PHYS 7810: Solar Physics with DKIST

Lecture 3: Diffraction (Theory)

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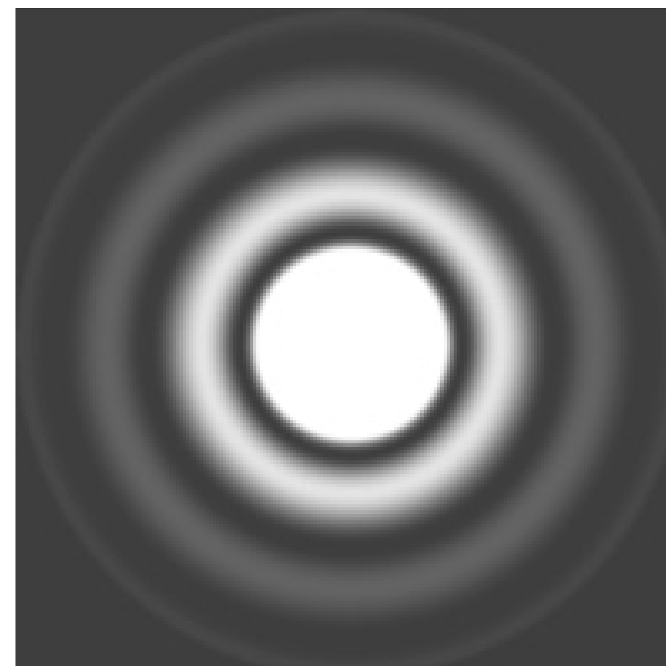


Previous classes

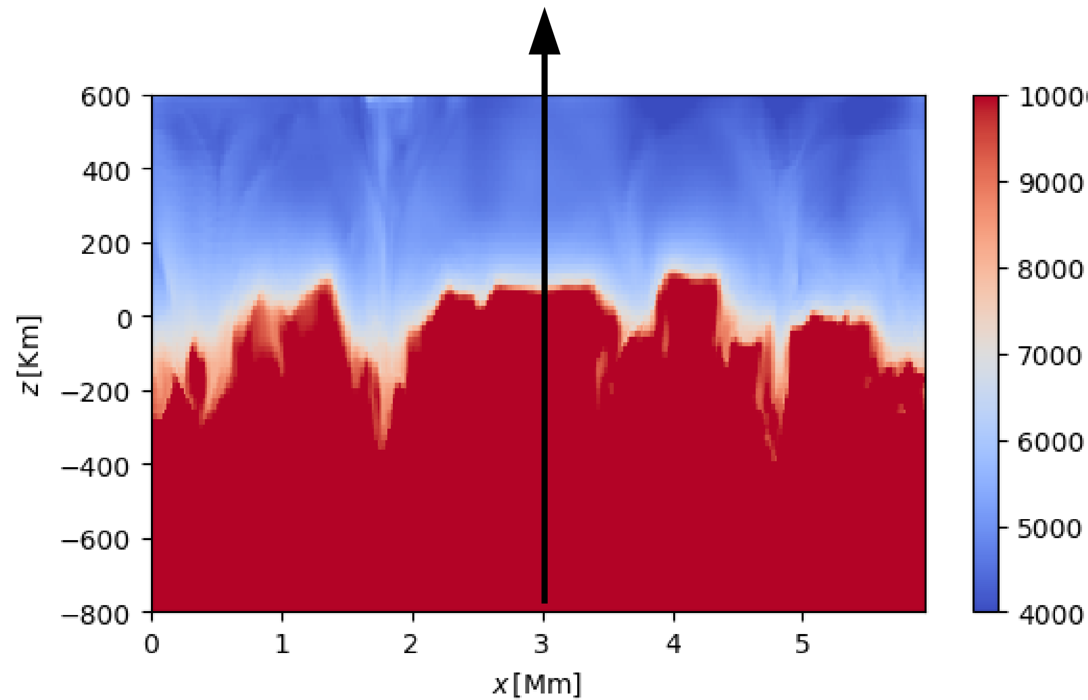
- We discussed where does the light come from – solar atmosphere
- Different wavelengths – different regions
- We want to measure specific monochromatic intensity of the light at different locations at solar surface, at different wavelengths
- Changing the size of spatial and wavelength chunks (and time chunks) changes amount of photons we get (sampling)
- Having bigger telescope allows us to have smaller chunks.
- We did not really answer the question: **Why can't we have infinitely small chunks? (why can't we sample as densely as we want).**
- After this, let's do a fast exercise we skipped on Thursday… (HINODE data).

Let's look at a datacube that can be used for science

- Open lites_qs.fits
- Visualize varios "slices" and familiarize yourself with the data
- You will need python with matplotlib, numpy and astropy

What is “the story” of the light we detect?

- Spectra formed on the way through the atmosphere.
- Light travels to the Earth’s atmosphere (we will leave this for a bit later)
- Light enters the telescope at the primary
- Light travels through our optical system
- Enters instruments, we disperse it
- CCD measures counts
- We infer from the data what we need.



Vertical temperature structure
MURAM quiet Sun simulation, courtesy of T. Riethmüller

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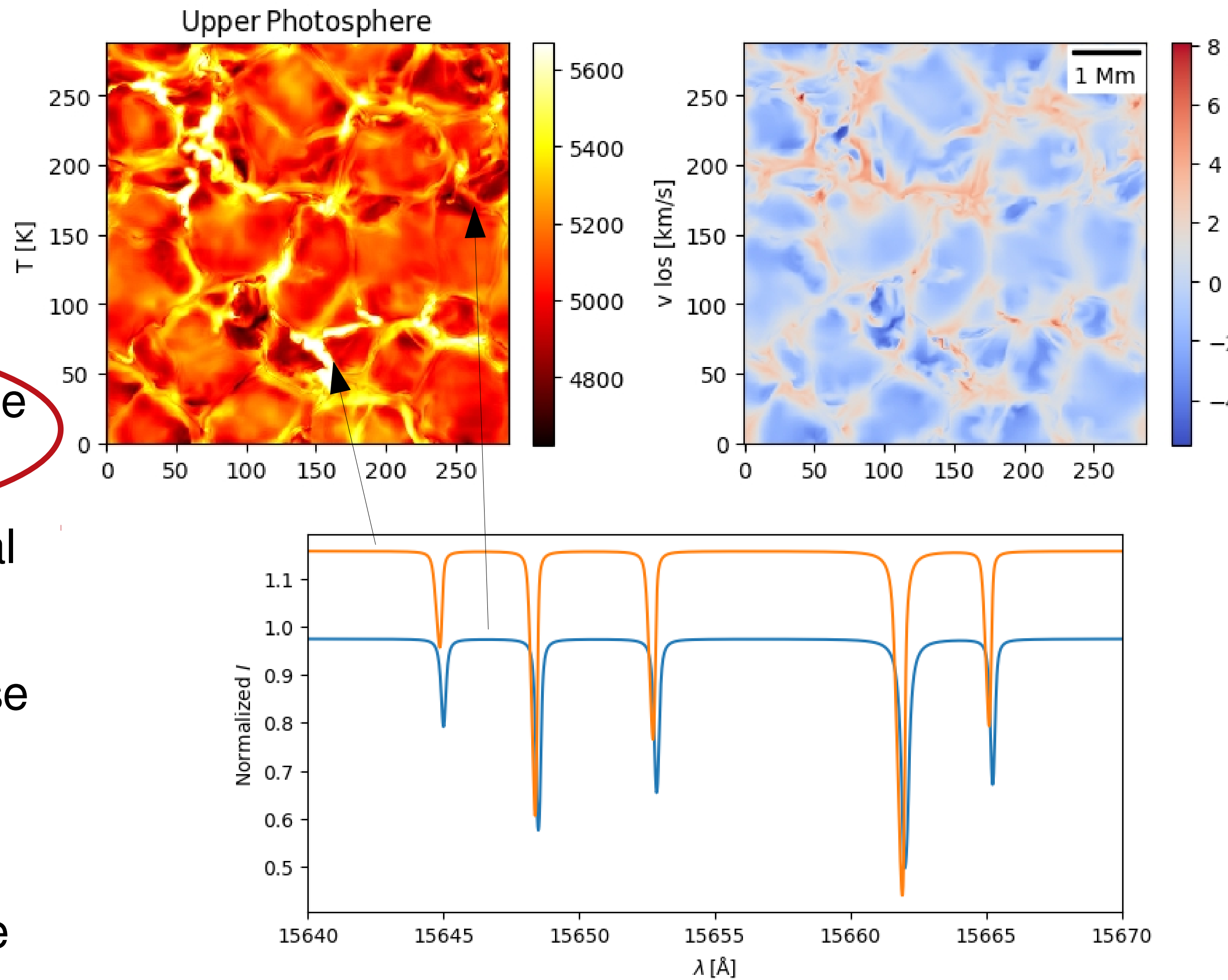


MURAM quiet Sun simulation, courtesy of T. Riethmüller

We are now going to look at the light as a wave

- We talked about telescopes in terms of **geometrical optics –** light treated as bundle of rays.
- Light is (classicaly speaking), a wave after all, and exhibits (not to say suffers) the wave properties.

$$\nabla \cdot \mathbf{E} = 0$$

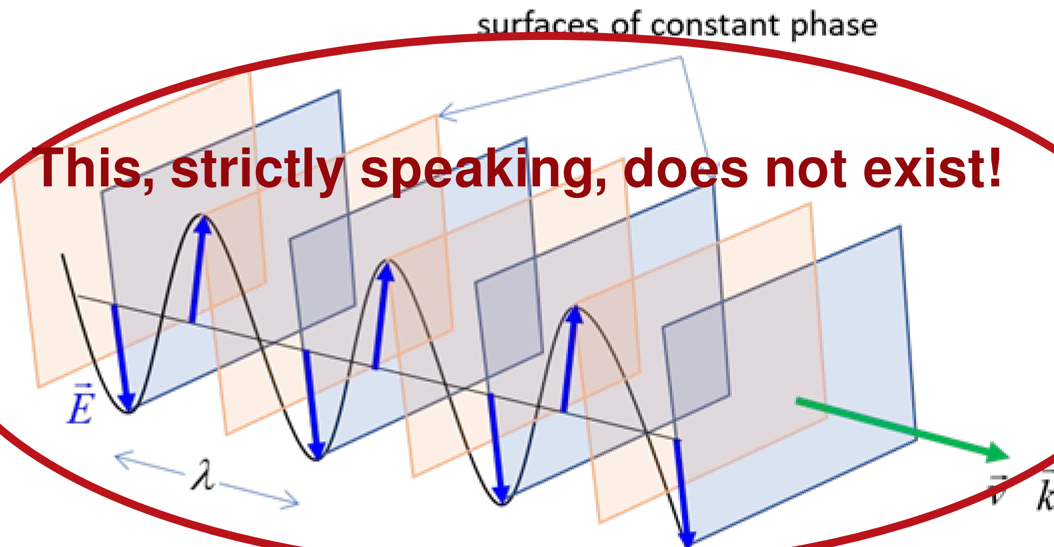
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$E_x(z) = E_{x,0} e^{i(kz-\omega t))}$$

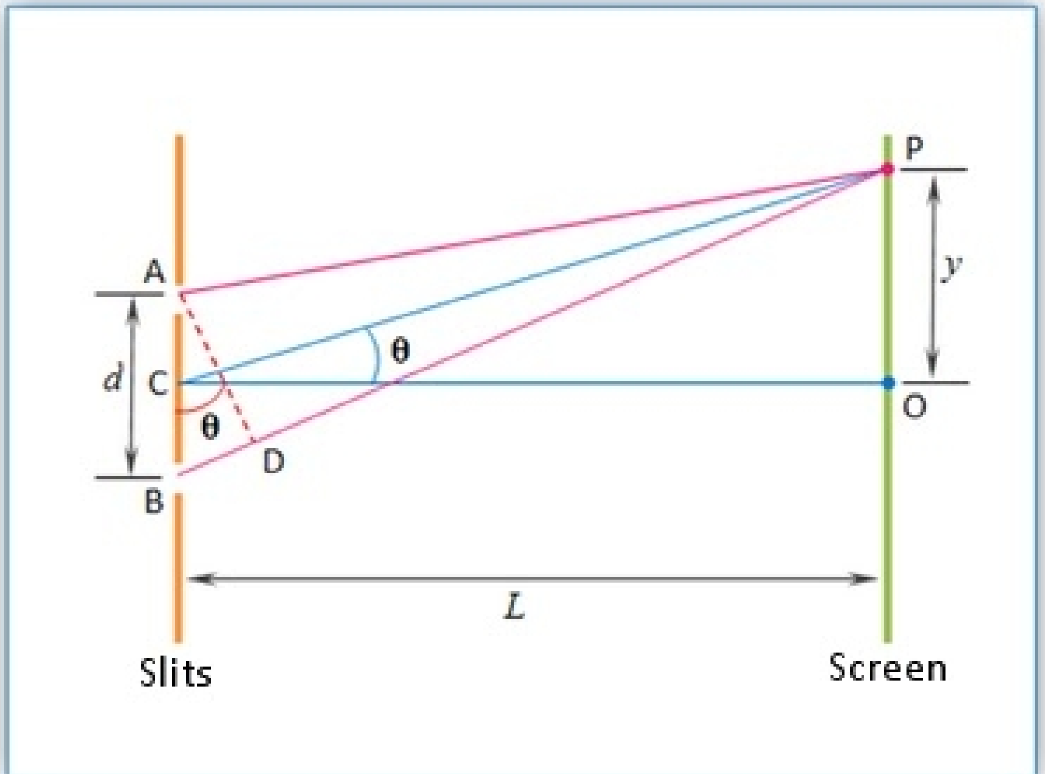
$$E_y(z) = E_{y,0} e^{i(kz-\omega t))}$$



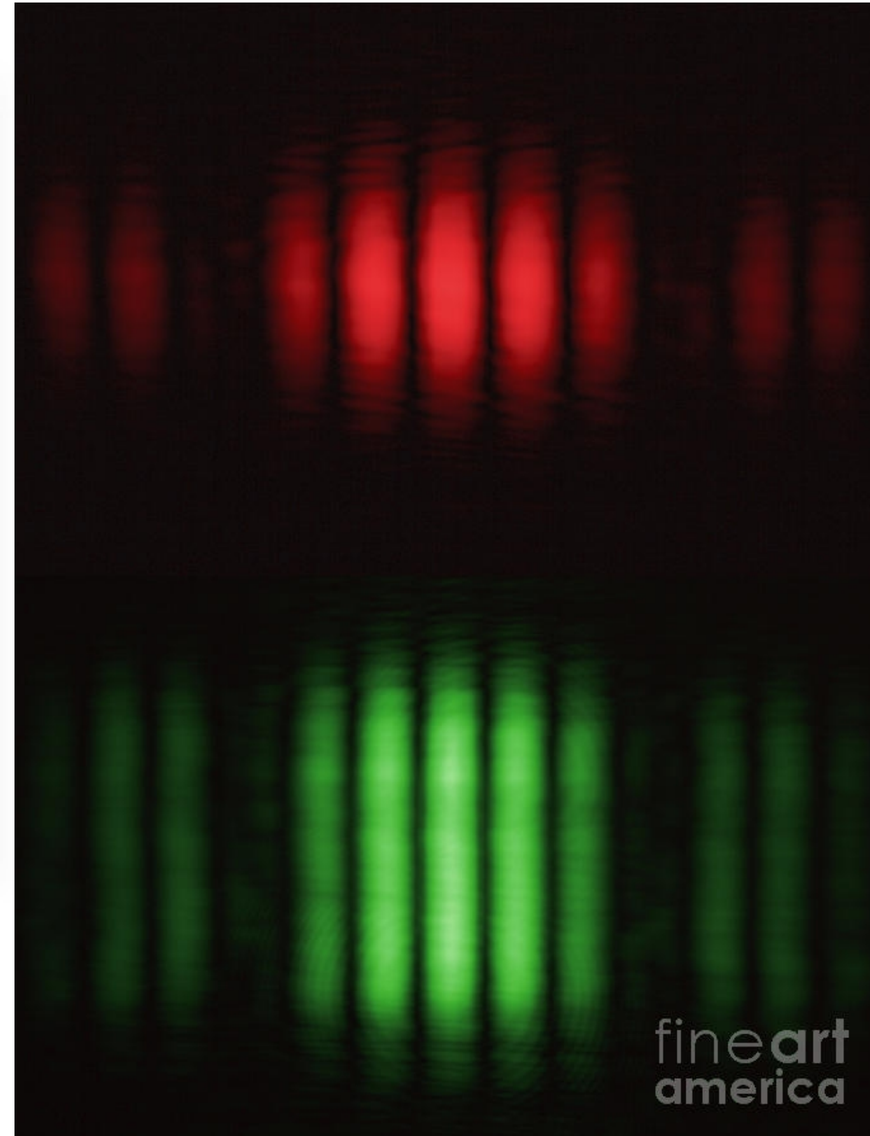
This, strictly speaking, does not exist!

Credits: University of Sydney

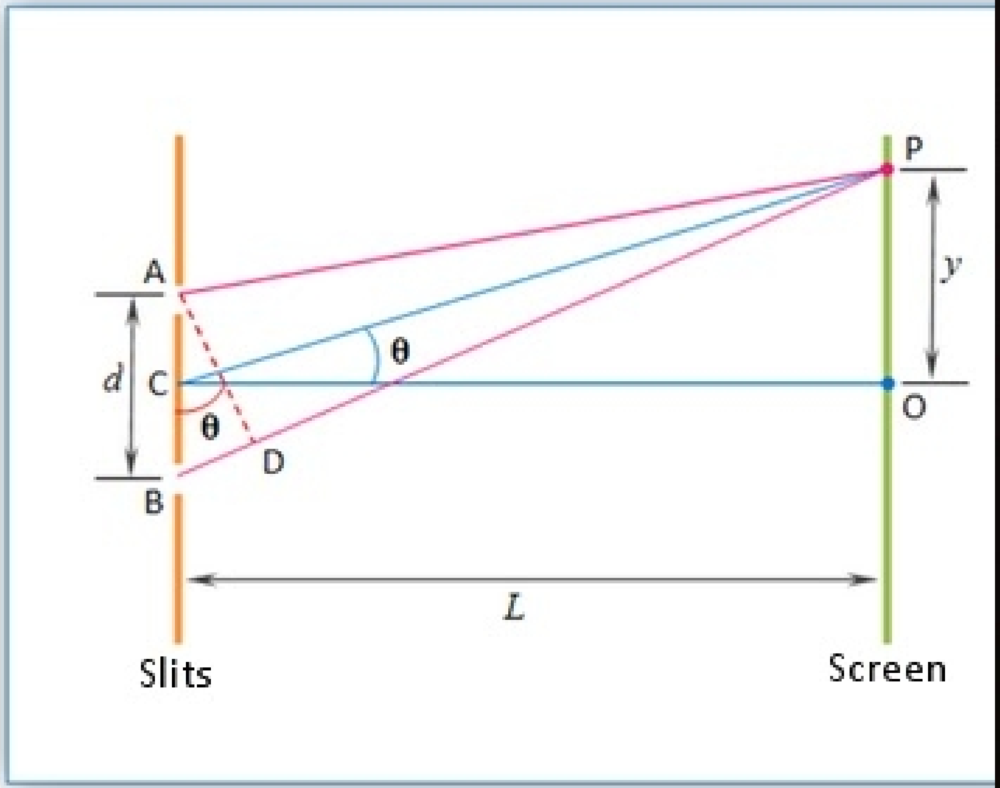
Interference (keep in mind for later)



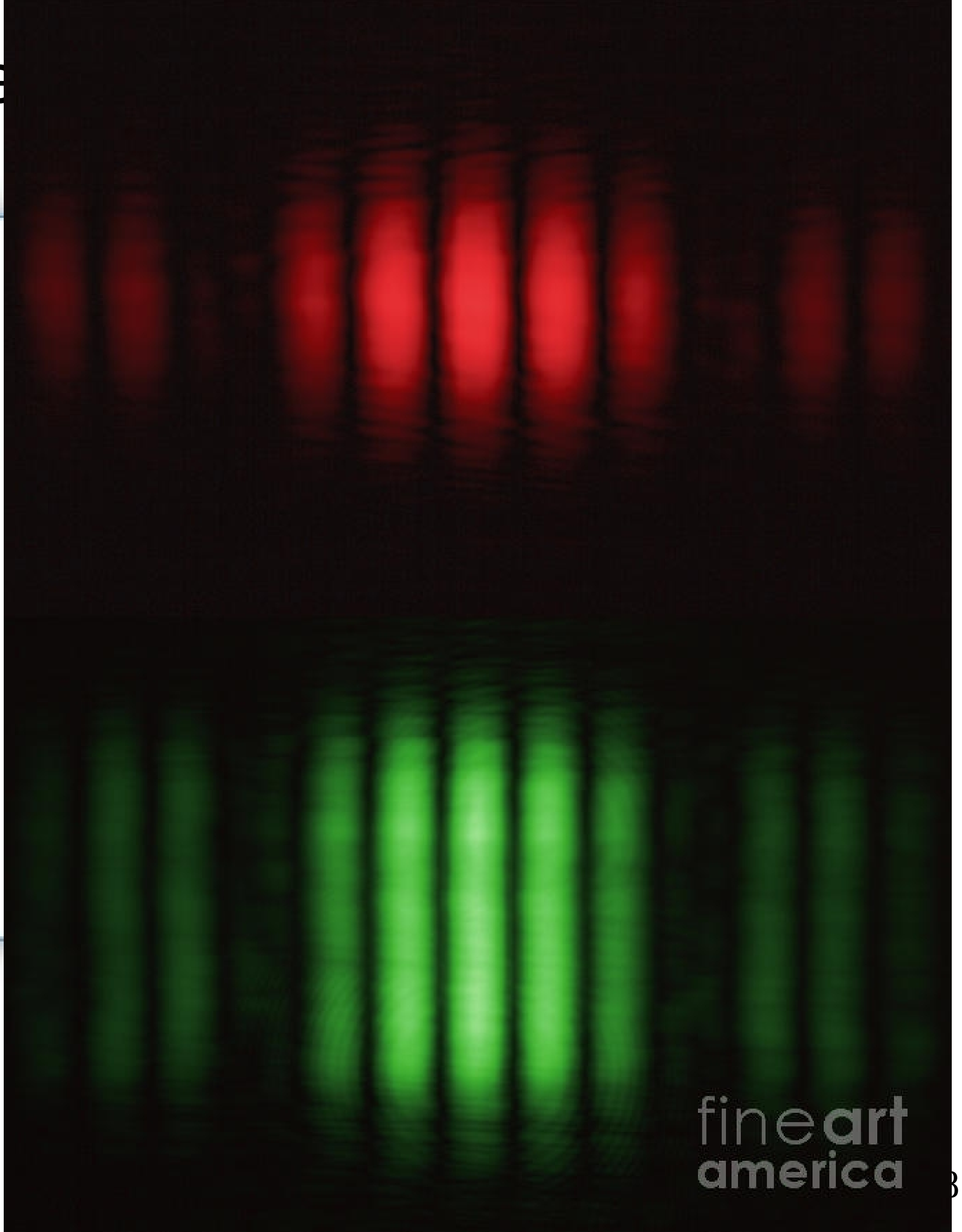
Young's double slit experiment



Interference (keep in mind for late

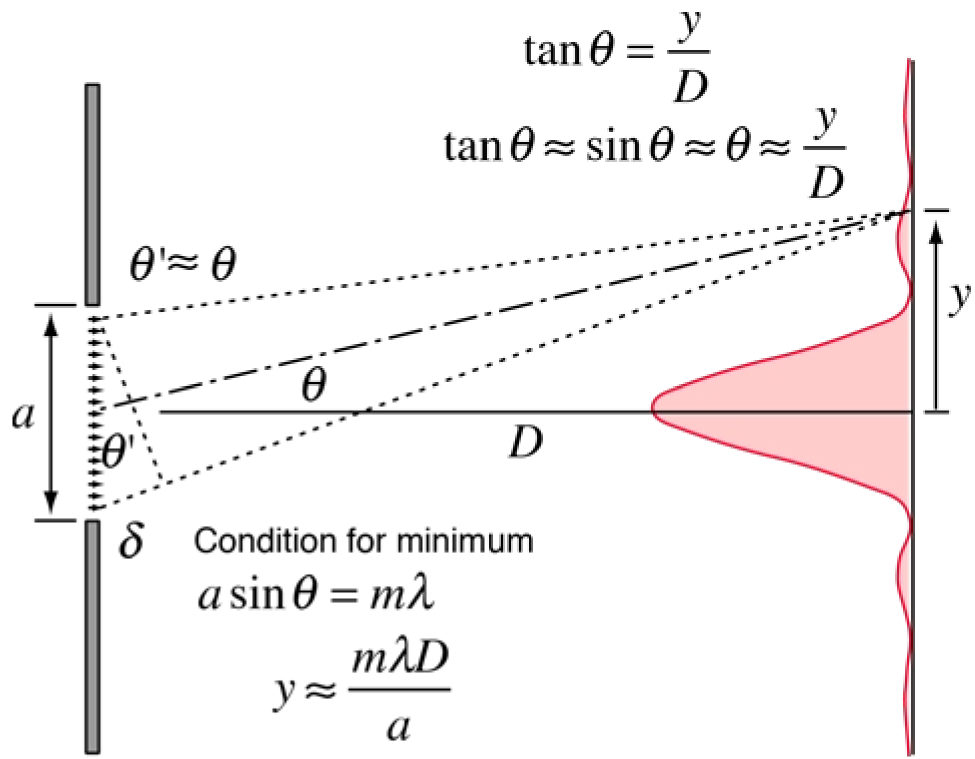


Young’s double slit experiment



Diffraction on a slit

- A coherent wavefront comes from the left.
- We divide the slit in many infinitesimally small emitters



Single slit diffraction, Fraunhoffer regime

Credits: hyperphysics.phys-astr.gsu.edu

$$dE(y') = \underbrace{\varepsilon \cdot dy'}_{dE_0} \cdot e^{i(kr-\omega t)}$$

$$r = R - y'\sin\theta \approx R - y'\theta$$

$$dE(y') = \varepsilon \cdot e^{i(kR-\omega t)} \cdot e^{-iky'\theta}\, dy'$$

$$E = \int_{-\frac{a}{2}}^{\frac{a}{2}} dE(y')\, dy'$$

$$E = \varepsilon \cdot e^{i(kR-\omega t)} \cdot \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-ik\theta y'}\, dy'$$

$$E \propto \frac{\sin\frac{k\theta a}{2}}{\frac{k\theta a}{2}} = \mathrm{sinc}\,\frac{k\theta a}{2}$$

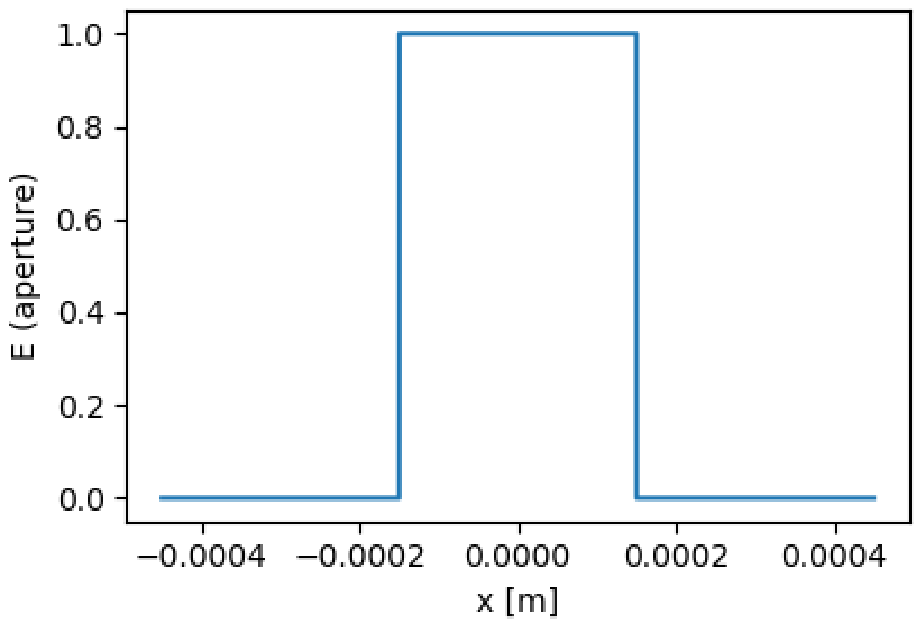
$$I \propto \mathrm{sinc}^2\,\frac{k\theta a}{2}$$

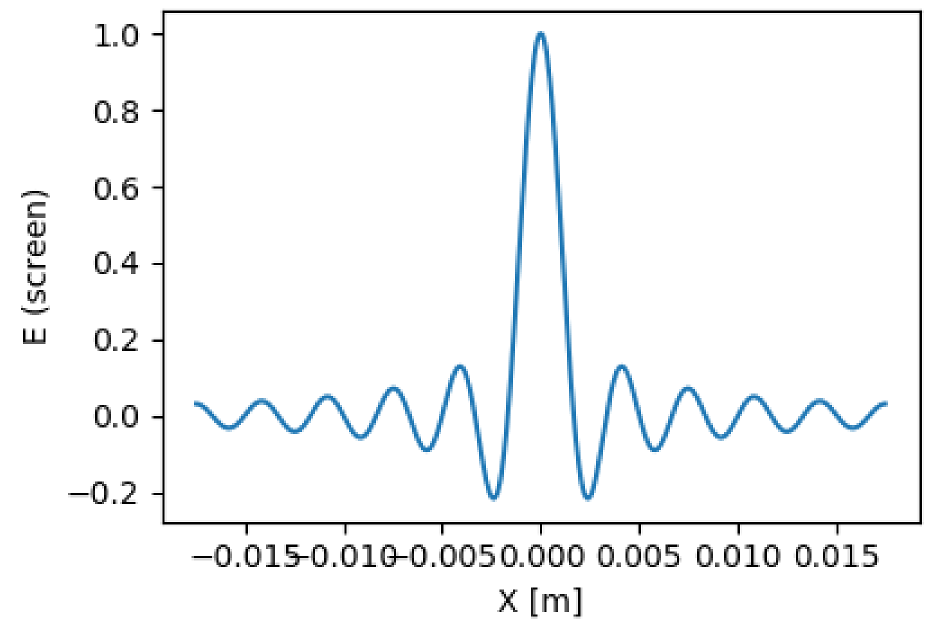
Diffraction as a Fourier transformation

$$E(\theta) = \text{const} \times \int_{\frac{a}{2}}^{\frac{a}{2}} e^{-ik\theta y'} dy'$$

- This integration is done over the slit (or multiple slits, or a square hole, or a circular hole, or any kind of a hole) – **the aperture**
- The result is the (scalar) Electric field dependent on the angle.
- So we have a Fourier transformation taking us from the physical space in the plane of the aperture to the angular distribution of the far field, as seen from the aperture.
- Alternatively, we can say that the electric field at **the screen** is the F.T. of the electric field at **the aperture**
- This simplifies many diffraction problems.

Diffraction as a Fourier transform

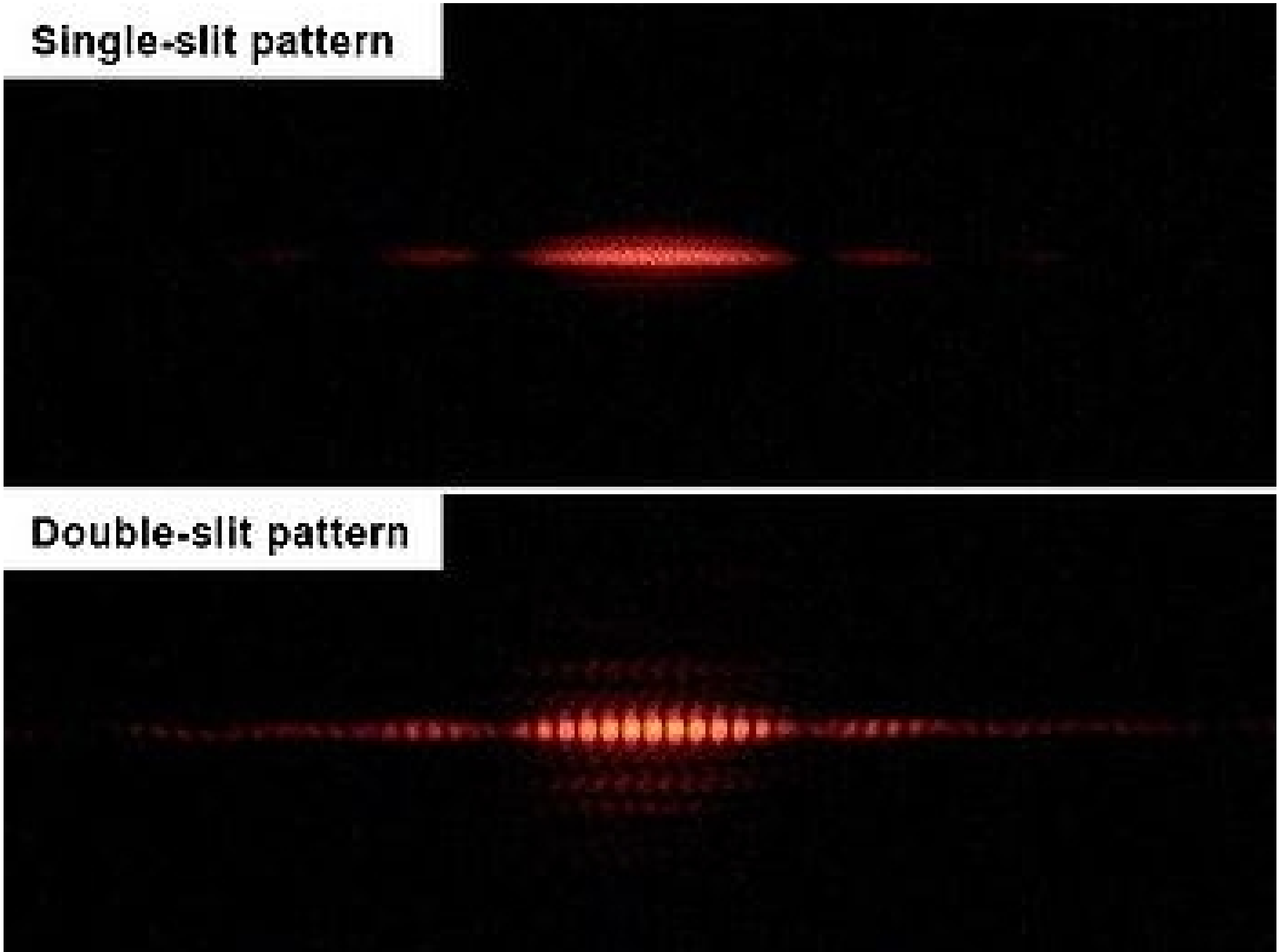




The diffraction on a single slit: Fourier Transform of a top-hat function is sinc function.

Can you try and work out the diffraction on two slits? *(take 5 minutes for this one)*

Diffraction on two slits

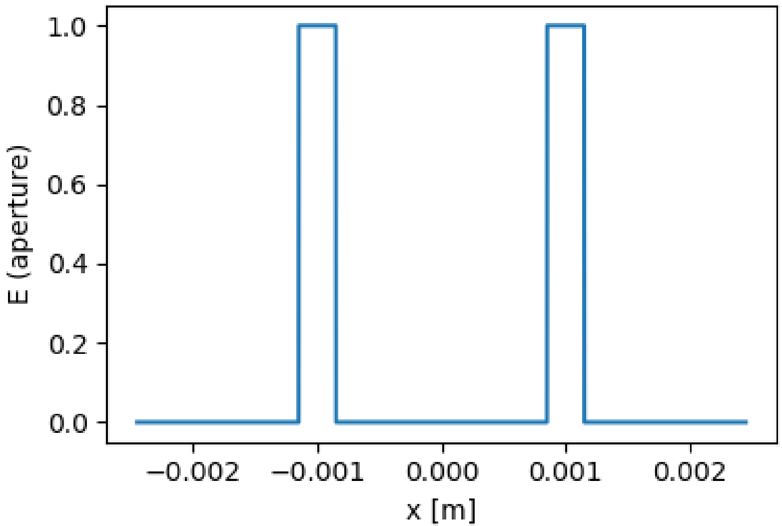


Properties of Fourier Transform 1

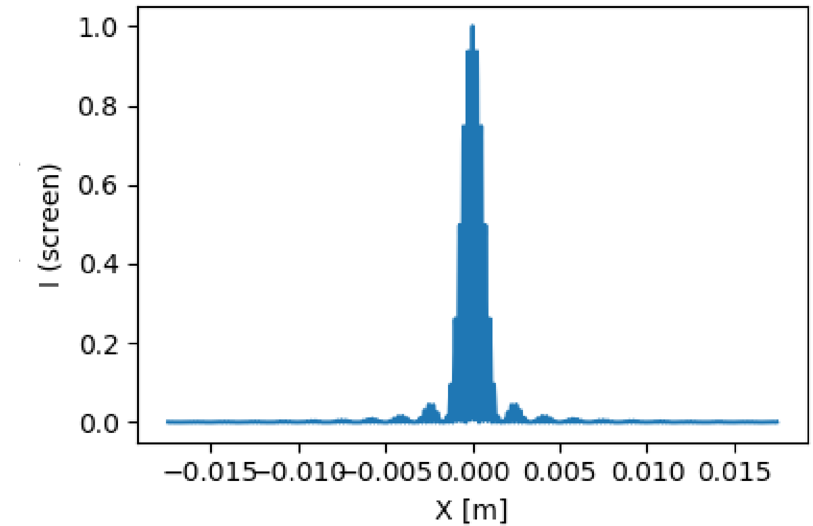
	Spatial Domain (x)	Frequency Domain (u)
Linearity	$c_1 f(x) + c_2 g(x)$	$c_1 F(u) + c_2 G(u)$
Scaling	$f(ax)$	$\frac{1}{\lvert a \rvert} F\left(\frac{u}{a}\right)$
Shifting	$f(x - x_0)$	$e^{-i2\pi u x_0} F(u)$
Symmetry	$F(x)$	$f(-u)$
Conjugation	$f^*(x)$	$F^*(-u)$
Convolution	$f(x) * g(x)$	$F(u)G(u)$
Differentiation	$\frac{d^n f(x)}{dx^n}$	$(i2\pi u)^n F(u)$

Note that these are derived using frequency ($e^{-i2\pi ux}$)

Two slit diffraction – solution



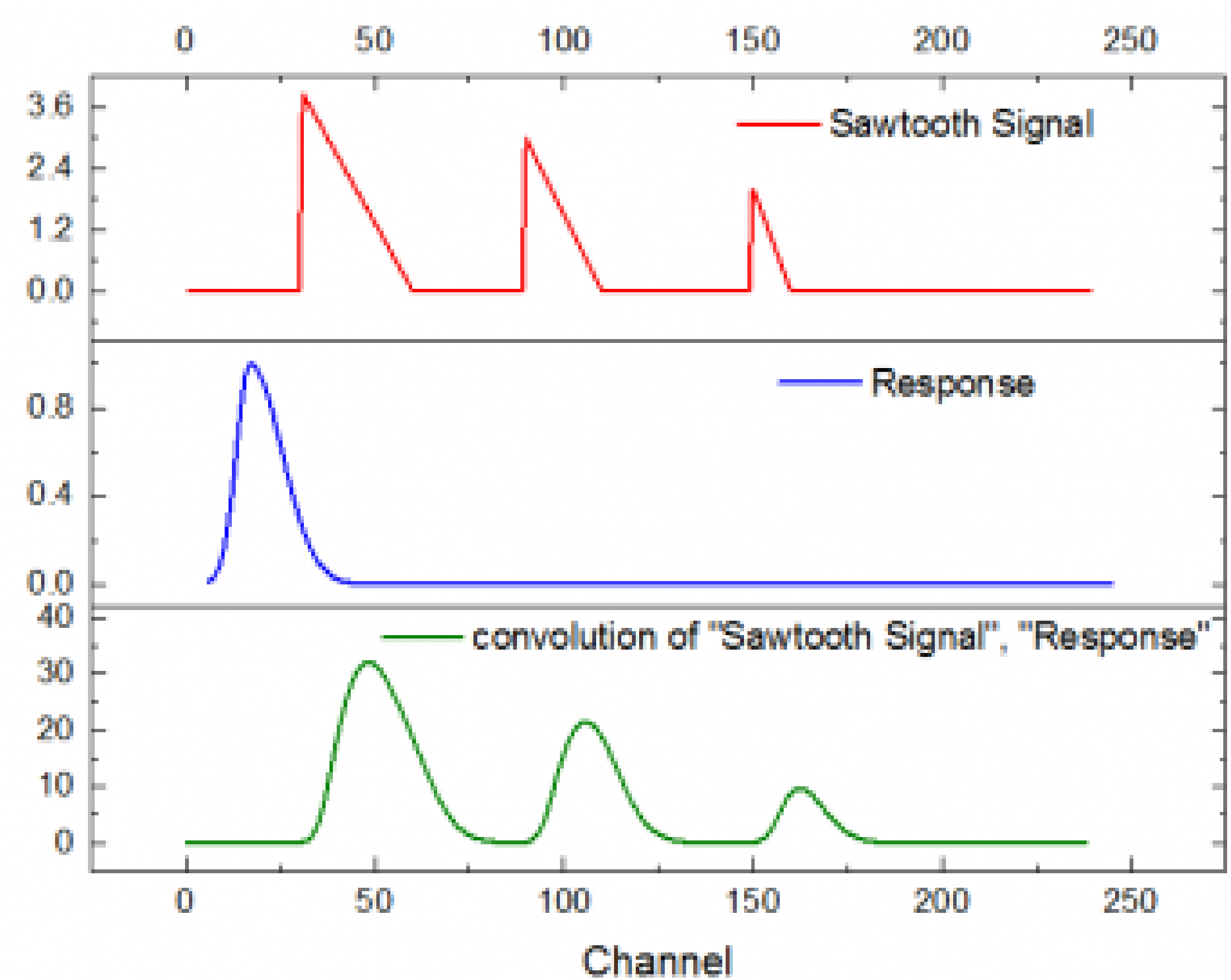
This is **the convolution** of a top hat and two delta functions.



This is **the product** of cosine function and sinc function.

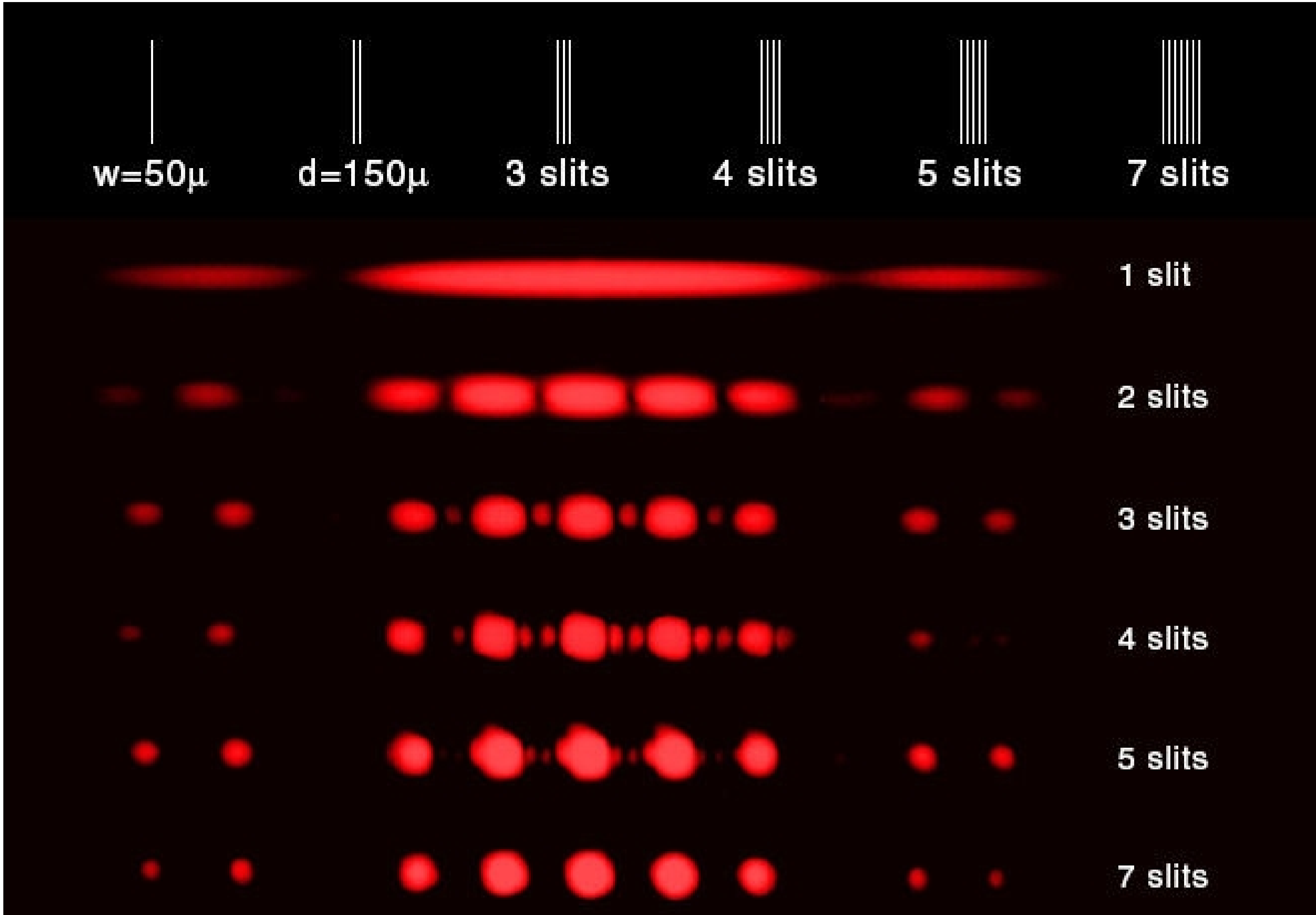
$$f \star g(x) = \int_{-\infty}^{\infty} f(x')g(x'-x)dx'$$

Convolution – in astronomy usually some sort of “smearing”

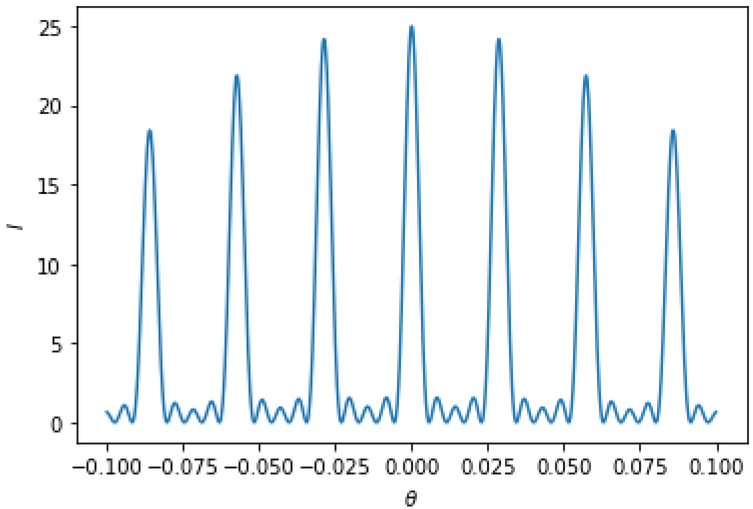


Credits: Origin Lab

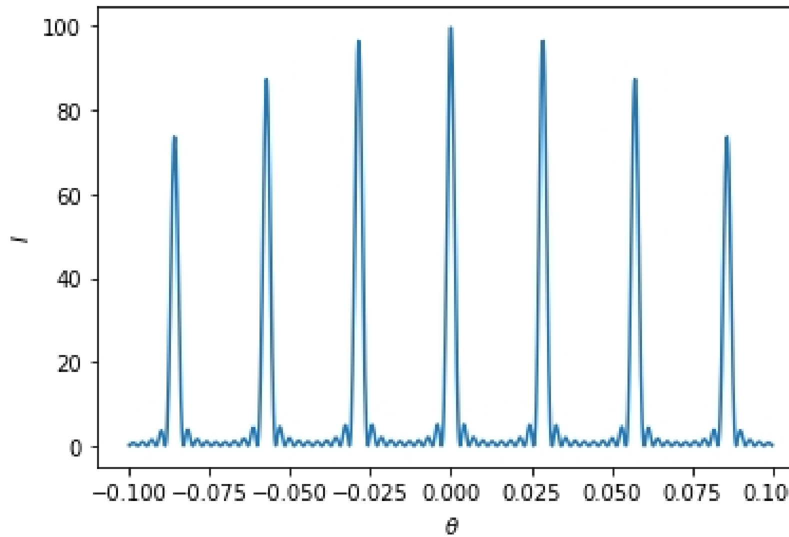
Diffraction on multiple slits



Diffraction on multiple slits



5 slits, each 0.1 mm wide, 1mm separate



10 slits, each 0.1 mm wide, 1mm separate

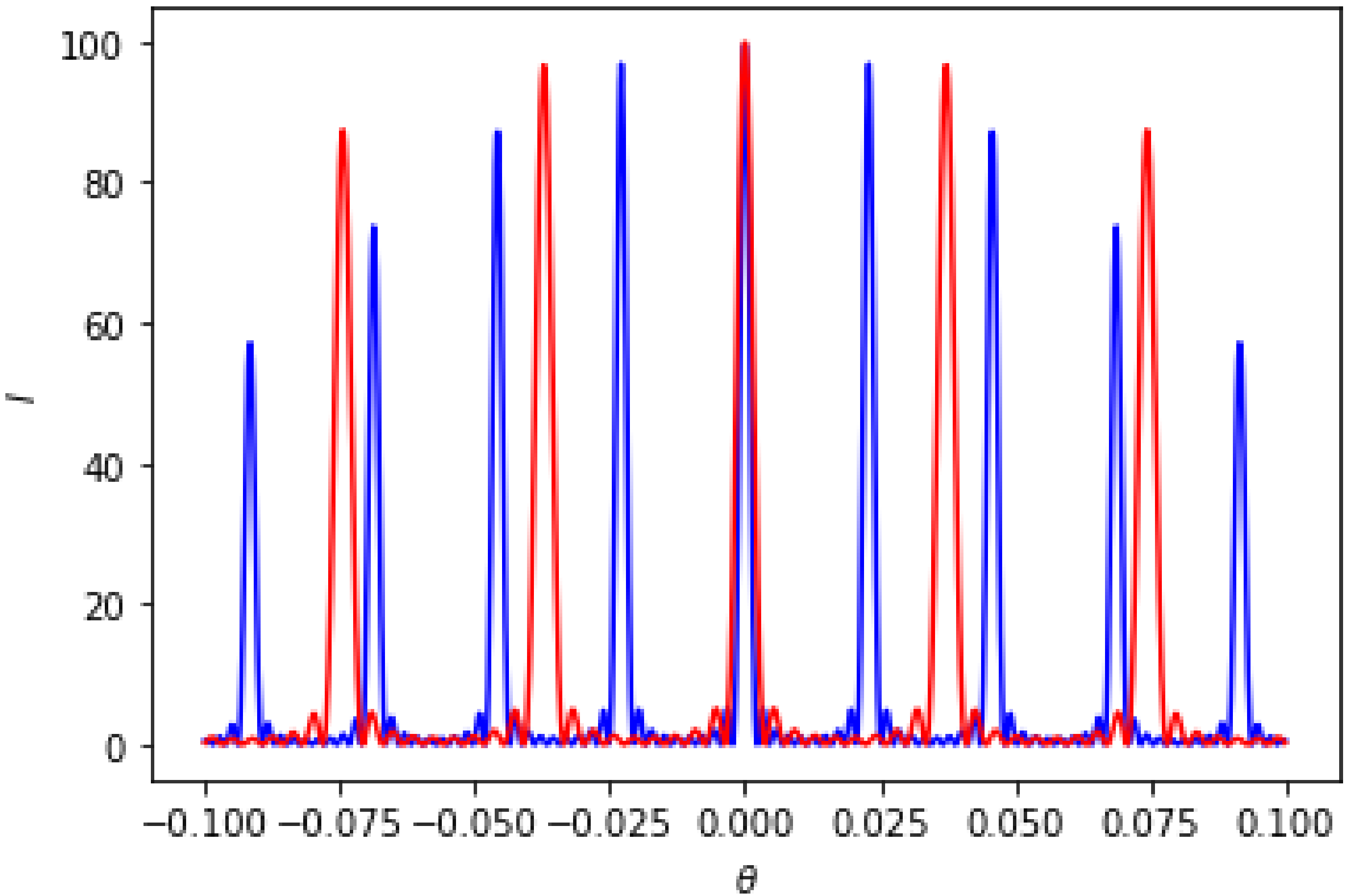
This is how diffraction grating works!

$$I \propto \text{sinc}^2\beta \frac{\sin^2(N\alpha)}{\sin^2\alpha}$$

$$\beta = kd\theta/2; \;\; \alpha = ka\theta/2$$

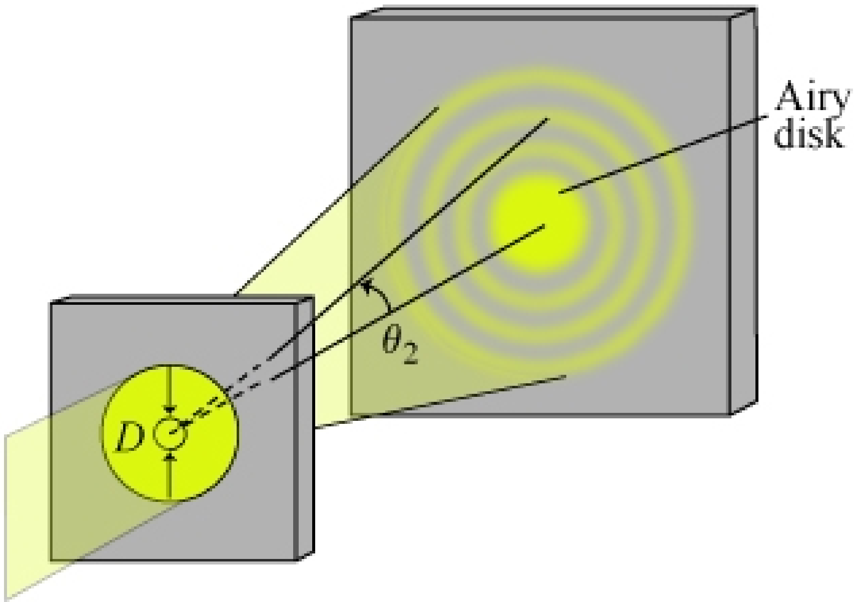
What do we use the diffraction grating for?
(Discuss)

That's right, to separate the wavelengths (more about this later)



Blue – 400 nm
Red – 650 nm

Diffraction on a circular aperture



Why is this relevant for us? - Because our primary (lens, mirror) is a circular aperture!

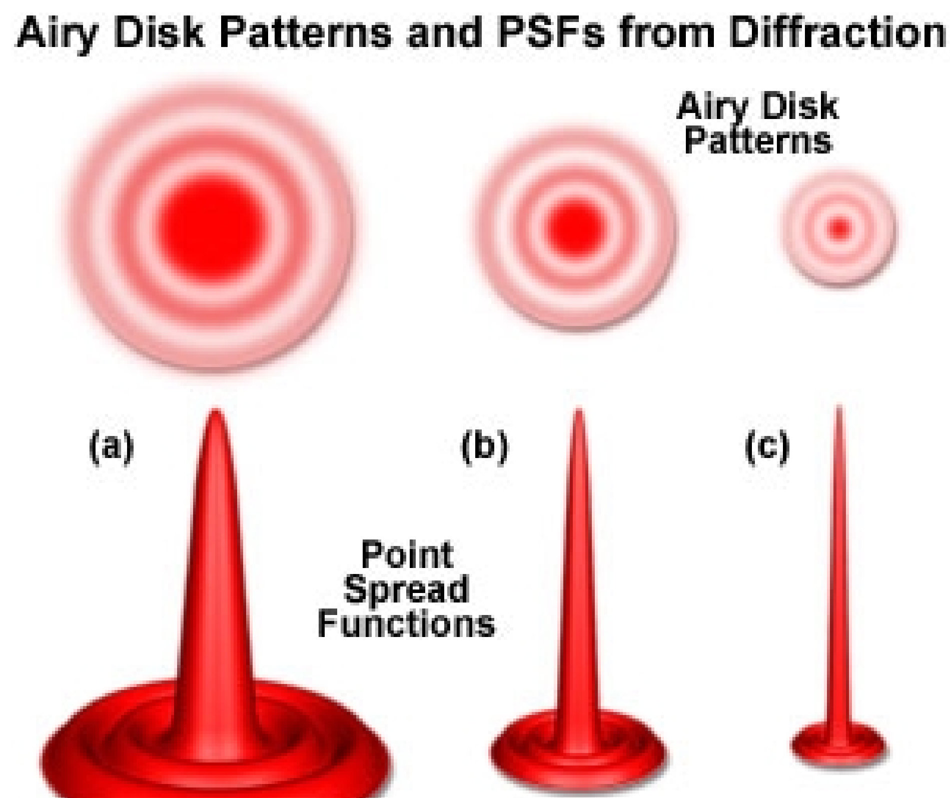
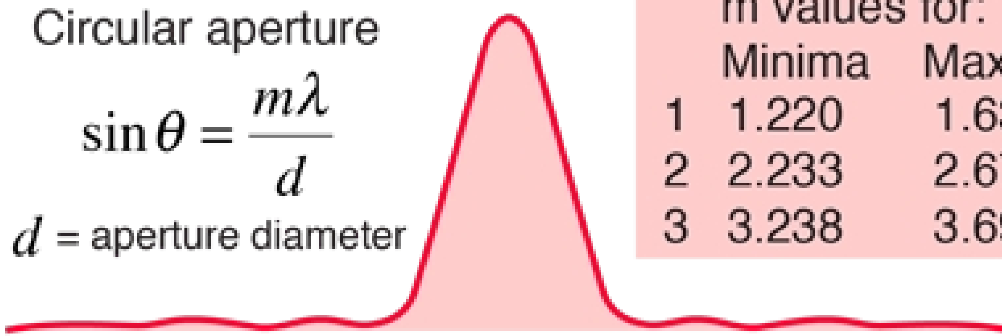


Figure 5

$$I \propto \left[\frac{J_1(\rho)}{\rho}\right]^2 ; \quad \rho = k\theta a/2$$

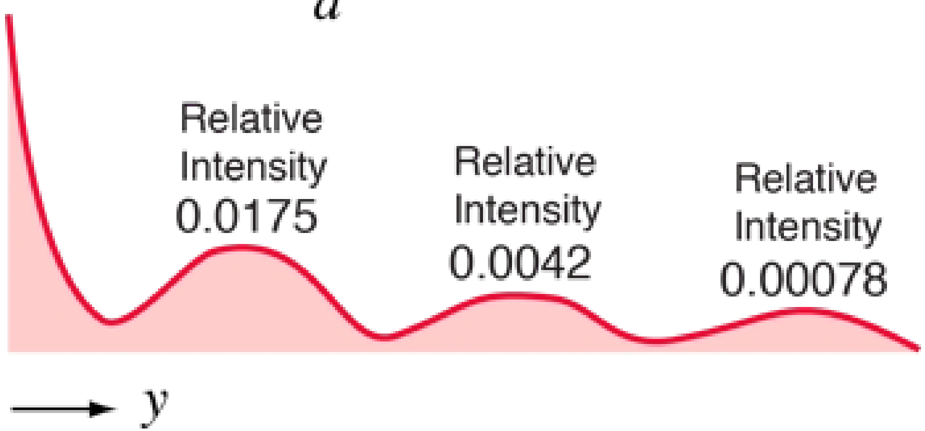
Airy disk

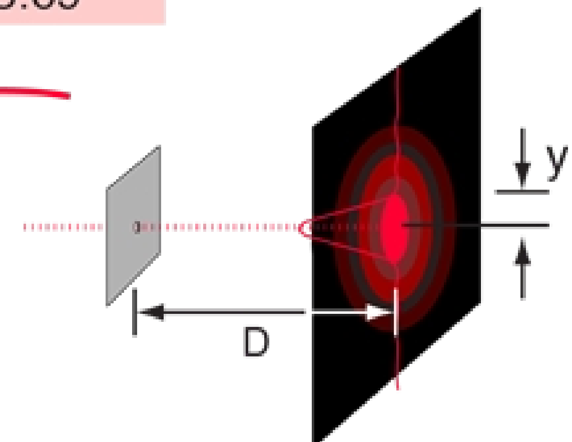
$$\theta_0 = 1.22\frac{\lambda}{D}$$



	m values for: Minima	Maxima
1	1.220	1.635
2	2.233	2.679
3	3.238	3.69

$$y \approx D\frac{m\lambda}{d}$$ for maxima and minima





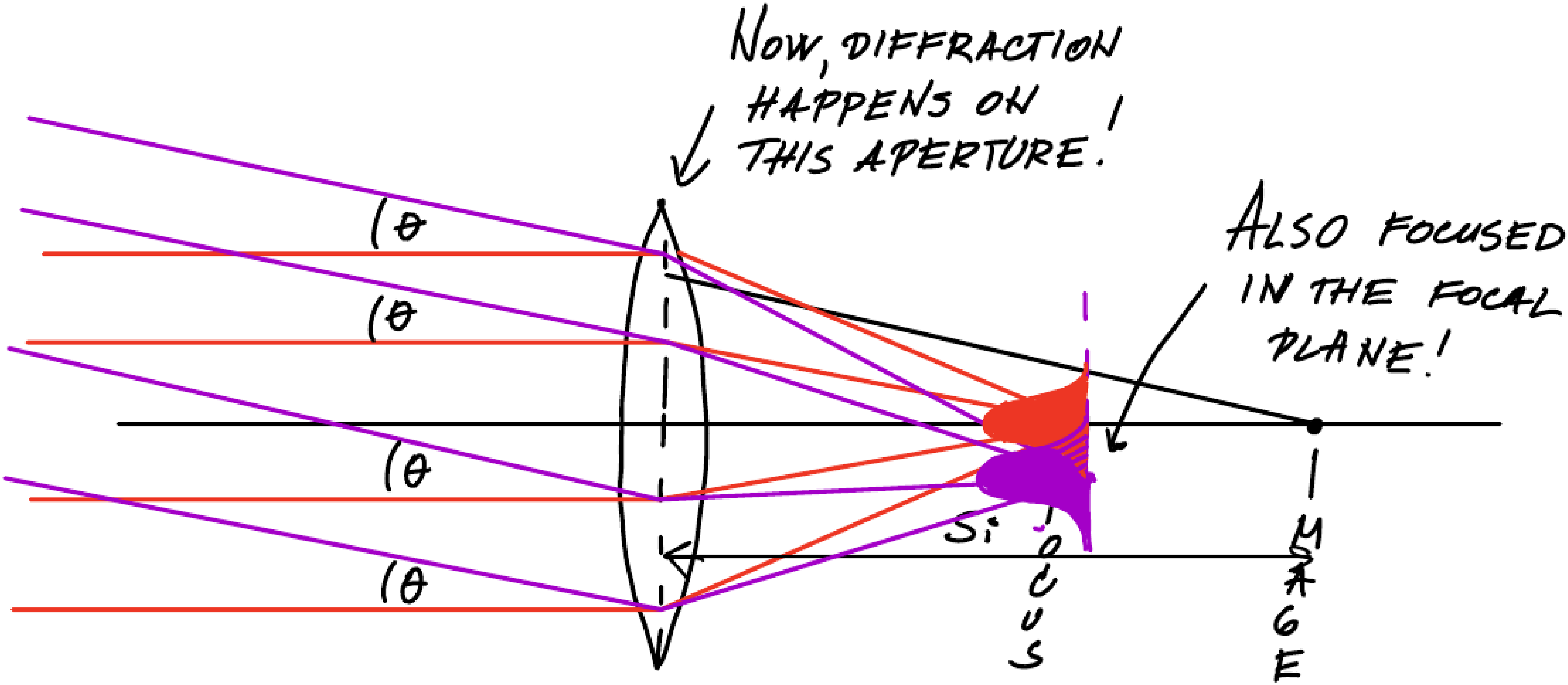
$$\frac{y}{D} = \tan\theta \approx \sin\theta \approx \theta$$

for small angles θ

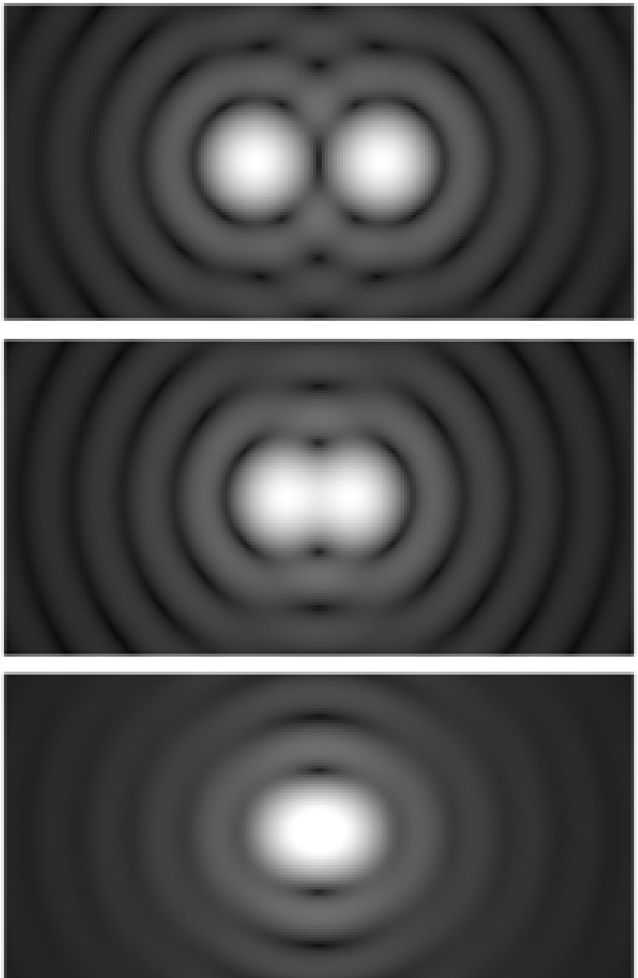
Credits: hyperphysics.phys-astr.gsu.edu

$$I \propto \left[\frac{J_1(\rho)}{\rho}\right]^2; \quad \rho = k\theta a/2$$

What happens when we have two sources (directions)



Rayleigh criterion



Resolved

Barely resolved

Unresolved

- We need the sources to have angular separation greater than the angular radius of the primary maximum in order to be able to resolve them!

$$\theta_0 = 1.22\frac{\lambda}{D}$$

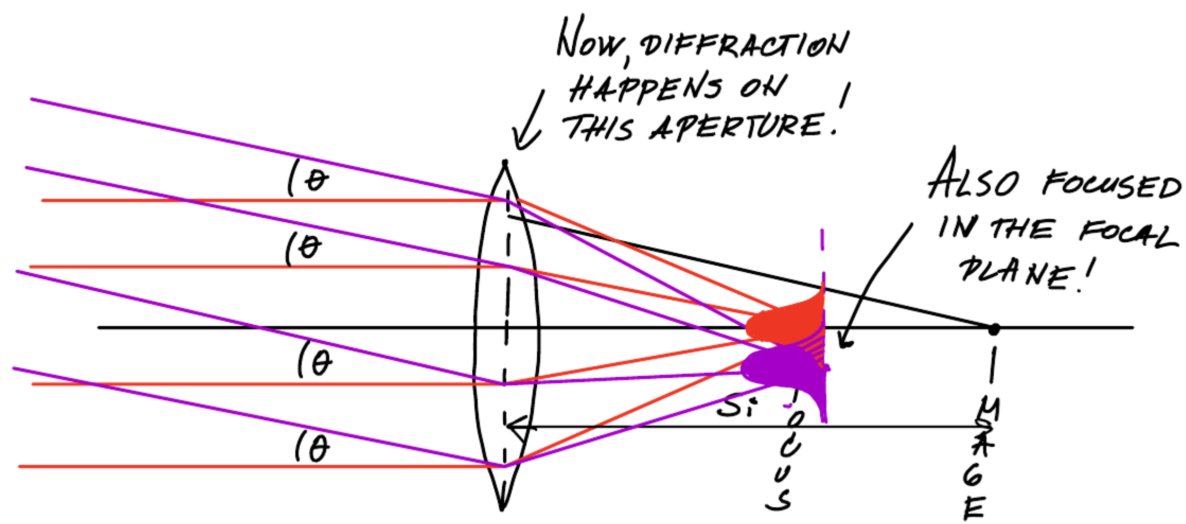
- Larger wavelength $\rightarrow$ worse resolution
- Larger primary $\rightarrow$ better resolution
- *This is why radio telescopes have extremely bad resolutions (But also easier to make an interferometer).*

AGAIN, RESOLUTION != SAMPLING!

What happens for finite sources?

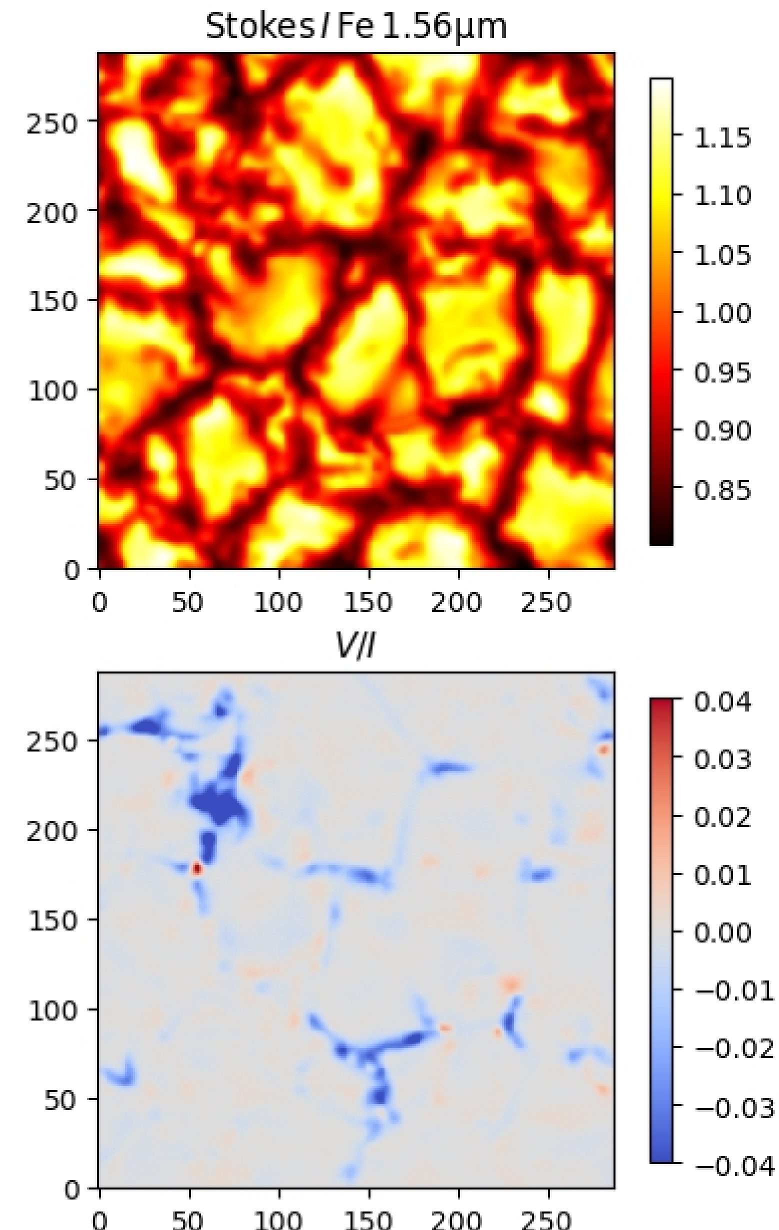
- Finite sources are “ensembles” of points in the sky
- Each point will be spread due to the finite angular resolution of the aperture
- The original “image” (distribution of the intensity with the angle), get’s smeared by the so called PSF (Airy disk in the case of circular aperture)
- In the case of two stars, we just added intensities (why is this allowed?)
- In the case of the finite sources, we will have a **convolution (2D one)**

Purple and red “wavefront” are not coherent with each other. They cannot interfere. So we add intensities and not the electric fields.

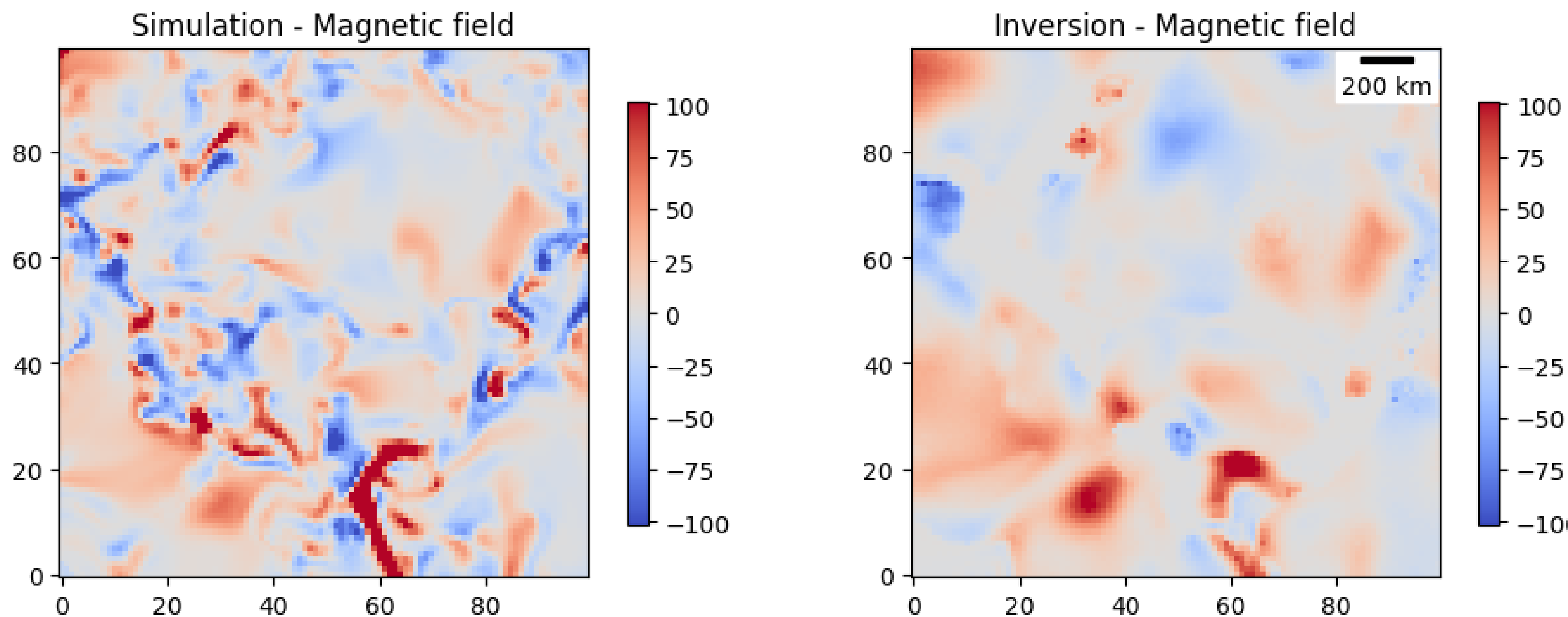


Convolution of the original image and the telescope PSF

- Luckily, different points in the solar surface are incoherent, we can just convolve (and we will do this exercise
- Signal is smeared
- In the case of the intensity – this decreases the contrast, or can completely erase small features
- In the case of the polarization (that can be negative) this can lead to cancelations.
- Since polarization tracks magnetic fields → magnetic fields become invisible.



Effects of finite spatial resolution on the inference

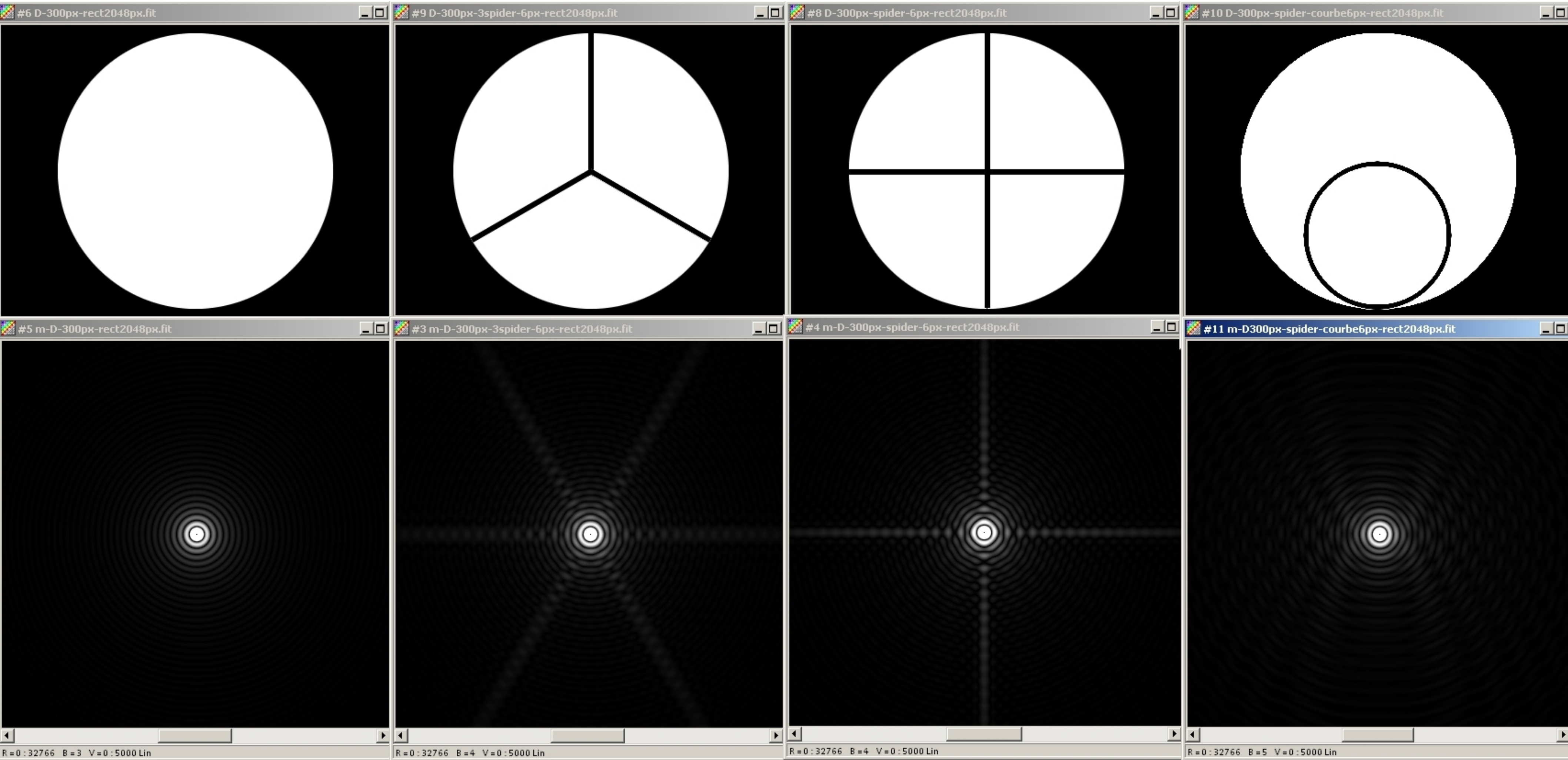


Because of finite angular resolution some part of spatial information is lost. There is a lot of interesting physics hiding behind small scale phenomena (magnetic reconnection, heating, flux emergence...) - **We want as high spatial resolution as "the nature" has!**

Resolution and sampling

- Nyquist theorem – if observed frequency is T, we need to sample at 2T.
- This means optimal sampling should be half of the resolution (true not only for angular / spatial resolution by also for wavelength resolution)
- Bigger telescope $\rightarrow$ better resolution smaller sampling
- Smaller chunks in the plane of the sky. (Surface scales as 1/D^2)
- Collected photons scale as D^2
- **Photon flux per spatial resolution element is constant!**
- What does phrase "**photon bucket**" mean.

PSFs are more complicated than Airy function



Credits: telescope-optics.net

Summary

- Light that enters our telescopes exhibits wave properties
- This leads to the smearing of our images due to (hopefully only) primary aperture
- Diffraction can also be exploited in the instruments to disperse the light (separate wavelengths) – diffraction grating.
- Or we can use interference to create interference filters (more on this later).
- **Next class:** We will try to convolve and de-convolve some simulated images of the solar surface.