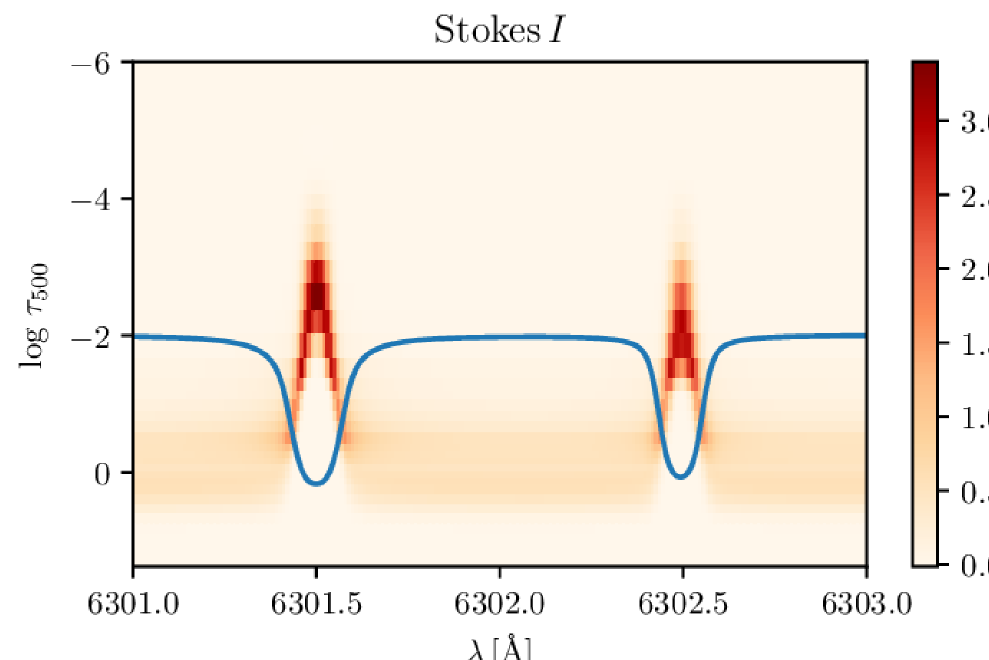
PHYS 7810: Solar Physics with DKIST

Lecture 16: Spectral Line diagnostics

Ivan Milic *ivan.milic@colorado.edu*



Previous lectures

- Our main motivation is to see how the spectrum is formed, how the intensity that we measure came to be
- That way we can hopefully estimate, quantitatively, what kind of atmosphere produced our measurement (**we “measure” magnetic fields, velocities, temperatures, etc…**)
- We derived the radiative transfer equation and attempted to solve it
- We saw how the opacity and emissivity in the spectral line can be calculated
- What now, what comes next?

I still owe you a few slides...

Final recipe for spectral synthesis:

- Find all the atomic data for the lines you want to consider.
- Calculate populations of relevant atomic levels (how? - discuss 5 mins)
- Calculate all the broadening effects, and thus the profiles
- Do this for each point in the atmosphere and obtain opacity and emissivity, at each wavelength.
- Add other opacity / emissivity sources (H-, electrons, Bound-free, Free-free)
- Solve radiative transfer equation and obtain spectrum.

Source function in a spectral line

Assume the line processes dominate:

$$\frac{j_\lambda}{\chi_\lambda} = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$$

In LTE, this needs to be equal to the Planck function:

$$\frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$$

From here we can derive relationships between A and B's.

Another important point: For optical domain, usually $n_l >> n_u$, and if n_l is the ground level, most of the electrons are there. So the value of the source function is set by n_u. More on this when we talk about NLTE.

Relationships between Einstein coefficients

From the previous slide, taking into account Boltzmann distribution, it is straightforward to derive:

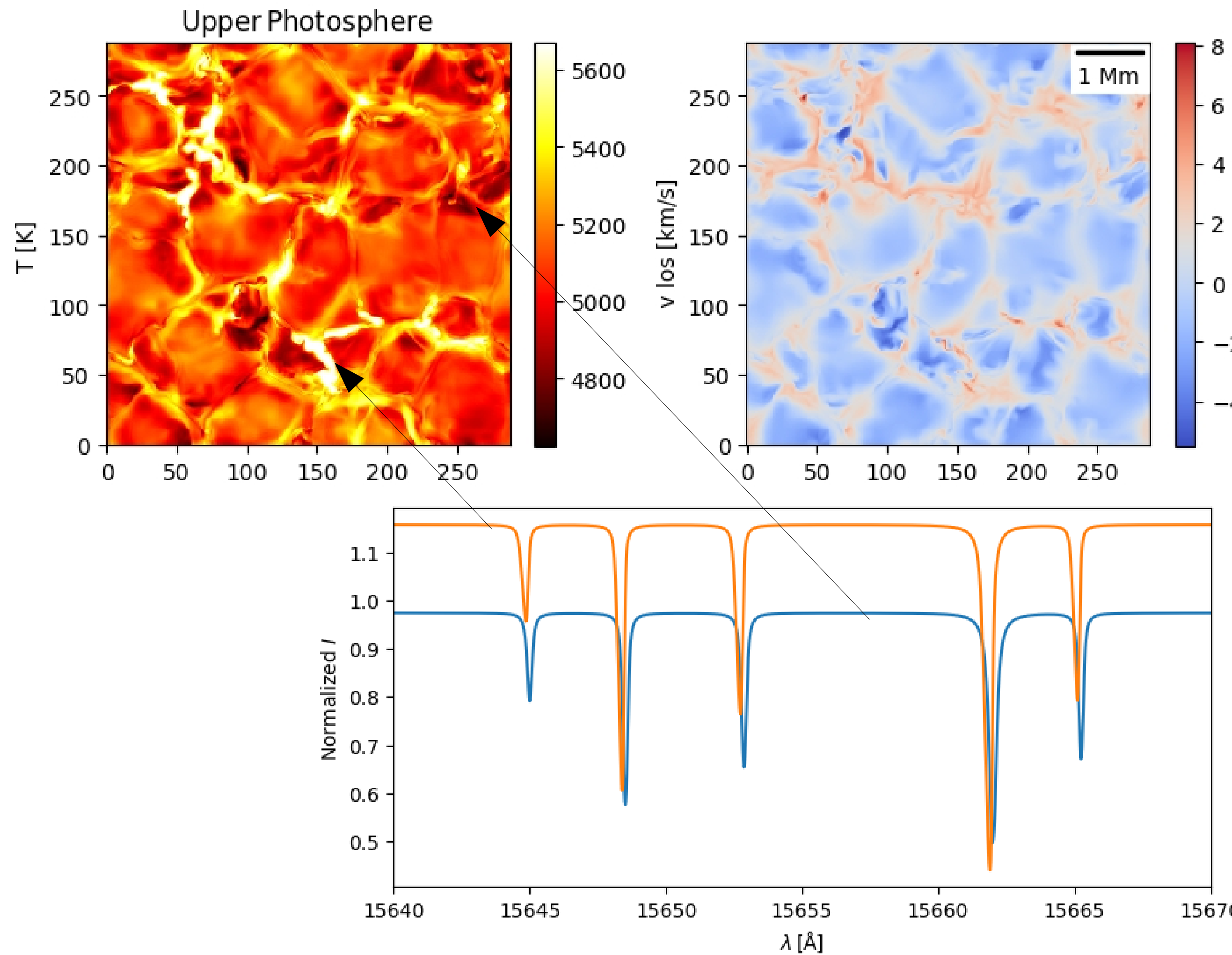
$$B_{ul} g_u = B_{lu} g_l$$

$$\frac{A_{ul} g_u}{B_{lu} g_l} = \frac{2hc^2}{\lambda^5}$$

We will need this for our hands-on next week!

Let’s look at a finished product and unpack

- What is going on here?
- Different points produce different spectra
- Why, how is this spectra generated?
- What influences how will each spectral line look like?
- **What depth is probed by what wavelength? That is something we all would like to know!**



MURAM quiet Sun simulation, courtesy of T. Riethmüller

Equation refresher - macroscopic

Our tool for spectra calculation 1D, time indepedent RTE:

$$\frac{dI(z,\theta,\lambda)}{dz} = -\chi(z,\lambda)I(z,\theta,\lambda) + j(z,\theta,\lambda),$$

that we usually cast as:

$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda.$$

And the formal solution (once we know opacity and emissivity):

$$I_\lambda^+ = I_\lambda^0 e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t)e^{-t}dt$$

We can generalize this to any upper and lower point.

Equation refresher - microscopic

$$j(z,\theta,\lambda) = n_u(z) A_{ul} \frac{hc}{4\pi\lambda_0} \phi(z,\lambda,\theta)$$

$$\chi(z,\theta,\lambda) = \left(n_l(z) B_{lu} - n_u(z) B_{ul}\right) \frac{hc}{4\pi\lambda_0} \phi(z,\lambda,\theta)$$

$$\phi(z,\lambda,\theta) = \frac{1}{\Delta\lambda_D \sqrt{\pi}} e^{(\lambda' - \lambda_0)^2 / \Delta\lambda_D^2} \star \phi_0^{\mathrm{damping}}(z)$$

- A **lot of equations.** We will use some of them at some point, but for now let's just investigate what is going on.

Question:

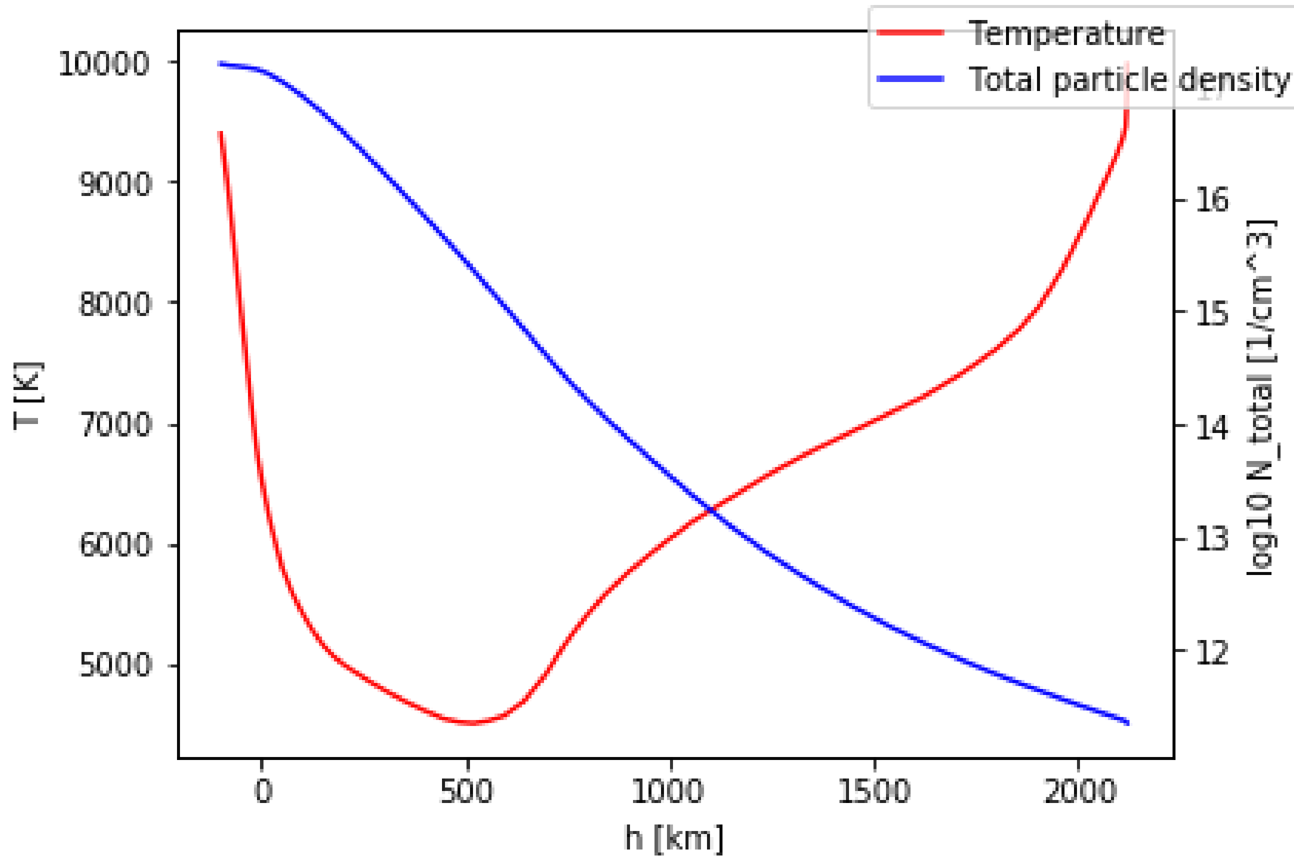
Say I infer something from these lines (temperature, Doppler shift, magnetic field)

To which “layer” of the solar atmosphere do I ascribe it to?

Outline

- Pick a line, say H alpha
- See how we obtain each of the quantities of interest in the spectral line
- Calculate the line – get the wrong result
- Switch the line
- Study the line formation
- Introduce a few useful quantities

An example atmospheric model – infamous FALC



Do these distributions make sense? What does total particle density distribution look like? What does it relate to?

H alpha, from second to third level of neutral hydrogen

Observed Wavelength Air (Å)	Unc. (Å)	Ritz Wavelength Air (Å)	Unc. (Å)	Rel. Int. (?)	A_{ki} (s^{-1})	Acc.	E_i (cm^{-1})		E_k (cm^{-1})	Lower Level Conf., Term, J			Upper Level Conf., Term, J			Type	TP Ref.	Line Ref.
6 562.70970	0.00004	6 562.70970	0.00003		5.3877e+07	AAA	82 258.9191133	-	97 492.319433	2*p*	$^2P^\circ$	$^1/_2$	3*d*	^2D	$^3/_2$		T8637	L2752
		6 562.714	0.004															c67
		6 562.722	0.009															c68
6 562.72483	0.00003	6 562.72483	0.00003		2.2448e+07	AAA	82 258.9543992821	-	97 492.319611	2*s*	^2S	$^1/_2$	3*p*	$^2P^\circ$	$^3/_2$		T8637	L6891c38
		6 562.75181	0.00003		2.1046e+06	AAA	82 258.9191133	-	97 492.221701	2*p*	$^2P^\circ$	$^1/_2$	3*s*	^2S	$^1/_2$		T8637	
		6 562.76701	0.00003				82 258.9543992821	-	97 492.221701	2*s*	^2S	$^1/_2$	3*s*	^2S	$^1/_2$	M1		
		6 562.770	0.006															c66
6 562.77153	0.00003	6 562.77153	0.00003		2.2449e+07	AAA	82 258.9543992821	-	97 492.211200	2*s*	^2S	$^1/_2$	3*p*	$^2P^\circ$	$^1/_2$		T8637	L6891c38
6 562.79	0.03	6 562.819	0.007	500000	4.4101e+07	AAA	82 259.158	-	97 492.304	2			3				T8637	L7400c29
		6 562.795	0.009															c69
6 562.85175	0.00007	6 562.85177	0.00003		6.4651e+07	AAA	82 259.2850014	-	97 492.355566	2*p*	$^2P^\circ$	$^3/_2$	3*d*	^2D	$^5/_2$		T8637	L2752
		6 562.8533	0.0003															c71
		6 562.854	0.003															c70
		6 562.86734	0.00003		1.0775e+07	AAA	82 259.2850014	-	97 492.319433	2*p*	$^2P^\circ$	$^3/_2$	3*d*	^2D	$^3/_2$		T8637	
		6 562.90944	0.00003		4.2097e+06	AAA	82 259.2850014	-	97 492.221701	2*p*	$^2P^\circ$	$^3/_2$	3*s*	^2S	$^1/_2$		T8637	

Problem one, how to obtain the amount of neutral H?

- Saha ionization equation – describes ratios between ions in an equilibrium state

$$\frac{n_{j+1}n_e}{n_j} = \frac{2Z_{j+1}}{Z_j\Lambda^3}e^{-E_{\mathrm{ion}}/kT}$$

$$\Lambda = \sqrt{h^2/2\pi m_e kT}$$

- If we asume only hydrogen matters, what can we do?

Problem one, how to obtain the amount of neutral H?

- Saha ionization equation – describes ratios between ions in an equilibrium state

Protons

$$\frac{n_{j+1} n_e}{n_j} = \frac{2 Z_{j+1}}{Z_j \Lambda^3} e^{-E_{\text{ion}}/kT}$$

Neutral hydrogen

$$\Lambda = \sqrt{h^2 / 2\pi m_e k T}$$

Three unknowns, three equations (nonlinear!)

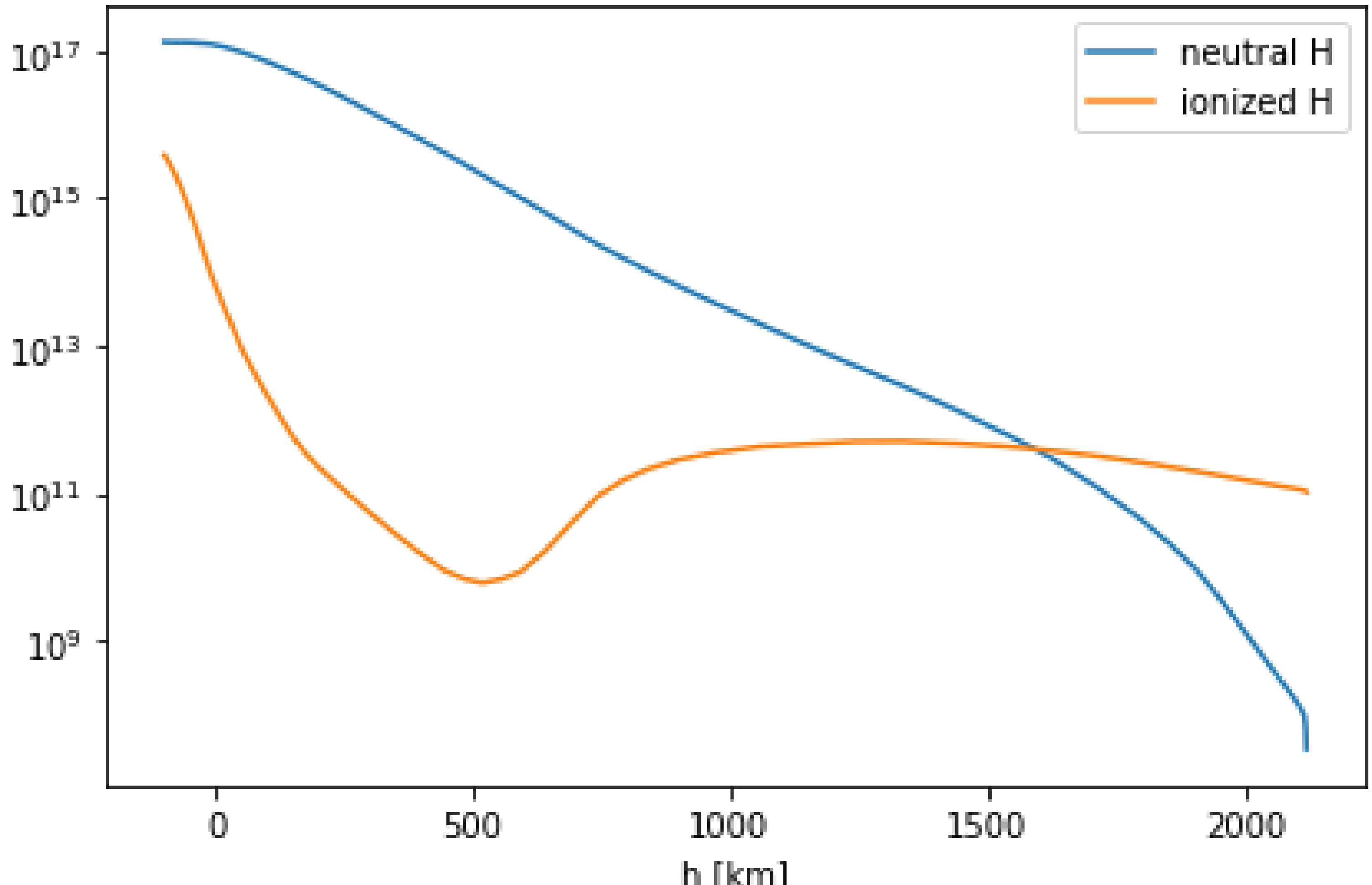
- If we asume only hydrogen matters, what can we do?

$$n_e = n_{j+1}$$

$$n_e + n_{j+1} + n_j = N_{total}$$

Provided by the atmospheric model!

Ah there it is - does it make sense?



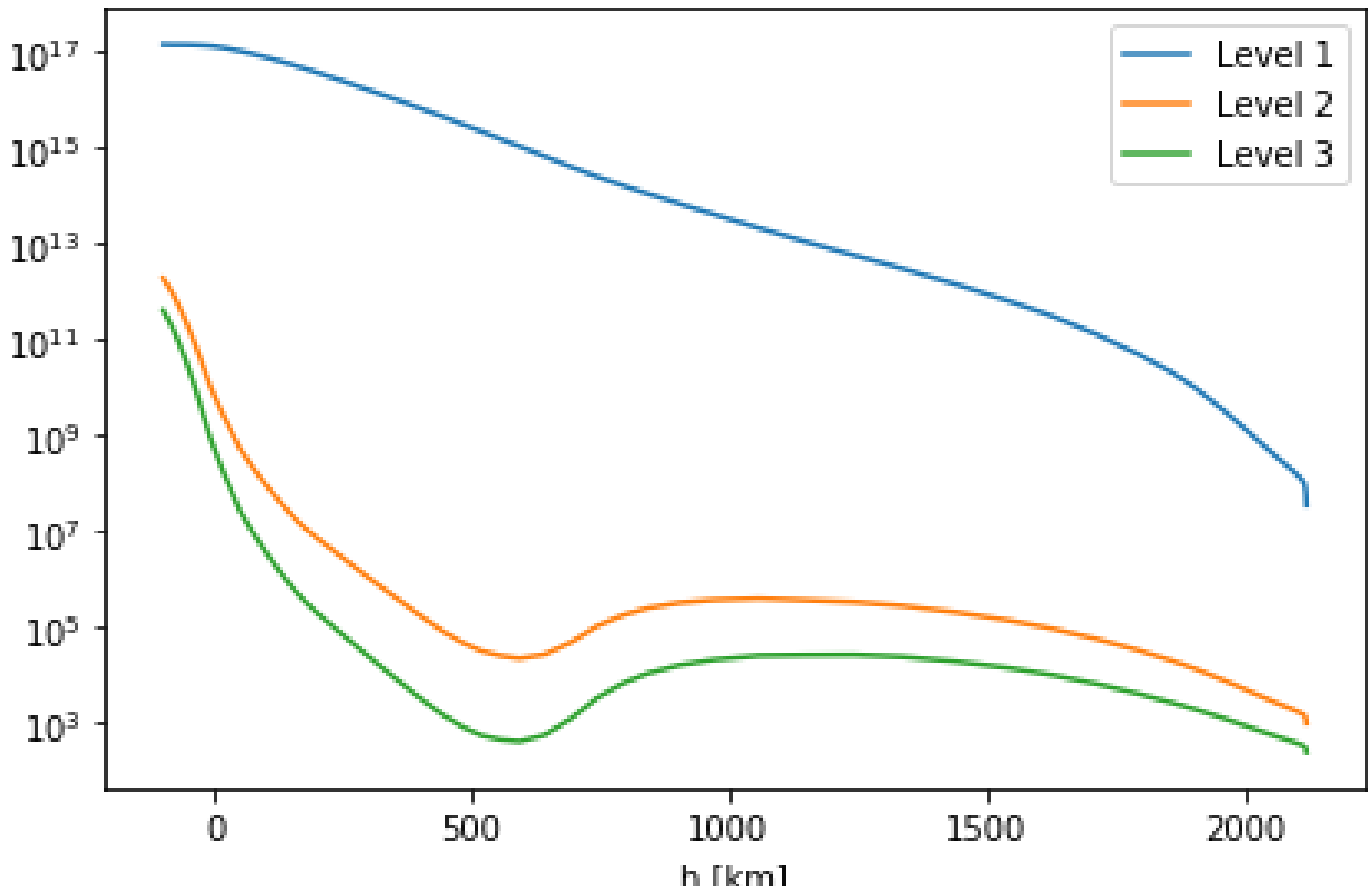
Problem two, how to calculate the level populations?

- Boltzmann distribution

$$n_{j,i} = n_j \frac{g_i e^{-E_i/kT}}{Z_j}$$

- So temperature will influence both the ionization state as well as the populations of the individual levels. Neat.
- Remember when I said the temperature is the most important physical parameter?

Ok, can you tell me a little bit about this graph here?



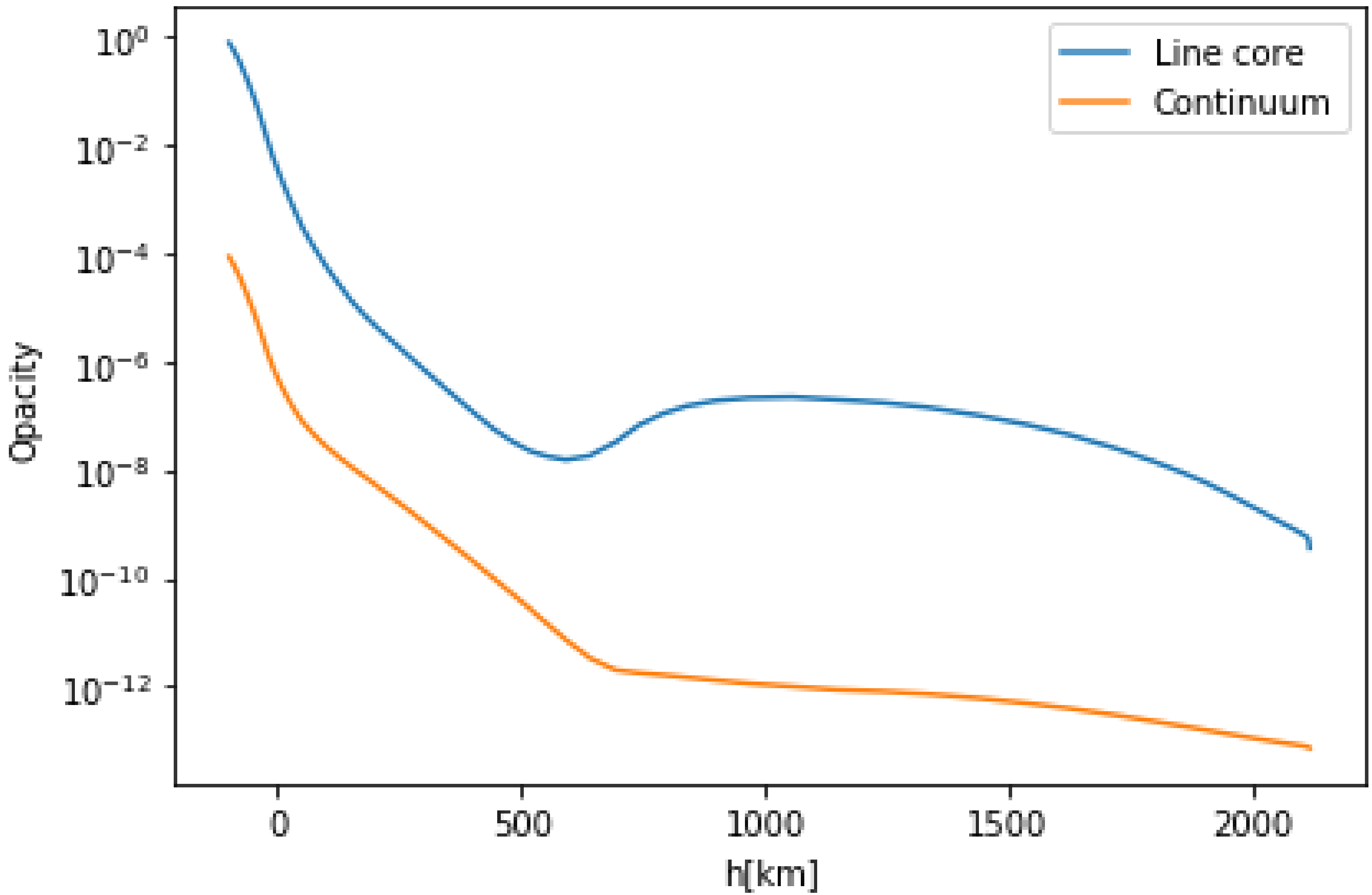
Let's calculate the opacity and emissivity

$$j(z,\theta,\lambda) = n_u(z) A_{ul} \frac{hc}{4\pi\lambda_0} \phi(z,\lambda,\theta)$$

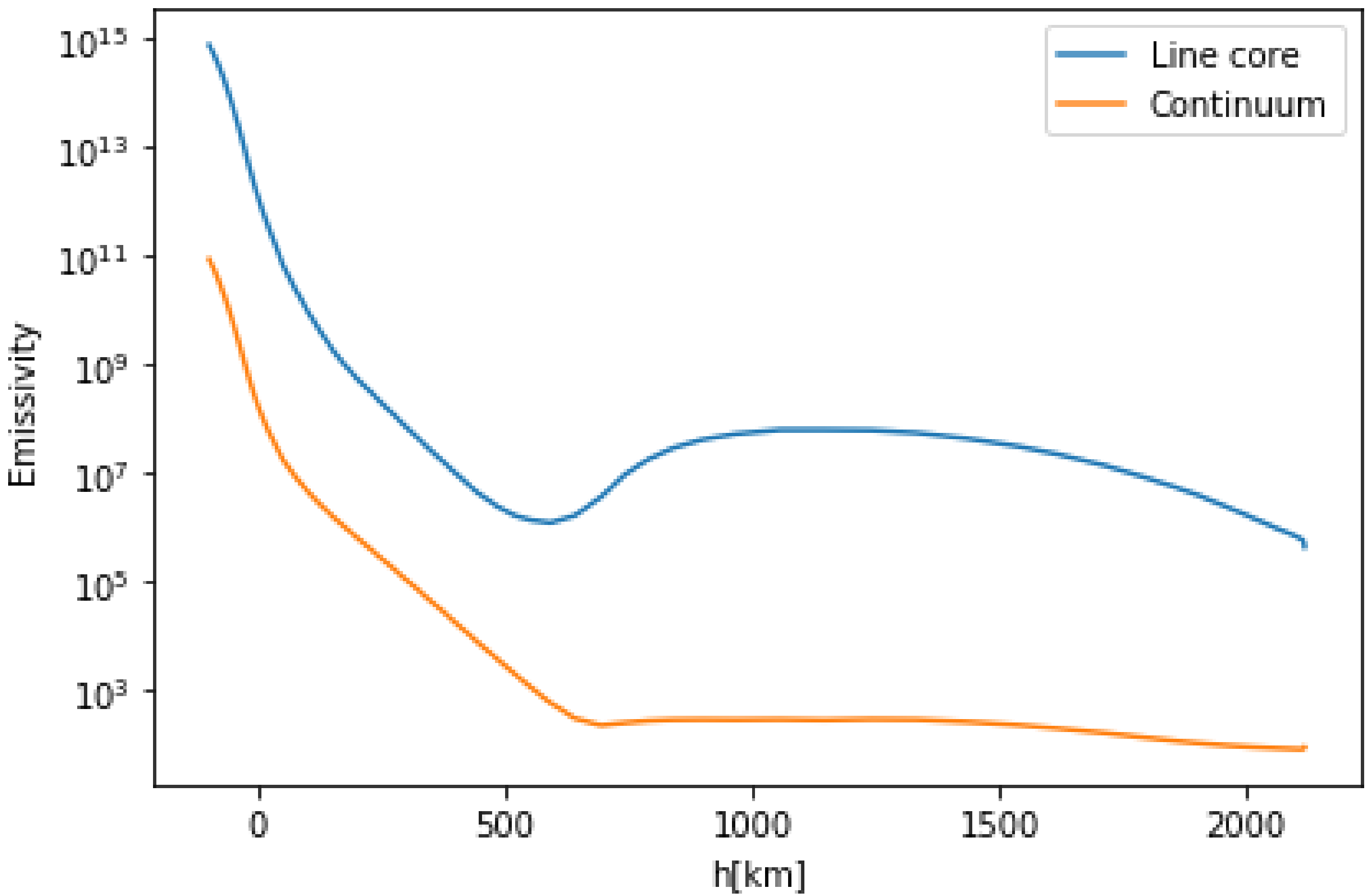
$$\chi(z,\theta,\lambda) = \left(n_l(z) B_{lu} - n_u(z) B_{ul}\right) \frac{hc}{4\pi\lambda_0} \phi(z,\lambda,\theta)$$

$$\phi(z,\lambda,\theta) = \frac{1}{\Delta\lambda_D\sqrt{\pi}} e^{(\lambda'-\lambda_0)^2/\Delta\lambda_D^2} \star \phi_0^{\mathrm{damping}}(z)$$

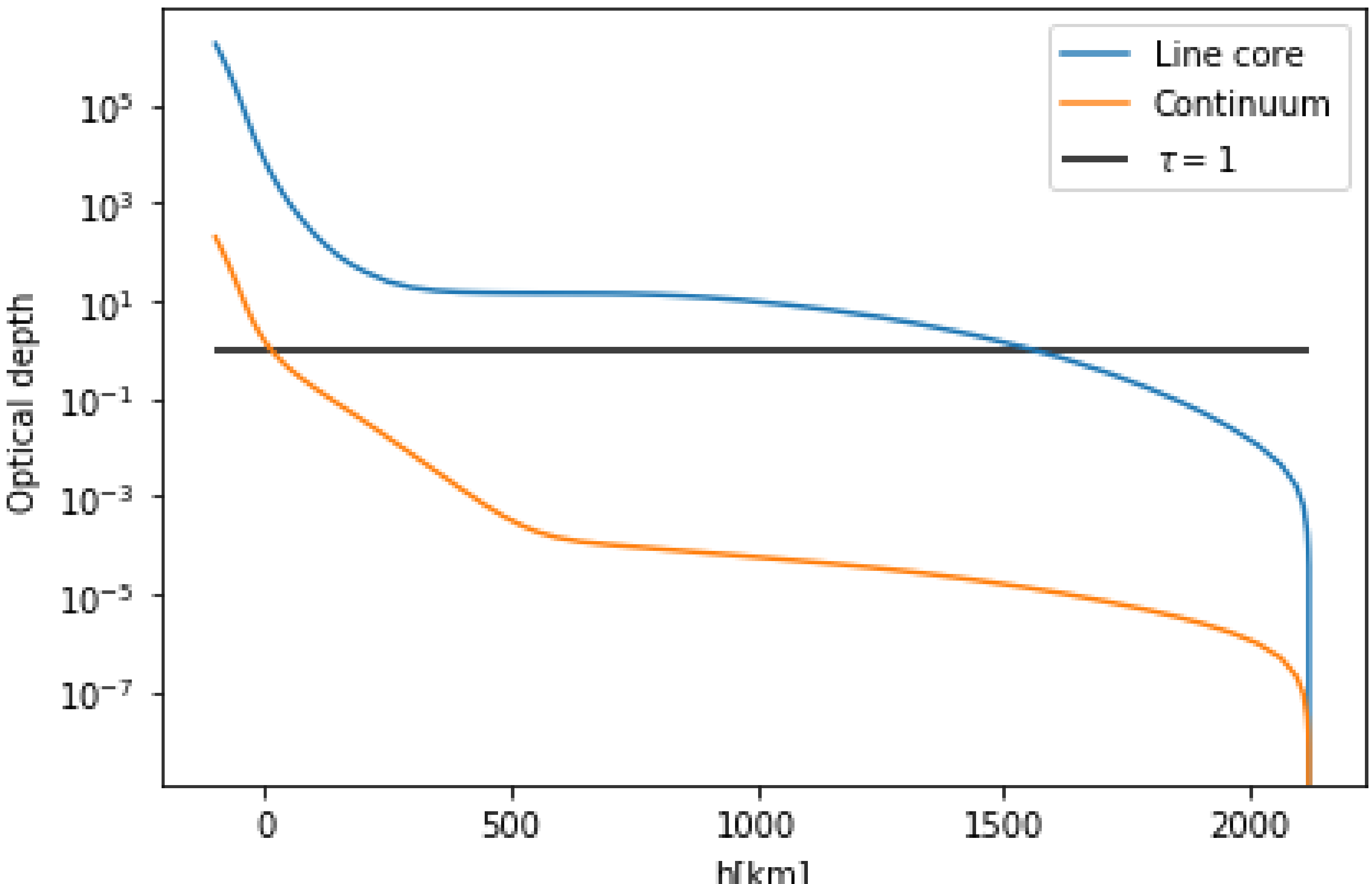
Here they are - opacity



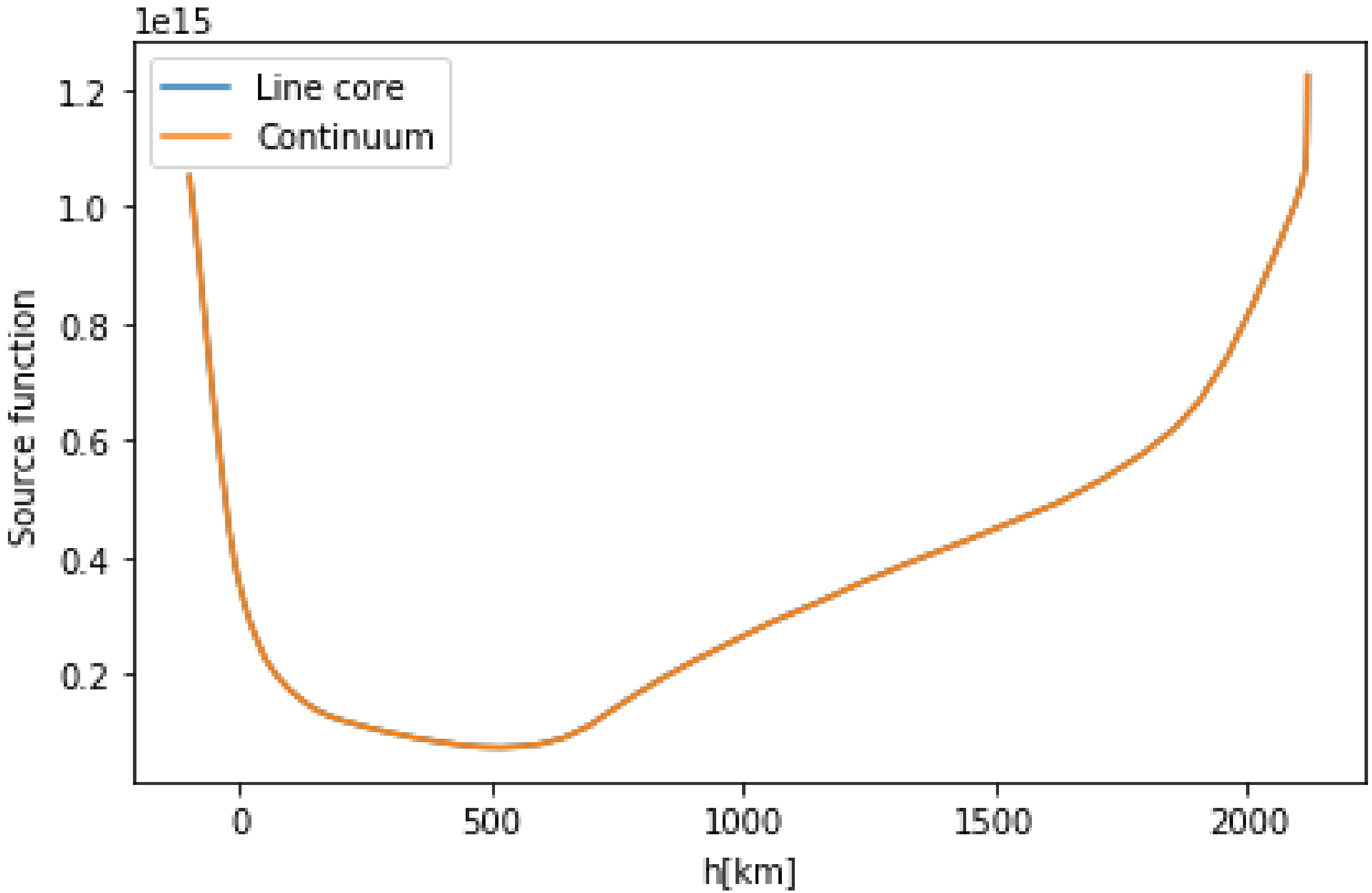
Here they are, emissivity



But we want optical depth and the source function



Source function



Can anyone tell me why does the source function look like the temperature?

Now we understand better plots such as this one

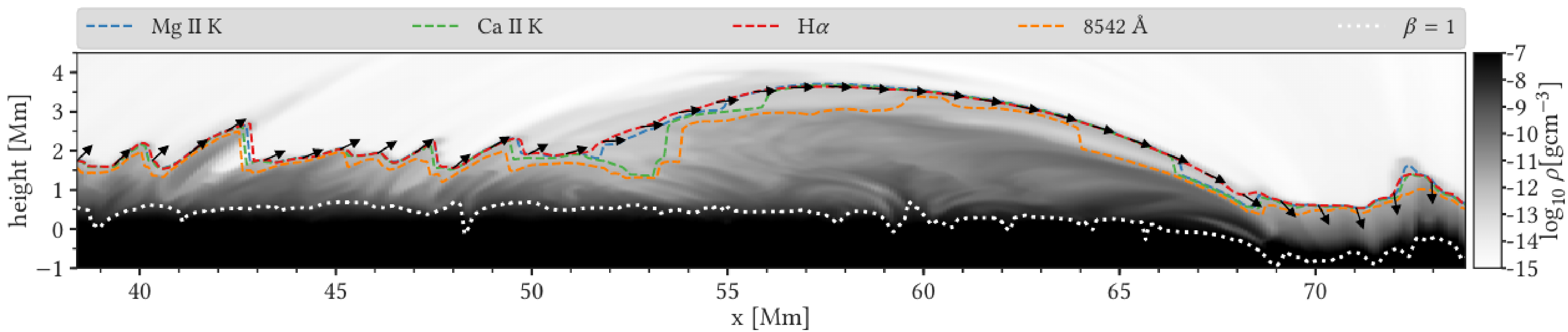
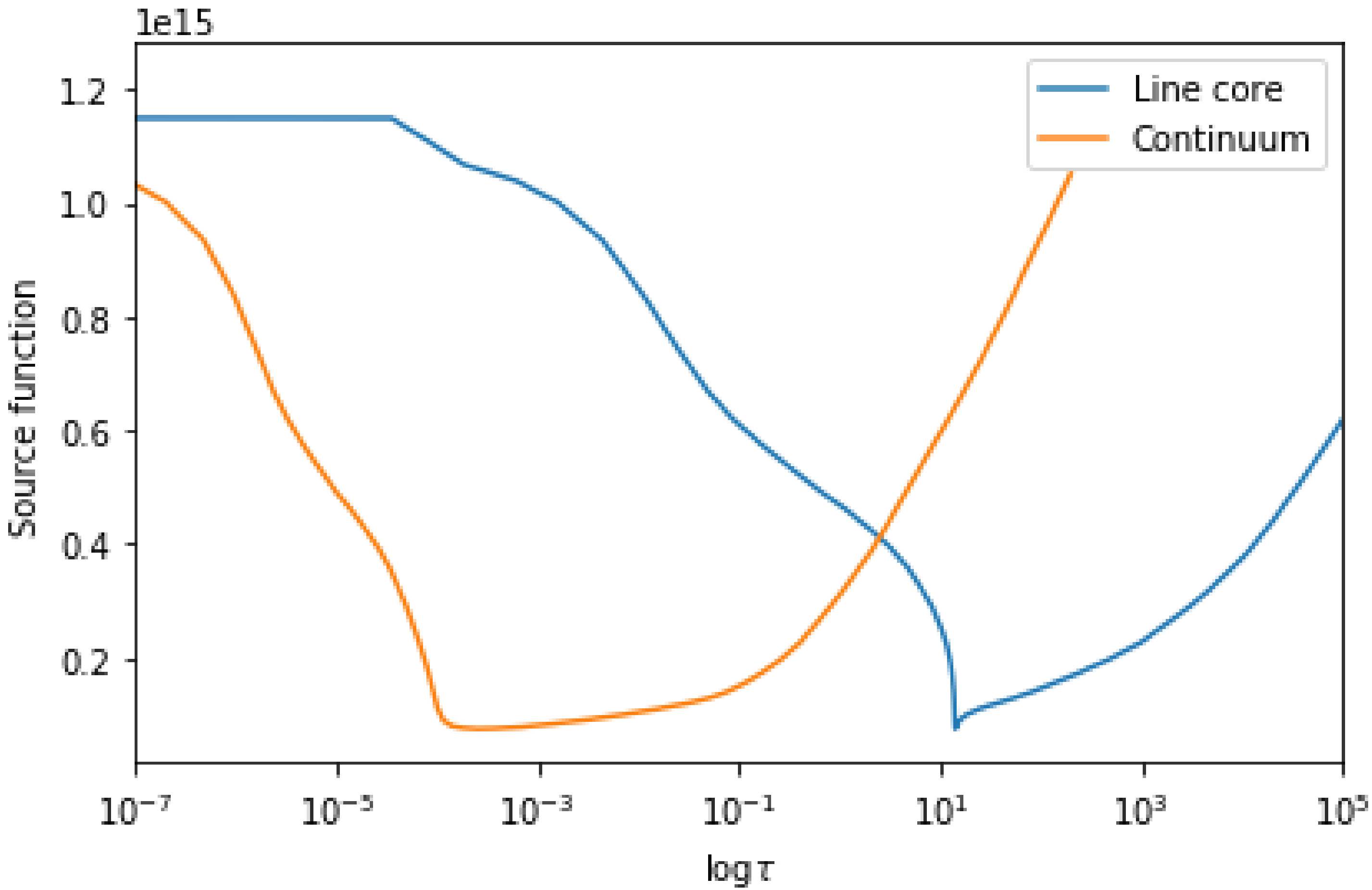


Fig. 9. Mass density in a vertical cut through the model atmosphere along the blue line in Fig. 8. For each spectral line mentioned in the legend, corresponding dashed curves show the maximum formation height at optical depth unity. The white dotted line shows the first occurrence from the upper convection zone where the magnetic pressure equals gas pressure ($\beta = 1$). The arrows show the direction of the magnetic field vector in the plane of the cut along the formation height curve of Hα. These arrows are plotted every tenth grid point.

From Bjorgen et al (2019)

Now it makes more sense, does it? (It does not :P)



Contribution function

- We can simply say that optical depth equal unity is the most important one. Is there a more sophisticated method?

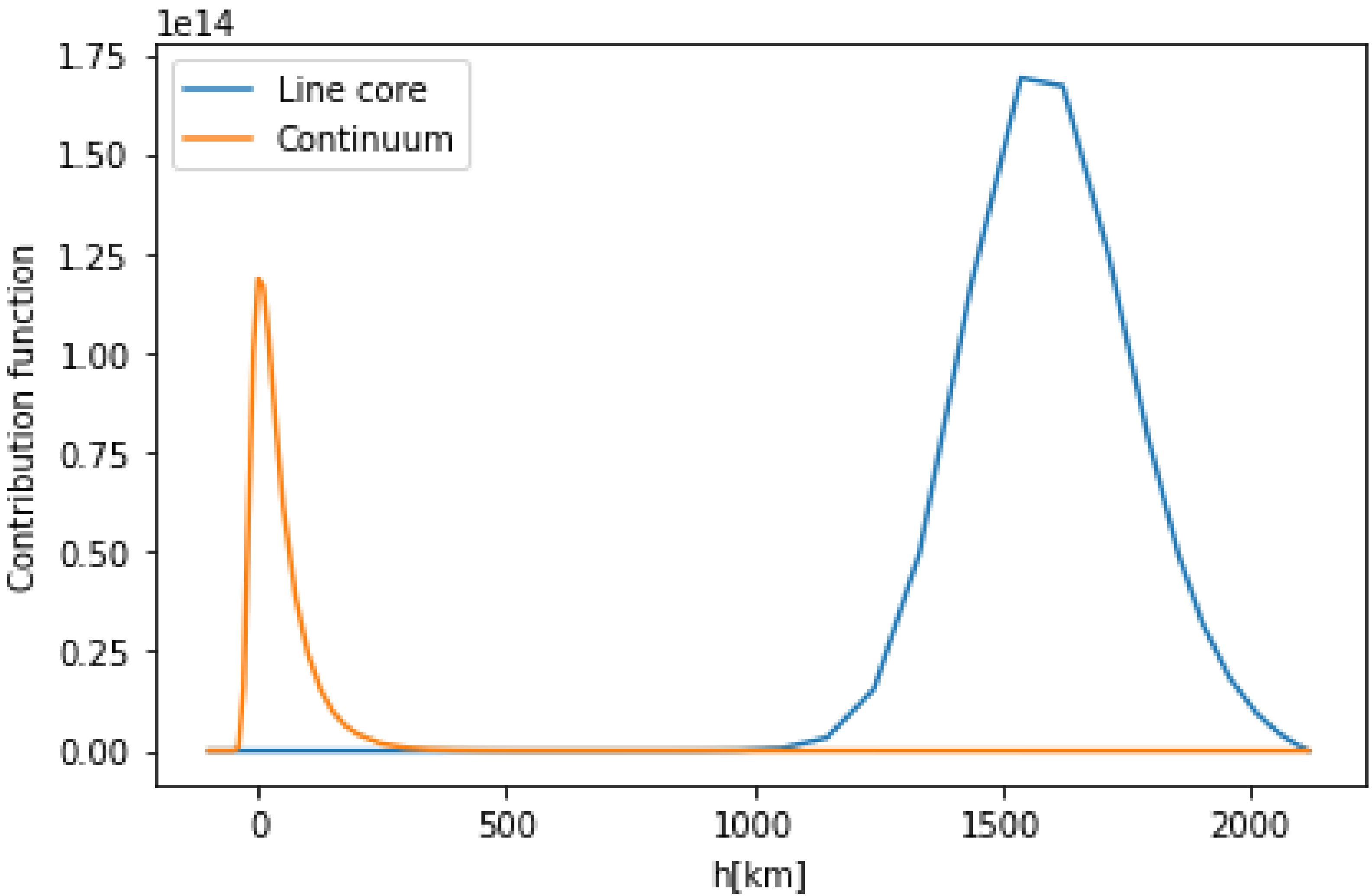
$$I_\lambda^+ = \int_0^\infty S(\tau_\lambda) e^{-\tau_\lambda} d\tau_\lambda = \int_{-\infty}^\infty S(\log\tau_\lambda) e^{-\tau_\lambda} \tau_\lambda d(\log\tau_\lambda)$$

- **Contribution function** is the function under the integral

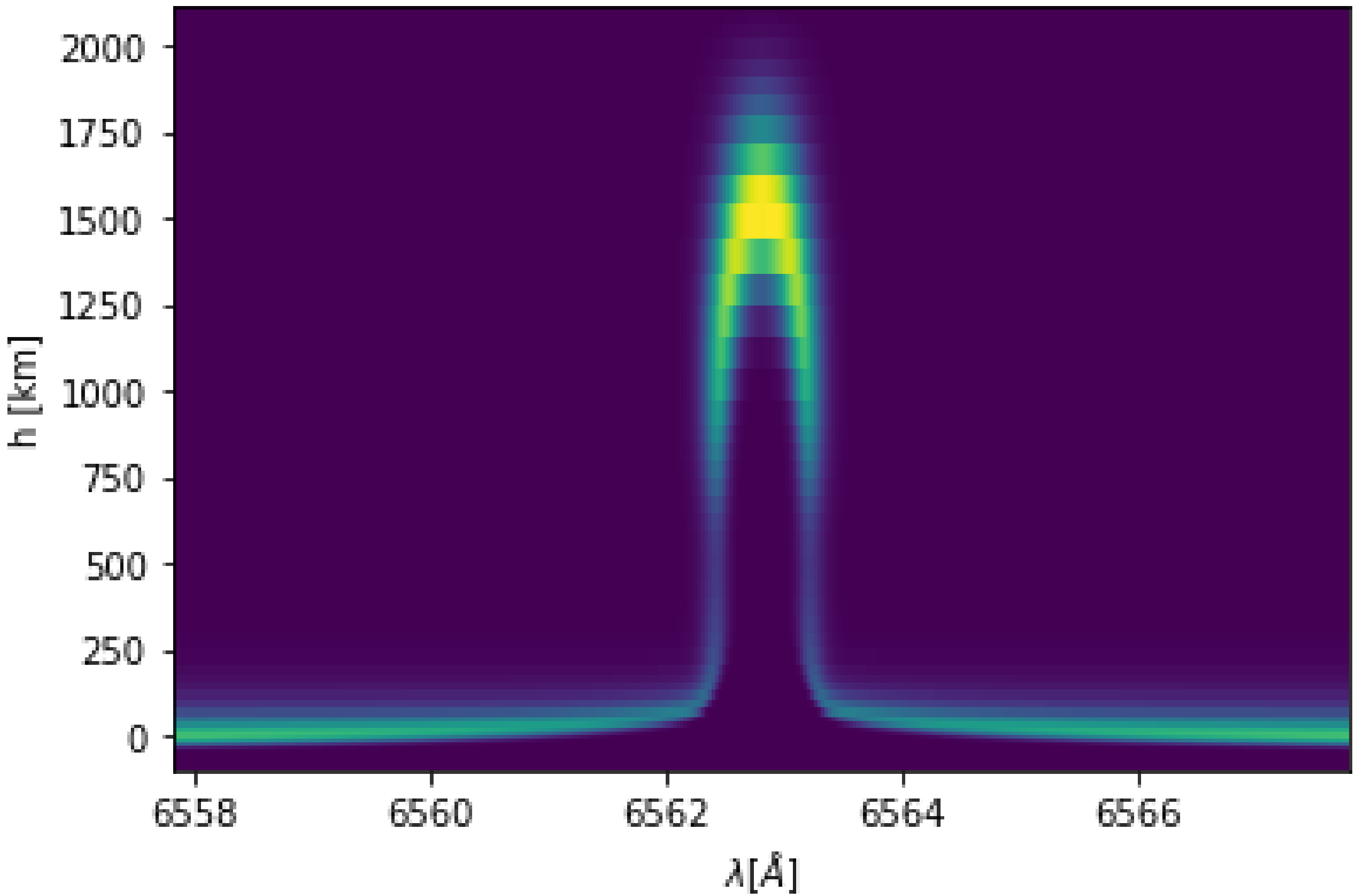
$$\mathcal{C}_\lambda(\tau_\lambda) = S(\tau_\lambda) e^{-\tau_\lambda} \tau_\lambda$$

- Line formation height is the one where contribution function peaks. Can you take 5' and try to infer optical depth unity rule from the contribution function?

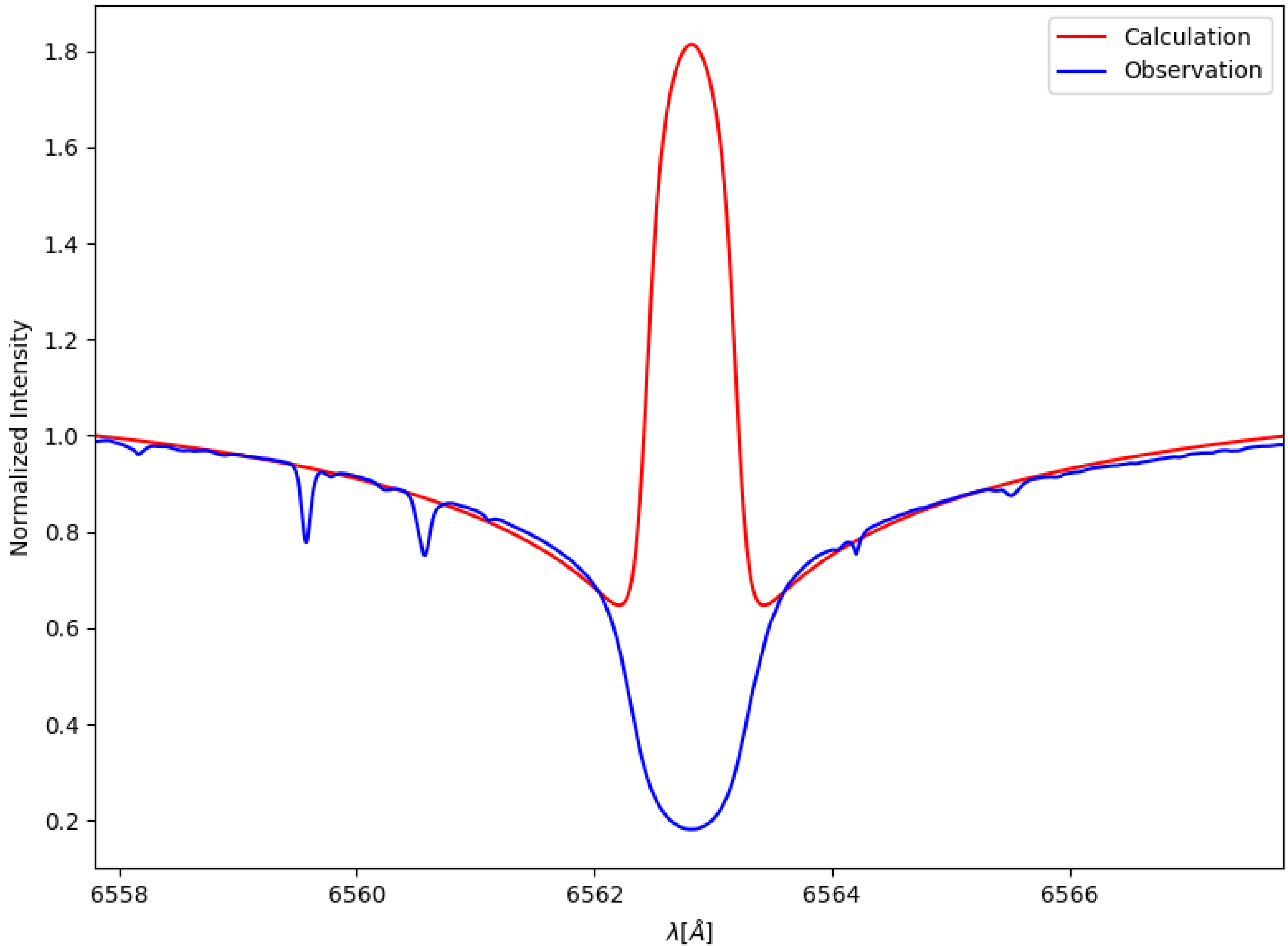
Now it makes more sense!



Contribution function in 2D



Calculate the spectrum... ooooooops!



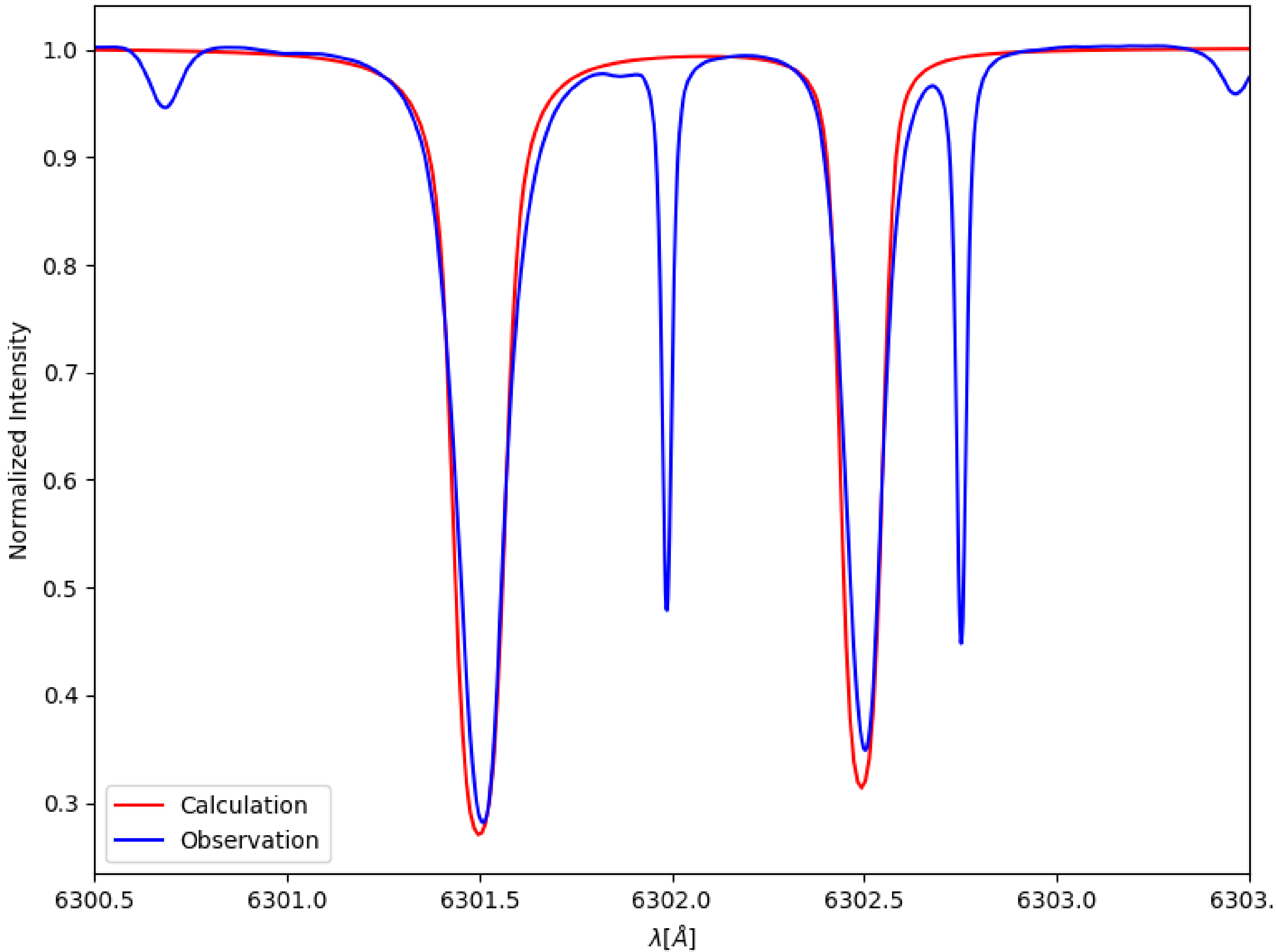
Ok, ok, something is wrong!

- One of the assumptions we made in the line formation is not appropriate
- Which one is it?

Ok, ok, something is wrong!

- One of the assumptions we made in the line formation is not appropriate
- Which one is it?
- Yes, it is the assumption that the Saha equation and Boltzmann equation are valid. They are not.
- H-alpha is an example of **scattering** line.
- There are many interesting aspects about NLTE. We will hopefully tackle some of these.
- What matters for us today, is that we have to change the spectral line. This one won't do.

Ok, let's try famous 6300 lines of neutral Iron ("HINODE lines")



Question:

Say I infer something from these lines (temperature, Doppler shift, magnetic field)

To which “layer” of the solar atmosphere do I ascribe it to?

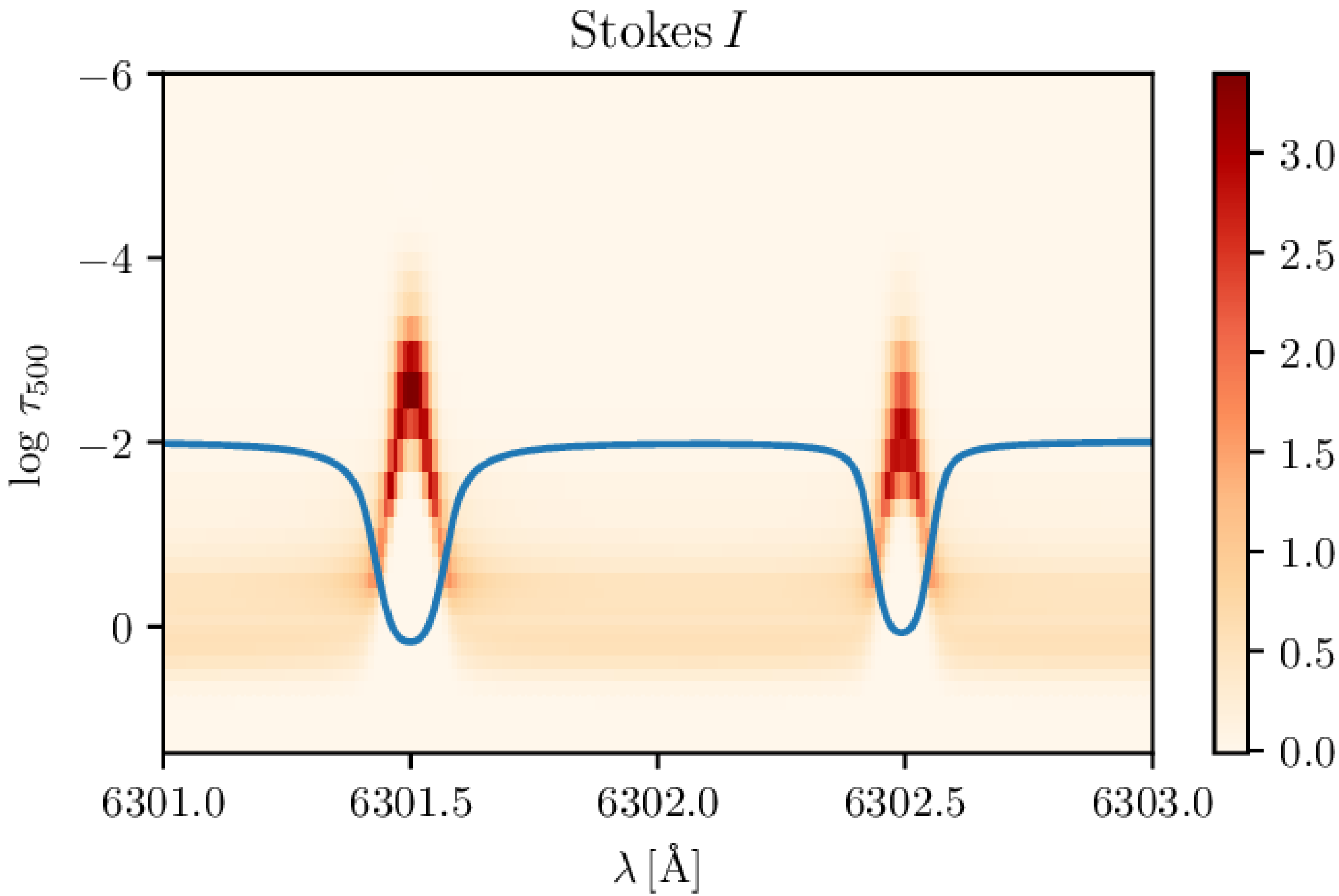
Response function

- Contribution functions measures what contributes to the integral.
- However, to quantify the “line formation height” it makes more sense to see what layer influence the line shape the most.
- So, for any physical quantity, *q,* at the depth point *d.* I can define a response function as:

$$\mathcal{R}_\lambda(h_d) = \frac{\partial I_\lambda^+}{\partial q_d}$$

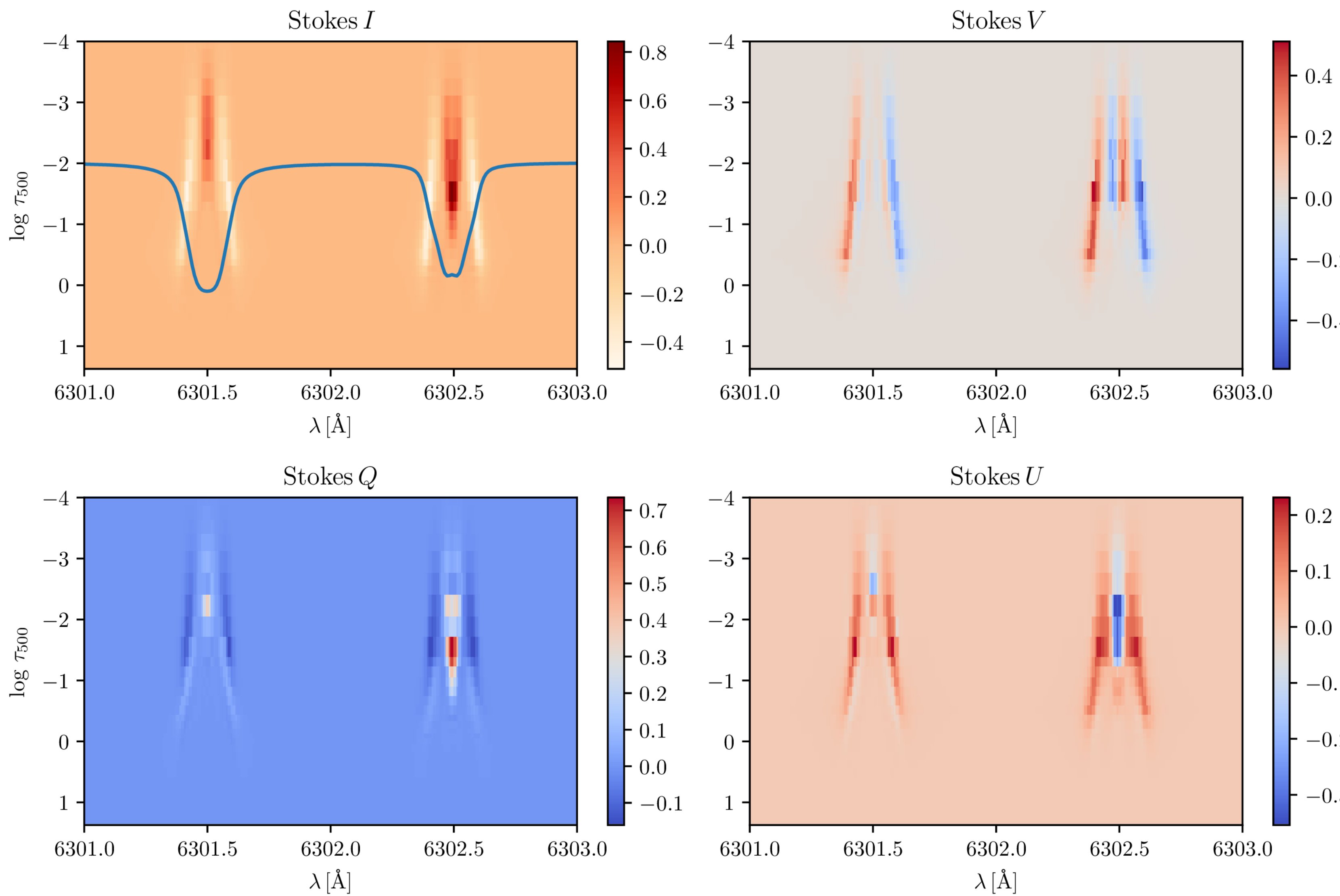
- A simple justification for that, can be: we can define contribution functions to any quantity, even if that quantity does not influence line formation. Response function shows us what matters!
- Can you come up with more justifications?

Response functions



$$\frac{\partial I(\lambda)}{\partial T_d}$$

Response functions to the magnetic field



How could we use the response functions?

- We use them in inversions, but that is better left for later
- Let's say that you used two different lines to measure LOS velocity
- You could calculate the response function to the LOS velocity, see where it peaks (or "weight" it properly), to relate these measurements to the heights in the solar atmosphere
- Question as old as the world itself is: What model to use it more (everything is model dependent)
- There is a lot to unpack and think about it, but at least we have some idea
- How do we calculate the response functions then?

How do you calculate the response functions?

- Say we have a model (tabulated values of the physical parameters), we can calculate the spectrum, what then?

$$I_\lambda^+ = f(T_1, T_2..., \vec{B}_1, \vec{B}_2..., \vec{v_1}, \vec{v_2}...)$$

$$R_\lambda(h_d) = \frac{\partial I_\lambda^+}{\partial q_d} = \frac{f(\mathbf{q} + \Delta q_d) - f(\mathbf{q})}{\Delta q}$$

- This is straightforward but cumbersome. Plus we do not gain much insight into this.

Response functions ("SIR approach")

- Say that we have changed the temperature in one point in the atmosphere. What is going to happen is that opacity and emissivity in that point are going to change.

$$j_\lambda = n_u \frac{hc}{4\pi\lambda_0} A_u l \phi(\lambda)$$

$$\frac{\partial j}{\partial T} = \left(\frac{\partial n_u}{\partial T n_u} + \frac{\partial \phi(\lambda)}{\partial T \phi(\lambda)} \right) n_u \frac{hc}{4\pi\lambda_0} A_u l \phi(\lambda)$$

This is super non-trivial to calculate

- And the same for opacity…
- But these are just emissivity and opacity changes, then we need to propagate them somehow.

Response functions ("SIR approach")

$$\frac{dI_\lambda}{dz} = -\chi_\lambda I_\lambda + j_\lambda$$

$$\frac{d(I_\lambda + \delta I_\lambda)}{dz} = -(\chi_\lambda + \delta\chi_\lambda)(I_\lambda + \delta I_\lambda) + j_\lambda + \delta j_\lambda$$

$$\boxed{\frac{d\delta I_\lambda}{dz} = -\chi_\lambda \delta I_\lambda + (\delta j_\lambda - \delta\chi_\lambda I_\lambda)}$$

This is a RTE for the perturbation

$$\delta I_\lambda^+ = \int_0^\infty \boxed{\frac{(\delta j_\lambda - \delta\chi_\lambda I_\lambda)}{\chi_\lambda}} e^{-\tau_\lambda} d\tau_\lambda$$

This is just a regular RTE

This is the Source function for the perturbation

Summary

- The spectrum and the spectral lines are formed over a range of heights. "Simulation" process is not straightforward.
- We want to know where the line "is formed"
- This means: what regions is the line sensitive to?
- Best estimator: response functions – hard to calculate
- Next step: Contribution function
- Next step: Optical depth unity
- Next (first?) step: Common sense
- All this is valid for canonical atmospheres. Things change with the atmosphere, and with the line (He 10830, Coronal lines…)
- Happy modeling!