PHYS 7810: Solar Physics with DKIST

Lecture 25: Scattering polarization and Hanle effect

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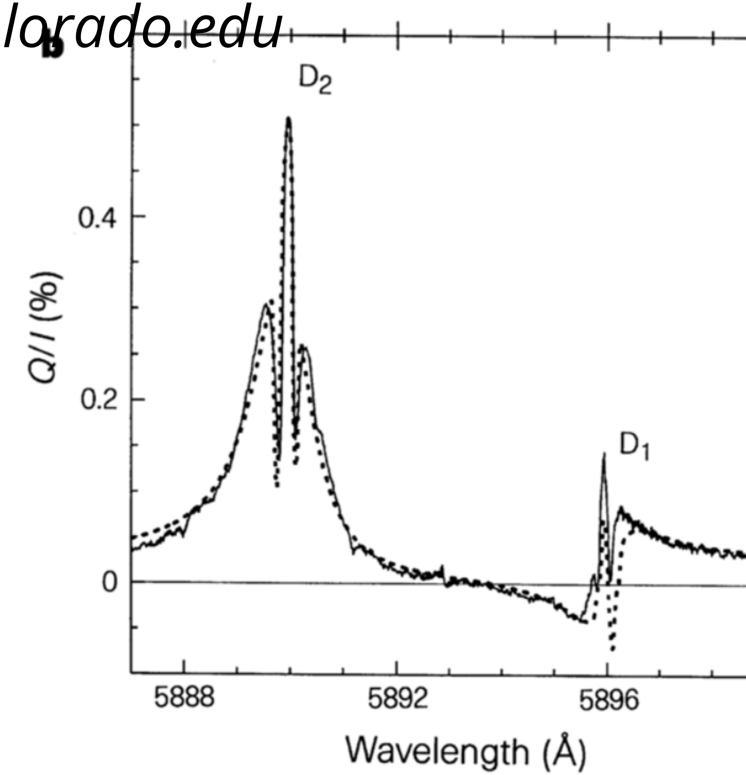
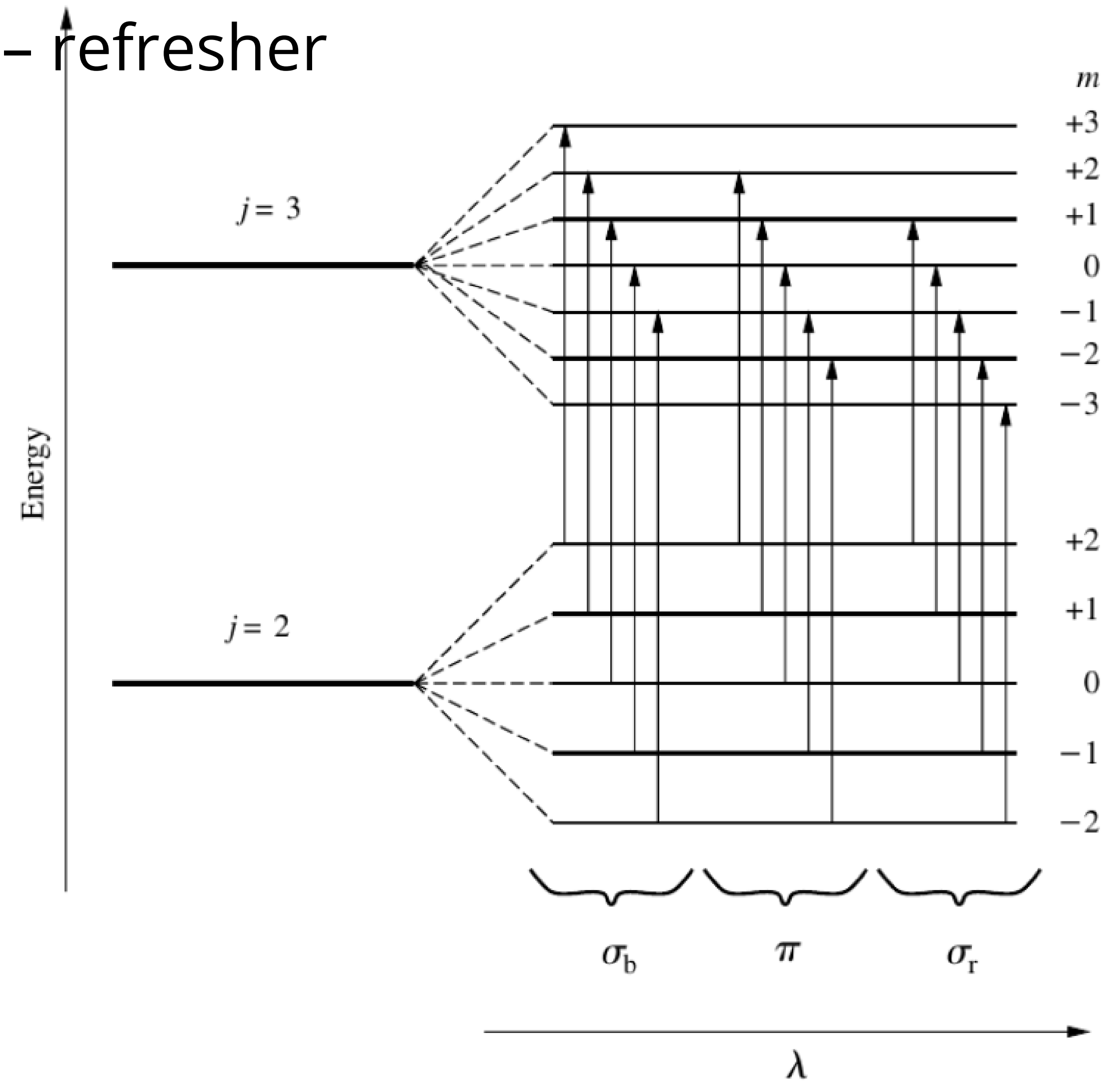


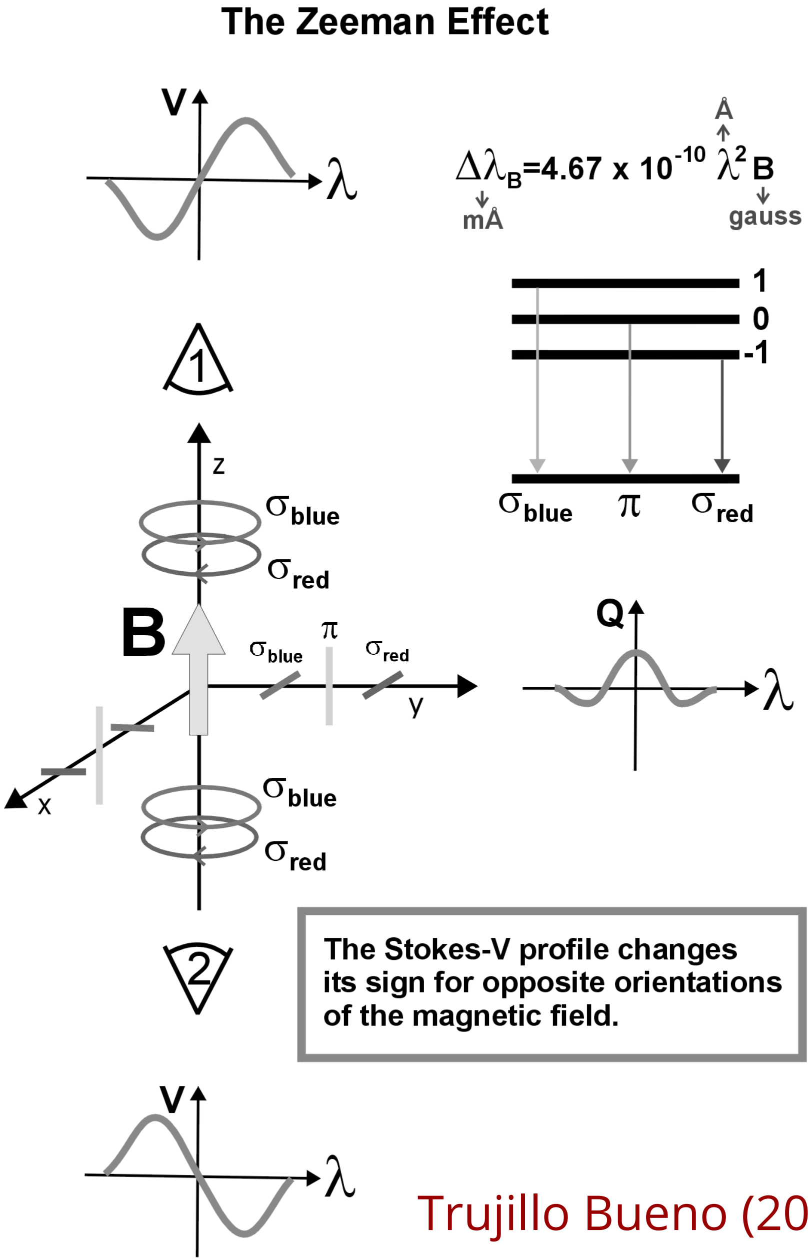
Previous lectures

- We started studying line formation
- Then the inversion procedures that allow us to retrieve properties of the solar atmosphere from the observations
- Then, with Professor Maria Kazachenko, we saw what further steps we can take to answer our scientific questions
- Somewhere in there we had to consider line polarizing mechanisms in order to diagnose the magnetic field – that was Zeeman effect
- Today we are going to talk about another physical mechanism, useful for diagnosing higher atmospheric layers – scattering polarization and Hanle effect

Zeeman effect – refresher



Zeeman splitting for the transition where upper level has J = 3 and lower J = 2. There are total of 15 sub-transitions. I sometimes call different *m* values – Zeeman sublevels.



Trujillo Bueno (2006)

Why the polarization?

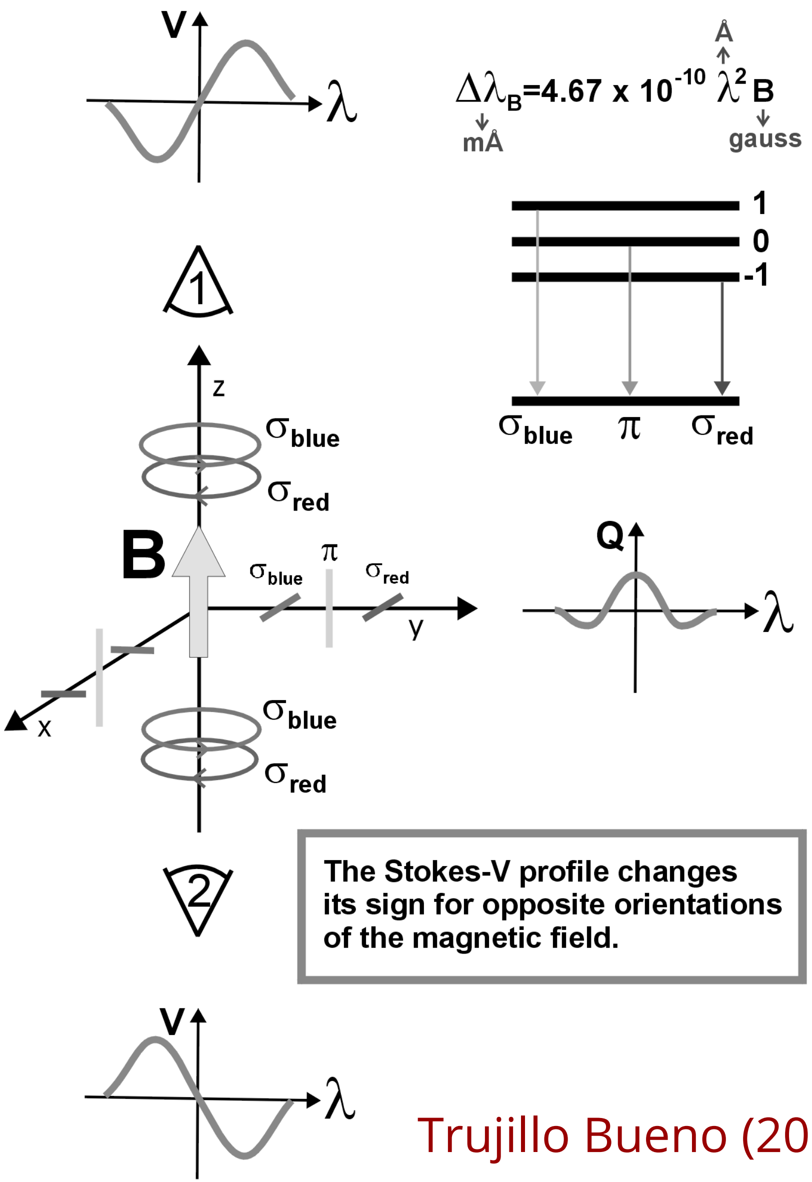
Individual photons are 100% polarized.

Different Δm transitions – different polarizations!

Parallel with B: only positive and negative circular polarization (σ_{blue},σ_{red})

Perpendicular to B: σ_{blue},σ_{red} seen as negative linear polarization, π as positive linear polarization

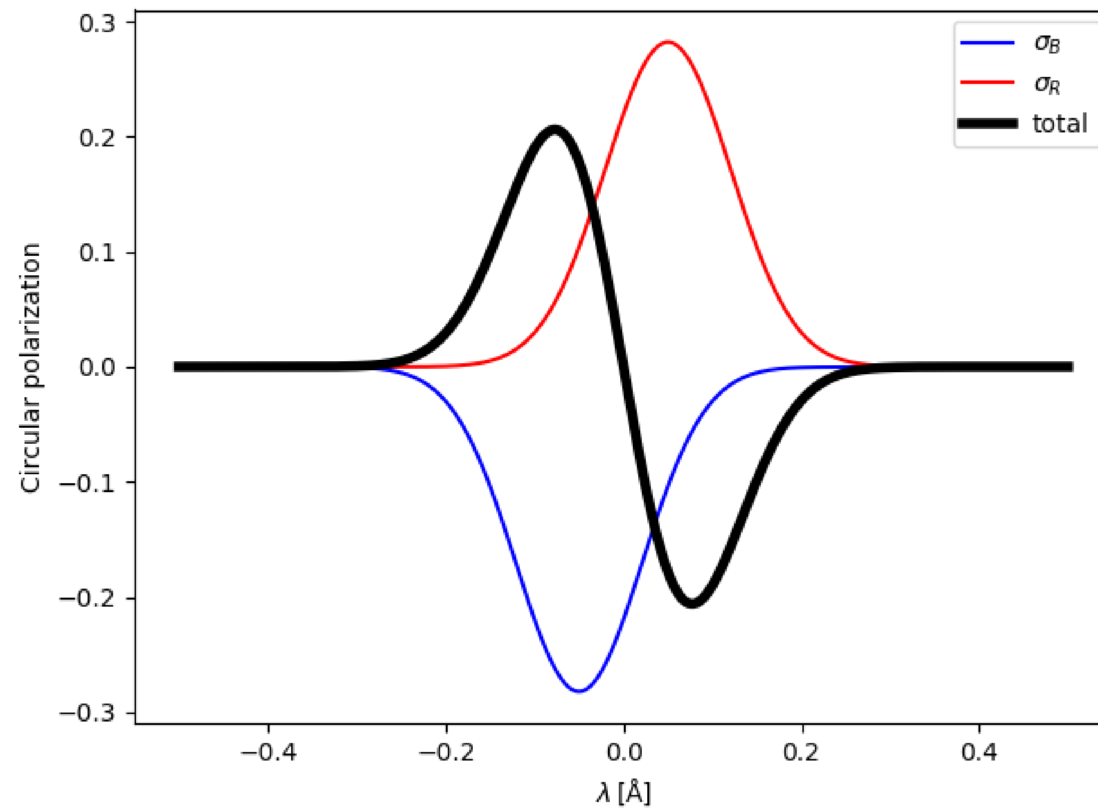
The Zeeman Effect

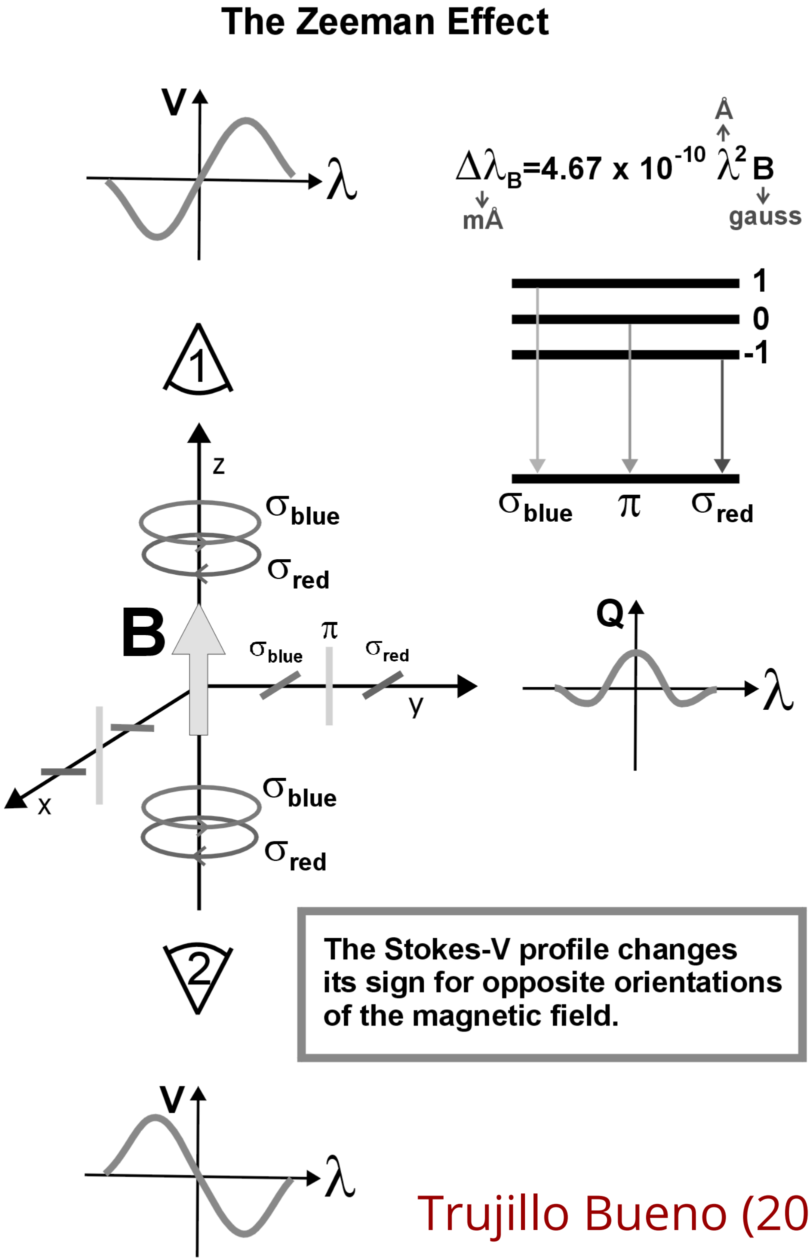


The Stokes-V profile changes its sign for opposite orientations of the magnetic field.

Trujillo Bueno (2006)

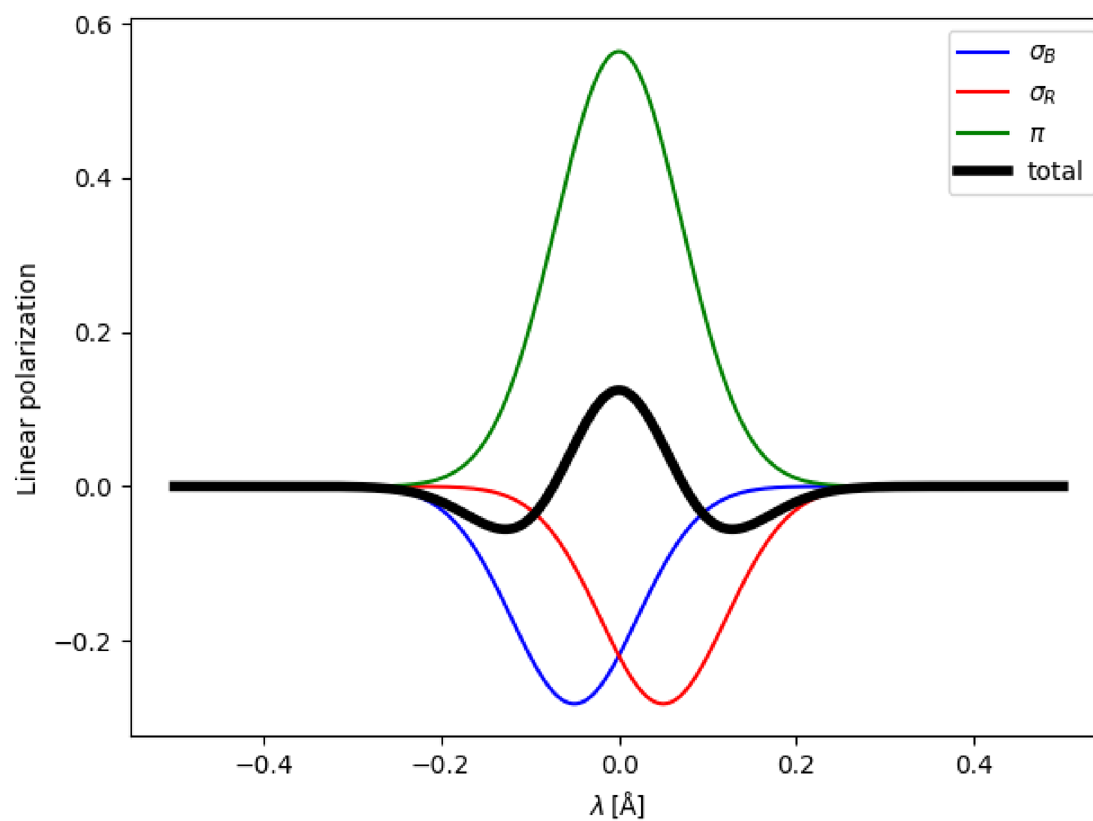
Parallel to **B**





Trujillo Bueno (2006)

Perpendicular to **B**



So, Zeeman effect is:

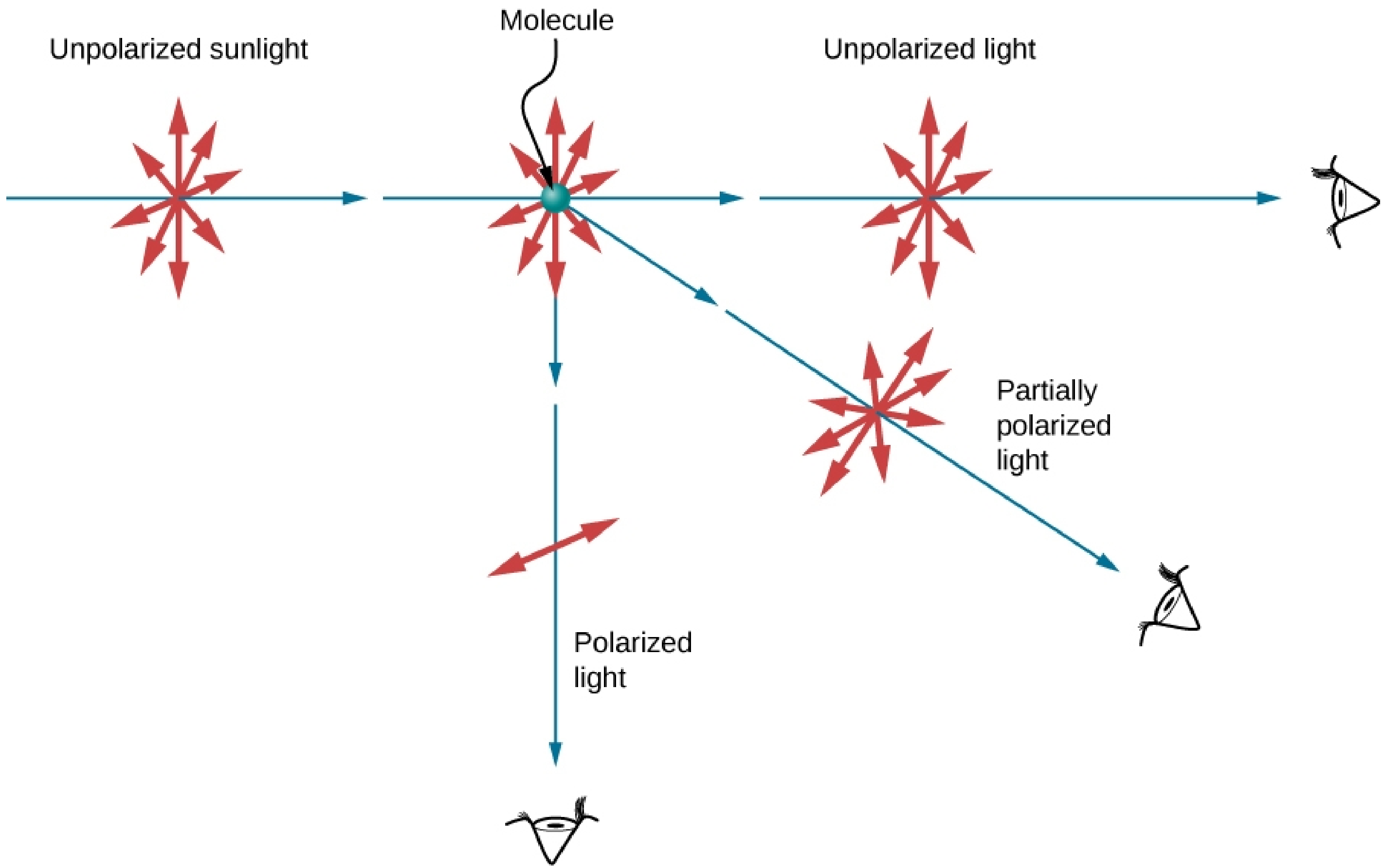
- **Wavelength shift of the completely polarized Zeeman sub-transitions, that leads to the net polarization of the light.**

- An implicit assumption here is that the population of Zeeman sublevels follows some equilibrium distribution.

- **In Scattering polarization / Hanle effect, everything is completely reversed.**

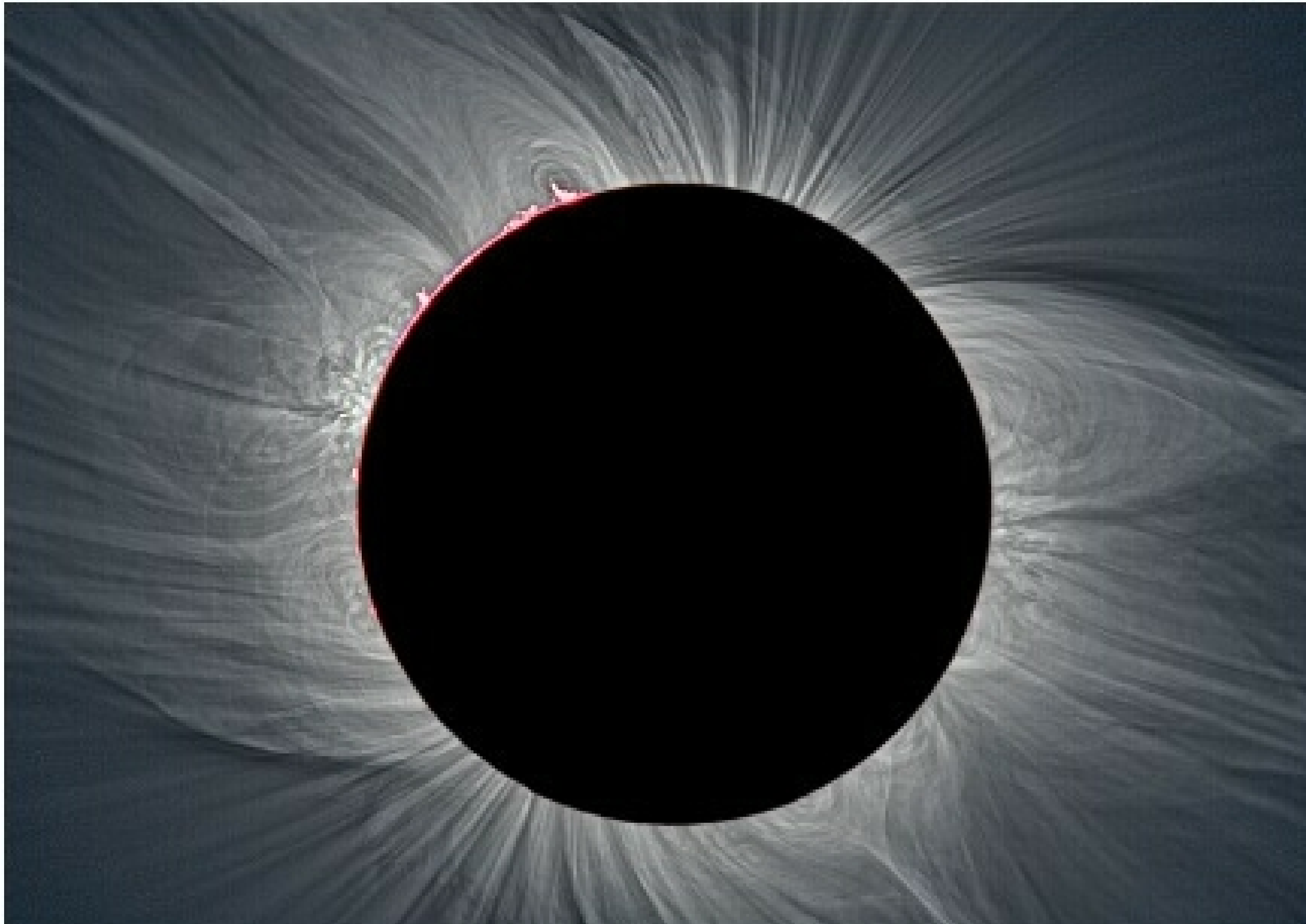
- But, let’s start from a classic case...

Classic scattering polarization: an EM wave scatters on a particle

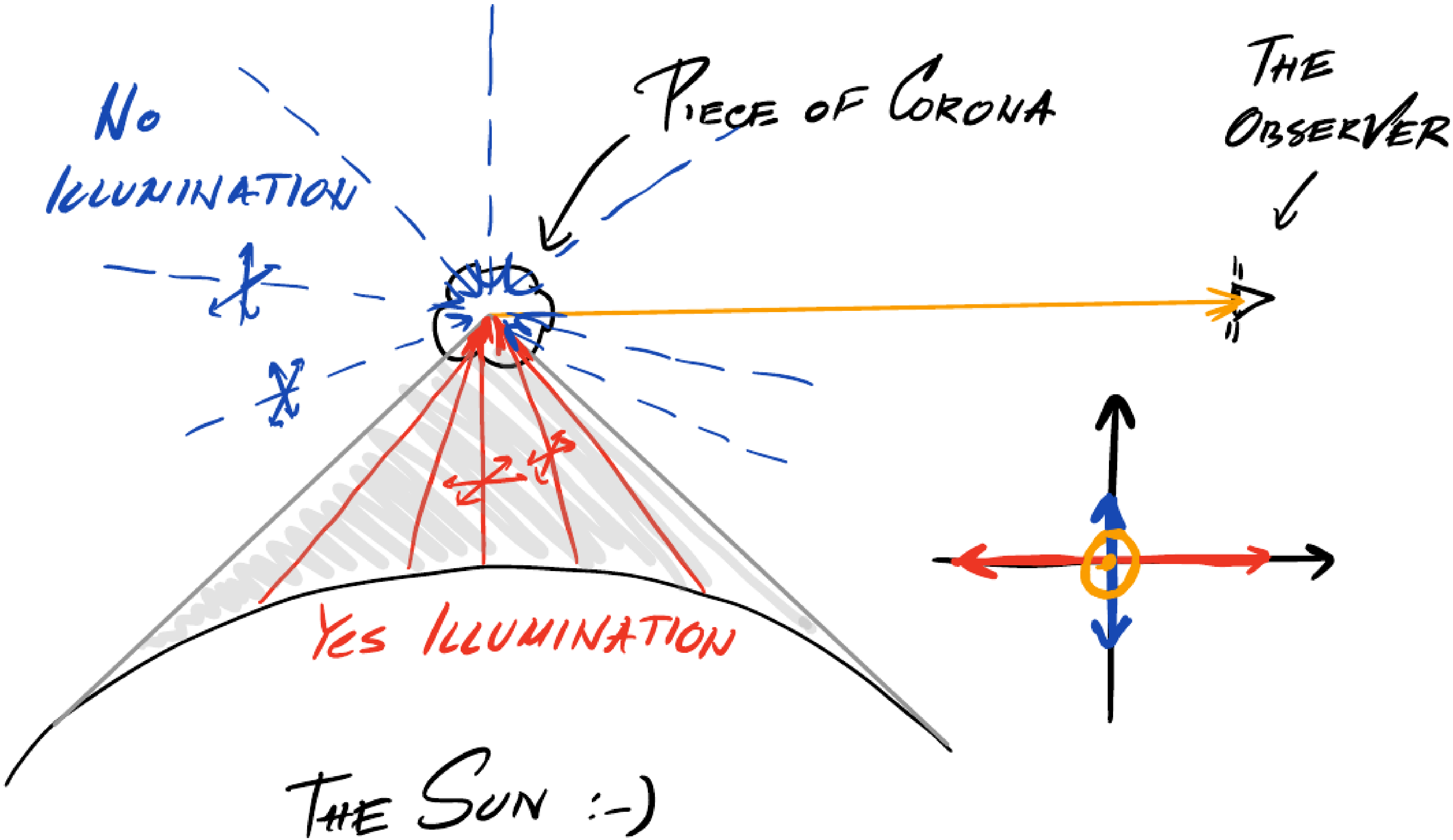


Think about this: a) Why is the polarization this way?
b) What would happen if illumination was isotropic?

Can this happen in the atmosphere of the Sun?



So, where does the anisotropy come from?



Ok, so how to formalize that?

- In the previous example, we have seen that the electric field component along *x* is stronger than along *y*.
- This will result in some non-zero Stokes Q (+ or – depend on how we define it, always check with your observers!)
- But, how to "formalize" this, how to calculate Q?
- We will have to embark on a modeling story to be able to calculate the intensity and the polarization of the scattered light.
- This will be a very gentle intro to non-local thermodynamic equilibrium radiative transfer (NLTE). Scattering is, in itself, a NLTE process.

For the polarization for now

- How to calculate the intensity of the scattered light?
- Ideally, we would solve RTE over the blob. Let's say we know the opacity.
- However, emissivity is not LTE one (Why? Discuss?)

For the polarization for now

- How to calculate the intensity of the scattered light?
- Ideally, we would solve RTE over the blob. Let’s say we know the opacity.
- However, emissivity is not LTE one (Why? Discuss?)
- Ok, well, let’s assume all the light is scattered. Isotropically (does not care where it came from).
- All intensity that is absorbed is going to be emitted, so:

$$\frac{dE_\lambda}{dV\,dt}^{\text{emitted}} = \frac{dE_\lambda}{dV\,dt}^{\text{absorbed}}$$

$$j_\lambda = \oint I_\lambda^{\text{inc}}(\theta, \phi)\chi_\lambda \, \sin\theta \, d\theta \, d\phi$$

There are some interesting aspects to this formula, so let's appreciate it a bit...

$$j_\lambda = \oint I_\lambda^{\mathrm{inc}}(\theta, \phi) \chi_\lambda \, \sin\theta \, d\theta \, d\phi$$

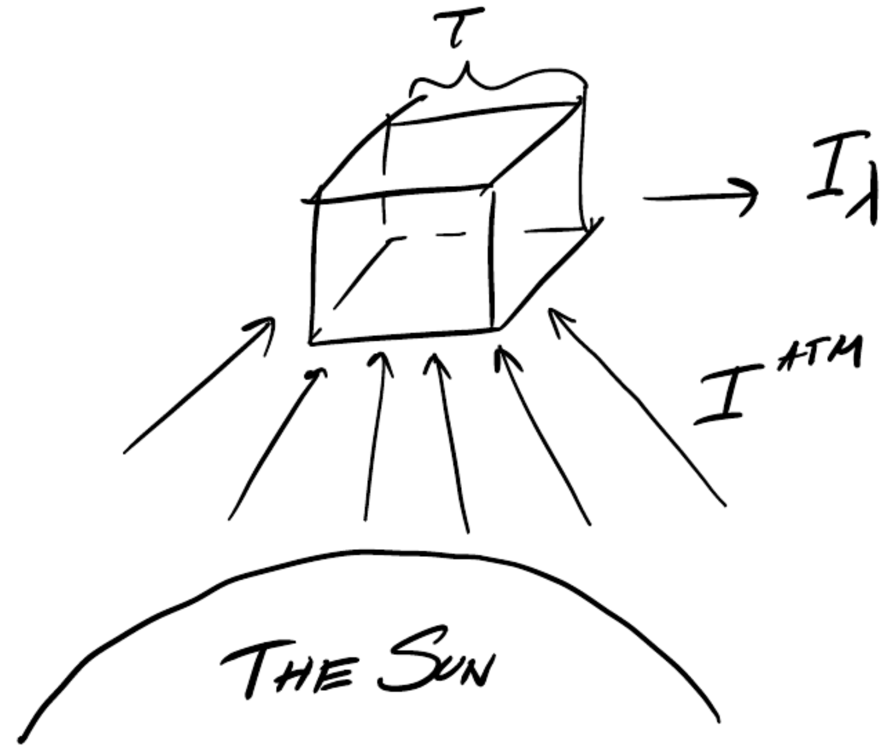
- Opacity is assumed to be isotropic (does not have to be)
- Emissivity too
- Incoming radiation, however, does not have to be isotropic
- We can divide both sides by the opacity and assume the axial symmetry...

One scattering approximation

$$S_\lambda = \frac{1}{2}\int I_\lambda(\mu)d\mu$$

$$\mu = \cos\theta$$

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda}) \approx S_\lambda \tau_\lambda$$



The emitted intensity is proportional to the illumination and to the number of absorbers (that are also emitters).

But we know that the radiation is not isotropic:

- Mueller matrix for Rayleigh scattering

$$\mathbf{P}(\Theta) = \frac{3}{2}\begin{bmatrix} \frac{1}{2}(1+\cos^2\Theta) & -\frac{1}{2}(1-\cos^2\Theta) & 0 & 0 \\ -\frac{1}{2}(1-\cos^2\Theta) & \frac{1}{2}(1+\cos^2\Theta) & 0 & 0 \\ 0 & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & \cos\Theta \end{bmatrix}$$

- Or, alternatively:

Mean intensity

$$J_0^0 = \frac{1}{2}\int_{-1}^{1} I(\mu')d\mu'$$

Anisotropy

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} I(\mu')(1-3\mu'^2)d\mu'$$

Anisotropy:

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} I(\mu')(1 - 3\mu'^2)d\mu'$$

Compare the anisotropy and the mean intensity for following cases:

- Isotropic radiation
- Radiation coming from below
- Radiation coming from the sides
- What is the polarization?

$$I(\mu') = \text{const} = 1$$

$$J_0^0 = \frac{1}{2}\int_{-1}^{1} d\mu' = 1$$

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} (1-3\mu'^2)\, d\mu' = \frac{1}{2}\left(2 - \mu'^3\Big|_{-1}^{1}\right) = 0$$

No POLARIZATION!

$$I(\mu') = \delta(1)$$ (RADIATION COMING FROM BELOW

$$J_0^0 = \frac{1}{2}\int_{-1}^{1} \delta(1)\, d\mu' = \frac{1}{2}$$

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} \delta(1)\left(1-3\mu'^2\right) d\mu' = \frac{1}{4} \times -2 = -\frac{1}{2}$$

$$\boxed{\frac{Q}{I} = -1}$$

PURELY HORIZONTAL!

$$I(\mu') = \delta(0)$$

$$J_0^0 = \frac{1}{2}\int_{-1}^{1} \delta(0)\, d\mu' = \frac{1}{2}$$

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} \delta(0)\left(1-3\mu'^2\right) d\mu' = \frac{1}{4} \times 1 = \frac{1}{4}$$

$$\frac{\Phi}{I} = \frac{1}{2}$$ PREDOMINANTLY VERTICAL!

Anisotropy:

$$J_0^2 = \frac{1}{4}\int_{-1}^{1} I(\mu')(1 - 3\mu'^2)d\mu'$$

Compare the anisotropy and the mean intensity for following cases:

- Isotropic radiation – **no polarization**
- Radiation coming from below – **Q positive**
- Radiation coming from the sides – **Q negative**
- What is the polarization? - **In first example 0% in second 100%, in last 50%**

This was for the single scattering continuum radiation

- Lines absorb/emit of a range of wavelength
- They are are also formed over a range of heights
- Lines are sensitive to the magnetic field
- There is also “collisional” depolarization
- And the so called intrinsic polarizability
- The expressions are going to be **much more complicated**

“Master equation”

$$
\begin{aligned}
\mathcal{R}_{ij}(\nu,\nu_1,\mathbf{\Omega},\mathbf{\Omega}_1;\boldsymbol{B}) = & \sum_{J_\mathrm{u}M_\mathrm{u}J'_\mathrm{u}M'_\mathrm{u}J_\ell M_\ell J'_\ell M'_\ell KK'Q} \int f(\boldsymbol{v})\mathrm{d}^3\boldsymbol{v}\,(-1)^Q\mathcal{T}^{K'}_{-Q}(j,\mathbf{\Omega}_1)\mathcal{T}^K_Q(i,\mathbf{\Omega}) \\
& \times 3\frac{2L_\mathrm{u}+1}{2S+1}(2J_\mathrm{u}+1)(2J'_\mathrm{u}+1)(2J_\ell+1)(2J'_\ell+1)\sqrt{(2K+1)(2K'+1)}\,(-1)^{M_\ell-M'_\ell} \\
& \times \begin{Bmatrix} J_\mathrm{u} & 1 & J_\ell \\ L_\ell & S & L_\mathrm{u} \end{Bmatrix}\begin{Bmatrix} J'_\mathrm{u} & 1 & J_\ell \\ L_\ell & S & L_\mathrm{u} \end{Bmatrix}\begin{Bmatrix} J_\mathrm{u} & 1 & J'_\ell \\ L_\ell & S & L_\mathrm{u} \end{Bmatrix}\begin{Bmatrix} J'_\mathrm{u} & 1 & J'_\ell \\ L_\ell & S & L_\mathrm{u} \end{Bmatrix} \\
& \times \begin{pmatrix} J_\mathrm{u} & 1 & J_\ell \\ -M_\mathrm{u} & p & M_\ell \end{pmatrix}\begin{pmatrix} J'_\mathrm{u} & 1 & J_\ell \\ -M'_\mathrm{u} & p' & M_\ell \end{pmatrix}\begin{pmatrix} J_\mathrm{u} & 1 & J'_\ell \\ -M_\mathrm{u} & p''' & M'_\ell \end{pmatrix}\begin{pmatrix} J'_\mathrm{u} & 1 & J'_\ell \\ -M'_\mathrm{u} & p'' & M'_\ell \end{pmatrix} \\
& \times \begin{pmatrix} 1 & 1 & K' \\ -p & p' & Q \end{pmatrix}\begin{pmatrix} 1 & 1 & K \\ -p''' & p'' & Q \end{pmatrix} \\
& \times \Bigg\{ \frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{R}+\Gamma_\mathrm{I}+\Gamma_\mathrm{E}+\frac{\mathrm{i}\Delta E_{M_\mathrm{u}M'_\mathrm{u}}}{\hbar}}\,\delta\left(\tilde{\nu}-\tilde{\nu}_1-\nu_{M_\ell M'_\ell}\right)\left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_\mathrm{u}M_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_\mathrm{u}M_\ell}-\tilde{\nu}_1\right)\right] \\
& + \left[\frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{R}+\Gamma_\mathrm{I}+\frac{\mathrm{i}\Delta E_{M_\mathrm{u}M'_\mathrm{u}}}{\hbar}} - \frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{R}+\Gamma_\mathrm{I}+\Gamma_\mathrm{E}+\frac{\mathrm{i}\Delta E_{M_\mathrm{u}M'_\mathrm{u}}}{\hbar}}\right] \\
& \times \left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_\mathrm{u}M_\ell}-\tilde{\nu}_1\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_\mathrm{u}M_\ell}-\tilde{\nu}_1\right)\right]\left[\frac{1}{2}\Phi_{ba}\left(\nu_{M'_\mathrm{u}M'_\ell}-\tilde{\nu}\right)+\frac{1}{2}\Phi^*_{ba}\left(\nu_{M_\mathrm{u}M'_\ell}-\tilde{\nu}\right)\right]\Bigg\}.
\end{aligned}
$$

Don’t worry, we will go simpler – let’s exclude **B**

- First notice tht he problem is axially symmetric (if there is no magnetic field). Axis of symmetry is atmospheric normal (that is why we use μ instead of the two angles)

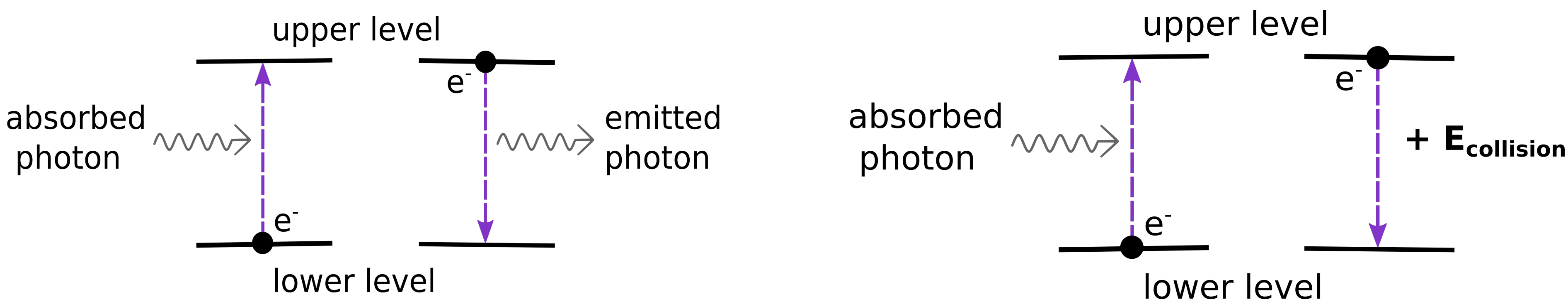
- There is no reason for *U* and *V* to exist. (Not obvious, let’s talk about it).

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda^I$$

$$\frac{dQ_\lambda}{d\tau_\lambda} = Q_\lambda - S_\lambda^Q$$

Let'se see why they call it line scattering – forget about polarization

- **Radiation** can alter the level populations (e.g. photoinization, optical pumping)



- Now, in two level atom case, there are two components to the source function

$$S_\lambda = \epsilon B + (1-\epsilon)\frac{1}{2}\int_0^\infty \int_{-1}^{1} I_\lambda(\mu')\phi_\lambda d\mu' d\lambda = \epsilon B + (1-\epsilon) J_0^0$$

This looks like the scattering we saw before (with an extra term that integrates over the wavelength – this is the famous complete frequency redistribution)

Now, with the polarization things become more complicated

- From Trujillo Bueno (2003) – Generation and Transfer of Polarized radiation, there is a lot to unpack here:

Collisional depolariation

Intrinsic line polarizability

$$S_\lambda^I = \epsilon B + (1-\epsilon)\left(J_0^0 + w^c w^H w^2 \frac{1}{2\sqrt{2}}(3\mu^2 - 1)J_0^2\right)$$

$$S_\lambda^Q = (1-\epsilon) w^c w^H w^2 \frac{3}{2\sqrt{2}}(\mu^2 - 1)J_0^2$$

Hanle depolarization

$$J_0^2 = \frac{1}{4\sqrt{2}}\int_0^\infty \int_{-1}^1 I_\lambda(\mu')(3\mu'^2 - 1)d\mu' \phi_\lambda d\lambda$$

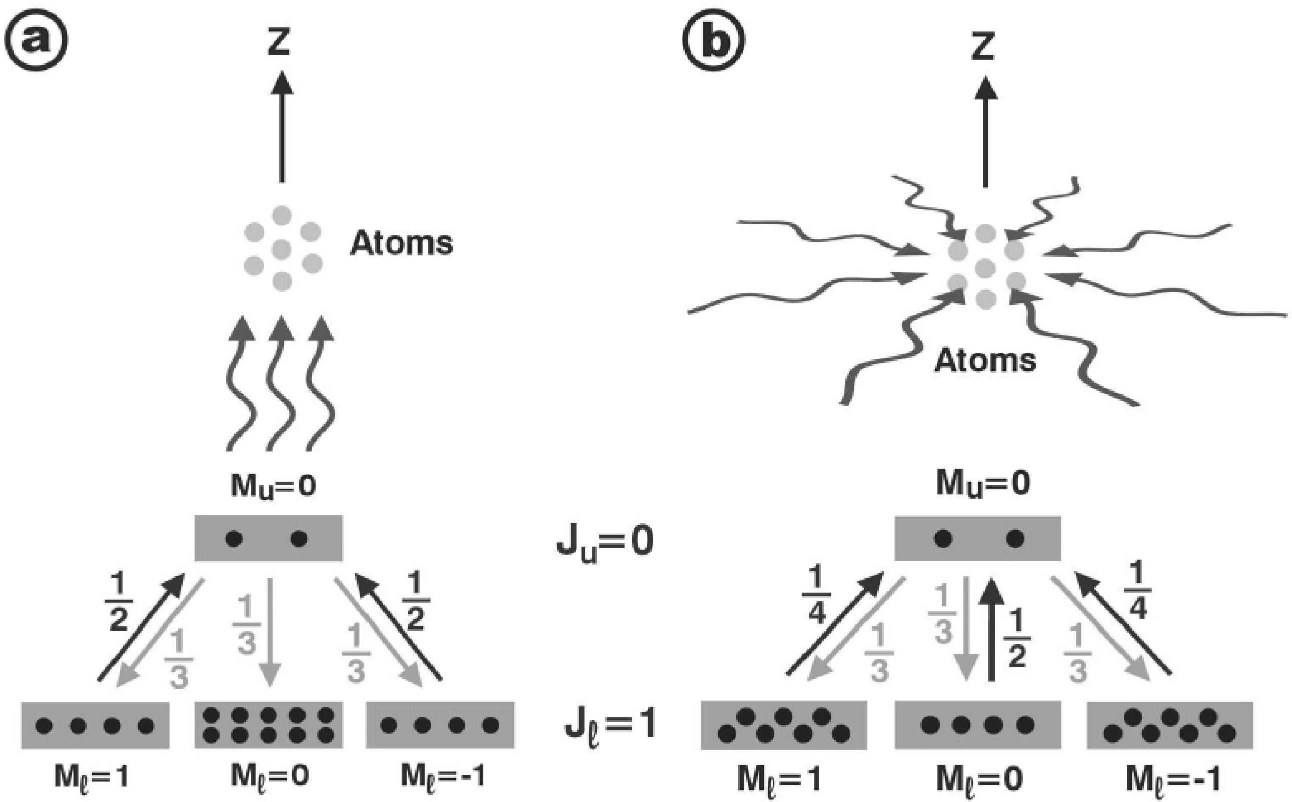
$$S_\lambda^I = \epsilon B + (1-\epsilon)\left(J_0^0 + w^c w^H w^2 \frac{1}{2\sqrt{2}}(3\mu^2 - 1)J_0^2\right)$$

$$S_\lambda^Q = (1-\epsilon) w^c w^H w^2 \frac{3}{2\sqrt{2}}(\mu^2 - 1)J_0^2$$

$$J_0^2 = \frac{1}{4\sqrt{2}} \int_0^\infty \int_{-1}^1 I_\lambda(\mu')(3\mu'^2 - 1)d\mu' \phi_\lambda d\lambda$$

- Source function is **anisotropic**
- Anisotropy modifies the “pure intensity” too!
- Sensitivity to the magnetic field
- More NLTE → more polarization
- Very very interesting and subtle

Scattering line polarization – QM picture



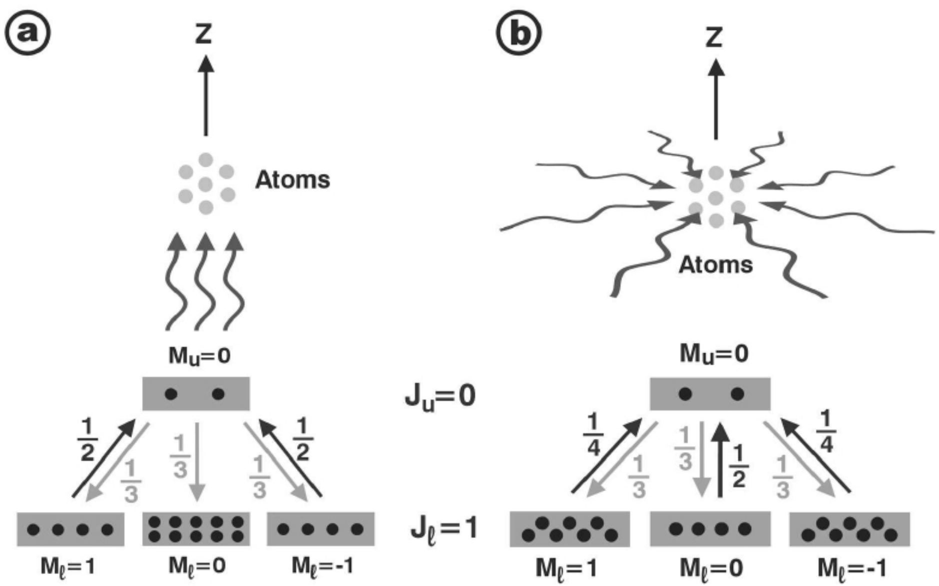
from Trujillo Bueno (2003)

„Selective absorption“

Uneven population of Zeeman subl-levels leads to the “polarization” of the atomic levels.

This leads to the net linear polarization of the light.

Scattering line polarization – analogy with Zeeman



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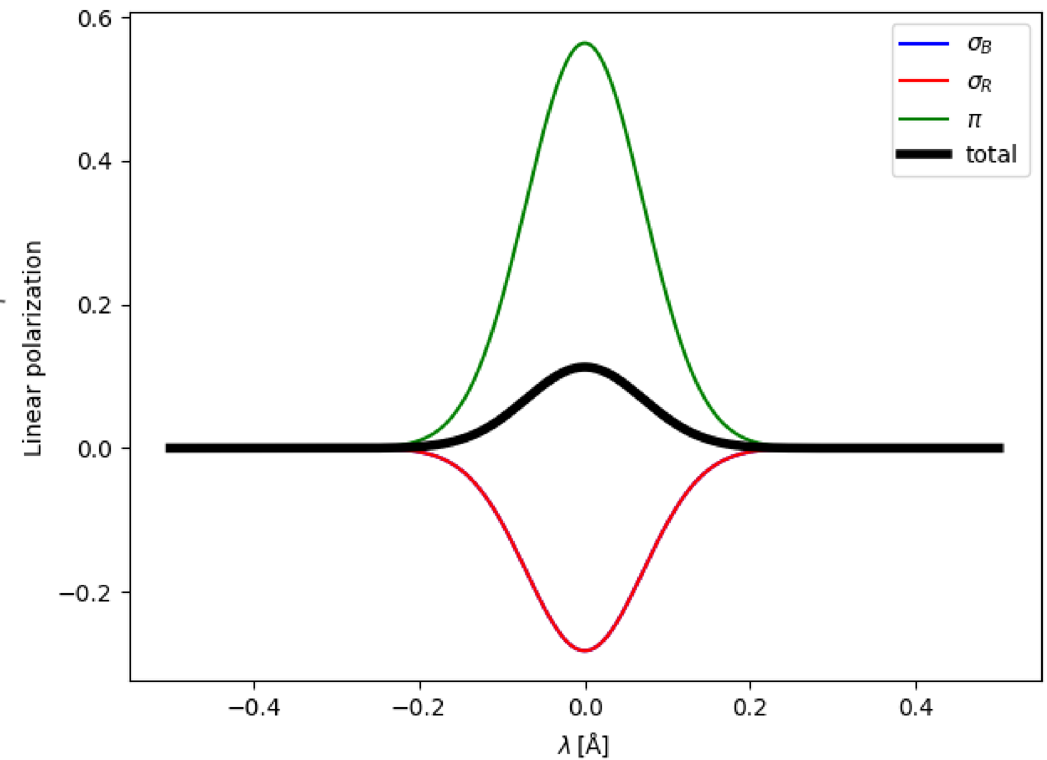


Why is the radiation in the atmosphere anisotropic?

- Let's write down RTE for inclined rays:

$$\frac{dI_\lambda}{ds} = \frac{dI_\lambda}{dz/\cos\theta} = \mu\frac{dI_\lambda}{dz} = -\chi_\lambda I_\lambda + j_\lambda$$

$$\mu\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

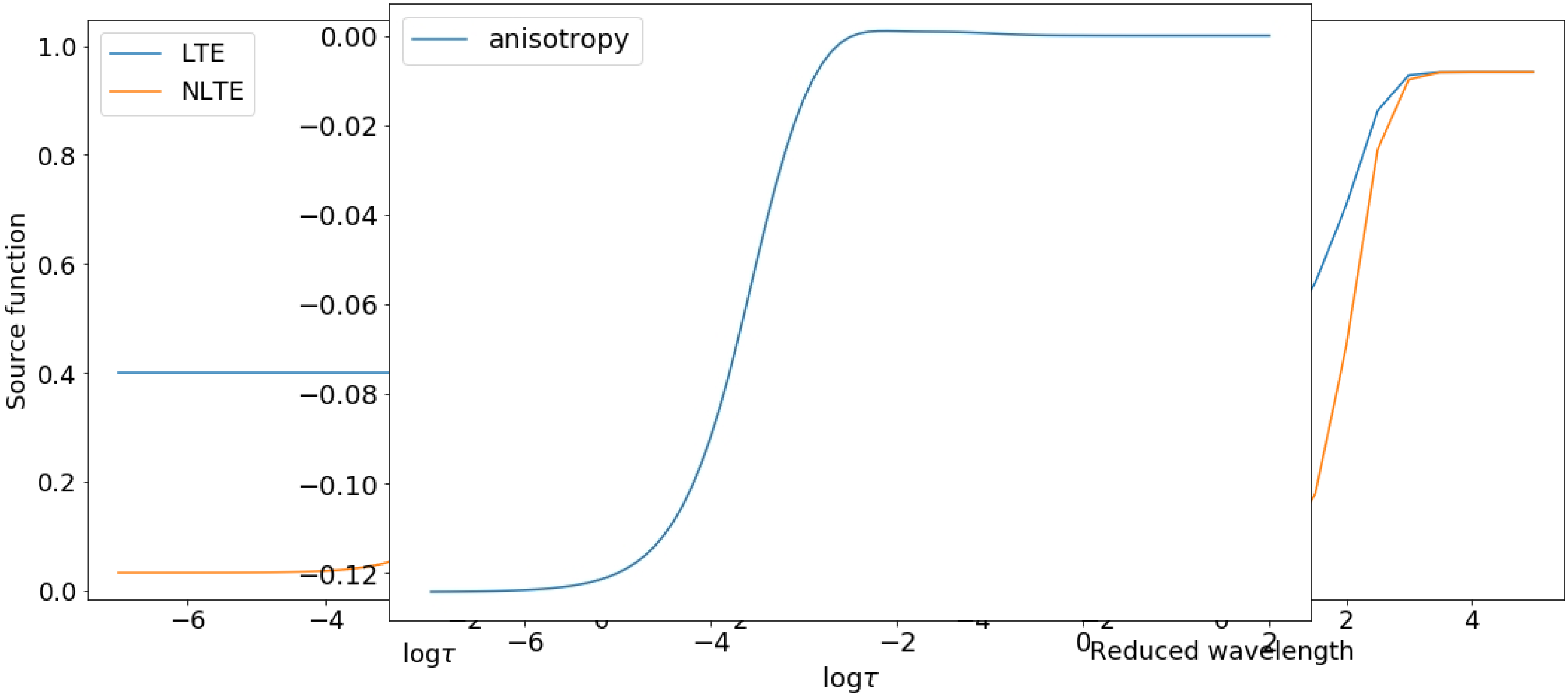
- Use Milne-Eddington approximation:

$$S = a + b\tau_\lambda$$

b is the Source function gradient. Larger gradient → more anisotropy

$$I_\lambda^+ = \int_0^\infty (a + b\tau_\lambda)e^{-\tau_\lambda/\mu}d\tau_\lambda/\mu = a + b\mu$$

NLTE gradients are larger:



“Microturbulent” Hanle effect

- Mixed polarity fields in a pixel would not be seen by Zeeman polarization (convince yourself of that)
- But, with Hanle:

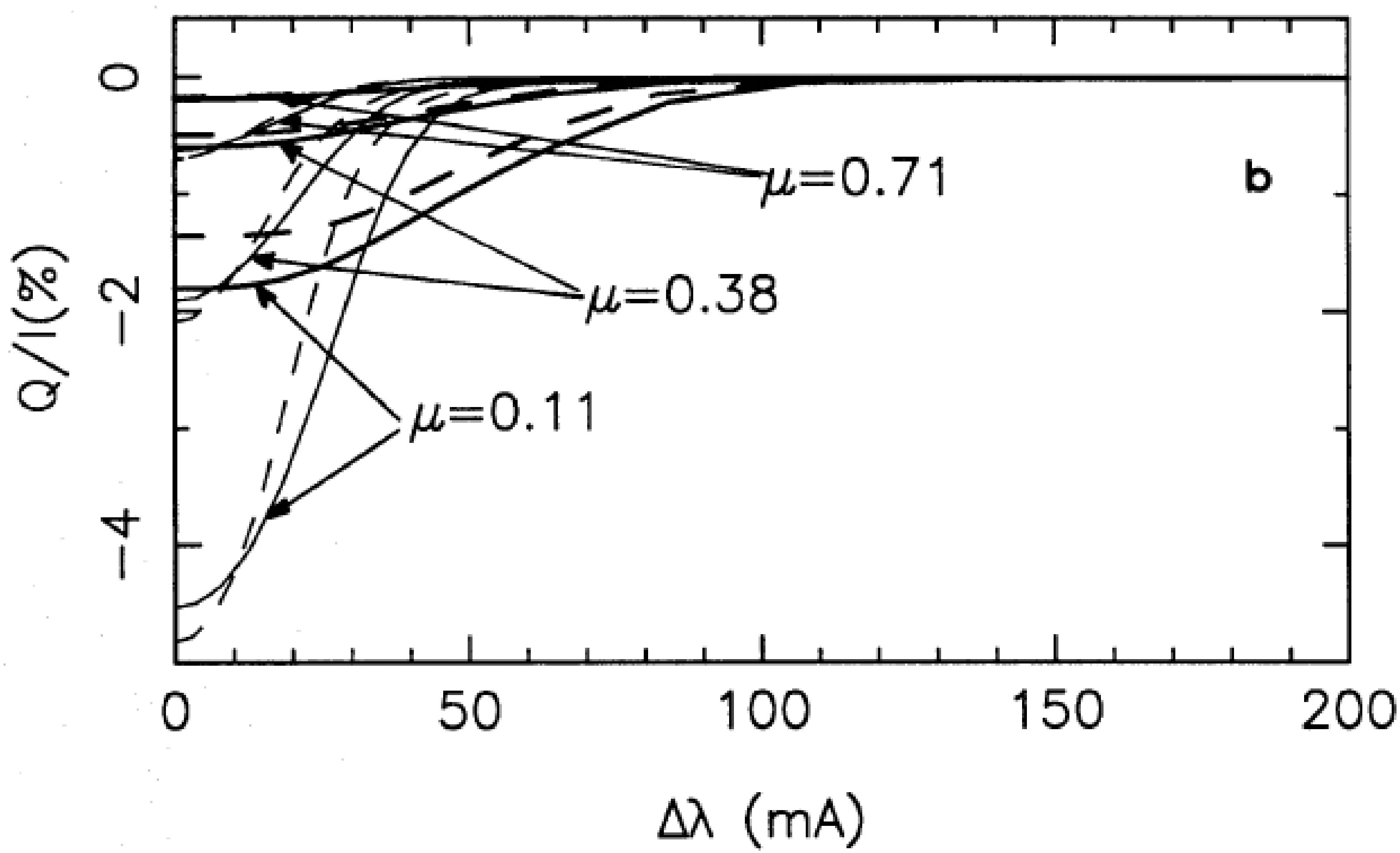
$$w^H = 1 - \frac{2}{5}\left(\frac{\Gamma_H^2}{1+\Gamma_H^2} + \frac{4\Gamma_H^2}{1+4\Gamma_H^2}\right)$$

$$\Gamma_H = 0.88\frac{gB}{A_{ul} + \Gamma_{\mathrm{depolarizing}}}$$

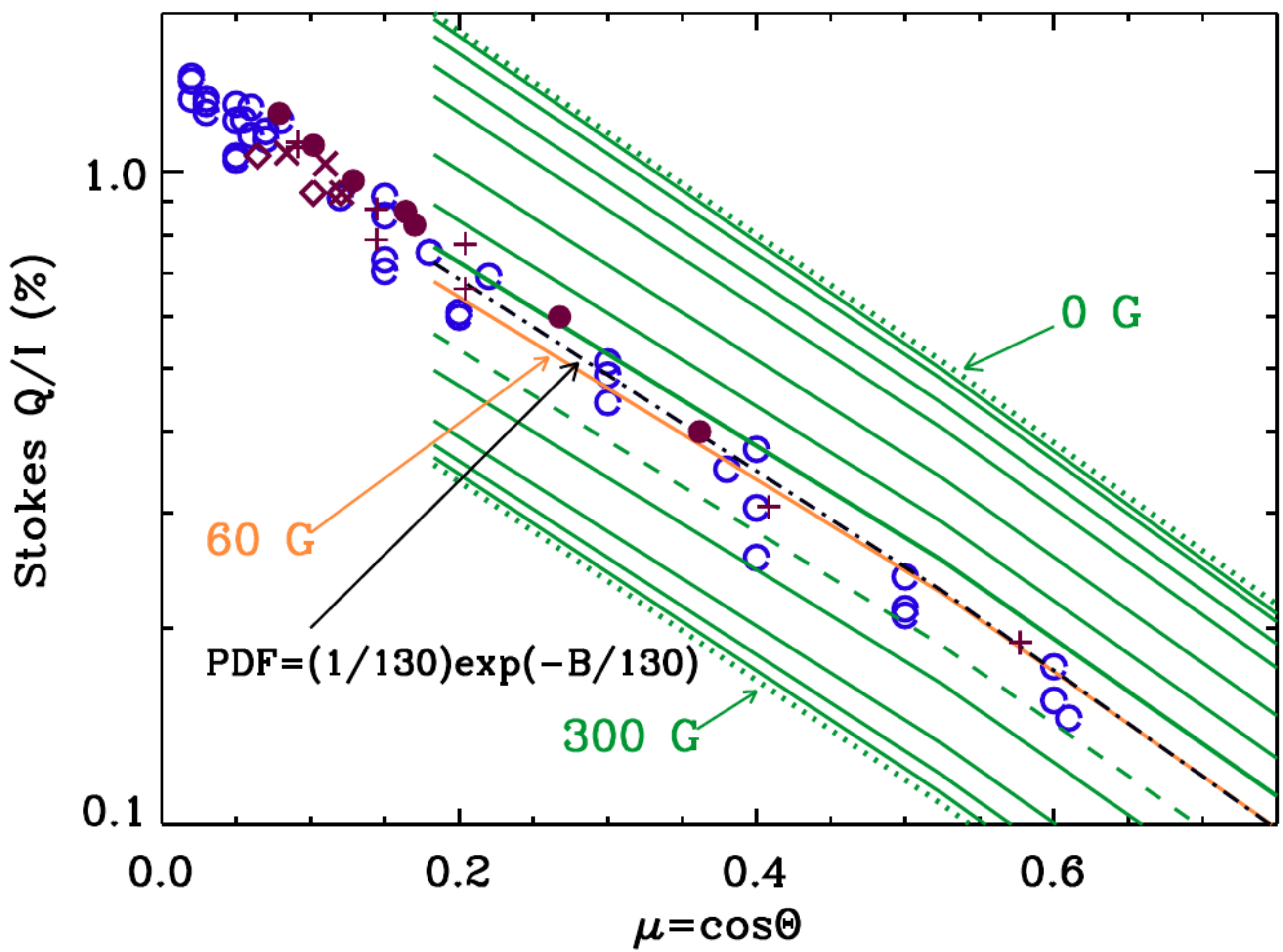
Ok, enough, let's see some results!

- How does Hanle diagnostic in the atmosphere work.
- First, polarization degrees are very small $\rightarrow$ high sensitivity needed $\rightarrow$ no spatial resolution
- That means we usually use some prototype atmosphere (e.g. FALC) to model anisotropy and then fit B to it.
- Usually we can do it at several heliocentric angles to get some more insight

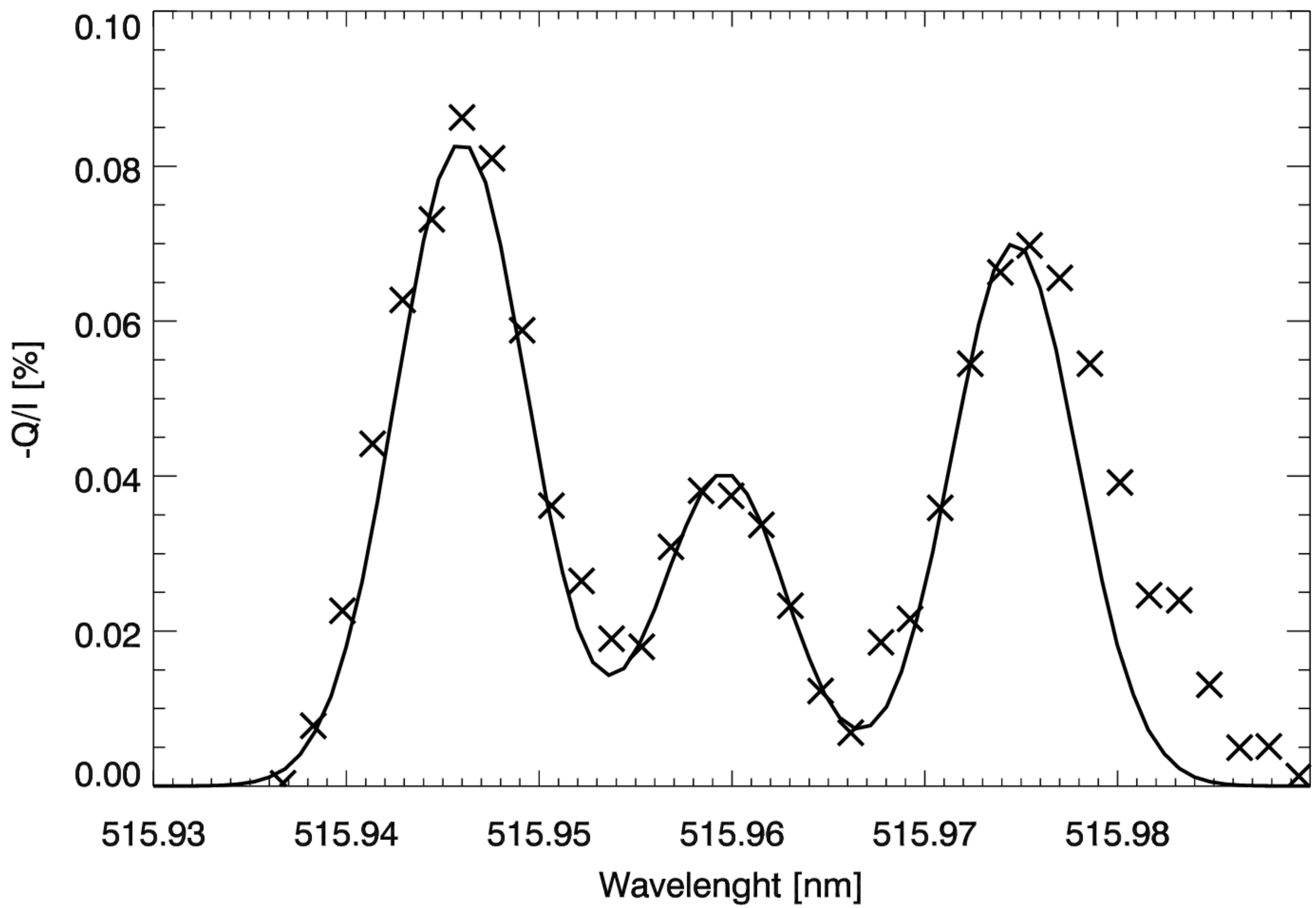
Faurobert-Scholl (1993), studying Sr 4607 polarization



The famous TB et al Nature paper, again using Sr 4607



Yours truly, playing with scattering polarization in molecules

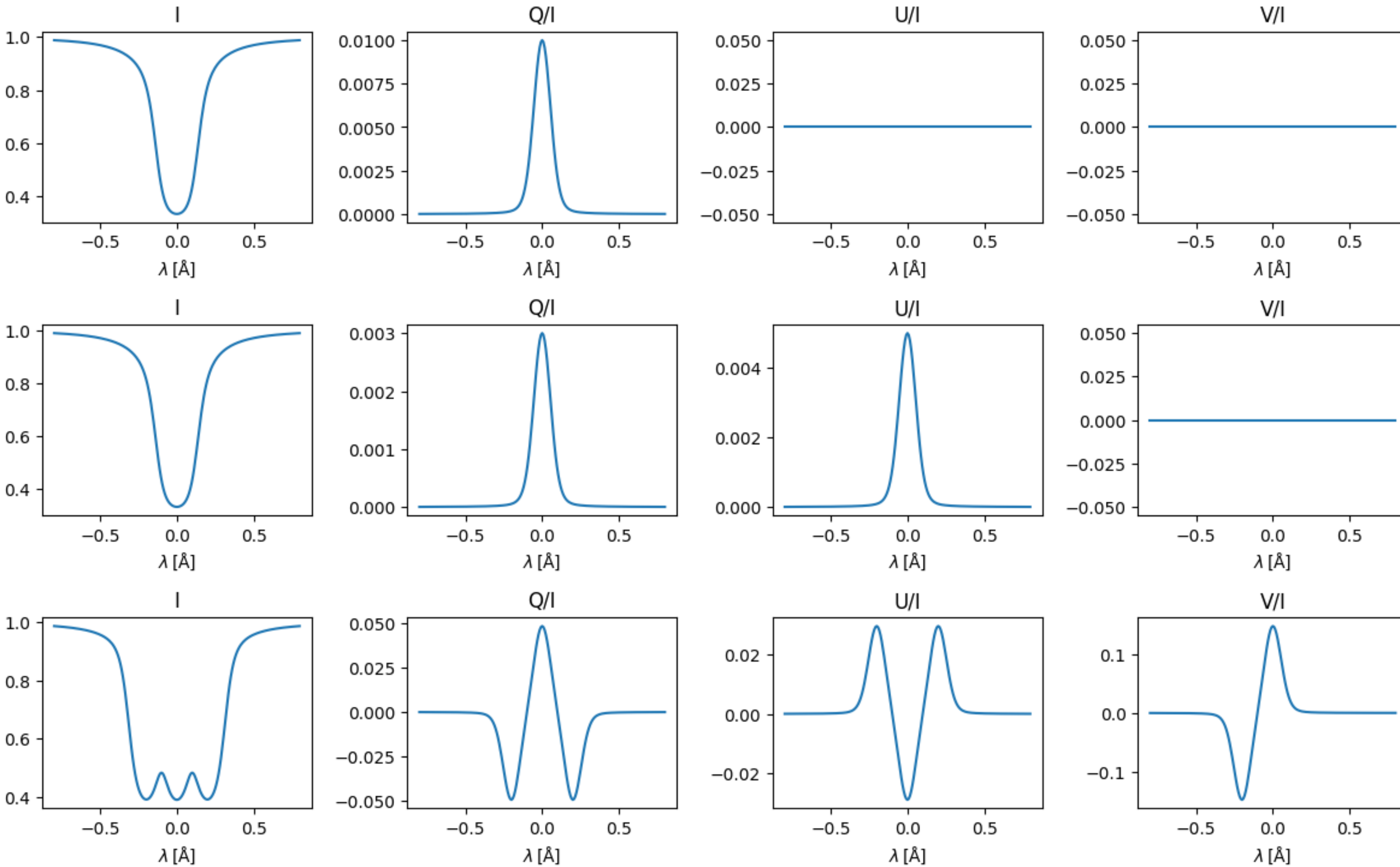


Milic & Faurobert (2012) – using depth dependent magnetic fields to fit multiple lines at multiple heliocentric distances

Summary

- Scattering polarization is a consequence of NLTE-ness
- Atomic physics involved is really complicated, but there are some analogies to be drawn to the classic case
- Hanle effect further modifies that polarization, rotates and depolarizes the lines
- It can see mixed polarity fields on small scales
- So far the analysis shows the magntude of that field in photosphere to be order of 100G, which is concordance with highest resolution Zeeman diagnotics
- With DKIST we aim to go a step further and also probe **horizontal anisotropies** (Ask Neeraj)

Zeeman vs Hanle



Pure scattering, no magnetic field

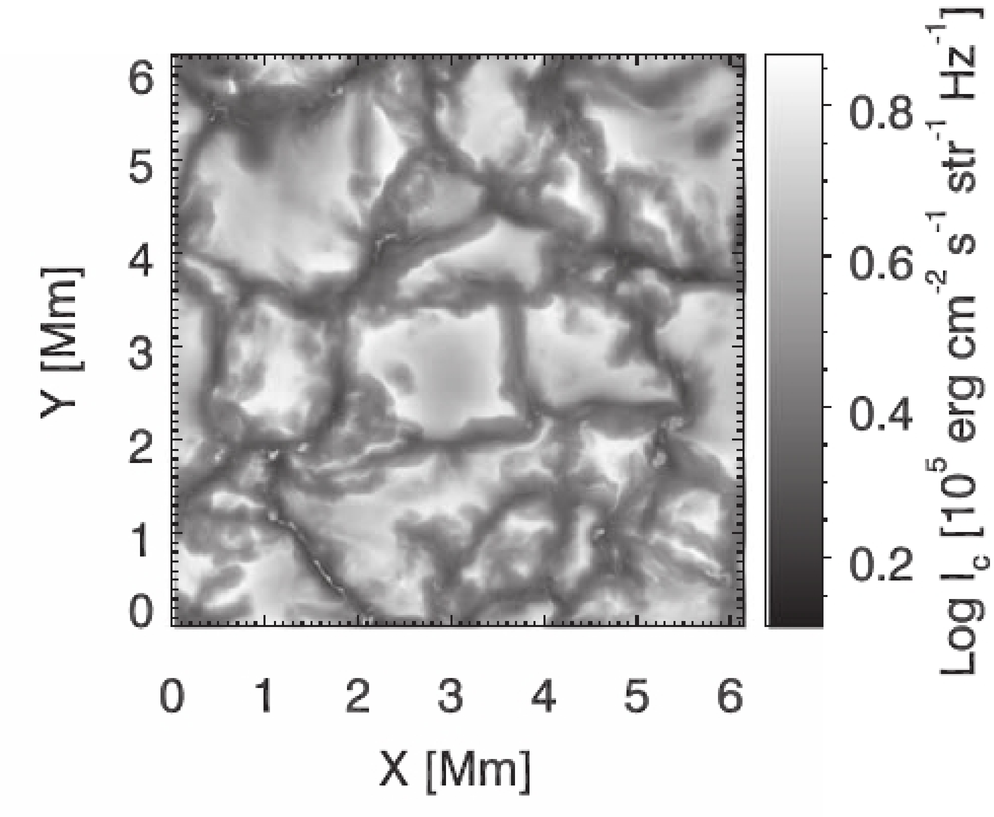
Weak magnetic field

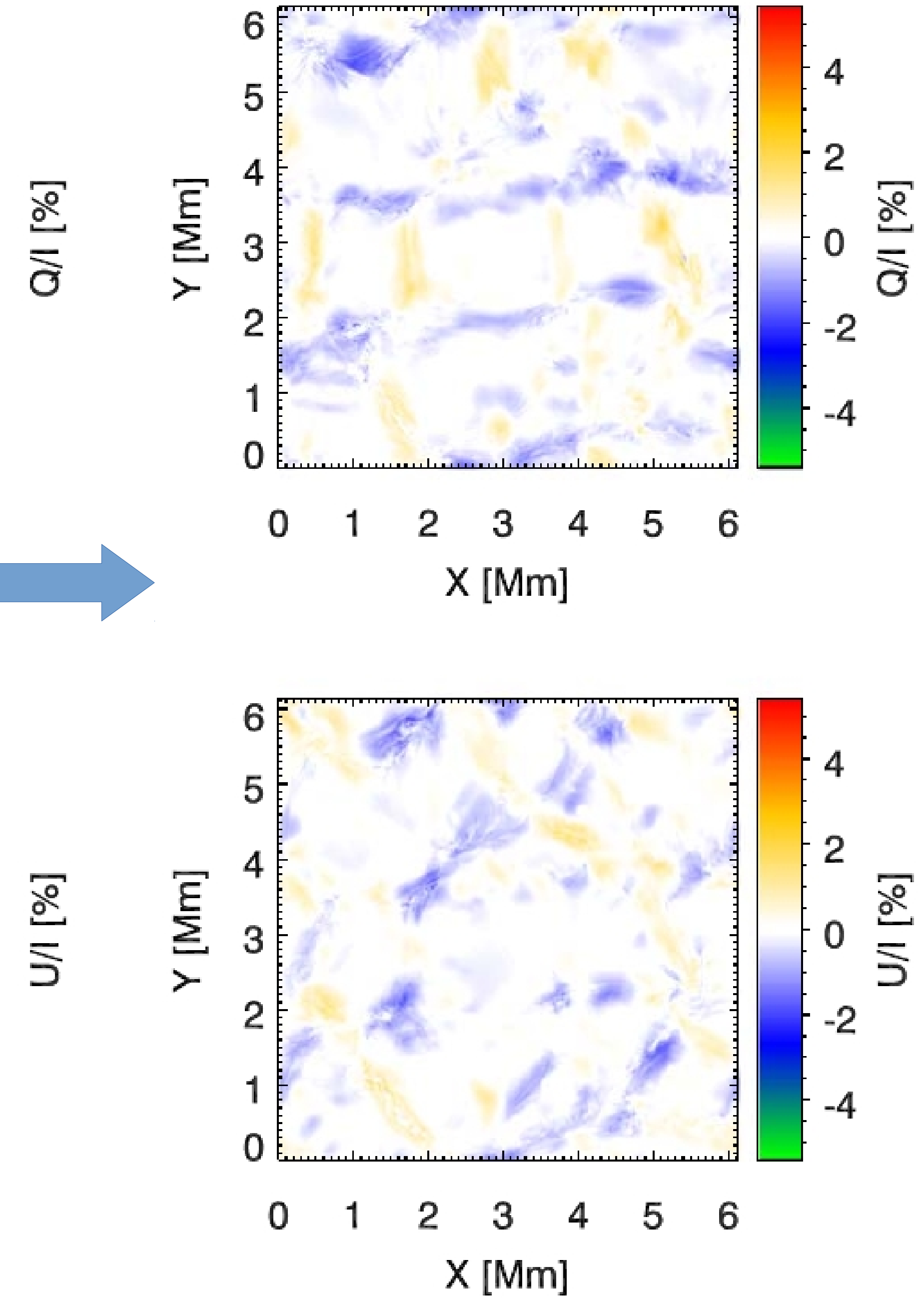
$$\frac{0.88 g B \times 10^5}{A_{ul}} \approx 1$$

Strong magnetic field

$$\Delta\lambda_{\mathrm{Zeeman}} > \Delta\lambda_{\mathrm{Dopp}}$$

Horizontal anisotropies – the future





From del Pino Aleman et al. (2018)