PHYS 7810: Solar Physics with DKIST

Lecture 17: Spectral Line Polarization

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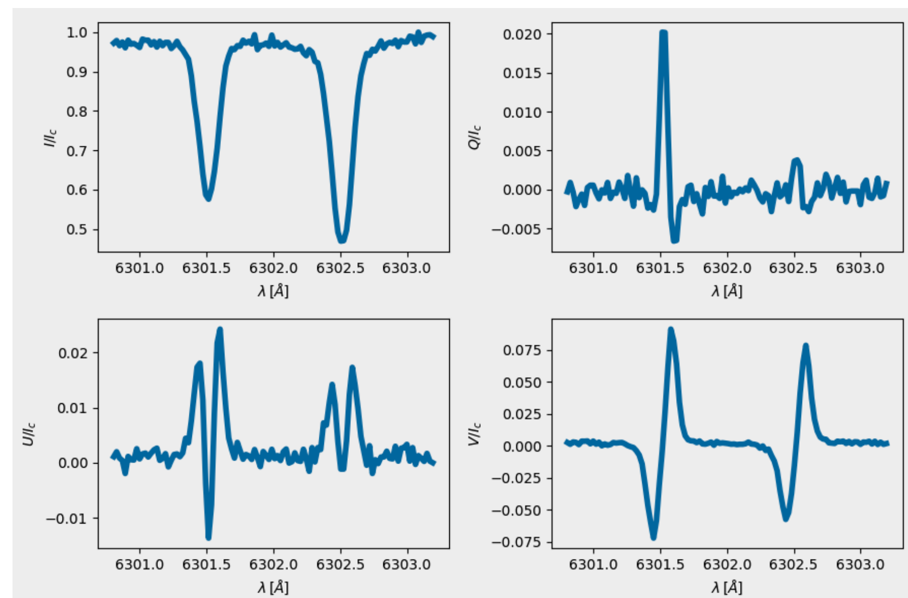


Previous lectures

- We have seen how to model spectral lines and investigate where they form

- We learned a bit about signatures of various quantities:

  temperature

  velocity

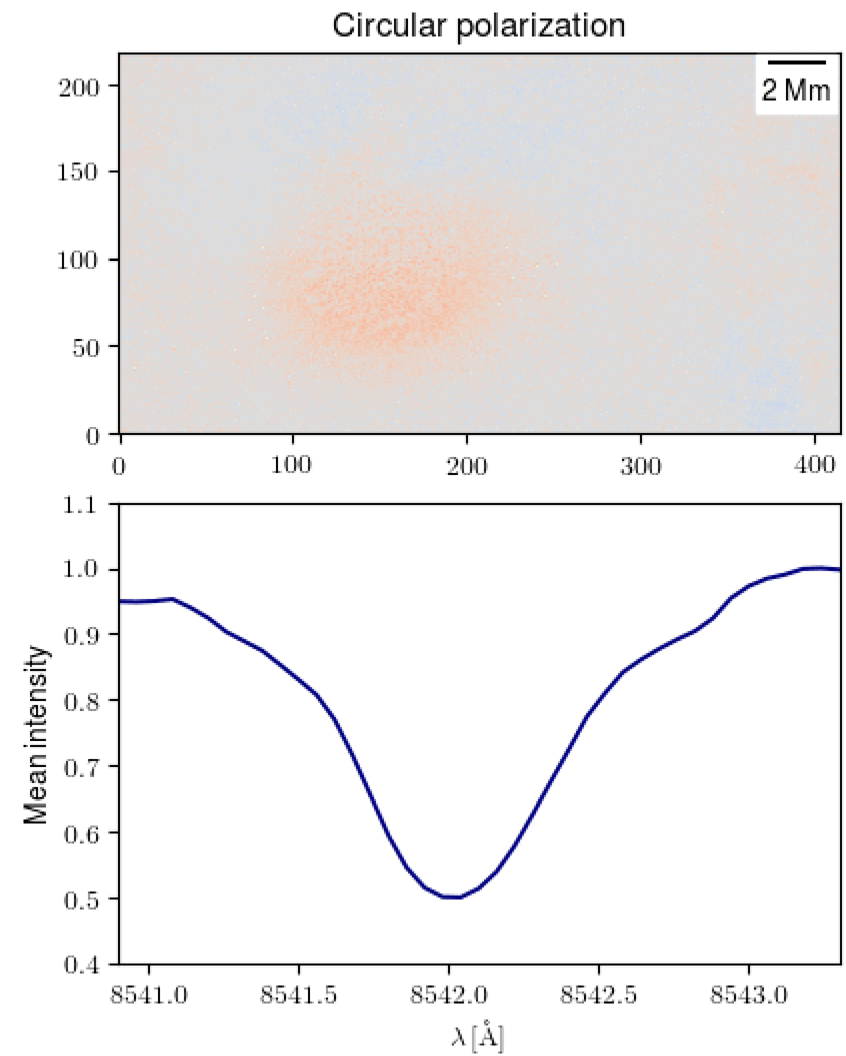
  turbulent velocities

  pressure

- All these leave their signature in the intensity spectrum

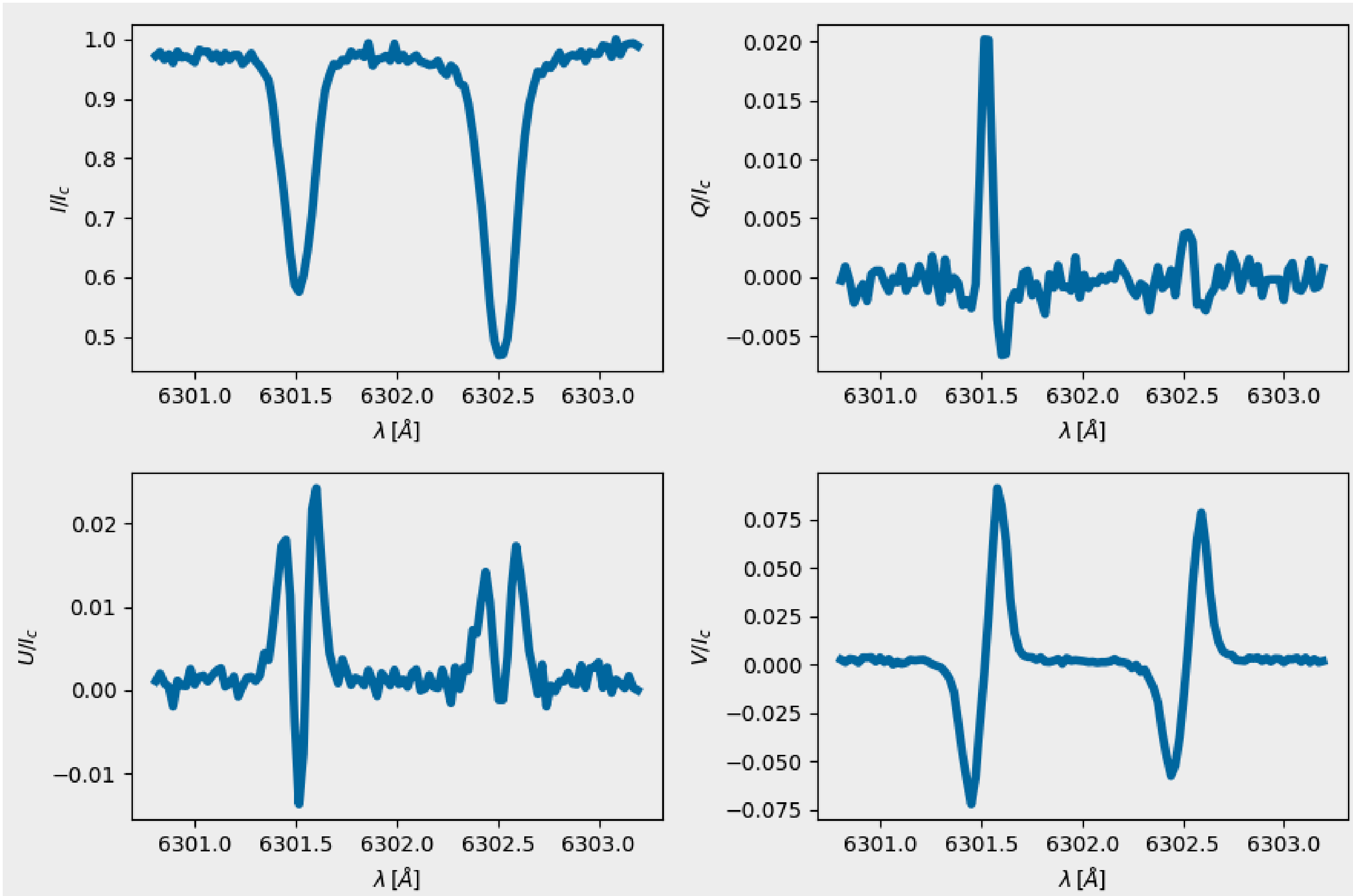
- Now we move on and we try to understand what kind of information is hidden in the polarization

So here is a cube, again



Time → Wavelength; Sunspot observations with IBIS at the NSO/Dunn Solar Telescope.
Courtesy of L. Kleint, K. Reardon, and A. Tritschler

Why do polarization profiles look like this?



"Quiet-ish" Sun observed with HINODE/SOT SP

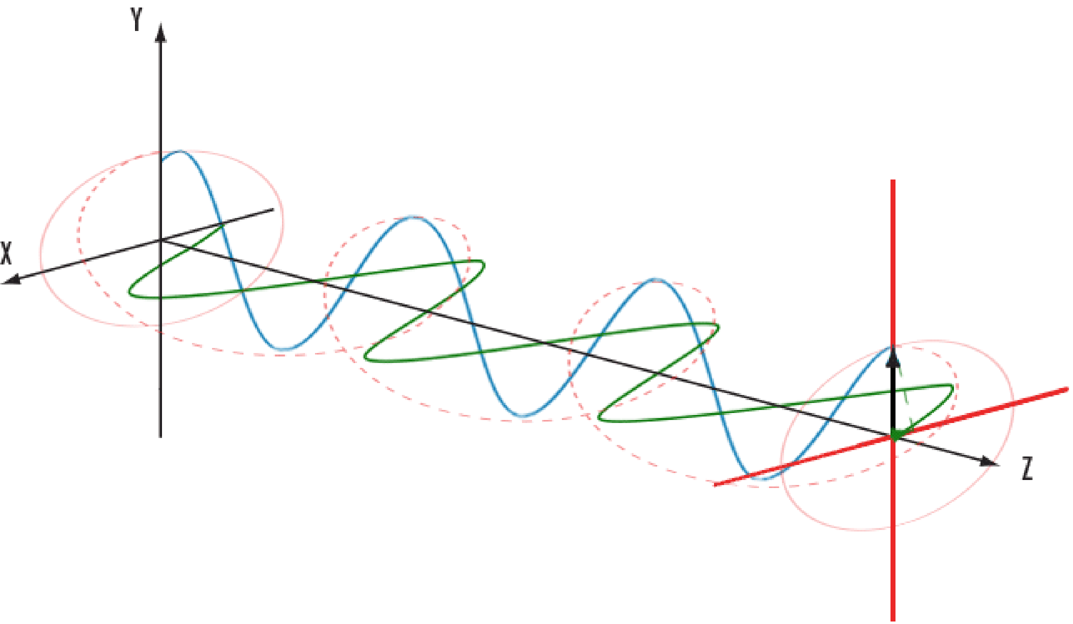
A classical approach

- Spectral line is treated as a harmonic oscillator
- Magnetic field interacts with the oscillator via Lorenz Force
- We solve differential equations for electron motion and we obtain the spectral distribution of the emitted light, absorption and dispersion coefficients
- It is based upon classical electrodynamics / EM
- You maybe embarked on fragments of this already
- Referent book “Introduction to Spectropolarimetry” - Jose Carlos del Toro Iniesta

A quantum approach

- We analyze the Hamiltonian of the atom in the presence of the magnetic field
- This explains Zeeman splitting
- We then employ quantum electrodynamics to understand photon-atom interaction and the absorption and emission of polarized light
- It explains well more “exotic” effects : scattering polarization, Hanle effect, orientation to alignment conversion, etc.
- Harder, but probably worth it if you are going to model anyway
- Referent book: “Polarization in Spectral Lines” by Egidio Landi Degl’Innocenti

Polarization of the light

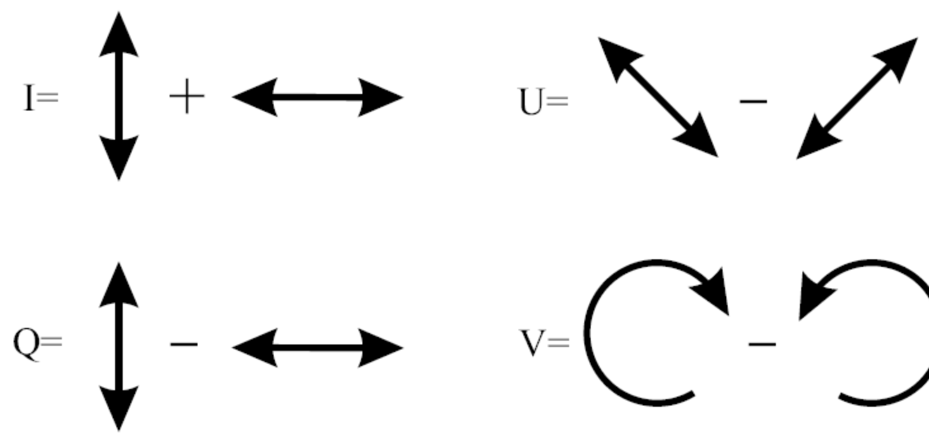


Credits: www.edmundoptics.com

Polarization = anisotropy

- **Zeeman:** magnetic field
- **Scattering:** geometry
- **Hanle:** both

Stokes parameters:



How do we model spectral line polarization

- Well, conceptually, the story is the same
- We look at this inhomogeneous slab of gas called solar atmosphere
- We assume some incident, unpolarized intensity impinging at the bottom
- And we see analyze how the intensity and the polarization (Stokes vector) change as they travel outward
- This is so called polarized radiative transfer
- Believe it or not, for that we will use the so called Mueller matrices

Mueller matrices

- We used them to represent changes of the Stokes vector, caused by a medium (optical element, or in our upcoming case, a slab of gas). A 4 x 4 matrix.

$$\boldsymbol{I}' = \hat{M}\boldsymbol{I}$$

For example, horizontal linear polarizer has:

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Solar atmosphere is a Mueller matrix too!

- We will call it the absorption matrix or the absorption tensor, so:

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = -\begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

- The coefficients follow from the nature of the microscopic processes involved in the polarization, we call them selective absorption and the dispersion of the light.

- Discuss: What do these mean? What would be non-selective absorption?

Why do off-diagonal elements exist?

- Magnetic field alters the Hamiltonian of the atom (most of the following plots are from the Introduction to Spectropolarimetry by J.C. del Toro Iniesta)

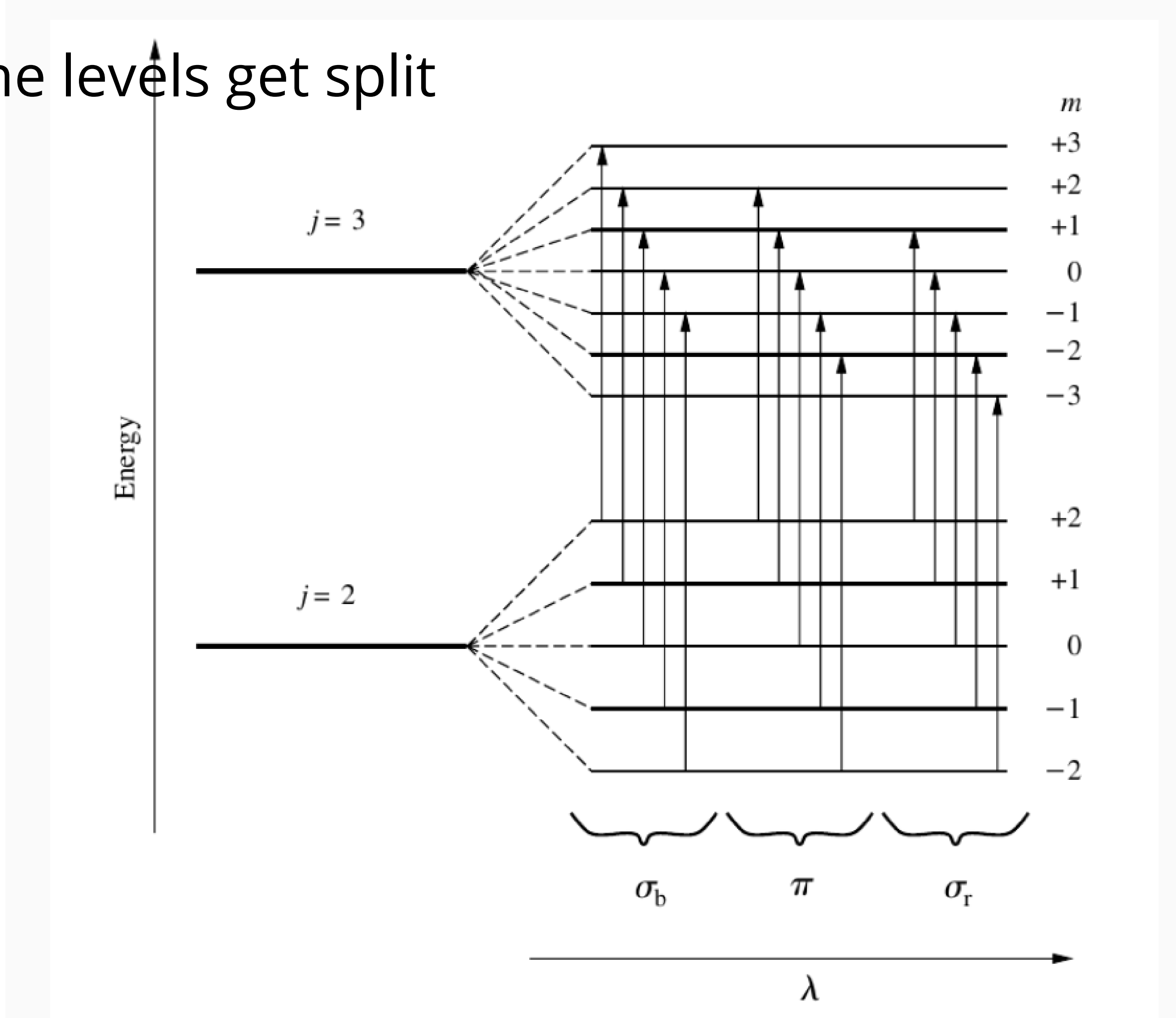
$$\mathbf{H}_B = \boldsymbol{\mu} \cdot \boldsymbol{B} + O\left(B^2\right) \qquad \boldsymbol{\mu} = \mu_0(\boldsymbol{J} + \boldsymbol{S})$$

- We look at the diagonal of this, which gives us different possible energy values

$$\langle lsjm \left| \mathbf{H}_B \right| \mathrm{sj}\, m\rangle = mg\mu_0 B = mgh\nu_{\mathrm{L}}$$

- g is the so called Lande factor, that describes the sensitivity of the level to the magnetic field.

And then the levels get split

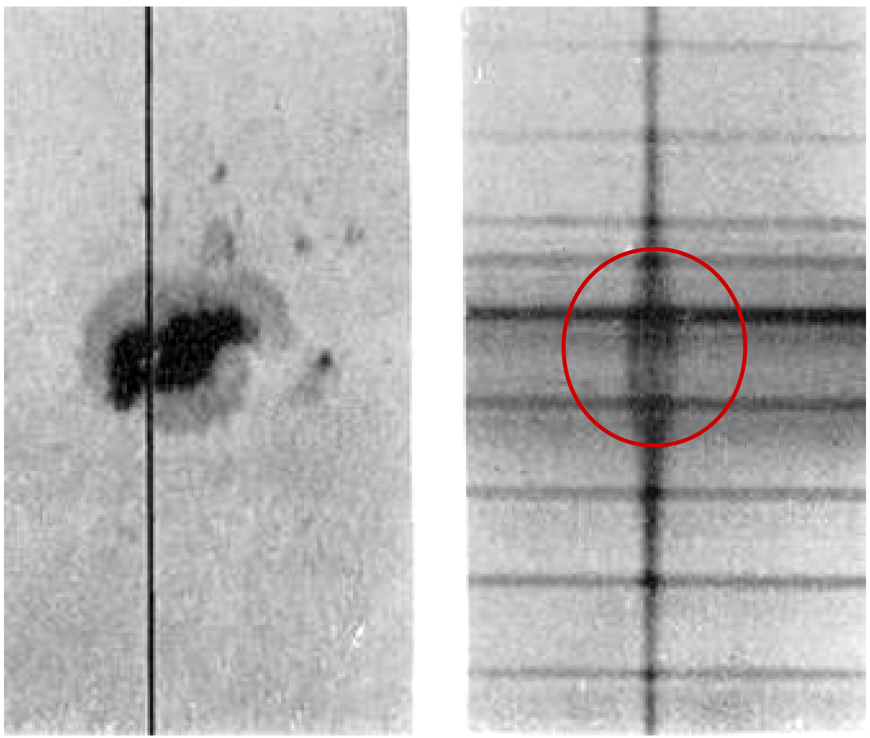


Zeeman splitting for the transition where upper level has J = 3 and lower J = 2. There are total of 15 sub-transitions. I sometimes call different *m* values – Zeeman sublevels.

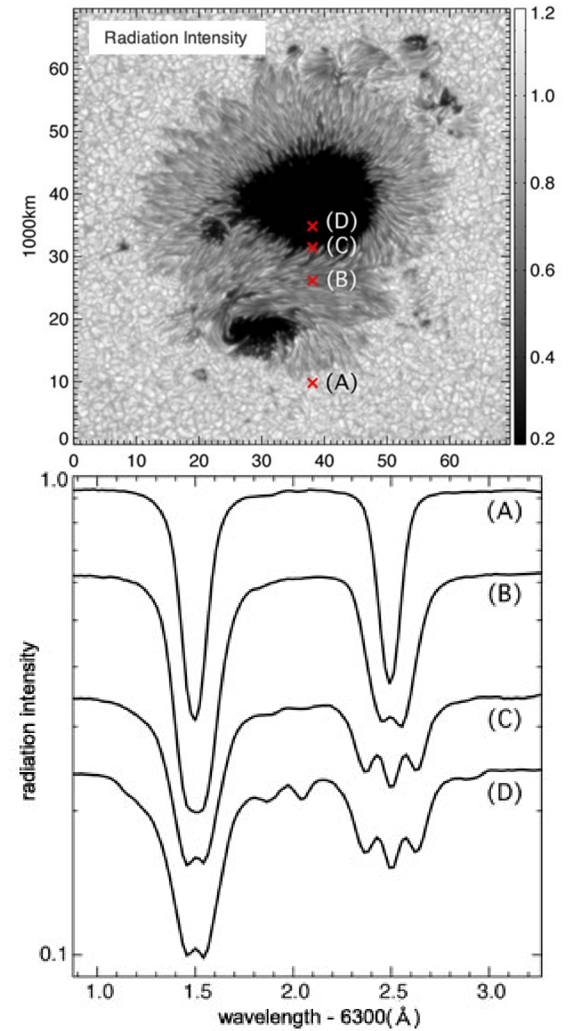
Few words about the transitions

- Different levels have different principal quantum number – **n**
- Levels with different orbital quantum number also generally have different energies (in case of Hydrogen they almost overlap) – **l**
- Then there is also spin quantum number and from the spin-orbit coupling we get the total angular quantum number **J**
- Degeneracy of a level is **2J + 1**
- Now we understand what that means, degeneracy are “hidden levels”
- We distinguish these (Zeeman sub-levels) by their magnetic quantum number - **m**

We know the Zeeman effect splits spectral lines

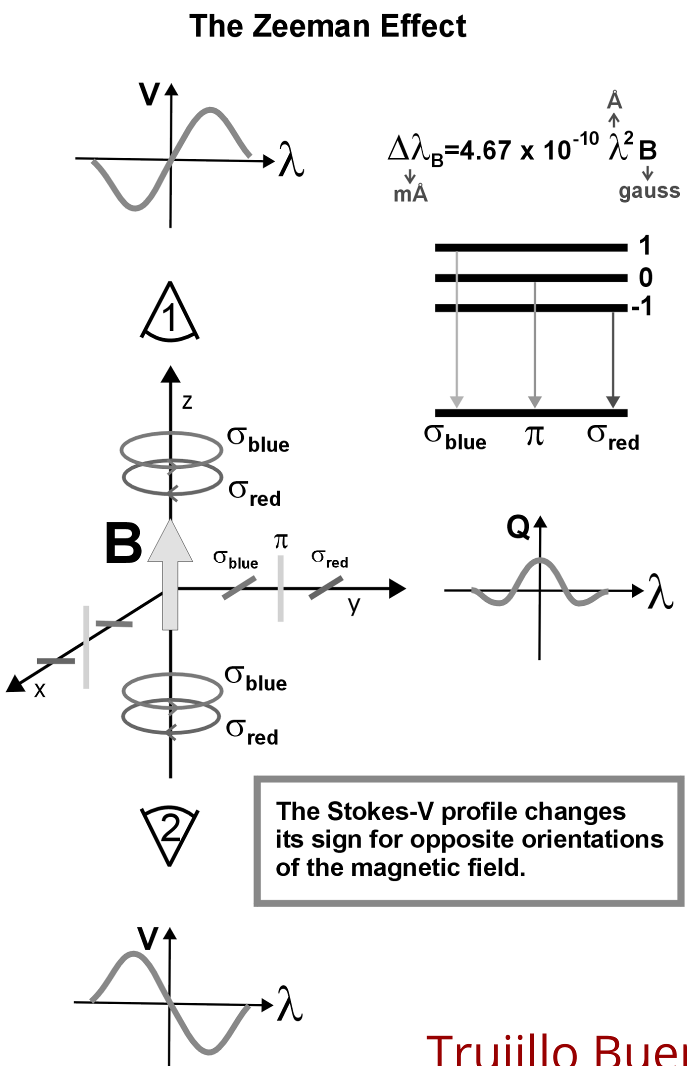


G.E. Hale, F. Ellerman, S.B. Nicholson, and A.H. Joy (ApJ, 1919)



Credits: Yukio Katsukawa

Can we see the splitting in the equations?



$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = -\begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

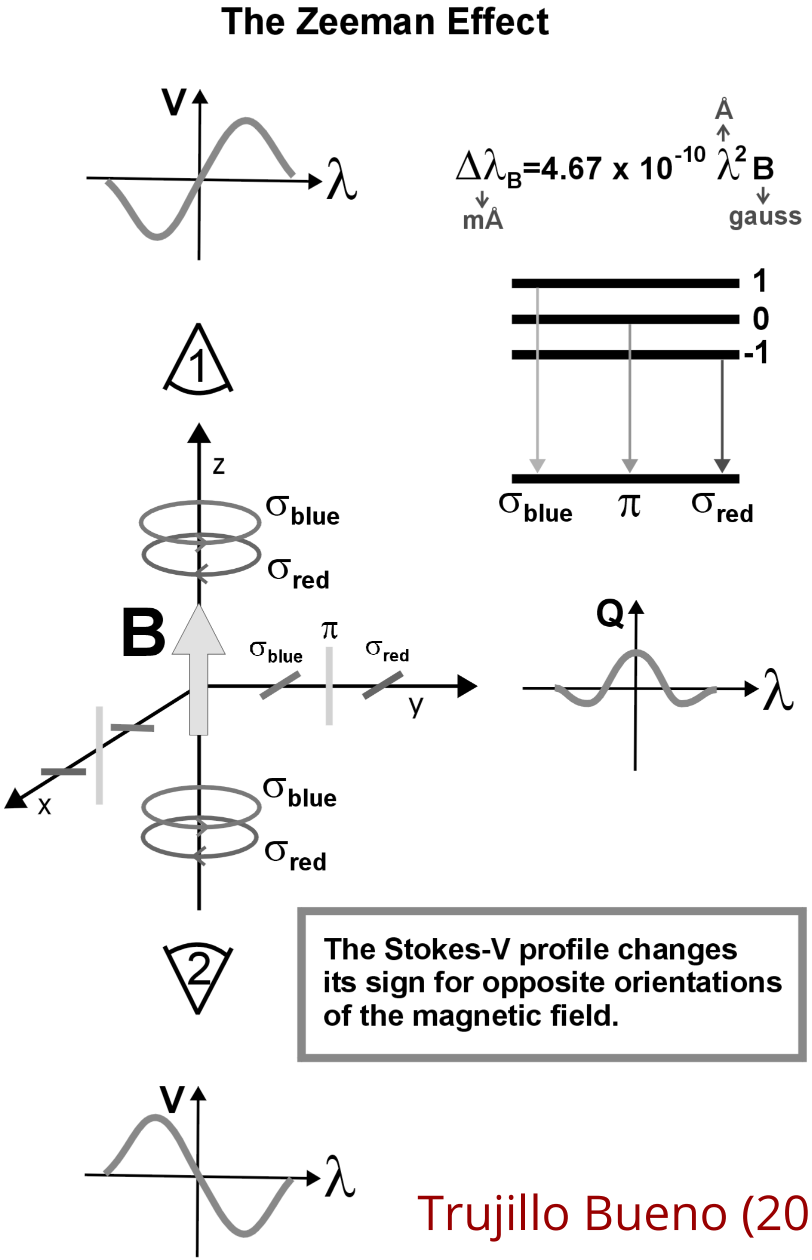
$$\eta_I = \frac{\eta_0}{2}\left\{\phi_0 \sin^2\theta + \frac{1}{2}\left[\phi_{+1} + \phi_{-1}\right]\left(1 + \cos^2\theta\right)\right\}$$

Yes, one profile is now three profiles, combined!

Trujillo Bueno (2006)

Ok, we know about that, why the polarization?

- Photons have spin one and two possible states, -1 and 1 (massless)
- Quantization axis for the angular momentum is this case is chosen to be the magnetic field director
- Projection of the angular momentum of the photon on the mag field can be -1,0,1
- However the angular momentum has to be conserved. If the photon travels paralelly to the mag field – only -1 and 1 transitions in *m* are allowd
- If it travels perpendicularly, then all three are allowed.



Trujillo Bueno (2006)

Why the polarization?

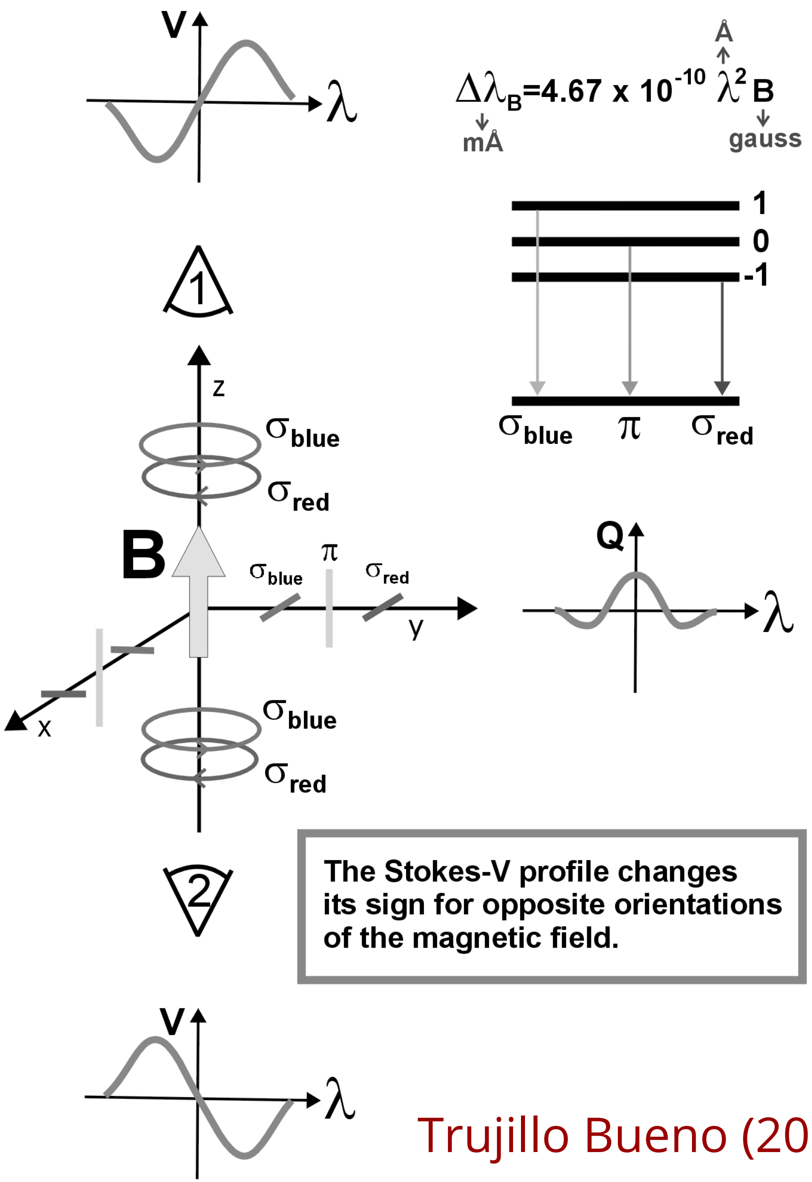
Individual photons are 100% polarized.

Different Δm transitions – different polarizations!

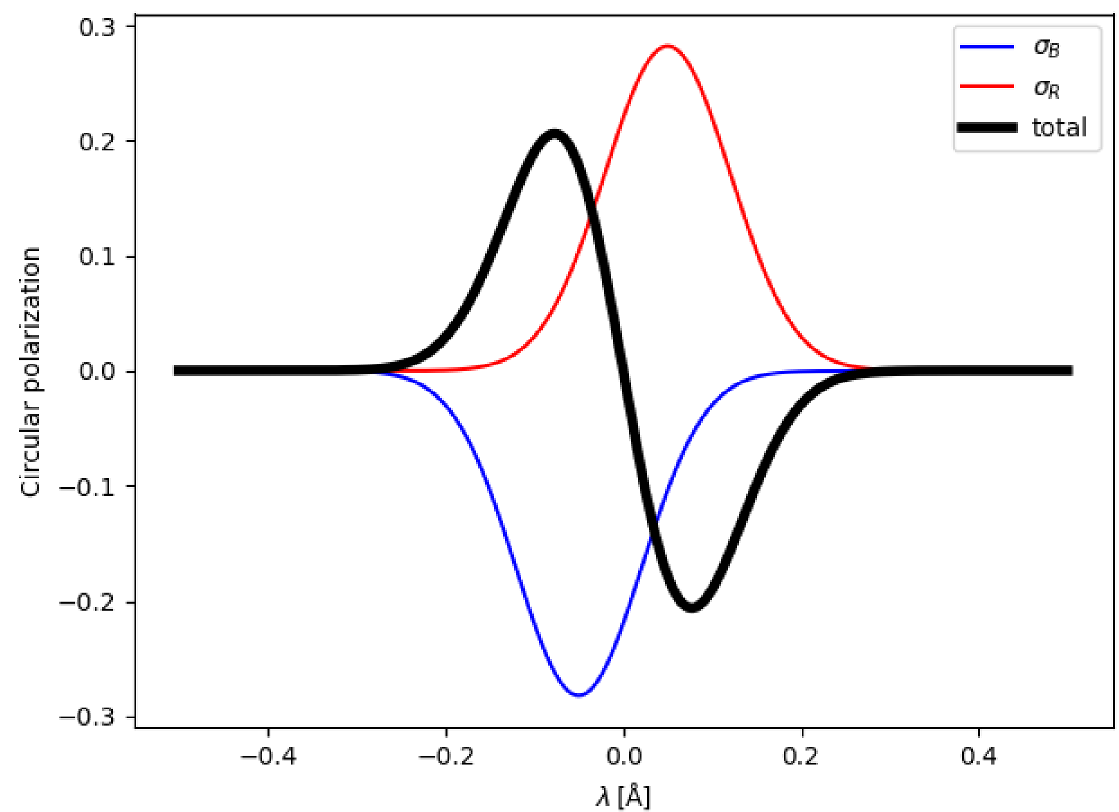
Parallel with B: only positive and negative circular polarization (σ_{blue},σ_{red})

Perpendicular to B: σ_{blue},σ_{red} seen as negative linear polarization, π as positive linear polarization

The Zeeman Effect

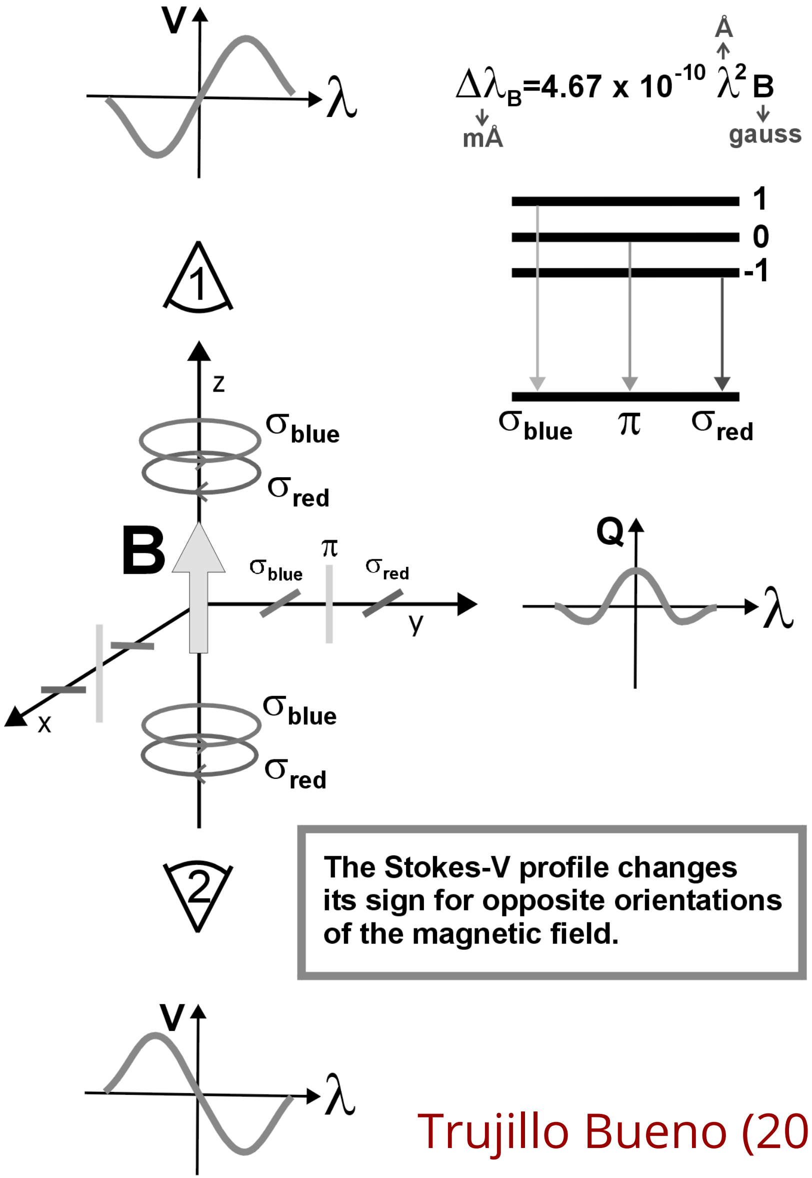


Parallel to **B**

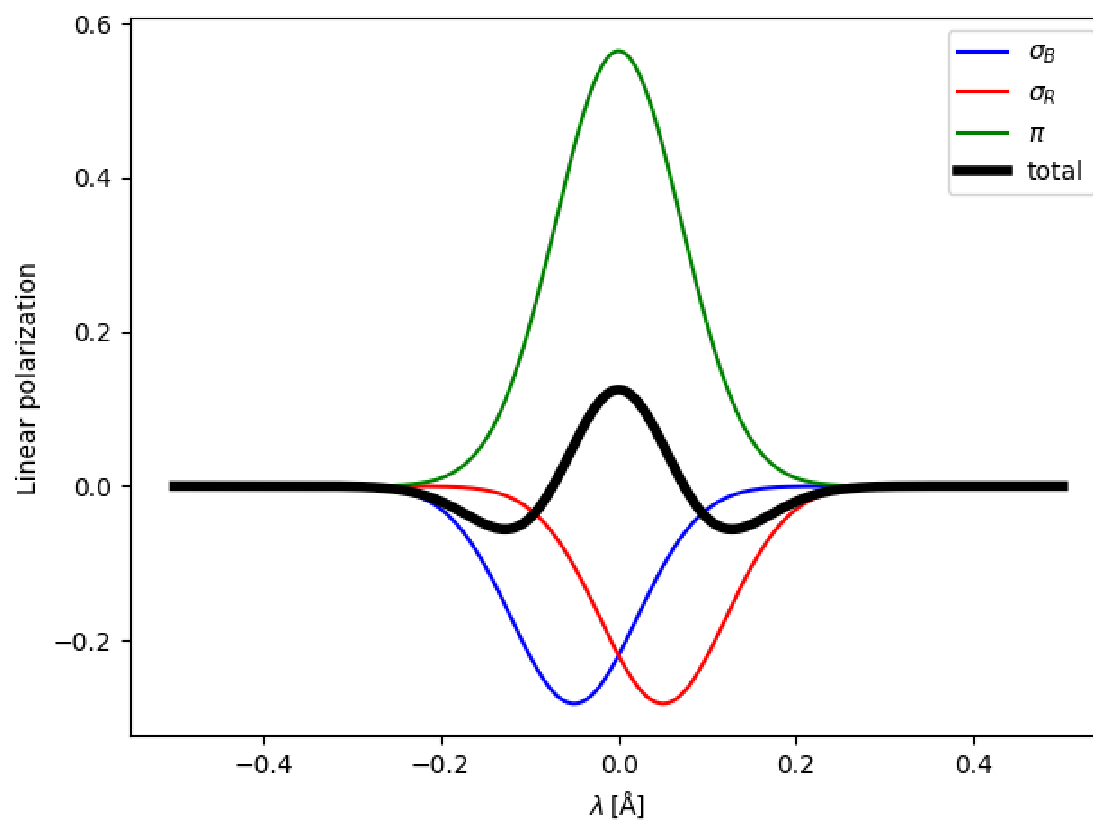


Trujillo Bueno (2006)

The Zeeman Effect



Perpendicular to **B**



Trujillo Bueno (2006)

When we put it all together

- These are all the absorption matrix elements.
- Each of the three profiles counts all the contributions (if we have multiple transitions with same change of *m*)
- Explicit dependency on the orientation of **B**
- Magnitude of **B** appears in the profiles (shifts them)

$$\eta_I = 1 + \frac{\eta_0}{2}\left\{\phi_\mathrm{p}\sin^2\theta + \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\left(1+\cos^2\theta\right)\right\}$$

$$\eta_Q = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

$$\eta_U = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

$$\eta_V = \frac{\eta_0}{2}\left[\phi_\mathrm{r} - \phi_\mathrm{b}\right]\cos\theta$$

$$\rho_Q = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

$$\rho_U = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

$$\rho_V = \frac{\eta_0}{2}\left[\psi_\mathrm{r} - \psi_\mathrm{b}\right]\cos\theta$$

Let's understand a simplified case, only I-V interaction

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & 0 & 0 & \eta_V \\ 0 & \eta_I & 0 & 0 \\ 0 & 0 & \eta_I & 0 \\ \eta_V & 0 & 0 & \eta_I \end{pmatrix}\begin{pmatrix} I - B_\nu(T) \\ Q \\ U \\ V \end{pmatrix}$$

And that turns into:

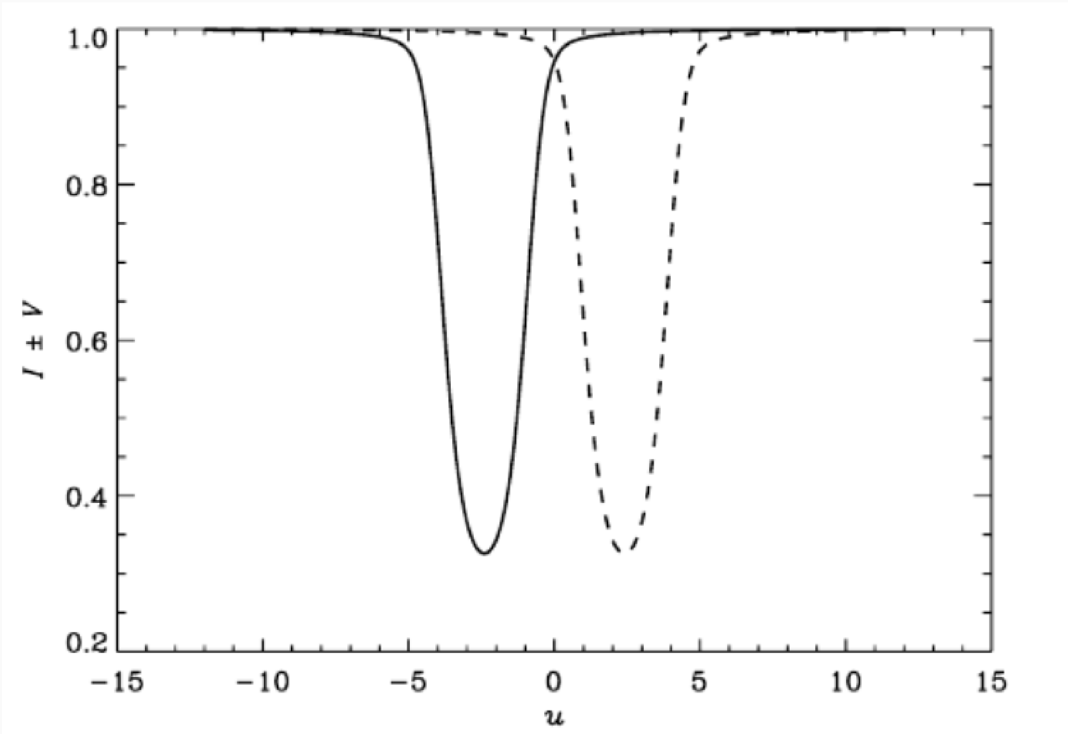
$$\frac{dI}{d\tau} = \eta_I I - \eta_I B + \eta_V V$$

$$\frac{dV}{d\tau} = \eta_V I - \eta_V B + \eta_I V$$

Do the good old, add, subtract thingie:

$$\frac{d(I \pm V)}{d\tau} = (\eta_I \pm \eta_V)(I \pm V - B)$$

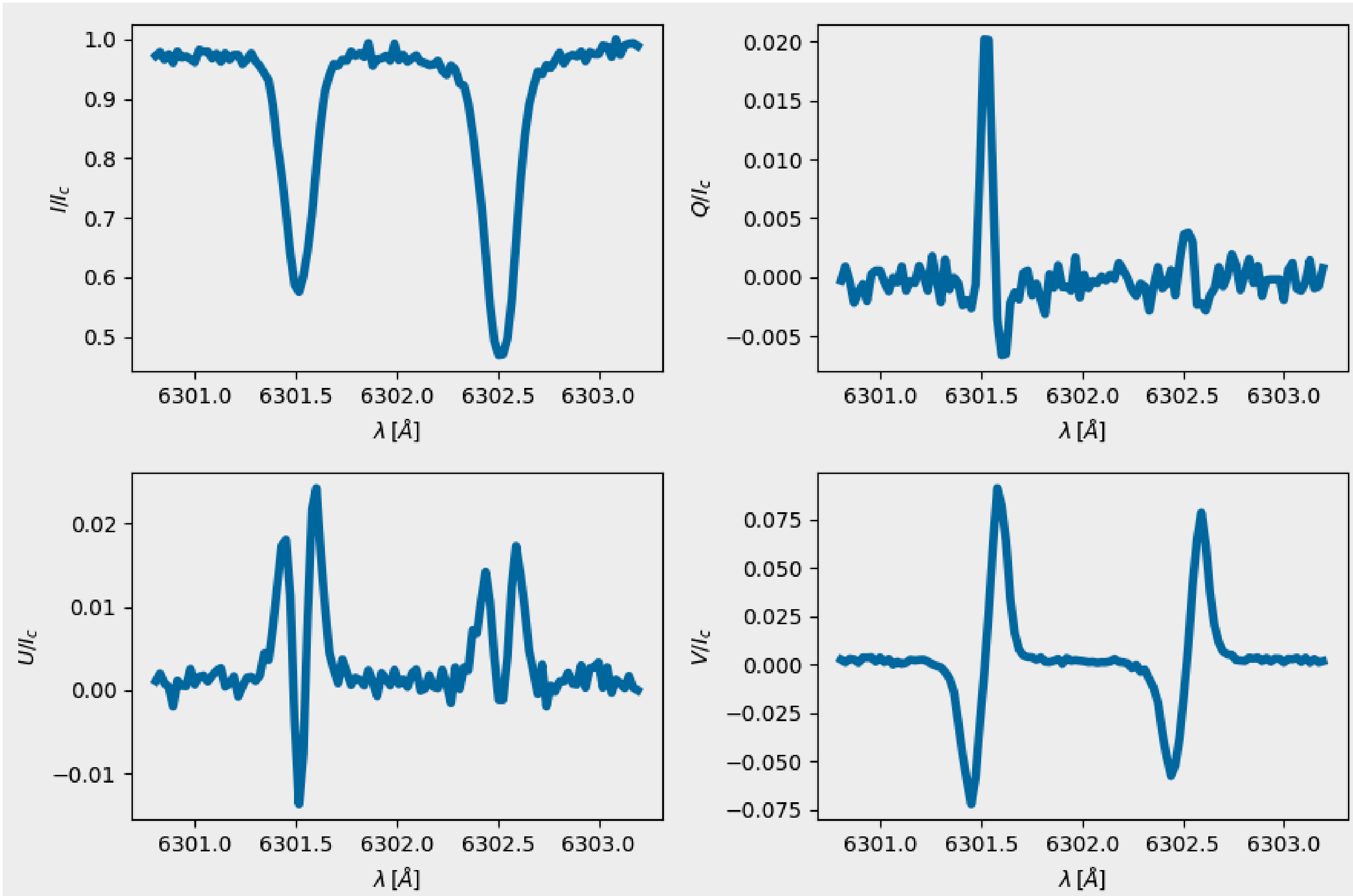
These are two RTE that we can solve separately. (Boundary contidion $I = B$, $V = 0$), and we get something like this:



$$\eta_I = \frac{\eta_0}{2}\left\{\phi_0 \sin^2\theta + \frac{1}{2}\left[\phi_{+1} + \phi_{-1}\right]\left(1 + \cos^2\theta\right)\right\}$$

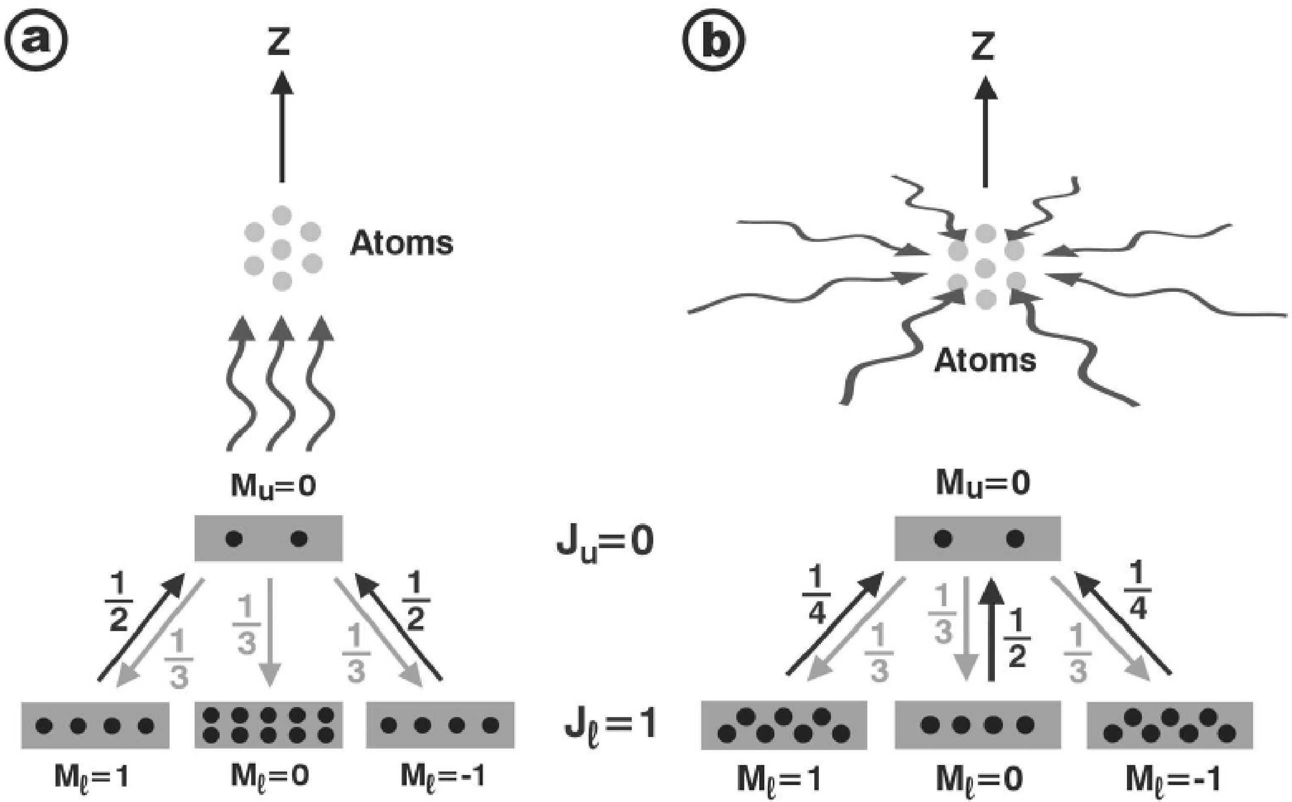
$$\eta_V = \frac{\eta_0}{2}\left[\phi_{-1} - \phi_{+1}\right]\cos\theta$$

Why do polarization profiles look like this?



“Quiet-ish” Sun observed with HINODE/SOT SP

Before we go to play, lets look into this



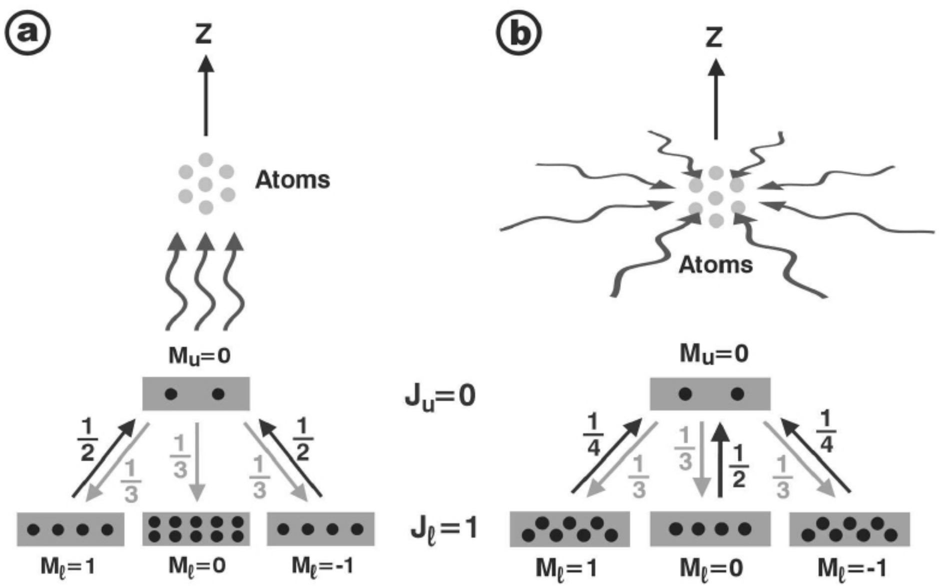
from Trujillo Bueno (2003)

„Selective absorption“

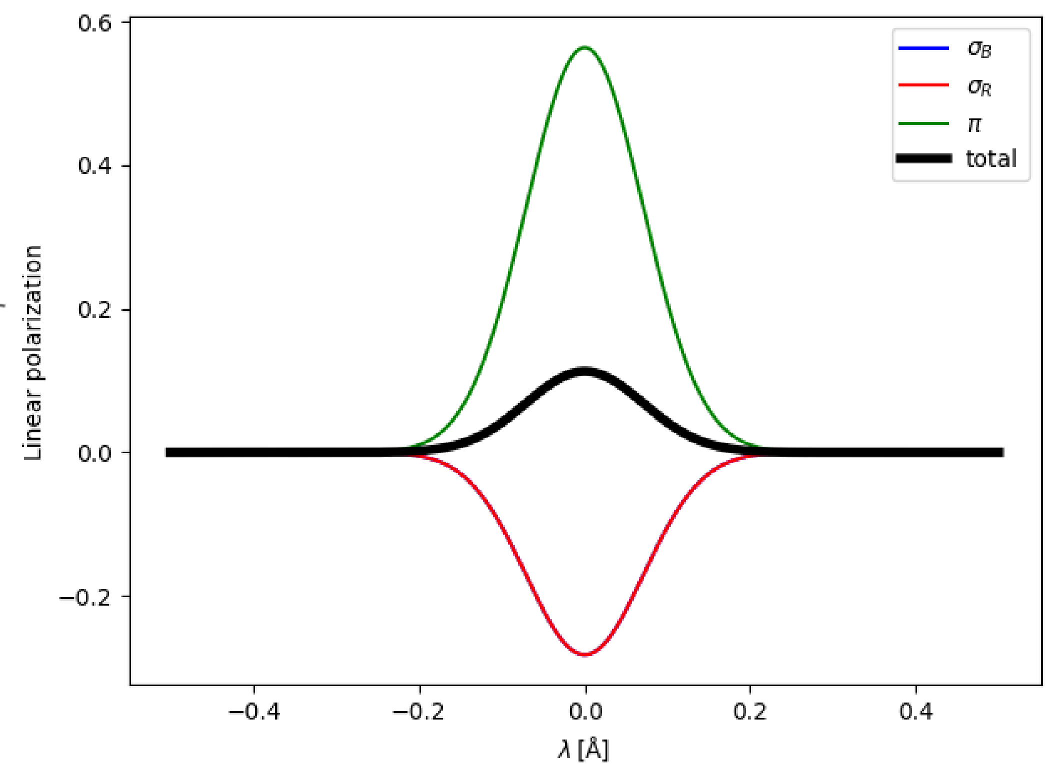
Uneven population of Zeeman subl-levels leads to the “polarization” of the atomic levels.

This leads to the net linear polarization of the light.

Scattering line polarization – comparison with Zeeman

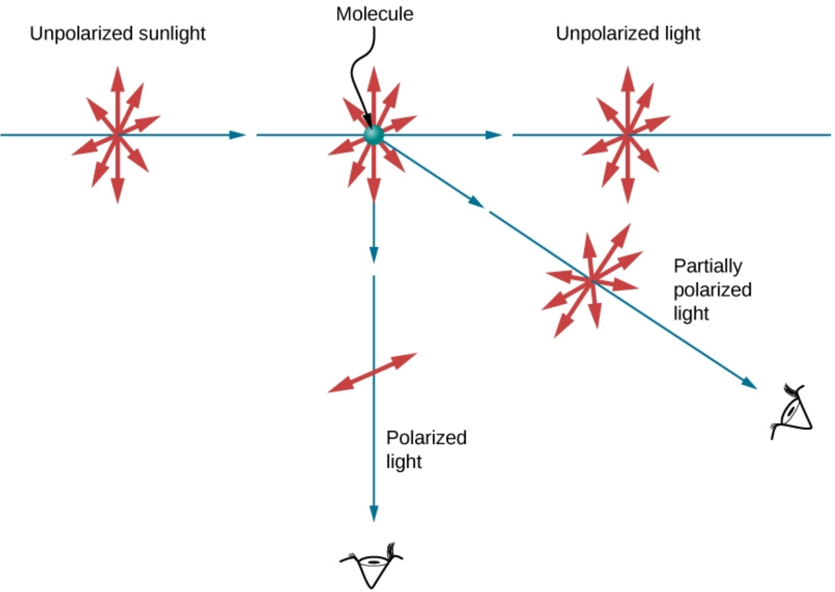


from Trujillo Bueno (2003)

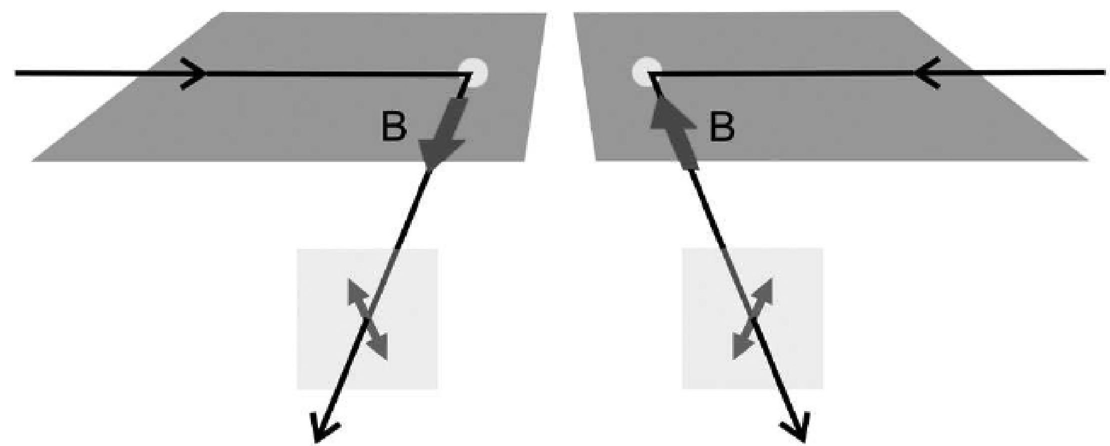


Scattering polarization and Hanle effect

In this case, the anisotropy is broken by the radiation field → **atomic polarization**

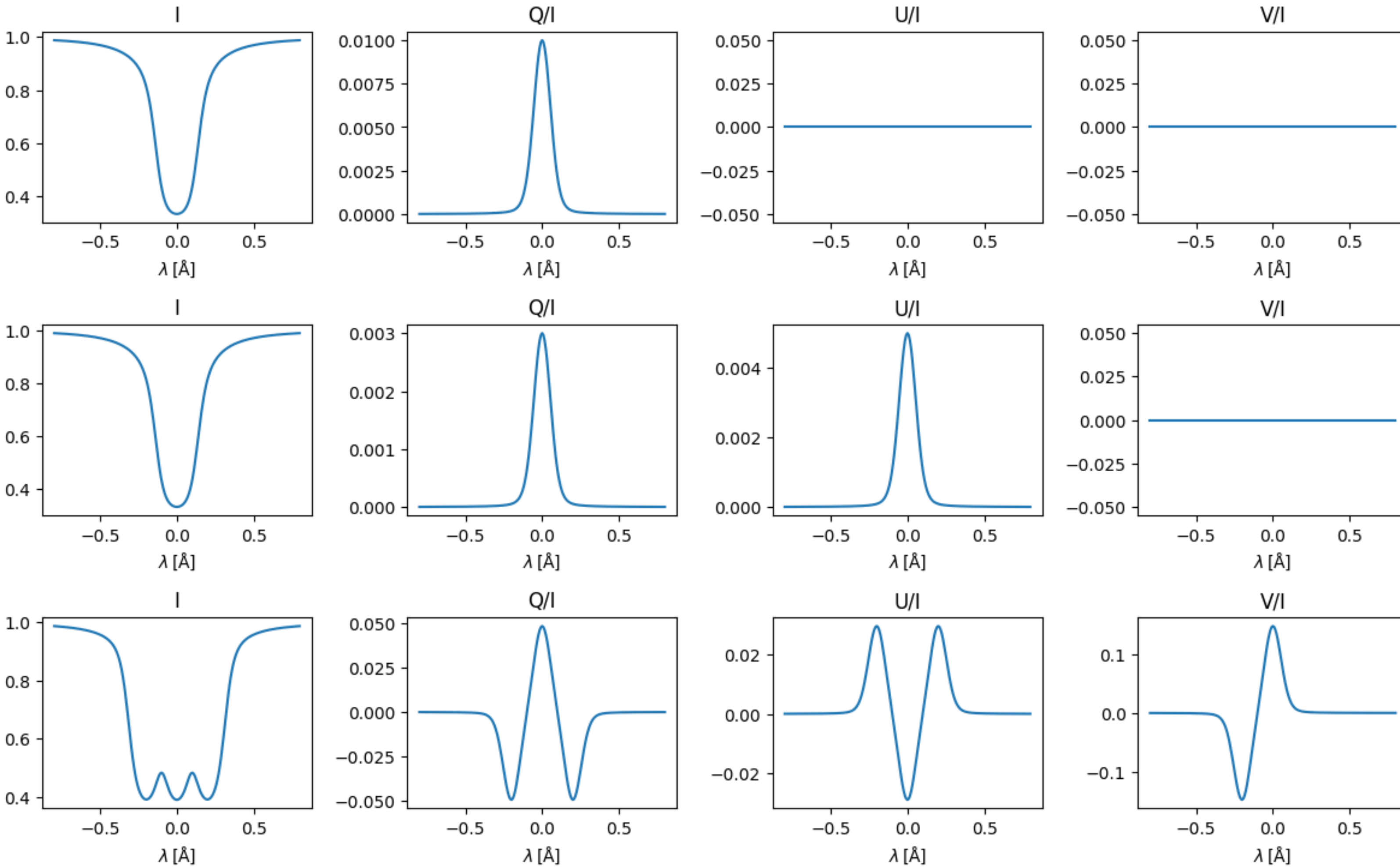


Hanle effect:
Magnetic field rotates the polarization plane



from Trujillo Bueno (2006)

Zeeman vs Hanle



Pure scattering, no magnetic field

Weak magnetic field

$$\frac{0.88 g B \times 10^5}{A_{ul}} \approx 1$$

Strong magnetic field

$$\Delta\lambda_{\mathrm{Zeeman}} > \Delta\lambda_{\mathrm{Dopp}}$$

Summary

- We describe the polarization with the Stokes vector and look how it changes throughout the atmosphere
- Matter influences the Stokes vector through processes of selective absorption, retardation etc. This are all microscopic in essence.
- We mostly use Zeeman effect
- Different Δm transitions have different polarization. It is always true. Breaking energy degeneracy makes them show up.
- There are different degrees of the realism when we want to model this.
- One of them is so called Milne-Eddington model. Let's play with that and we will use it in the hands-on next week.
- http://research.iac.es/proyecto/magnetism/pages/codes/milne-eddington-simulator.php