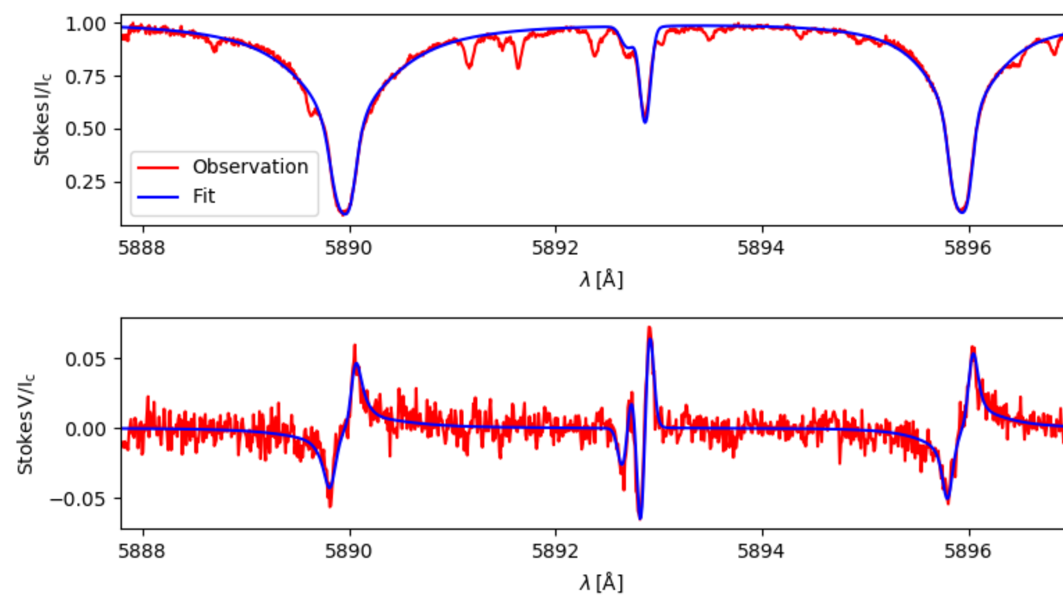
PHYS 7810: Solar Physics with DKIST

Lecture 21: Inversions and how to do them

Ivan Milic *ivan.milic@colorado.edu*



Previous lectures

- We have been introduced to line formation (model atmosphere → spectrum)
- We have been introduced to model fitting
- Once you choose your **model** and have your **observables** you can perform inference
- We have seen that method of choice is **posterior maximization** that with uniform priors reduces to **maximum likelihood**, which under the assumption of gaussian errors reduces to **chi-squared minimization.**

$$\chi^2 = \sum_i \sum_s (I_{i,s}^{\mathrm{obs}} - I_{i,s}^{\mathrm{synth}})^2 / \sigma_{i,s}^2$$

What we usually omit is what synthetic intensity depends on

- Our model is our choice of the approach to model polarized line formation.

$$I_{s,i}^{\mathrm{synth}} = \mathcal{F}\left[T(h), p_g(h), v_{los}(h), \zeta(h), \vec{B}(h)\right]$$

Some approaches
(increasing complexity, let’s discuss each one separately)

- Weak field approximation (what we did last class)
- Milne-Eddington approximation (Everything constant, the source function is linear)
- ***Stratified atmosphere, with all species in LTE***
- ***Stratified atmosphere, with species in NLTE***
- Stratified atmosphere, NLTE, with Partial Frequency Redistribution
- “Non atmospheric” models, e.g. slab model for hazel

We are going to focus on middle two

- ***Stratified atmosphere, with all species in LTE***
- ***Stratified atmosphere, with species in NLTE***

- We have observed spectra, we want to know atmospheric structure.
- But that is a function.
- Simplest is to discretize somehow. Then we can solve forward problem using methods from previous classes.

LTE line synthesis flowchart:

Atmosphere model: temperature, density, velocity, magnetic field

↓

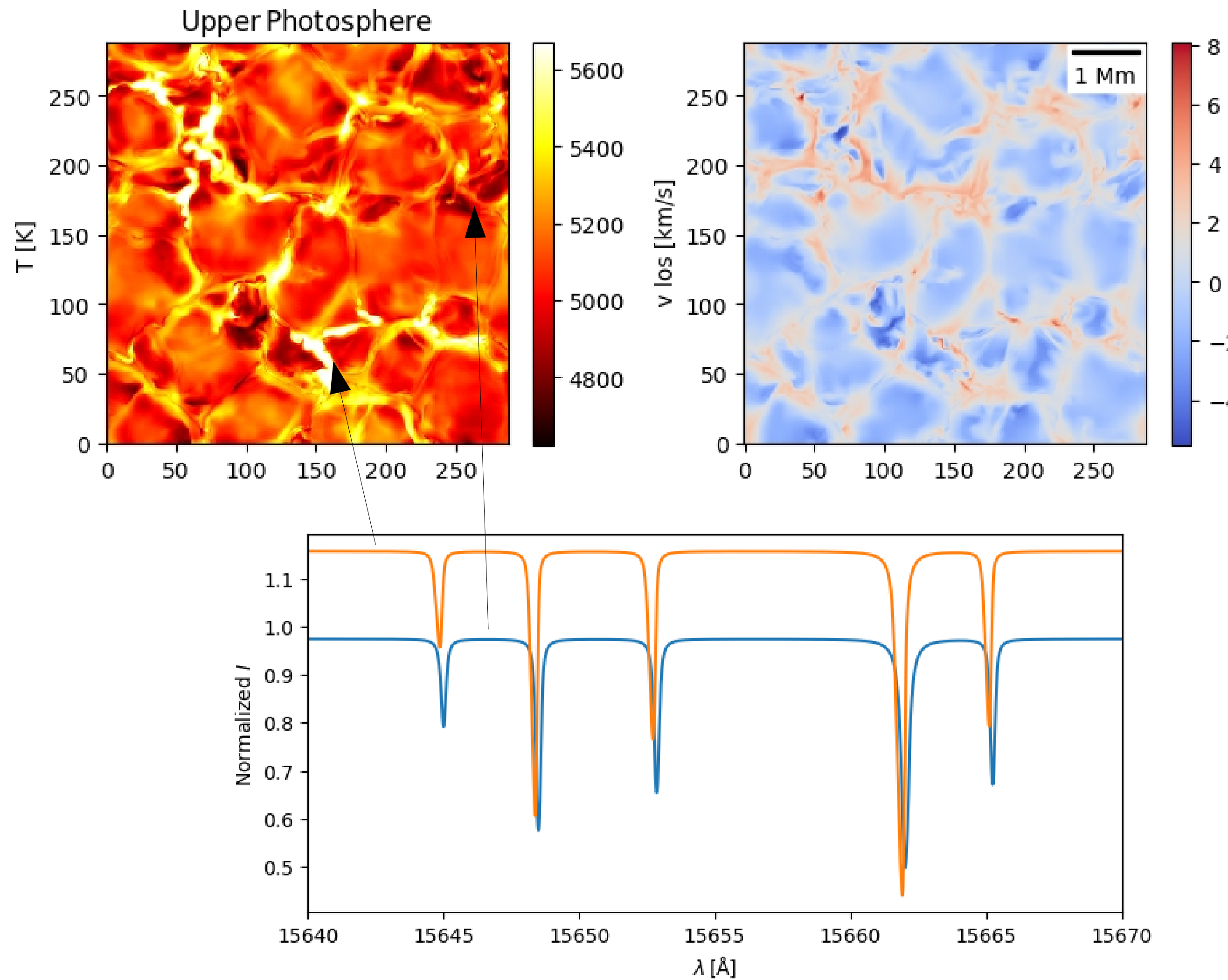
Use Saha-Boltzmann to calculate all relevant level populations

↓

Calculate opacity/emissivity everywhere in the atmosphere

↓

Solve Radiative Transfer Equation → Get the spectrum



MURAM quiet Sun simulation, courtesy of T. Riethmüller

What now, what comes next?

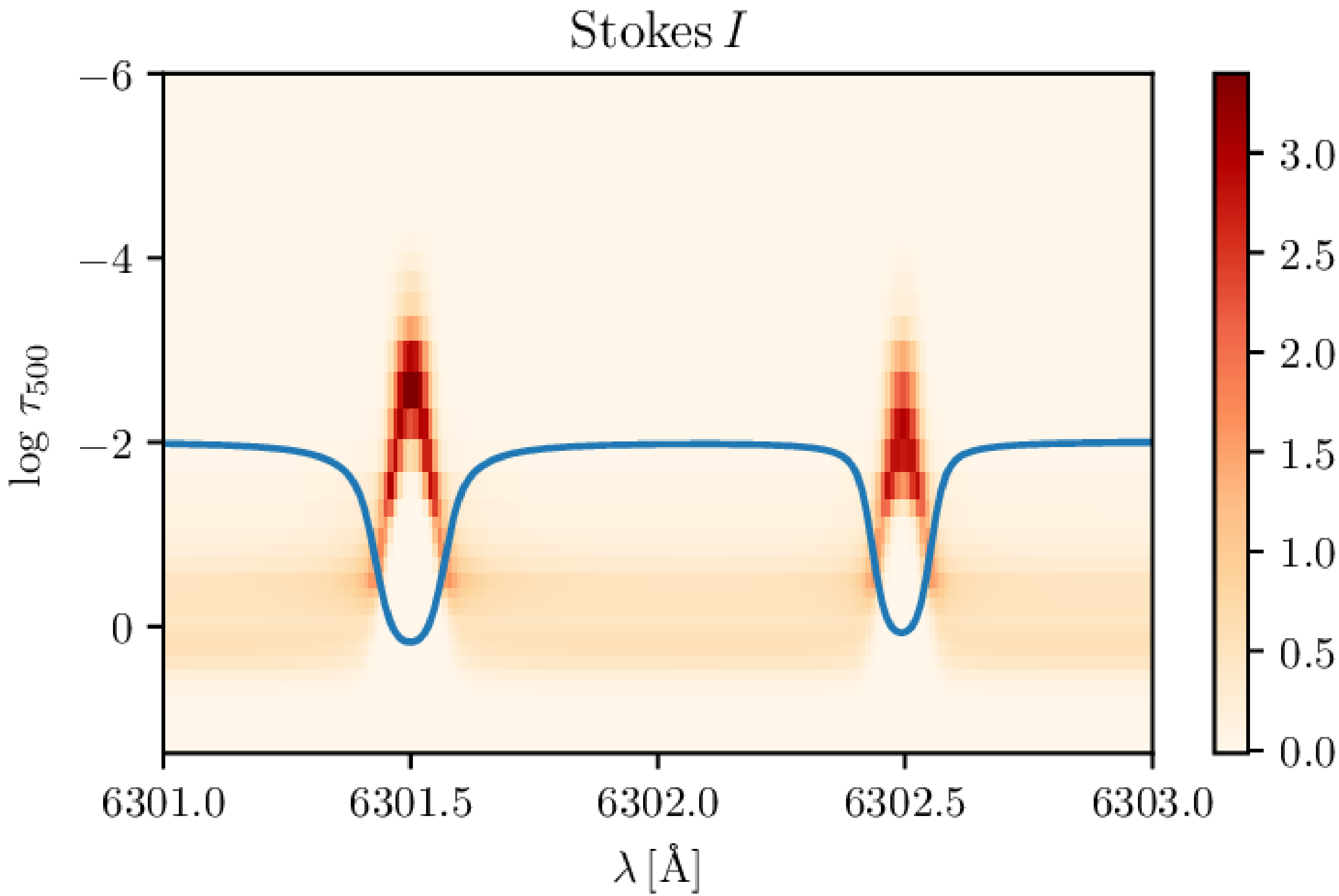
- We have to change the parameters **p** of the atmosphere to get a fit

For us it would be:

$$\sum_{j}^{NM} \frac{\partial I_{s,i}(\boldsymbol{p})}{\partial p_j} = (I_{s,i}^{\mathrm{obs}} - I_{s,i}^{\mathrm{synth}}) = \delta I_{s,i}$$

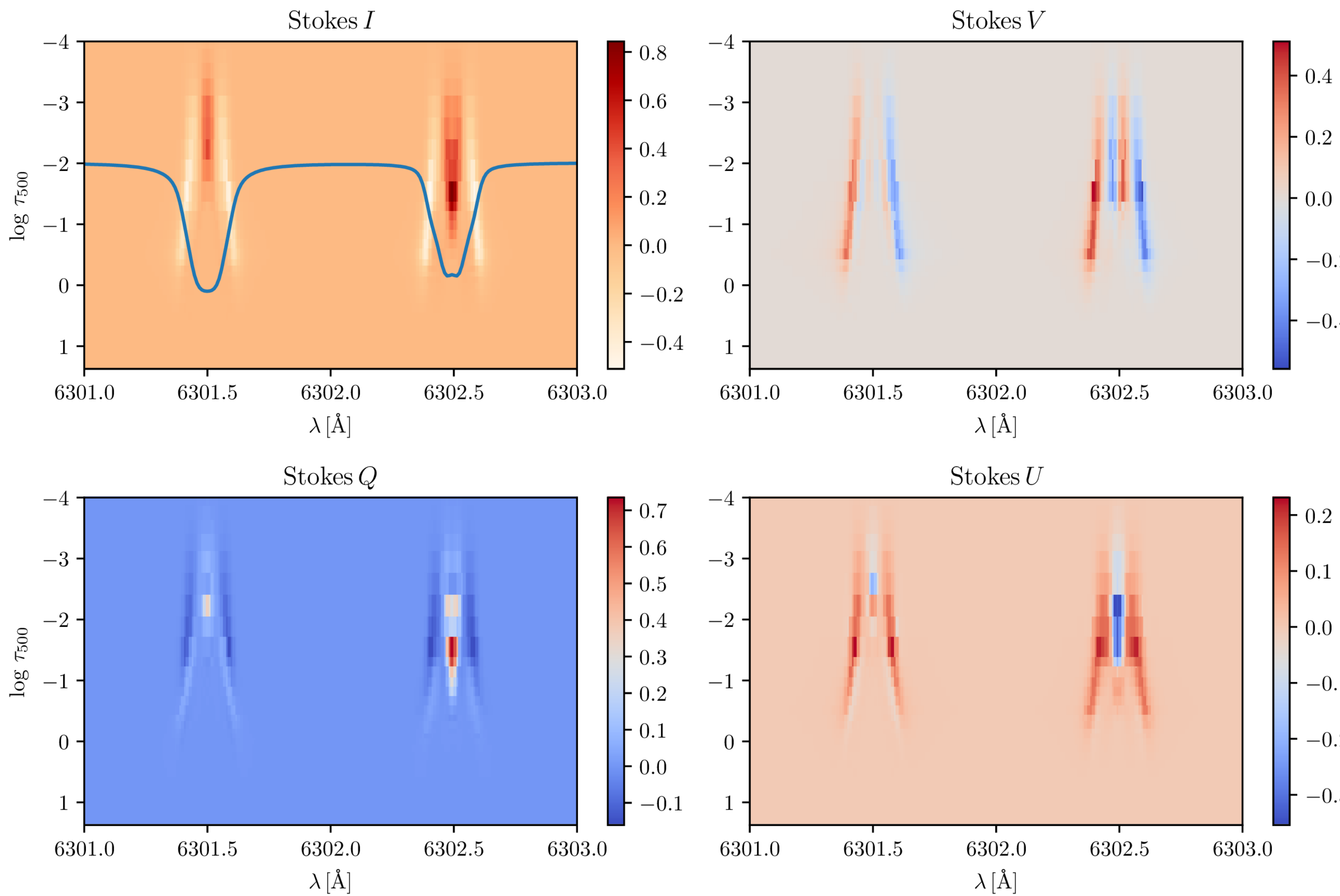
We will call the partial derivative of the specific polarized intensity with respect to a physical quantity at given depth **response function.**

Again, the response function, let's talk a lot about these



$$\frac{\partial I(\lambda)}{\partial T_d}$$

They look different for different parameters, eg. B



Levenberg – Marquardt

$$\sum_{j}^{NM} \frac{\partial I_{s,i}(\boldsymbol{p})}{\partial p_j} = J_{j,i}\delta p_j = \delta I_i$$

(I absorbed s into i to be more concise, as we did few slides back).

Scheme:

- Assume a model atmosphere, i.e. vector $\boldsymbol{p}_0$.
- Calculate $\boldsymbol{I}(\boldsymbol{p})$ and evaluate $\hat{J}$ and $\delta\boldsymbol{I}$.
- Calculate correction to the parameter vector as by multiplying both sides with $\hat{J}^T$:

$$\hat{J}^T\hat{J}\delta\boldsymbol{p} = \hat{J}^T\delta\boldsymbol{I}$$

- Correct the model and re-start the loop. As soon as $\boldsymbol{I}^{\mathrm{calc}}$ is good enough, stop.

How to calculate the response functions?

We imagine we perturbed a parameter at some depth. Everything changes.

What we want to know is how intensity changes

$$\frac{dI}{dz} = -\chi I + j$$

$$\frac{dI + \delta I}{dz} = -(\chi + \delta\chi)(I + \delta I) + (j + \delta j)$$

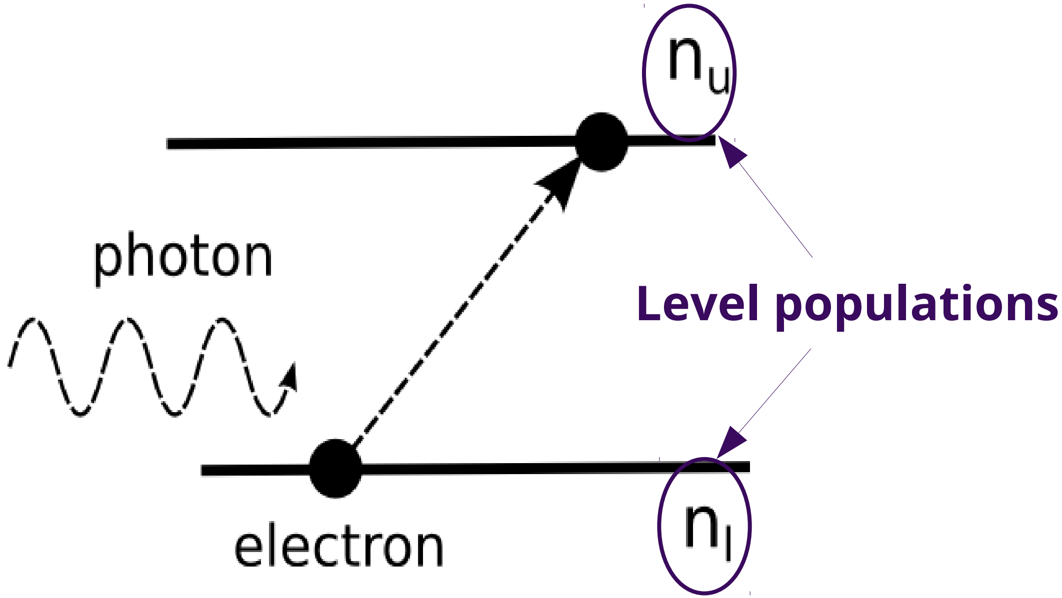
Change (response) in the intensity (what we need)

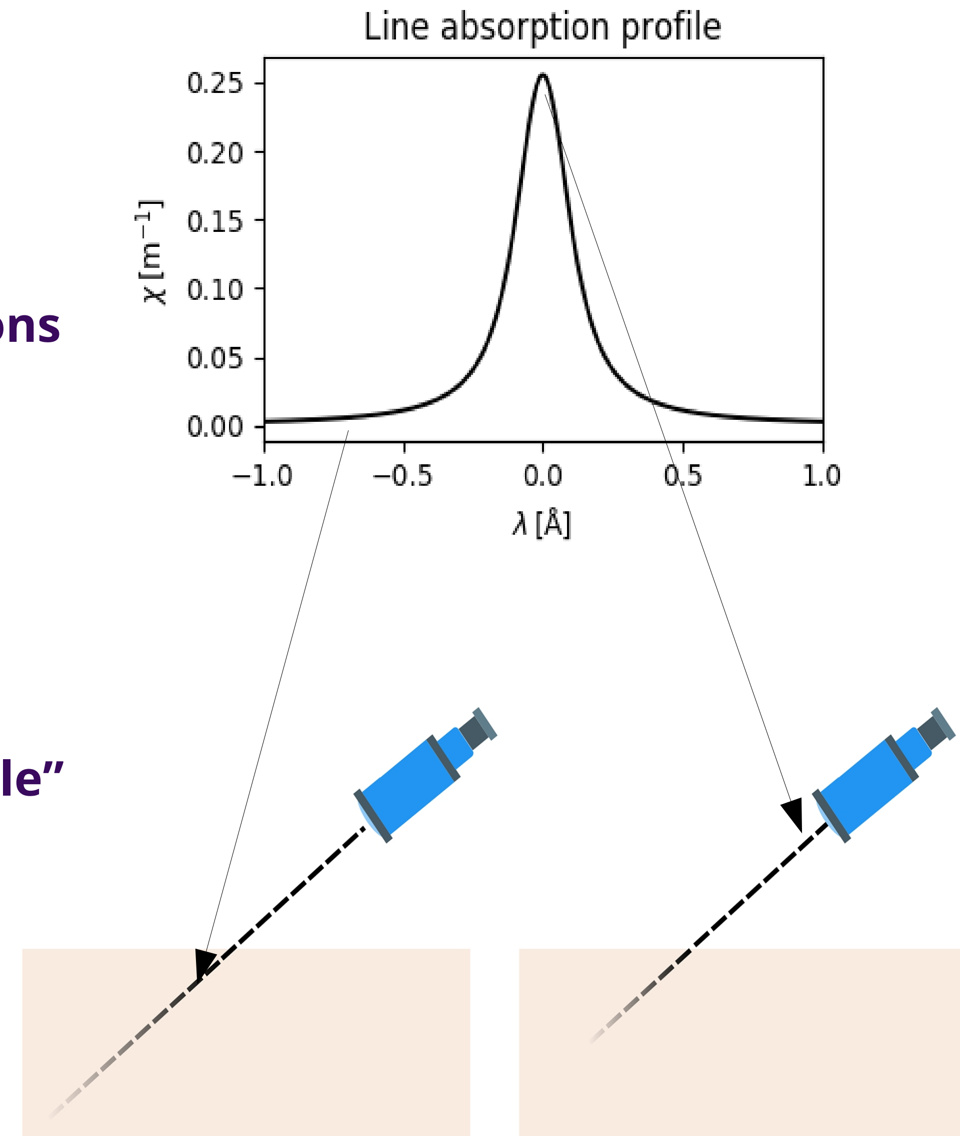
$$\frac{\boxed{\delta I}}{dz} = -\chi\delta I + (\boxed{\delta j} - \boxed{\delta\chi} I)$$

Changes in opacity and emissivity (what we have to move on to)

This looks like RTE!

Opacity and emissivity responses follow from here





"Line profile"

$$j(\lambda) = \frac{hc}{4\pi\lambda} n_u A_{ul} \phi(\lambda)$$

$$\chi(\lambda) = \frac{hc}{4\pi\lambda} (n_l B_{lu} - n_u B_{ul}) \phi(\lambda)$$

So, we need “responses” of opacity and emissivity

- There are two parts. **“Obvious one”,** changes in the line profile (splitting, shifting, broadening, polarization)

- **Non-obvious one: changes in the level populations (extremely complicated in NLTE)**

$$\frac{n_{j+1}n_e}{n_j} = \frac{2Z_{j+1}}{Z_j\Lambda^3}e^{-E_{\rm ion}/kT}$$

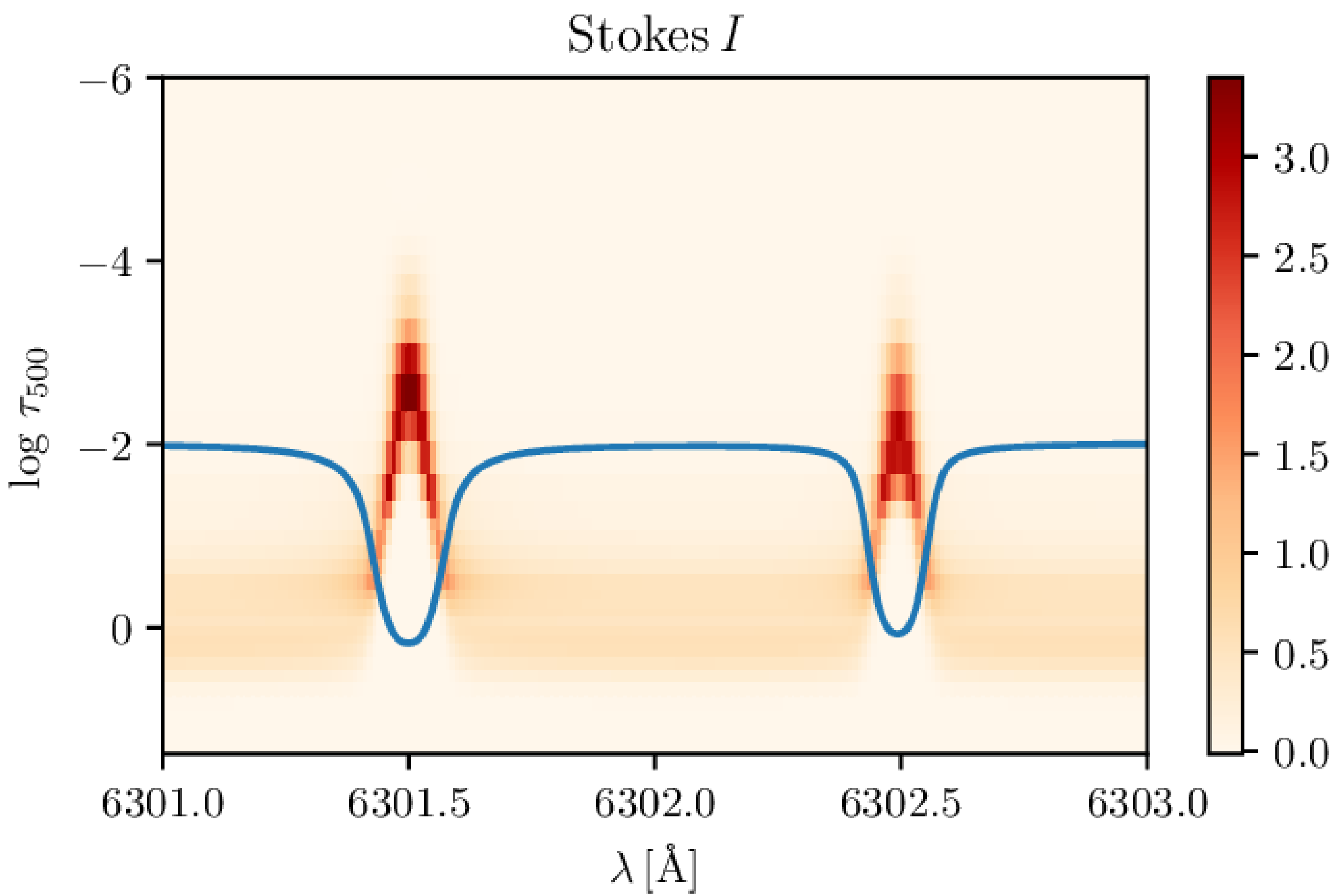
$$\Lambda = \sqrt{h^2/2\pi m_e kT}$$

$$n_{j,i} = n_j\frac{g_i e^{-E_i/kT}}{Z_j}$$

Response function calculation recipe

- Using Saha equation, calculate how the ionization changes and molecular number densities change when we change parameters (temperature, density)
- Using Boltzmann equation, see how excitation changes (temperature)
- Using your choice for damping see how damping changes (temperature, density)
- See how the line shift (los velocity)
- And broaden (temperature, microturbulence)
- And split / polarize (magnetic field vector)
- Put them all together, and “propagate” to get response function.

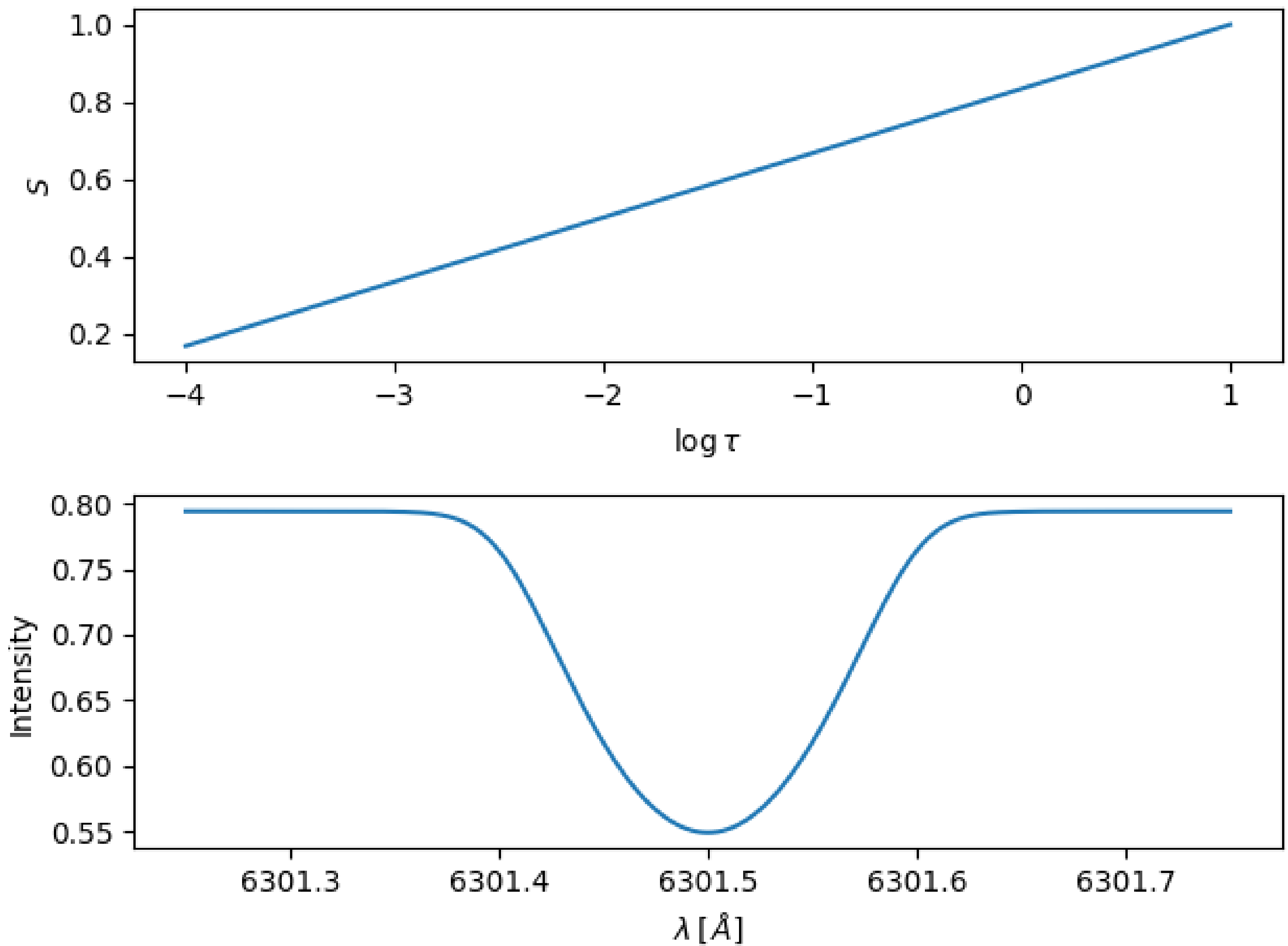
You need to do this once for each depth point to get this



$$\frac{\partial I(\lambda)}{\partial T_d}$$

But now we just plug this into the L-M and we are done, right?

A very simple linear line formation model to illustrate this



My model is:

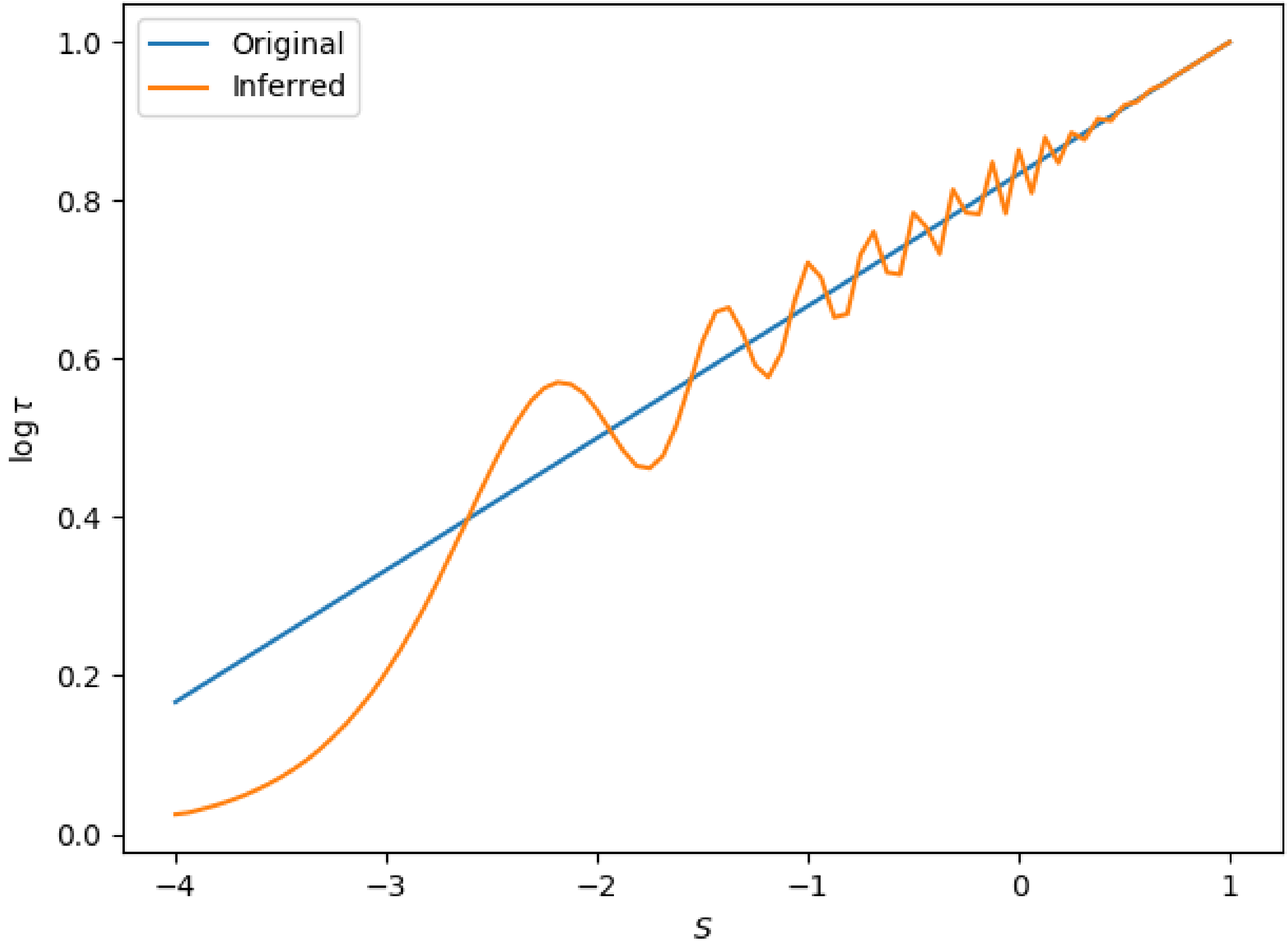
Measurement

Uknown

$$I_i = \sum_d w_{d,i} S_d$$

Weights, we assume we know (we don’t in general!)

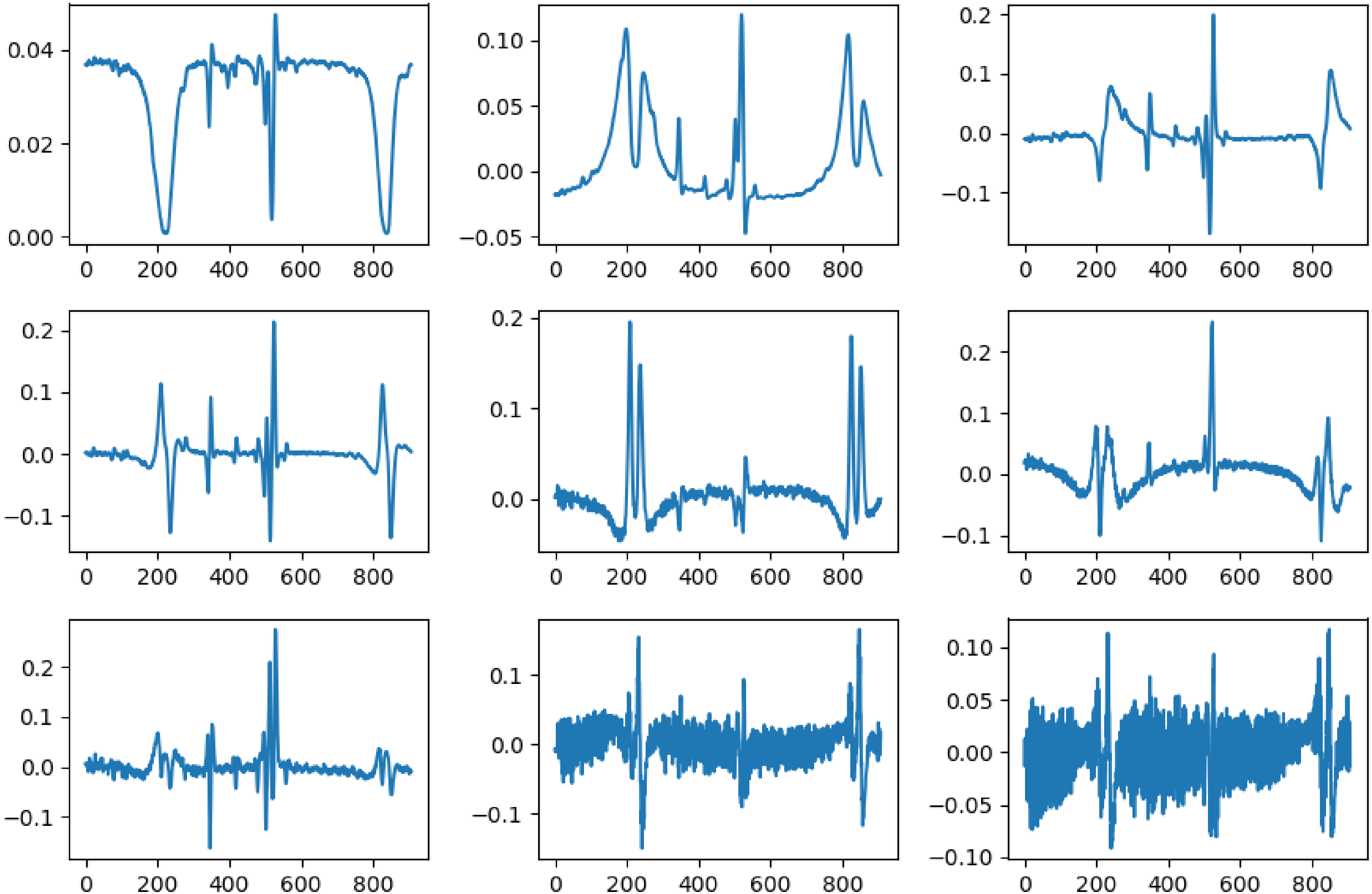
Does not work



Why is this so?

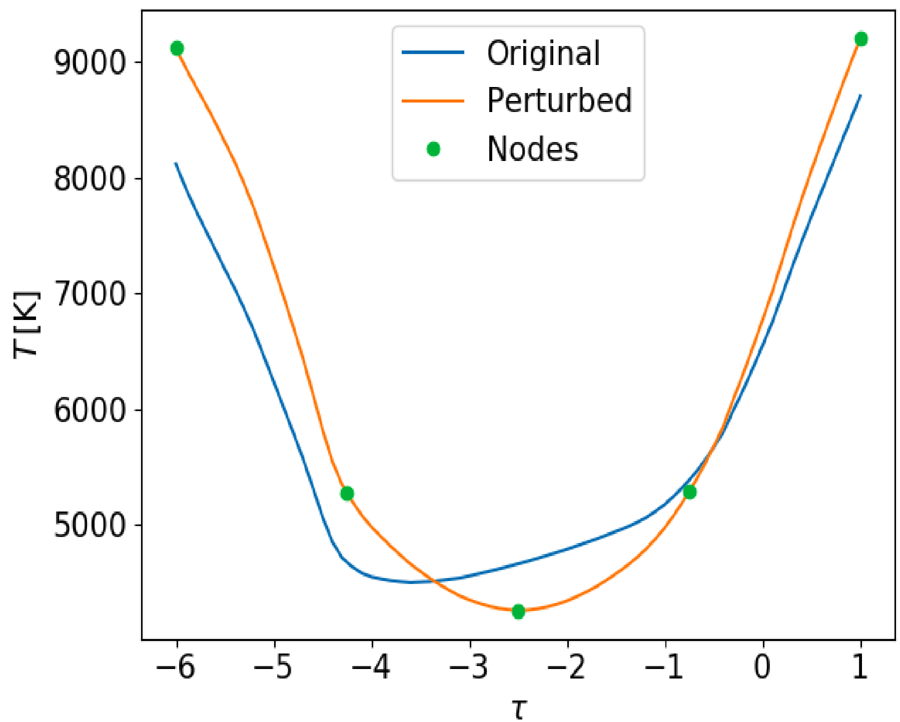
- Well, first of all, we can discretize our atmosphere as much as we want
- Does that mean all these values can be inferred? **No**
- Some points are simply not important.
- Some points are degenerate with other points.
- This means that inversion process is, actually, underdetermined.
- One way to think about this is that dimensionallity of our data is low.

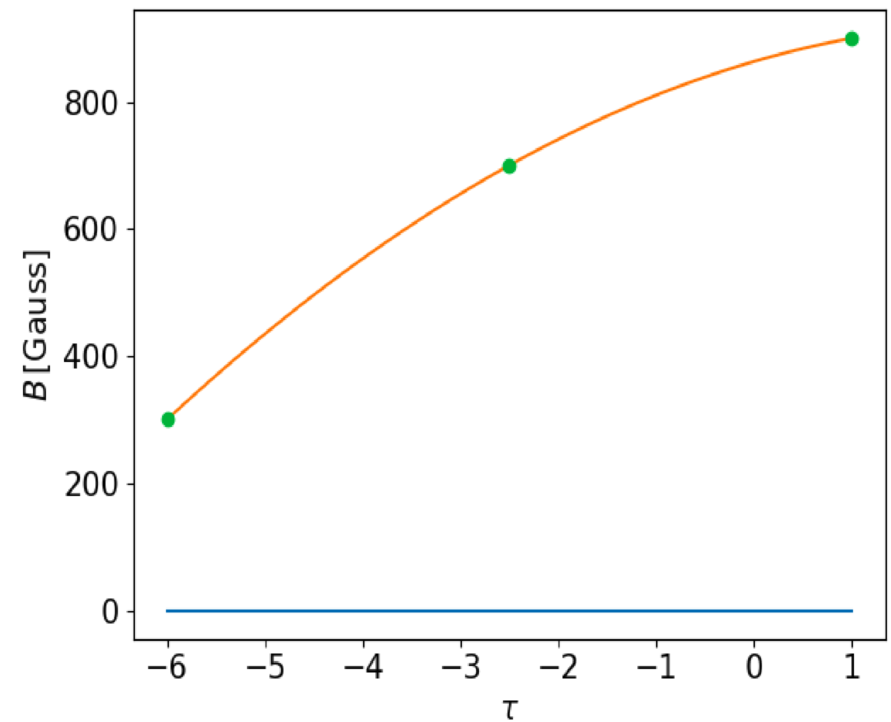
Our data is low dimensional: First 9 PCA components of our dataset



And that is why we use nodes

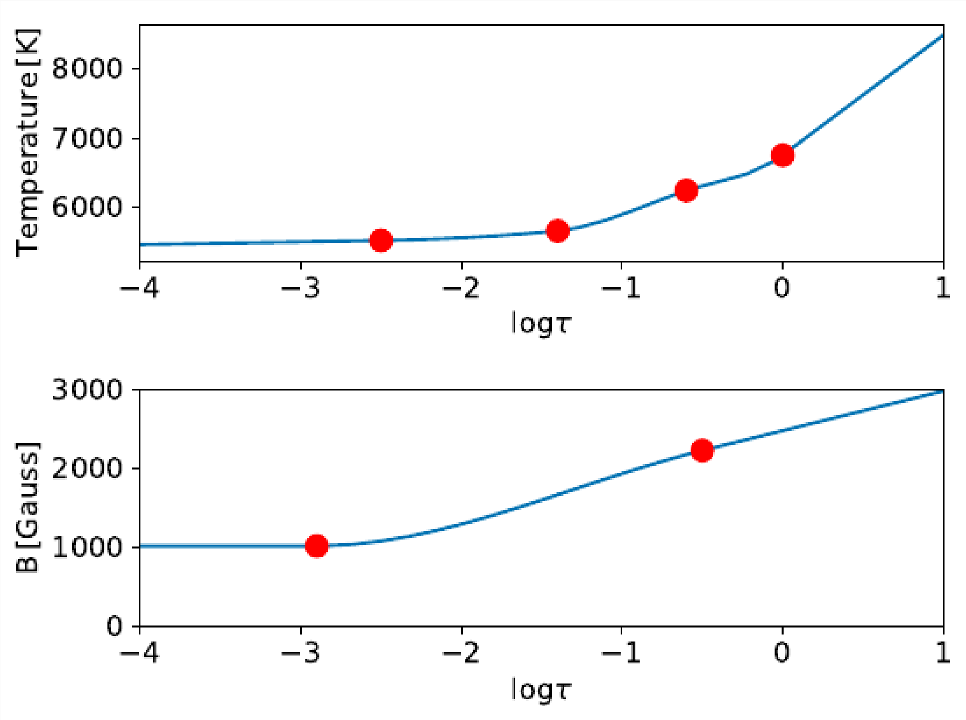
- You usually “reduce” number of parameters by including so called nodes. Still, the positions of the nodes / their number, come from some undefined scientific processes (personal preference, experience, etc.)



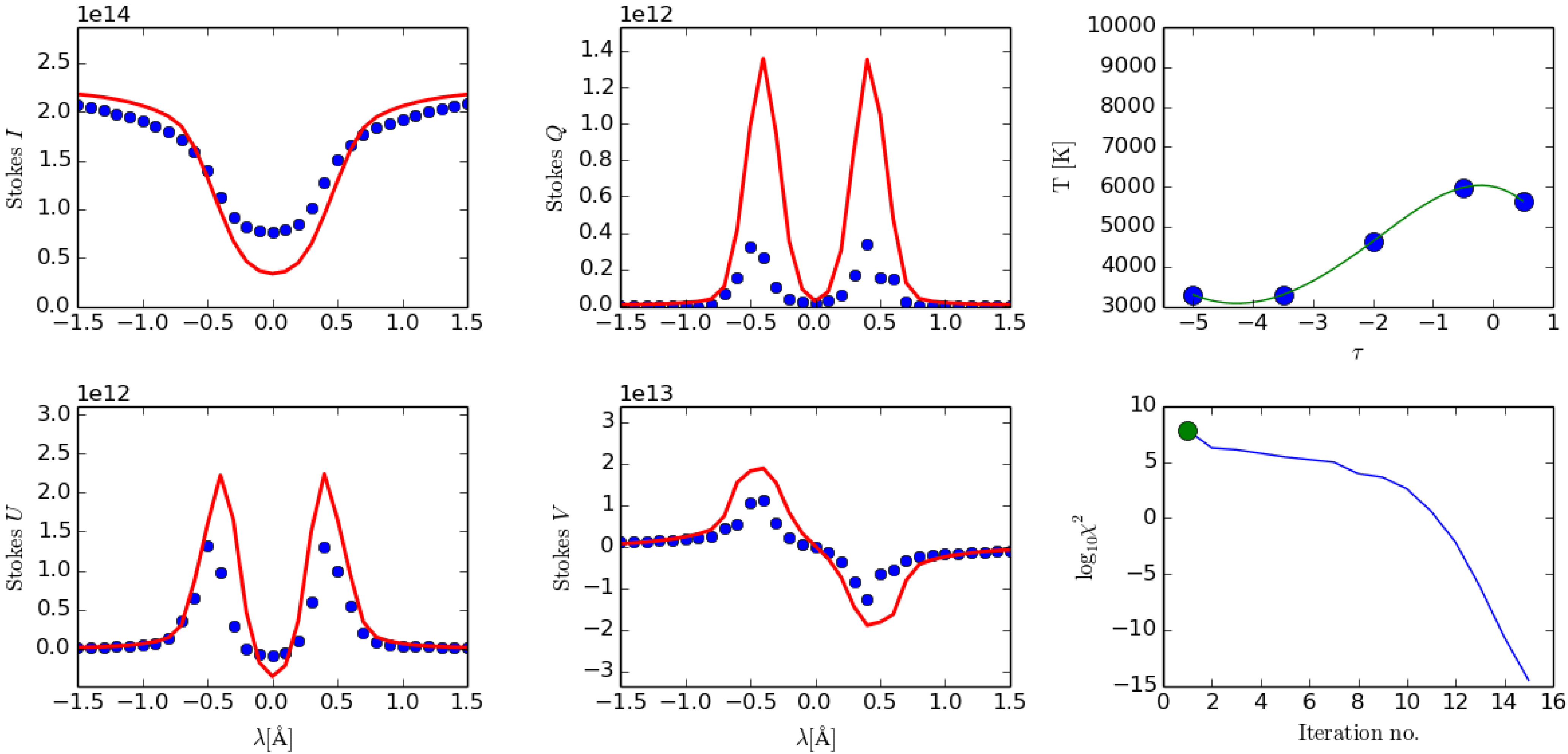


Instead of representing the full depth dependence of the atmosphere, we restrict it to few points. We map $\boldsymbol{p}$ to $\boldsymbol{\alpha}$, where dimension of $\boldsymbol{\alpha}$ is much smaller. Obviously, we need new response functions:

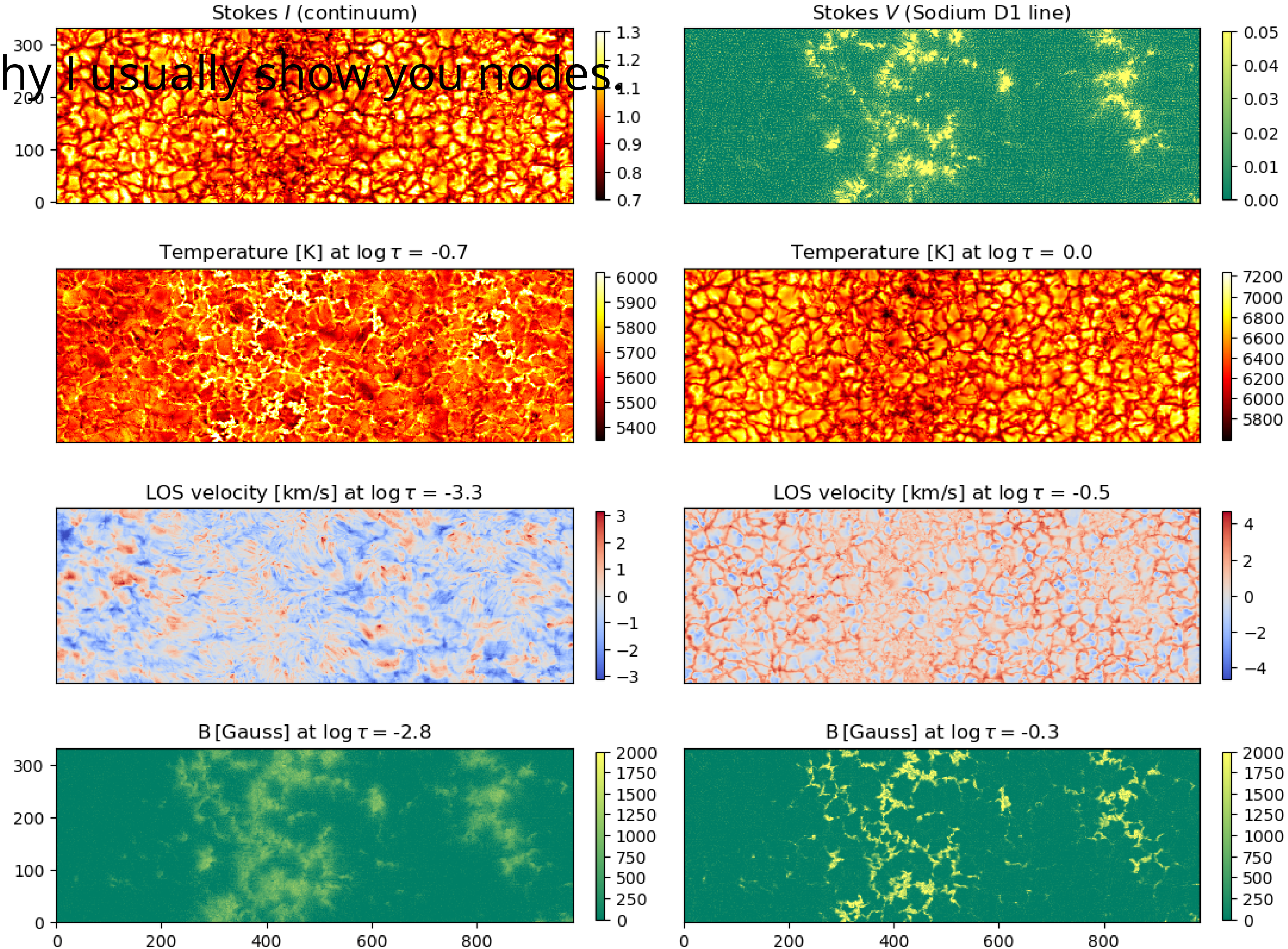
$$\frac{\partial I_i}{\partial \alpha_j} = \sum_{j'} \frac{\partial I_i}{\partial p_{j'}} \frac{\partial p_{j'}}{\partial \alpha_j}$$



So now we can understand what happens here



That is why I usually show you nodes.



What does this mean for you

- If you want to use DKIST, you will have to deal with inversions in some way
- Maybe you will do them yourself (several options)
- Or we will do them for you (then you will get a *cube* of your parameter models)
- So you will get a **model atmosphere** for each of your **observed spectra**
- It is good to know what came into that model atmosphere
- Keep eye out for **DKIST workshops** and **HAO Spectropolarimetry school**