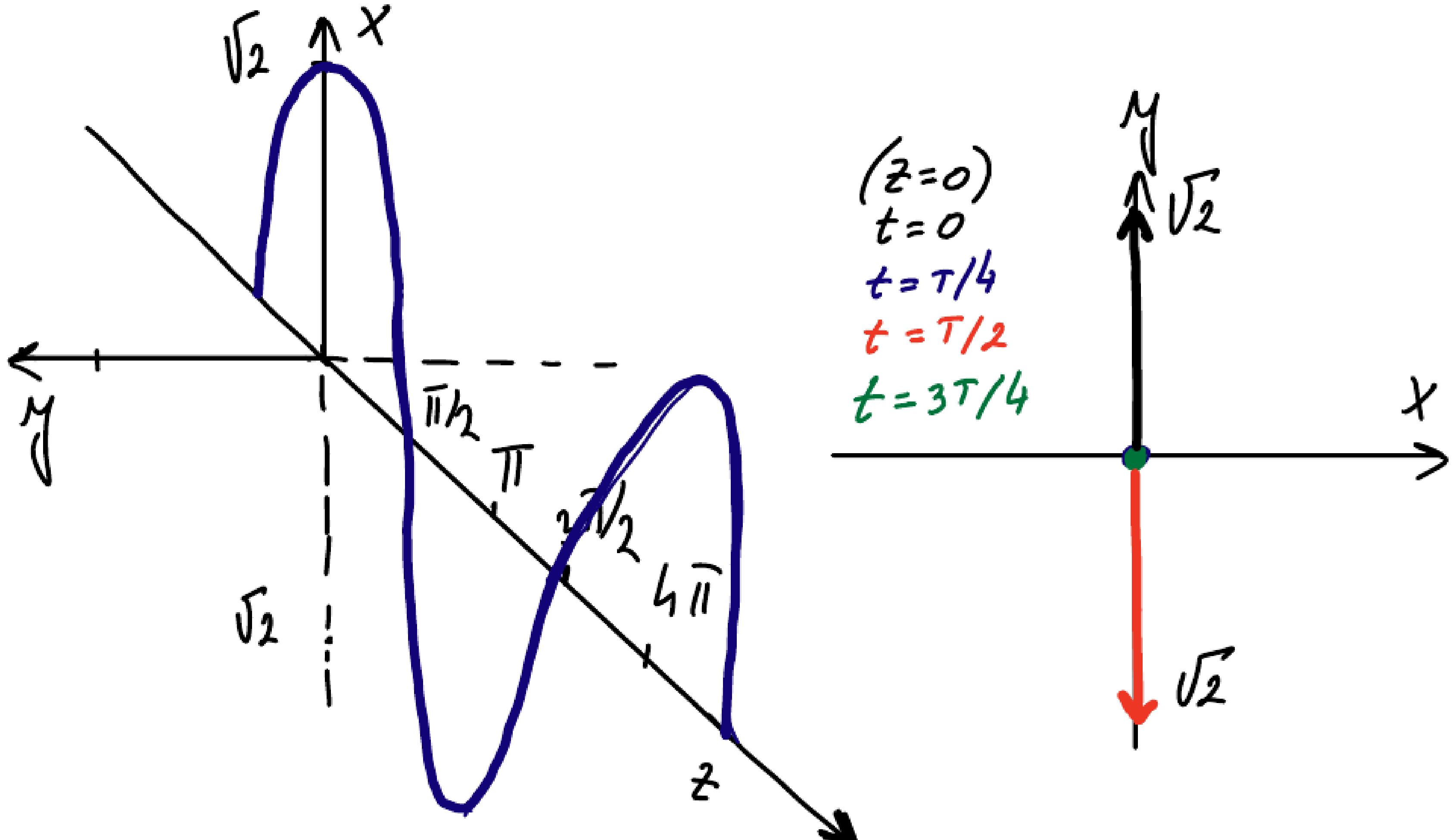
$$\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t+\pi/2)}\vec{e}_y$$

Let's sketch them $\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t)}\vec{e}_y$

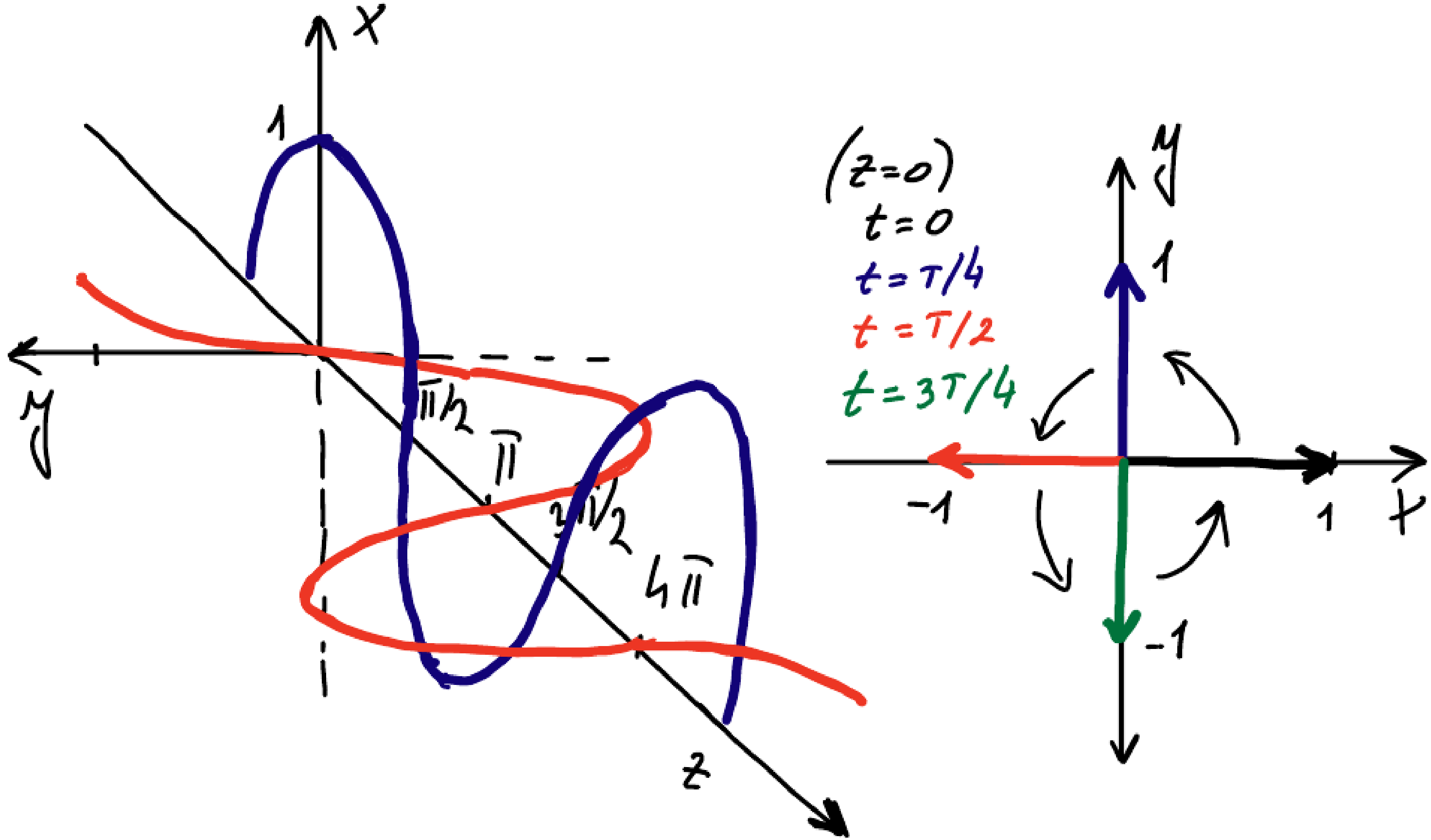




Let's sketch them $\vec{E} = \sqrt{2}e^{i(kz-\omega t)}\vec{e}_x$

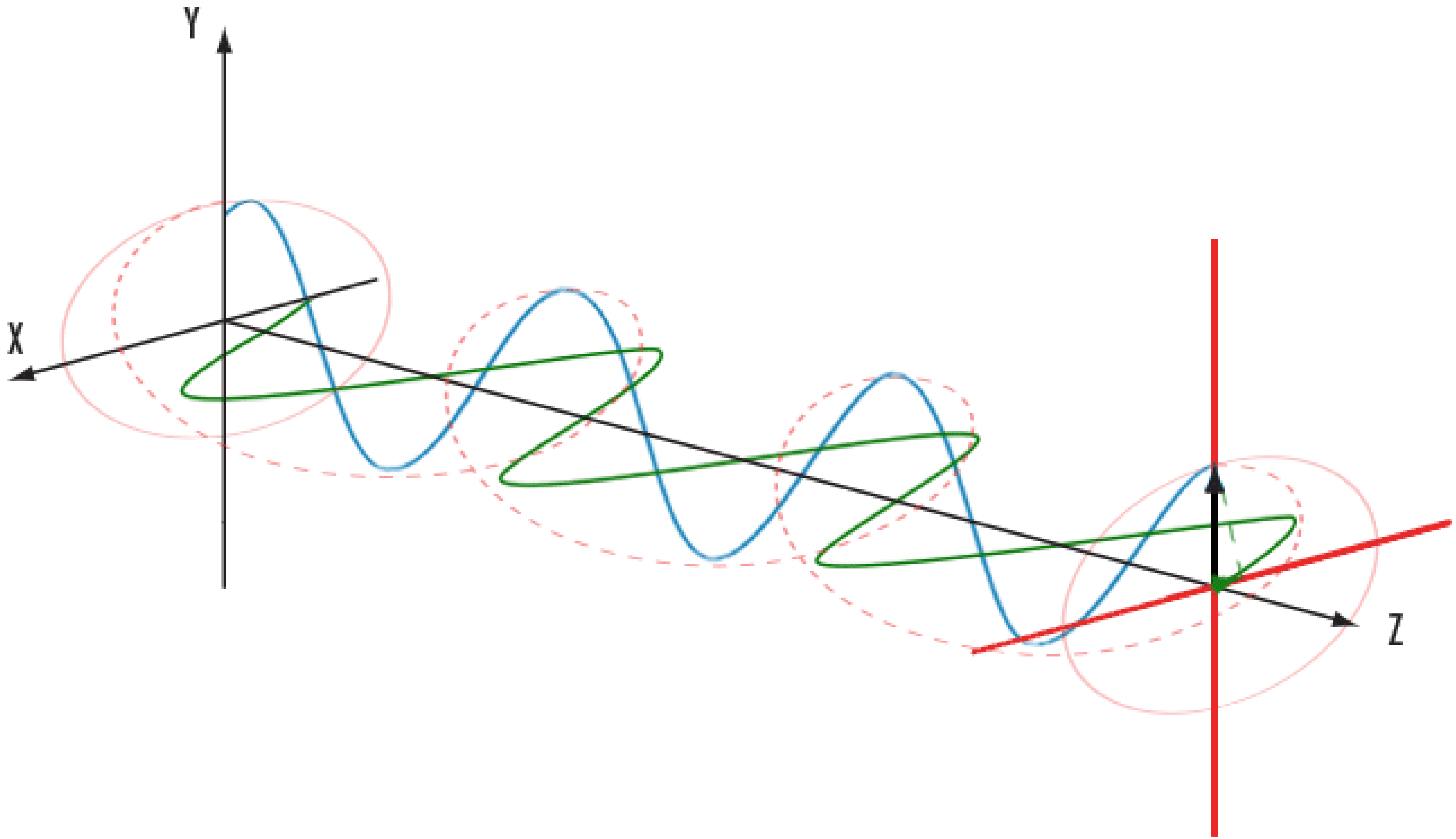


Let's sketch them $\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t+\pi/2)}\vec{e}_y$



This is what we call 'circular' polarization!

So now this part is a bit clearer...



Jones formalism

- Description of a harmonic, monochromatic plane wave (quite ideal situation)
- We don’t really have these in nature
- We don’t care about *t* and *z*, we only care about the amplitude and the phase (akka complex amplitude)

$$\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t)}\vec{e}_y = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

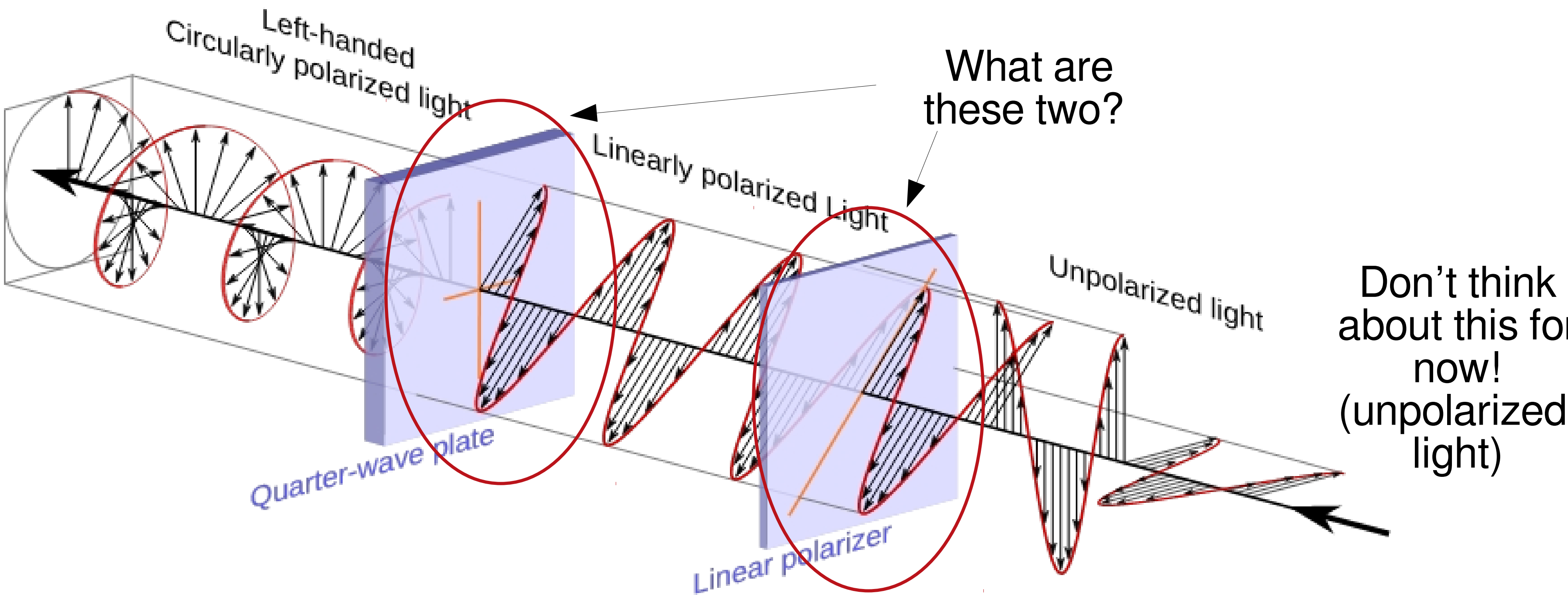
$$\vec{E} = \sqrt{2}e^{i(kz-\omega t)}\vec{e}_x = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}$$

$$\vec{E} = e^{i(kz-\omega t)}\vec{e}_x + e^{i(kz-\omega t+\pi/2)}\vec{e}_y = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

It’s pretty cool, because it teaches us to stop caring about things we can’t measure!

How do we make a circularly polarized wave?

How do we make a circularly polarized wave?



Credits: Wikipedia

Since amplitude and the phase are what matters

- Important optical elements are the ones that can change these two
- **Linear polarizers:** Only transmit electric field along the given direction (i.e. "project" the electric fields on the plane of the polarizer)
- **Retarders:** Add some amount of phase to one component but not the other one (quarter wave plate, half-wave plate).
- Slow axis : one that gets some amount of extra phase.
- So, to get from linearly polarized at 45 degrees to circularly polarized you really do only need extra pi/2 in the complex amplitude?

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Quarter-wave plate in Jones formalism

So to summarize what we got so far

- Light is a vector
- Characterized by three numbers (at least for harmonic, plane, EM wave)
- Specific combination of amplitude and phase carries some physical information
- Therefore, it is in our interest to measure these, how to do that?
- First, let’s change formalism, so we don’t have to learn everything twice

Stokes formalism applied to monochromatic waves

Total Intensity $$I = \frac{1}{2}(A_x^2 + A_y^2)$$

Linear polarization
$$Q = \frac{1}{2}(A_x^2 - A_y^2)$$
$$U = \frac{1}{2}(2A_x A_y \cos\delta)$$

Circular polarization $$V = \frac{1}{2}(2A_x A_y \sin\delta)$$

$$I^2 = Q^2 + U^2 + V^2$$

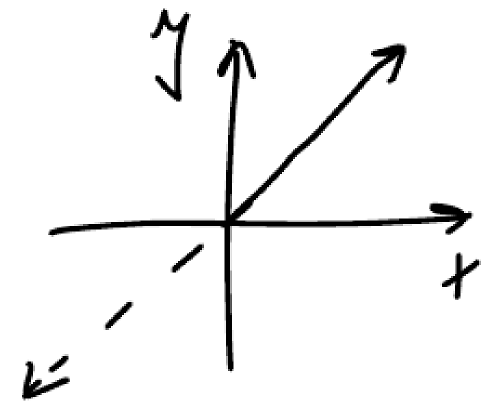
So harmonic, monochromatic, plane wave is 100% polarized, as we say

Nice, let's calculate I,Q,U,V of our test wave!

$$E_x = e^{i(kz-\omega t)}$$
$$E_y = e^{i(kz-\omega t)}$$

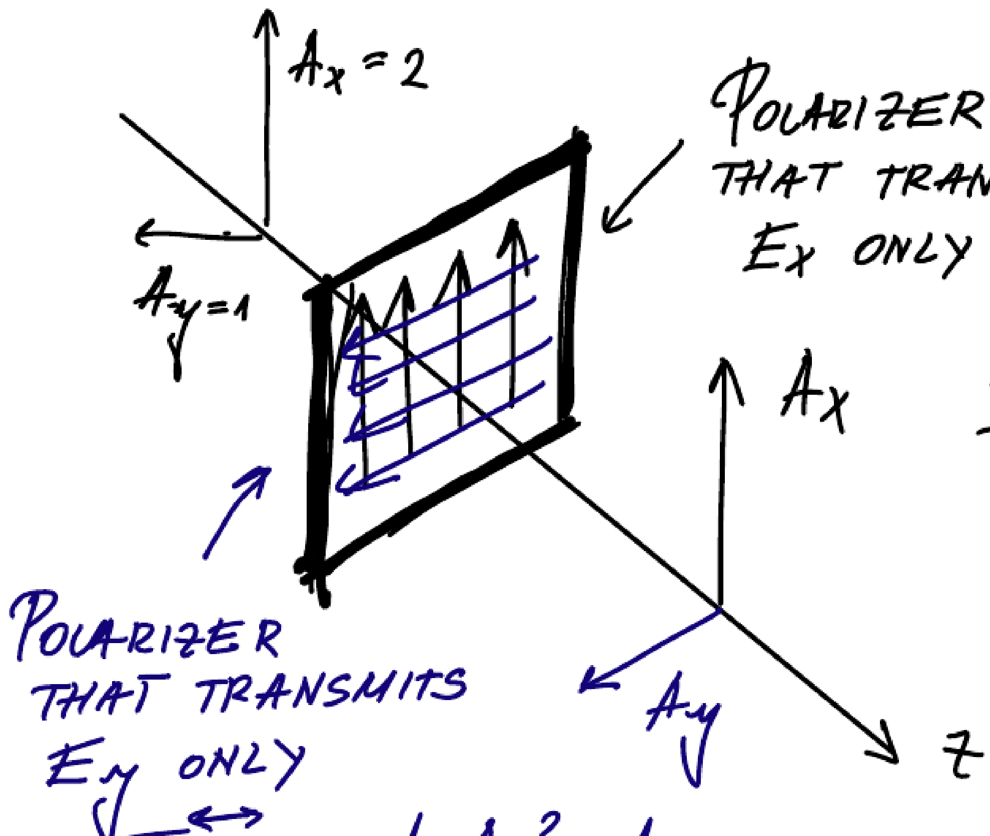
$I, Q, U, V = ?$

$$I = \frac{1}{2}(1^2 + 1^2) = 1$$
$$Q = \frac{1}{2}(1^2 - 1^2) = 0$$
$$U = \frac{1}{2} \cdot 2 \cdot 1 \cdot 1 \cdot \cos(\emptyset) = 1$$
$$V = \frac{1}{2} \cdot 2 \cdot 1 \cdot 1 \cdot \sin(\emptyset) = 0$$



(RECALL THIS!)

Ok, now, let's measure linear polarization...



$$I^{\updownarrow}_{meas} = \frac{1}{2} A_x^2 = 2$$

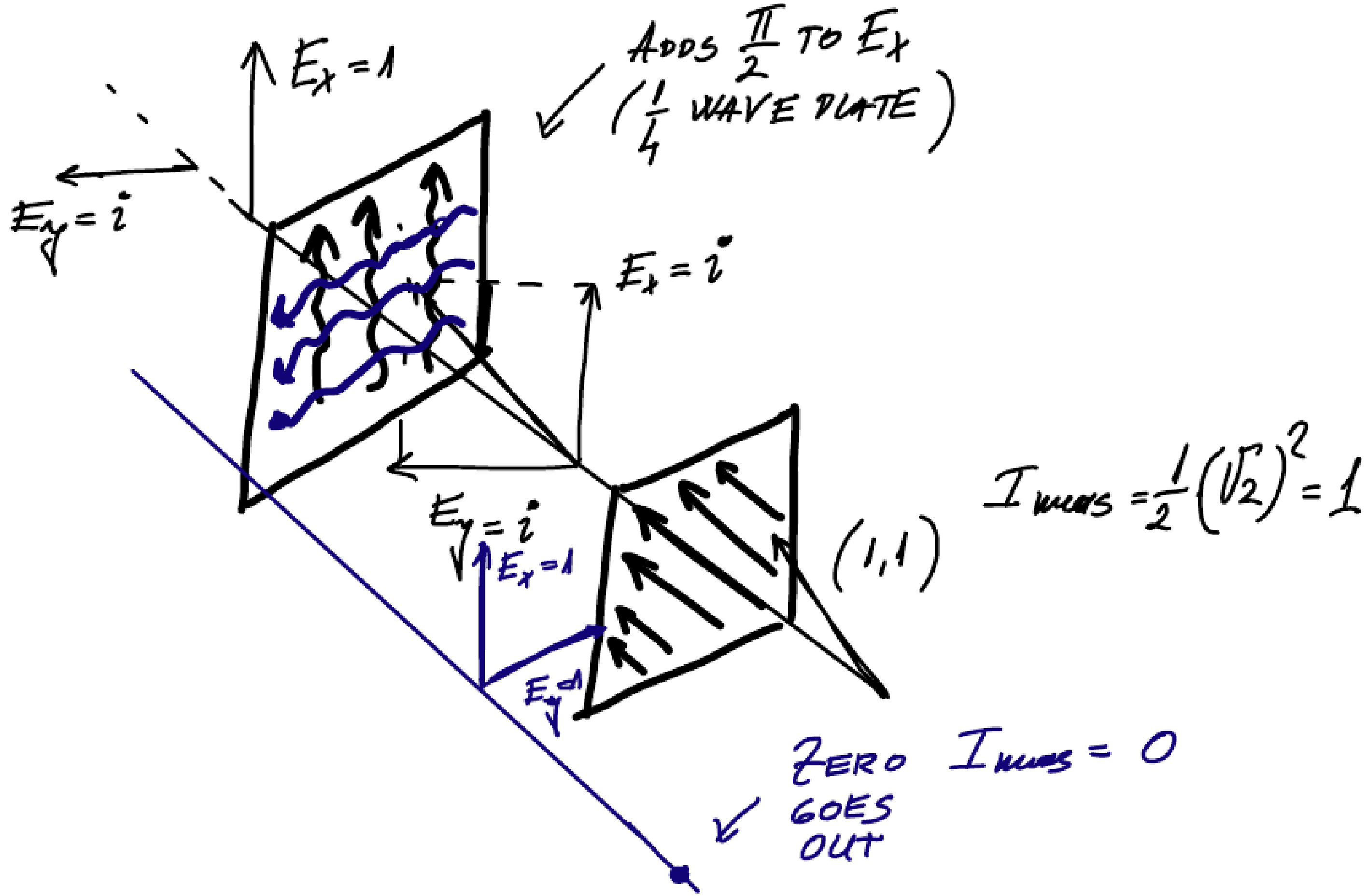
$$I^{\leftrightarrow}_{meas} = \frac{1}{2} A_y^2 = \frac{1}{2}$$

$$I = I^{\updownarrow}_{meas} + I^{\leftrightarrow}_{meas} = \frac{1}{2}\left(A_x^2 + A_y^2\right) = 2.5$$

$$Q = I^{\updownarrow}_{meas} - I^{\leftrightarrow}_{meas} = \frac{1}{2}\left(A_x^2 - A_y^2\right) = 1.5$$

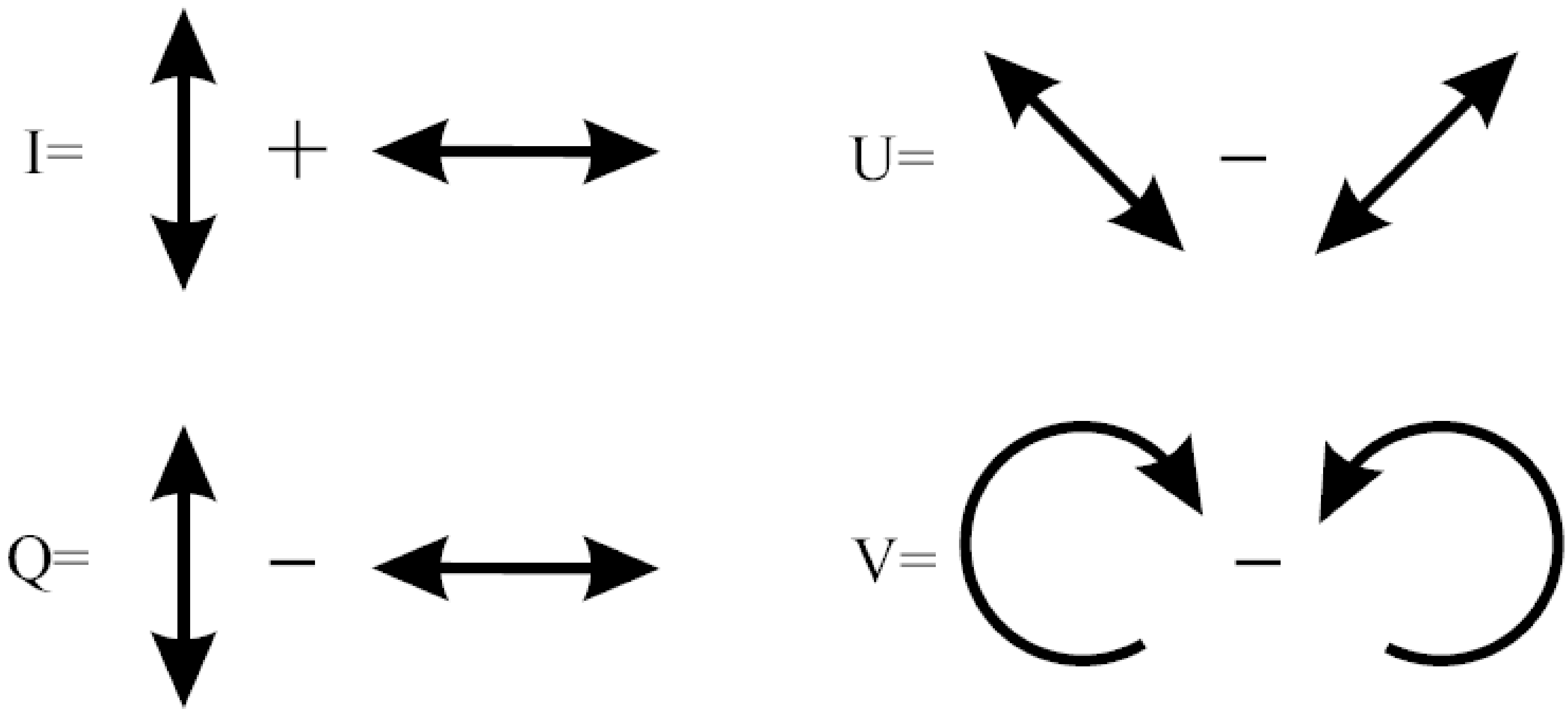
How about measuring circular polarization?

$$E^0 = \begin{pmatrix} 1 \\ i \end{pmatrix}$$



$V = \circlearrowright - \circlearrowleft = -1$ (IN THIS CASE)

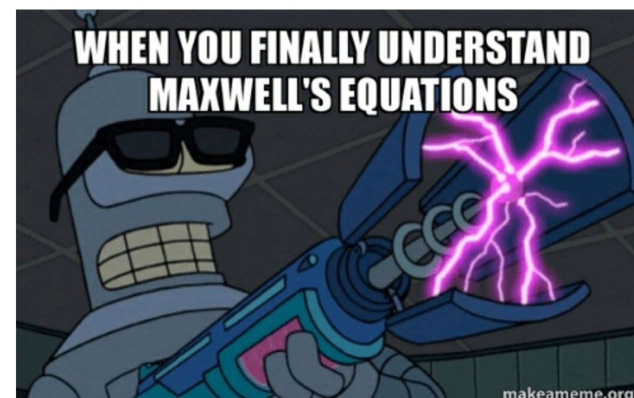
Wow, so we understand this!



Well, we do not completely

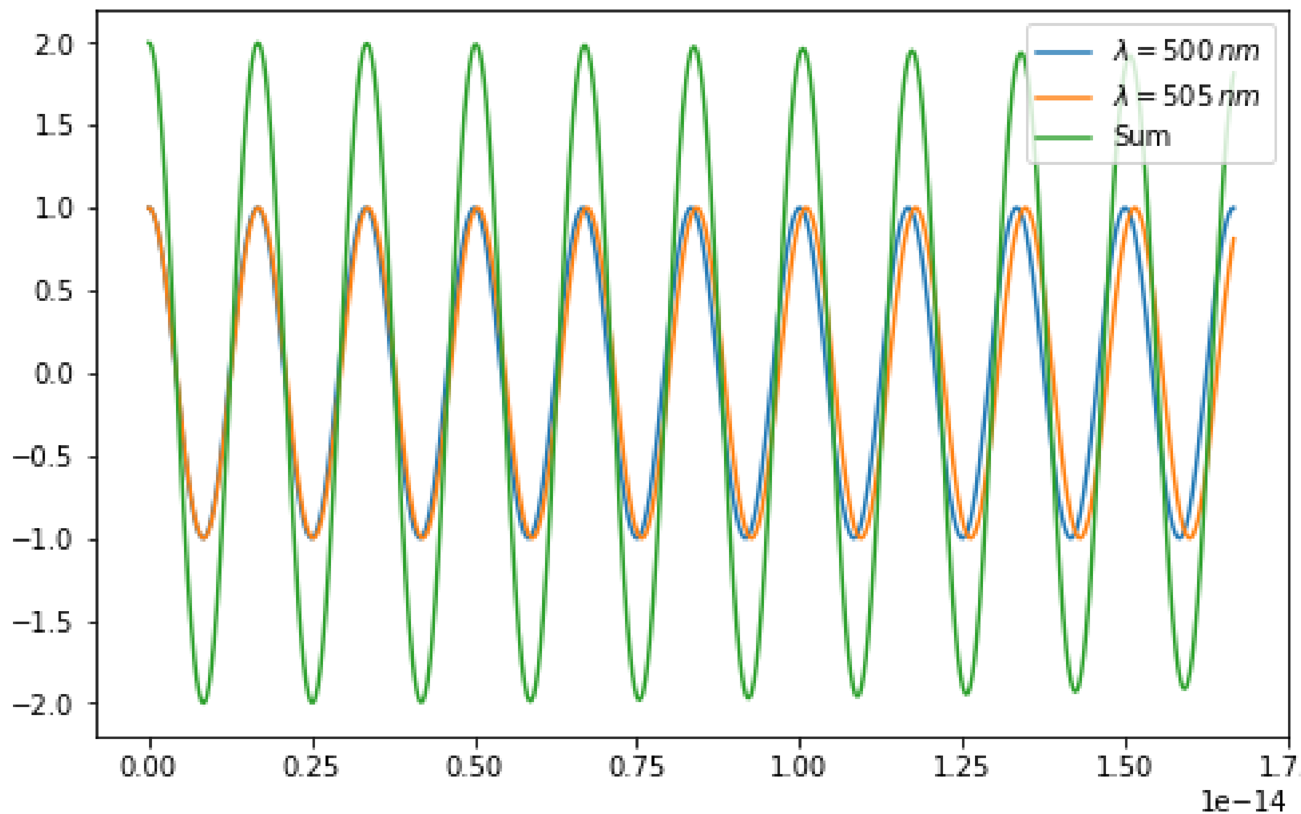
- As you might have heard, the light we measure is not 100% polarized
- We usually throw some fairly small numbers around when we talk about polarization
- Why is that?
- **Because we measure in-coherent superposition of many waves.**
- Remember, coherent – add electric fields, incoherent – add intensities
- But why Ivan, can't I just add electric fields? Maxwell Equations are linear!

Yes you can, but it does not matter since you can't measure Electric field



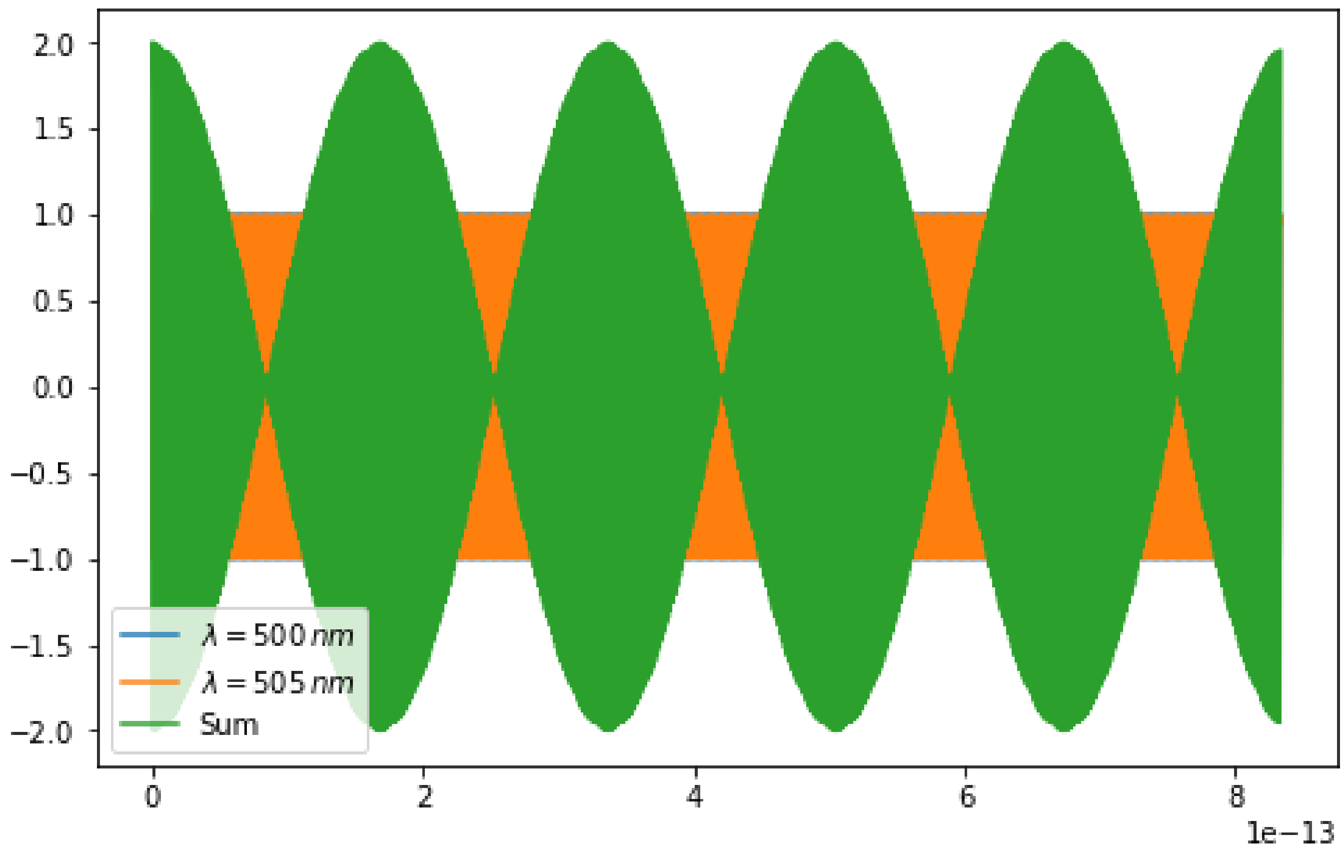
$\nabla \times E = -\partial B/\partial t$

Let’s add two waves of very similar wavelength



Intensity of the sum is ~ 2
Intensity of each individual wave is ~ 0.5
On short timescales these two behave **coherently**

Let's now go to a jupyter notebook and test what happens when we "integrate longer"



Intensity of the sum is ~ 1
Intensity of each individual wave is ~ 0.5
On **long** timescales these two behave **incoherently**

So, what is an unpolarized wave then?

(Take 5’ or more to discuss)

“Unpolarized” light

- It would be easy to suggest it is an ensemble of plane, harmonic, monochromatic waves with random phases
- You can convince yourself it is not true, as the above wave would average out
- You can think of unpolarized light either as an ensemble of plane harmonic monochromatic waves with random phases and somewhat different wavelengths
- Or, maybe even better as a set of plane, harmonic, monochromatic waves with limited durations, each duration (coherence time), is much shorter than measuring period, and random phases.
- In a way, you can think of these waves as **photons.**

So what do I,Q,U,V measure now?

- Now they measure some “average” amount of polarization

$$I = \langle A_x^2 + A_y^2 \rangle$$
$$Q = \langle A_x^2 - A_y^2 \rangle$$
$$U = \langle 2A_x A_y \cos\delta(t) \rangle$$
$$V = \langle 2A_x A_y \sin\delta(t) \rangle$$

- Where < > denotes time averaging. Now the degree of polarization is between 0 and 1.

So to summarize

- Stokes formalism describes polarization state of completely incoherent set of waves.
- Everything is linear.
- If I have few more circularly polarized waves, my V is non zero
- If I have a bit more waves oscillating in this or that plane, my Q or U are non zero
- It is basically as we measured the polarization state of each of the waves separately and then added them together.
- Is this justified? What are the typical timescales? EM wave period? Coherence time? Measurement time?

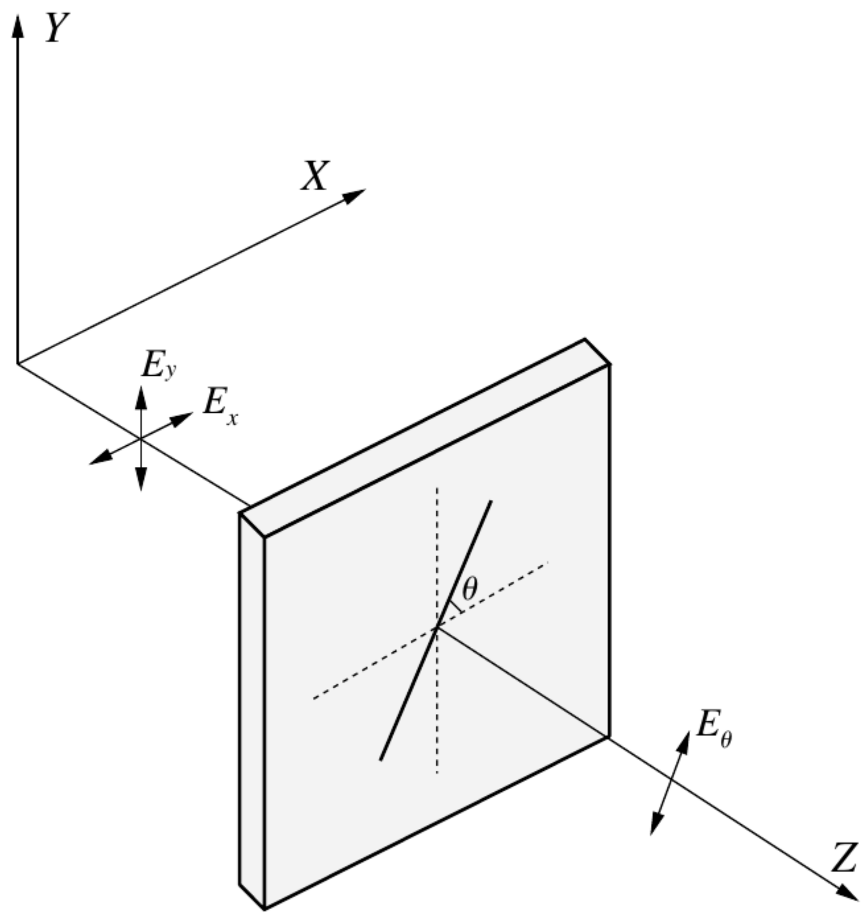
As a refresher

Polarization state	Stokes vector
Natural	$(1, 0, 0, 0)^T$
Linear at 0°	$(1, 1, 0, 0)^T$
Linear at 90°	$(1, -1, 0, 0)^T$
Linear at 45°	$(1, 0, 1, 0)^T$
Linear at 135°	$(1, 0, -1, 0)^T$
Right-handed circular	$(1, 0, 0, 1)^T$
Left-handed circular	$(1, 0, 0, -1)^T$

Credits : Introduction to Spectropolarimetry
(del Toro Iniesta, 2003)

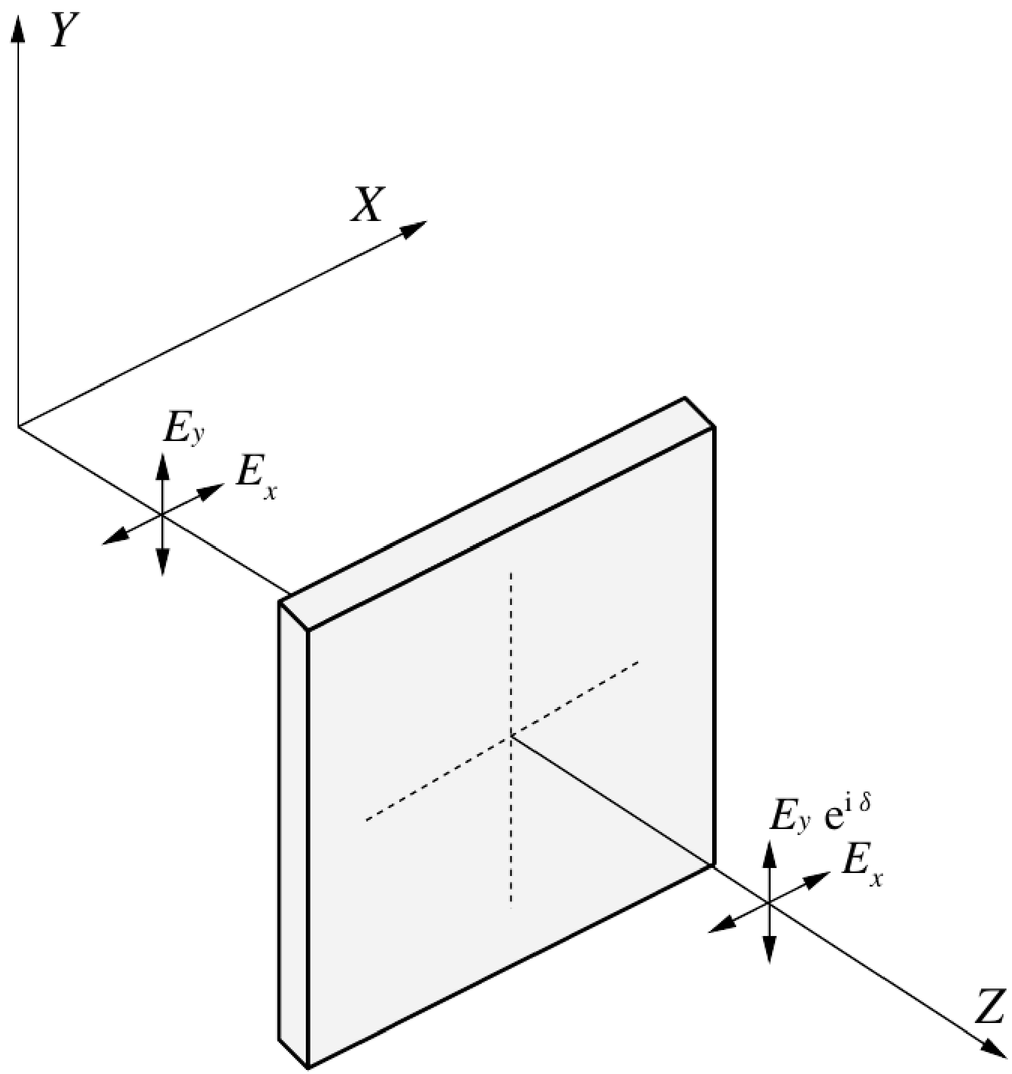
How to linear optical elements act on Stokes vector?

- Let's look at the general combination of linear analyzer and a retarder (from now on I am following Introduction to Spectropolarimetry, 2003):



$$E'_x = E_x \cos\theta; \quad E'_y = E_y \sin\theta$$

How do linear optical elements act on Stokes vector ?



$$E'_x = E_x; \quad E'_y = E_y \, e^{i\delta}$$

How do linear optical systems act on the Stokes vector?

$$I_{\text{meas}}(\theta,\delta) = \langle E_x E_x^* \cos^2\theta + E_y E_y^* \sin^2\theta + \frac{1}{2} E_x E_y^* \sin 2\theta\, \mathrm{e}^{-\mathrm{i}\delta} + \frac{1}{2} E_x^* E_y \sin 2\theta\, \mathrm{e}^{\mathrm{i}\delta} \rangle$$

$$I_{\text{meas}}(\theta,\delta) = \frac{1}{2}(I + Q\cos 2\theta + U\cos\delta\sin 2\theta + V\sin\delta\sin 2\theta).$$

By changing the angles and measuring the total intensity,
we can calculate I,Q,U,V – **modulation!**

Modulation (more in Valentin's talk)

- We change angle of the linear polarizer and the retardance of the quarter wave plate (in practice, we often rotate the waveplate) and record images that are a linear combination of the Stokes parameters. We want at least four measurements so we can solve the linear system.
- Obvious solution is to have one camera and change these angles in time – **temporal modulation**
- But we can also split our beam and do different things to the two parts – **spatial modulation**
- Hence the expressions single beam, dual beam, etc...

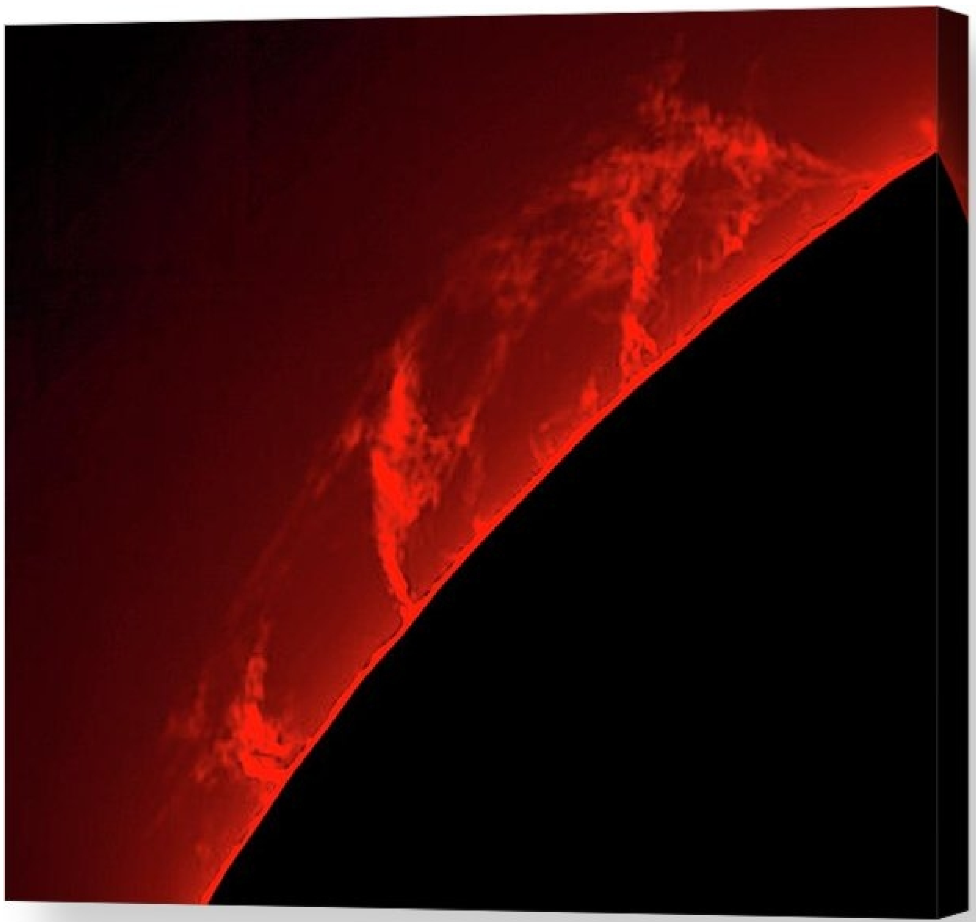
Mueller Matrices

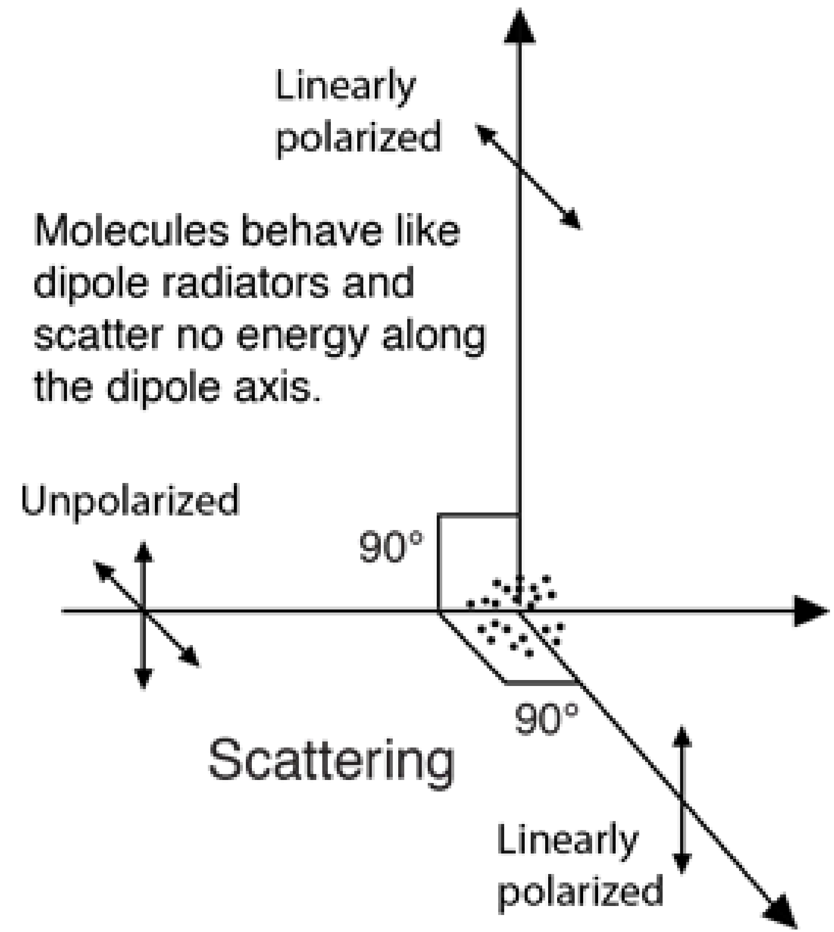
- These describe transformations of Stokes vector after interaction with matter

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \hat{M} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

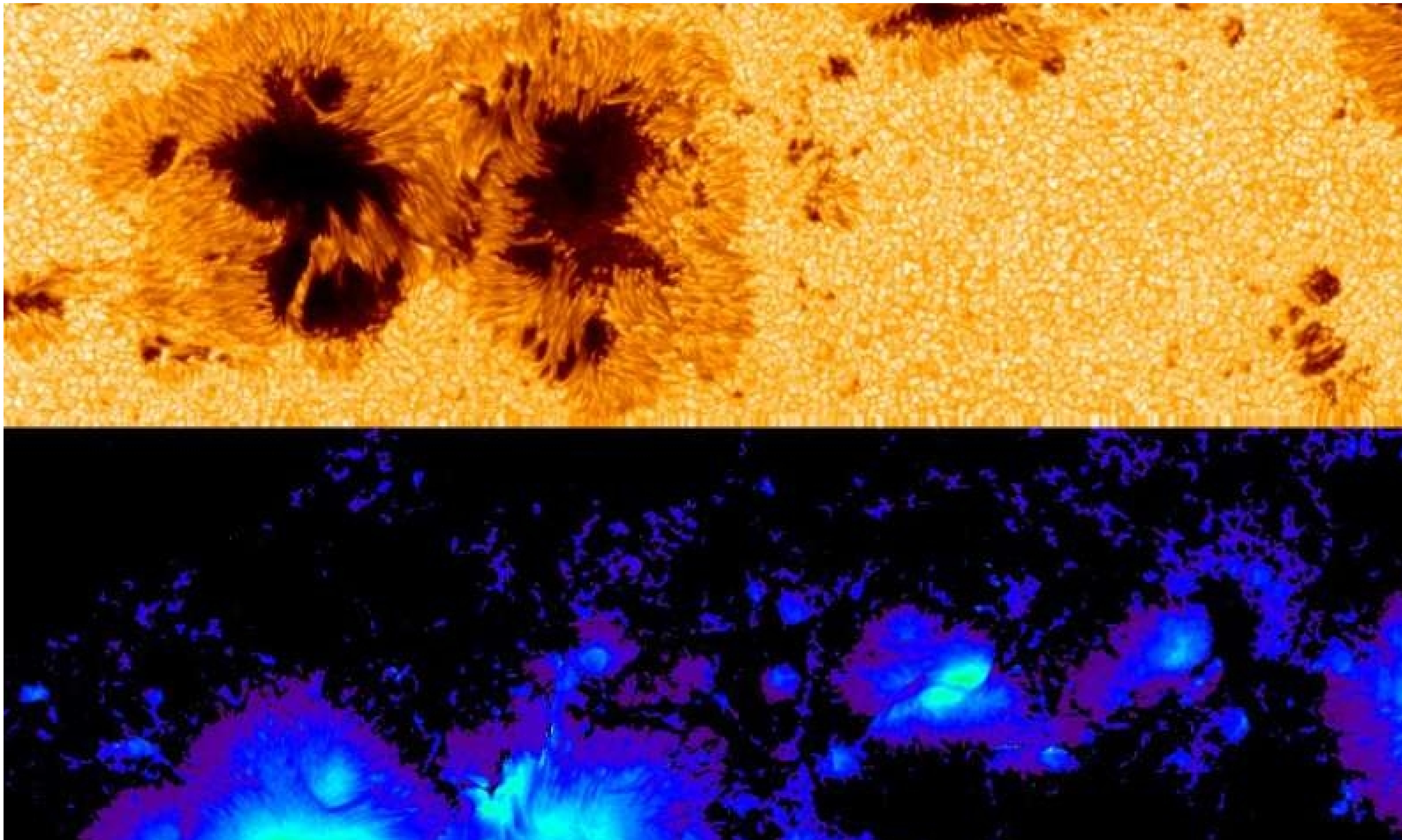
- Must transform Stokes vector to Stokes vector, hence, they must satisfy certain amount of constraints.
- You can define a Mueller matrix for your telescope, optics, even for solar atmosphere (Stokes vector changes as the light gets transported through the atmosphere)

Examples of polarization – how should a prominence be polarized?

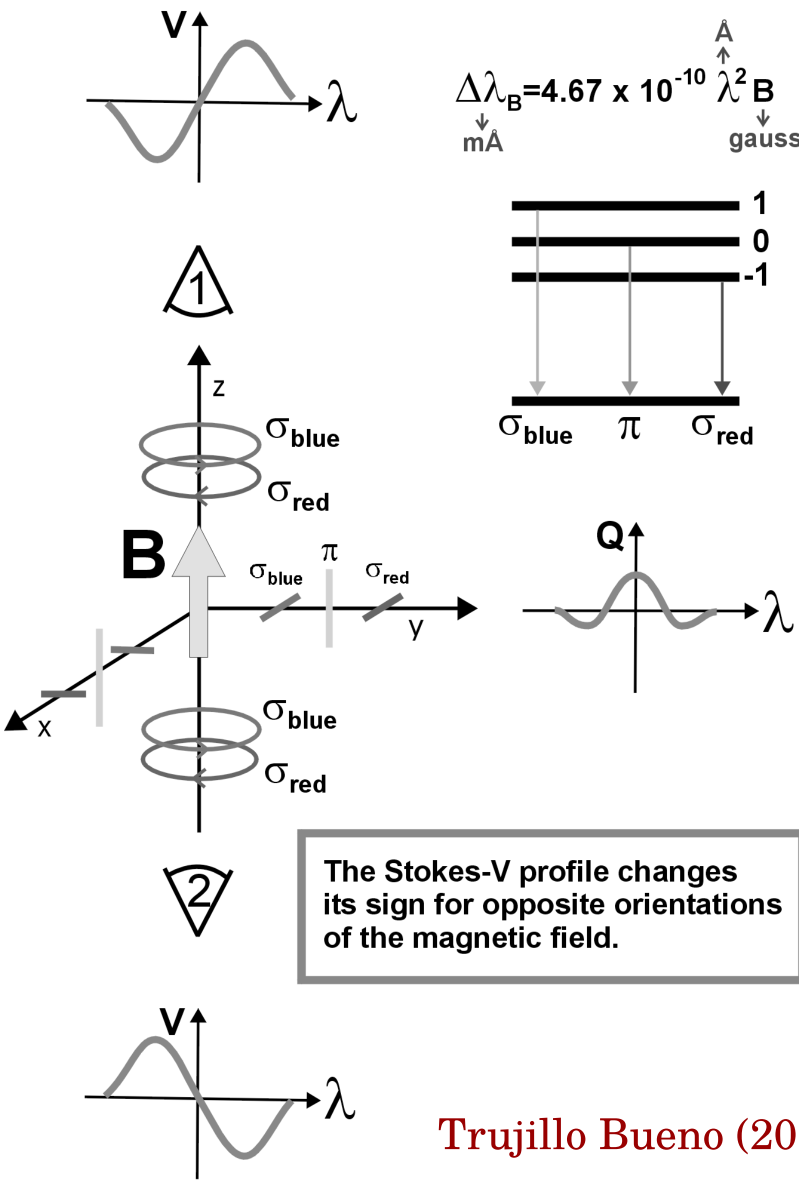




How about a sunspot?

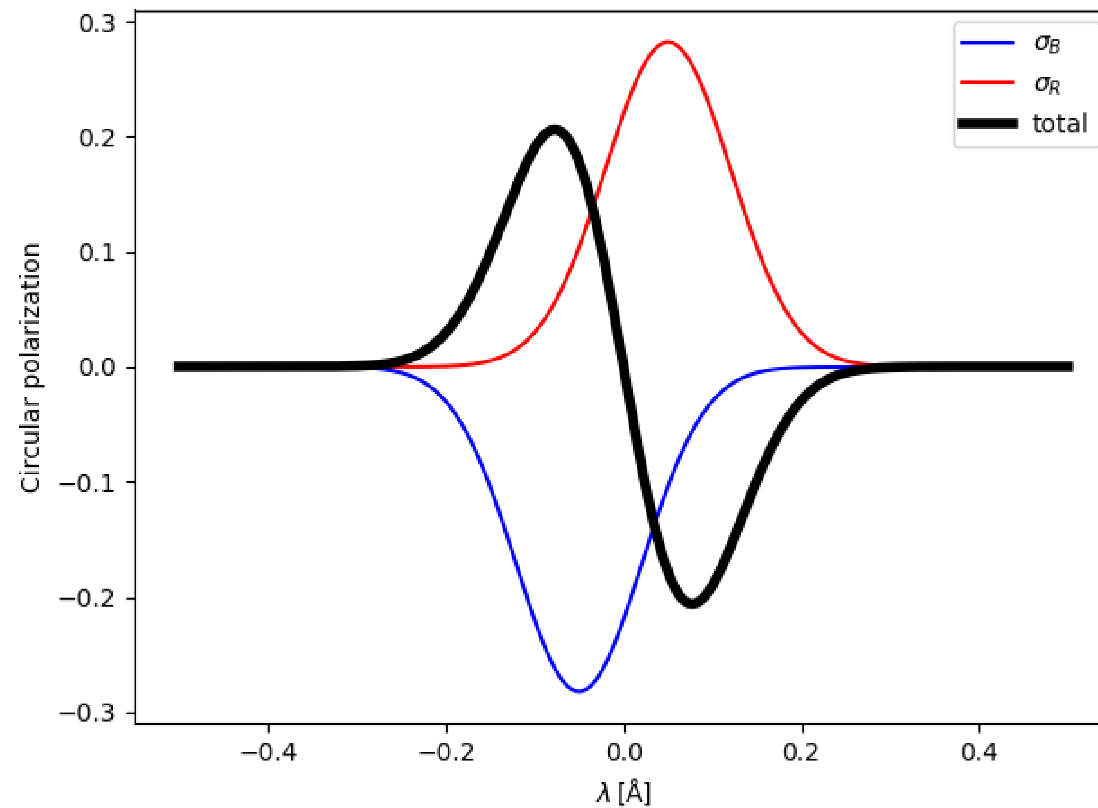


The Zeeman Effect



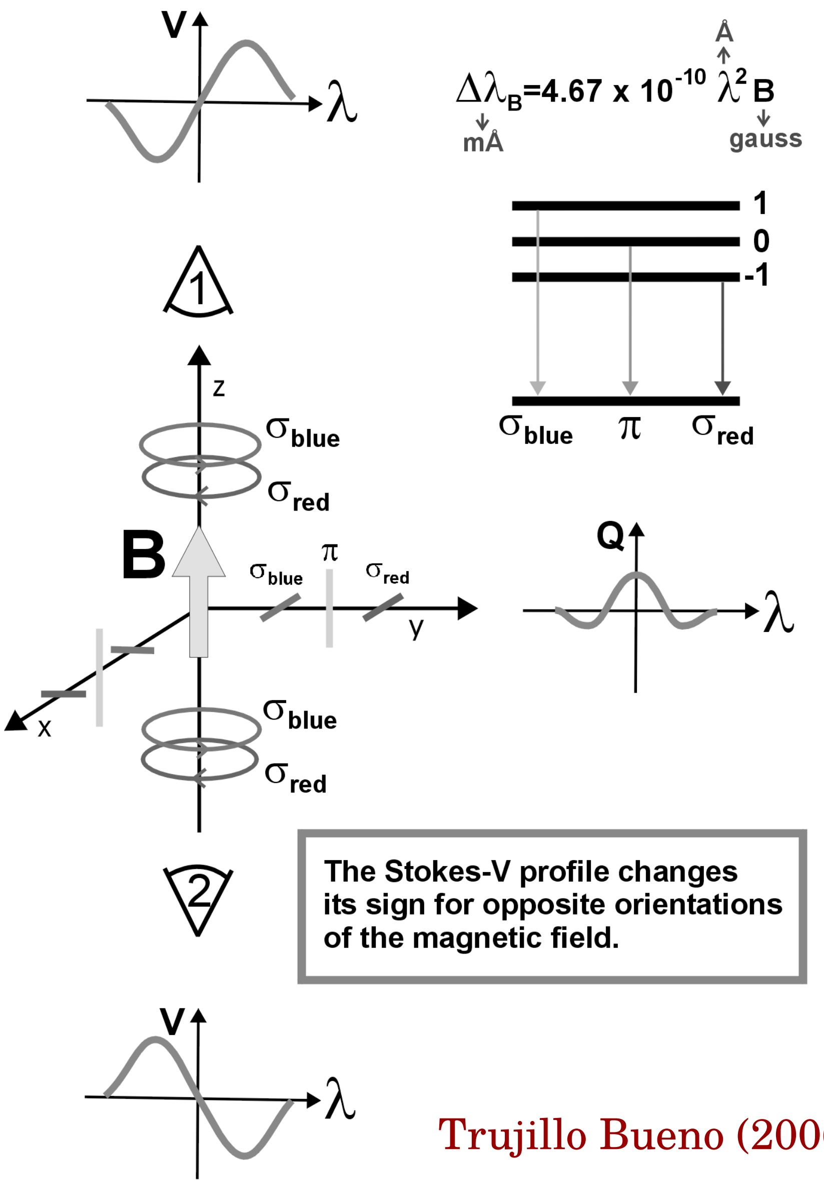
Trujillo Bueno (2006)

Why the polarization?



Why the polarization?

The Zeeman Effect



Trujillo Bueno (2006)