PHYS 7810: Solar Physics with DKIST

Lecture 14: Intro to Radiative Transfer

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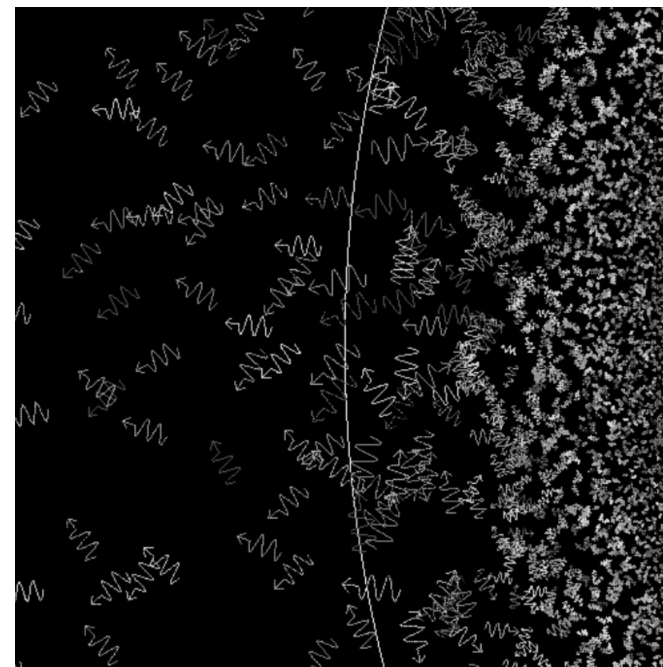
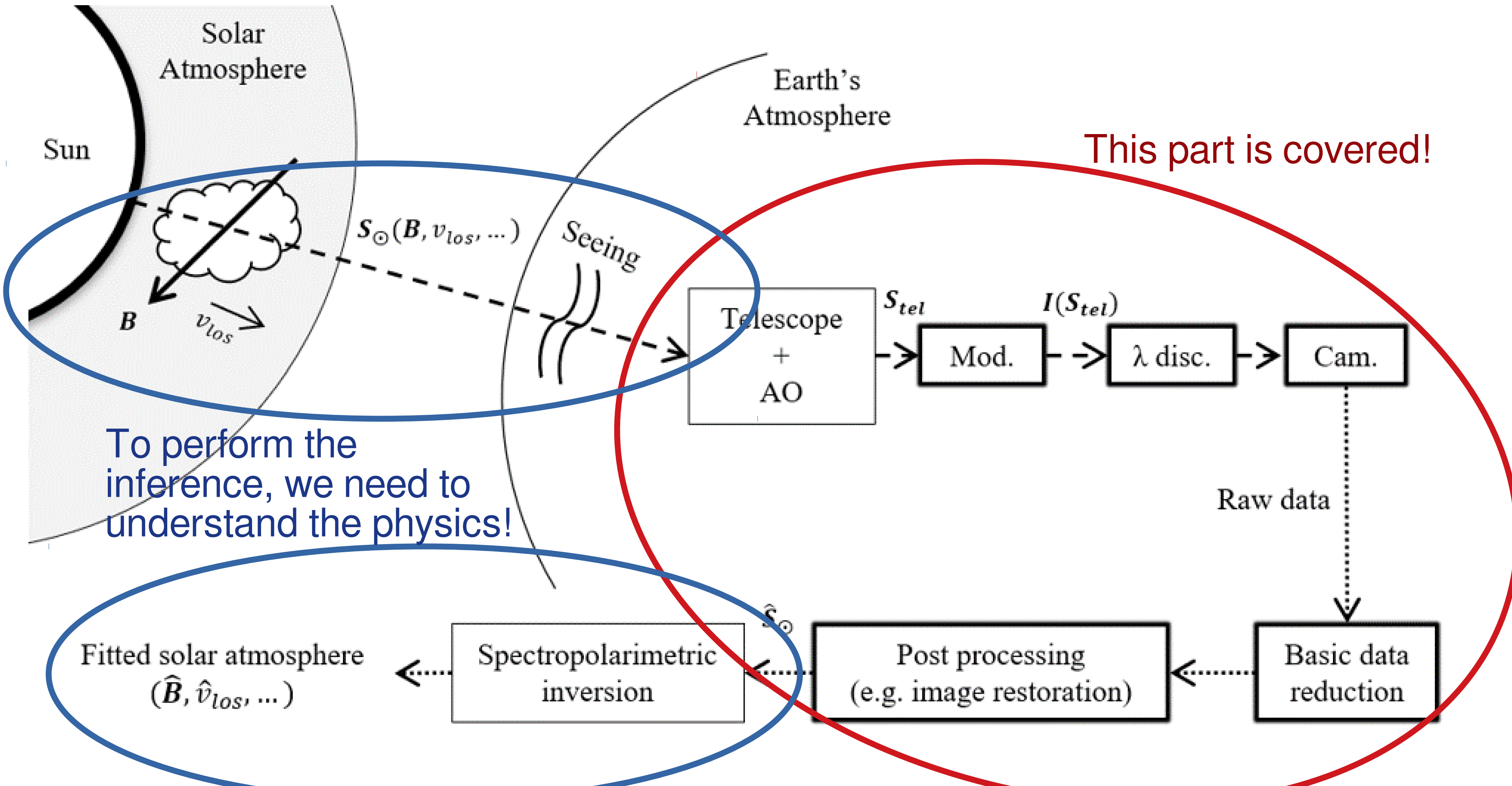


Chapter II of our course

- First, we wanted to measure stokes vector and its temporal, spatial and wavelength distribution
- We did not ask why (we hinted though)
- Now we want to approach the problem from a different perspective. Connect the measurements (counts $\rightarrow$ Stokes vector), to the physical properties of the solar atmosphere
- We detect radiation , radiation interacts with matter
- Even if we don’t want to model it, we need to understand what is going on

From the bottom of the solar atmosphere to our detector



“All models are wrong, but some are useful”

1st level: We cannot model the nature in sufficient detail

2nd level: When performing inference, we can’t account for all the physics, even if we know it, because it is just too overwhelming

Plan for the next part of the course

- Why is radiation important?
- What are the units, quantities and technicalities involved?
- What equations govern the radiative transfer?
- How do physical conditions in the atmosphere influence the observed radiation?
- How to fit the model to the data and extract what we need (or think we need) ?
- **More in-depth examples:** Photospheric magnetometry and local correlation tracking (Maria Kazachenko, CU), Off-limb spectroscopy/spectropolarimetry (Steven Cranmer, CU)

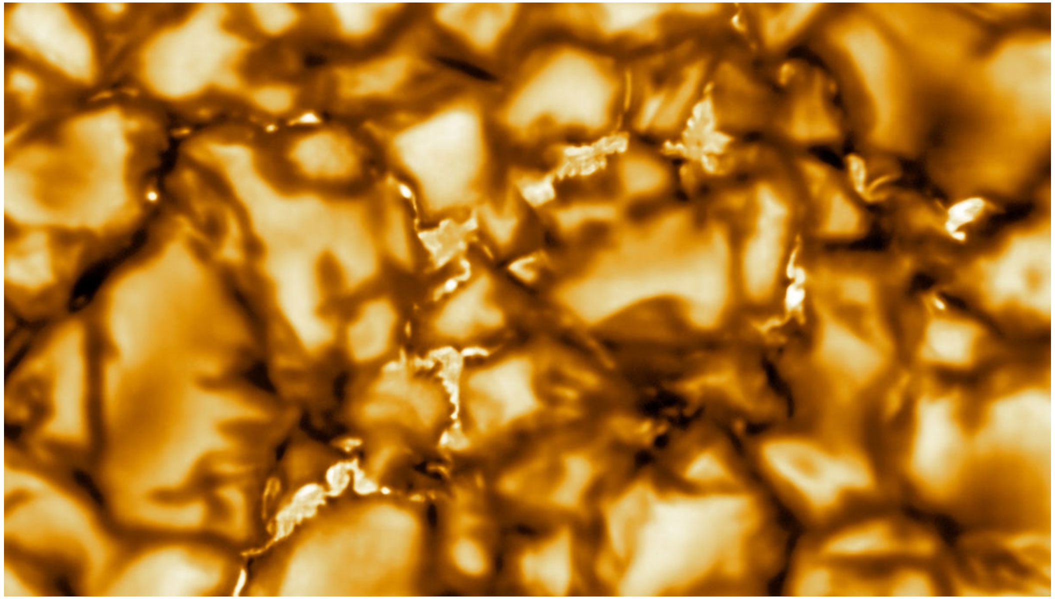
So what is radiation and why is it important

- Radiation is a type of a particle – photon. They are bosons, they have spin one but only (-1,1) are allowed.
- These correspond to two opposite polarizations. (Recall your QM classes).
- They pass through each other. This creates the most important difference w.r.t. massive particles.
- Their number is not conserved (Recall Bose's derivation of blackbody law)
- The photon distribution over wavelengths and the net polarization carries information about the object:
- **Temperature, density, velocity, magnetic field, geometry (3D structure)**

But also, an “active” role (we won’t go into this further)

- Photons transport energy and exert pressure
- They can help the medium cool (Kinetic energy $\rightarrow$ Radiation)
- Or heat it (Radiation $\rightarrow$ Kinetic energy)
- They can also significantly alter the ionization or excitation state of the gas
- It naturally feels we should focus on the interaction between the photons and individual atoms / molecules as the Sun is the gas.
- (In our optical systems it is different)
- *“Radiation is a constituent of astrophysical objects, yet the objects are probed only by the radiation”* (Hubeny, 2013)

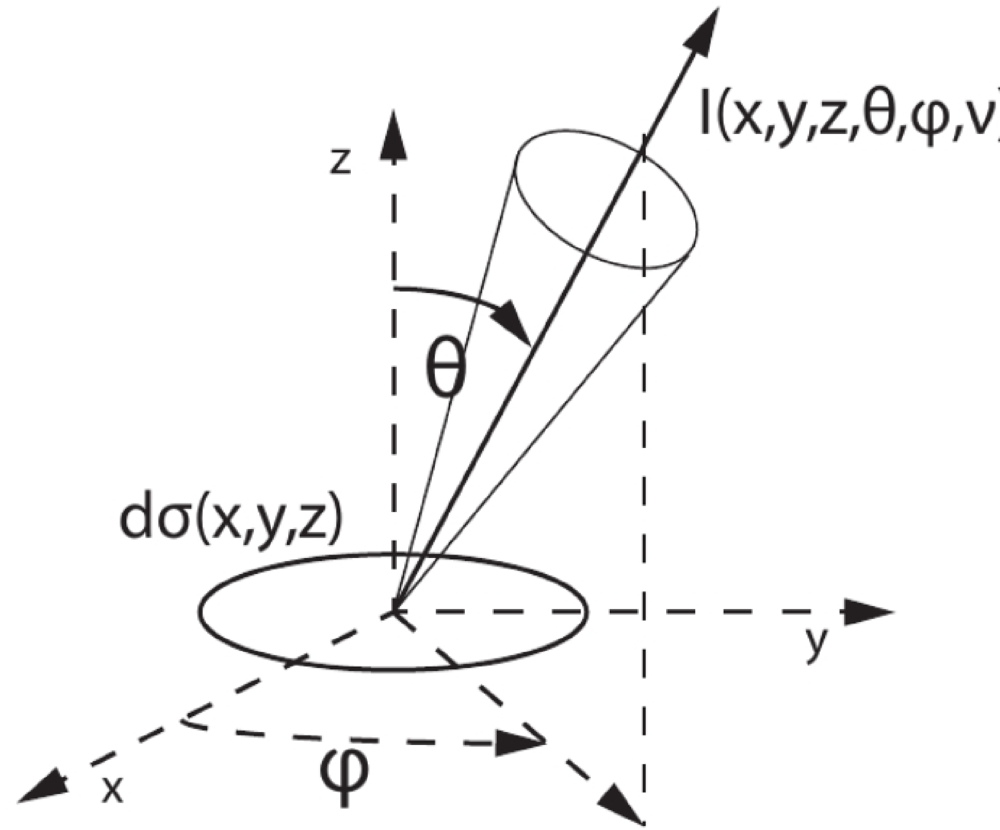
Basic quantities – Specific monochromatic intensity



$$N \propto \Delta\sigma \Delta t \Delta\Omega \Delta\lambda$$

$$I(x, y, z, t, \lambda, \theta, \phi) = \frac{d^4 E}{d\sigma(x, y, z) \cos\theta \, d\Omega(\theta, \phi) \, d\lambda \, dt}$$

Always remember this figure!



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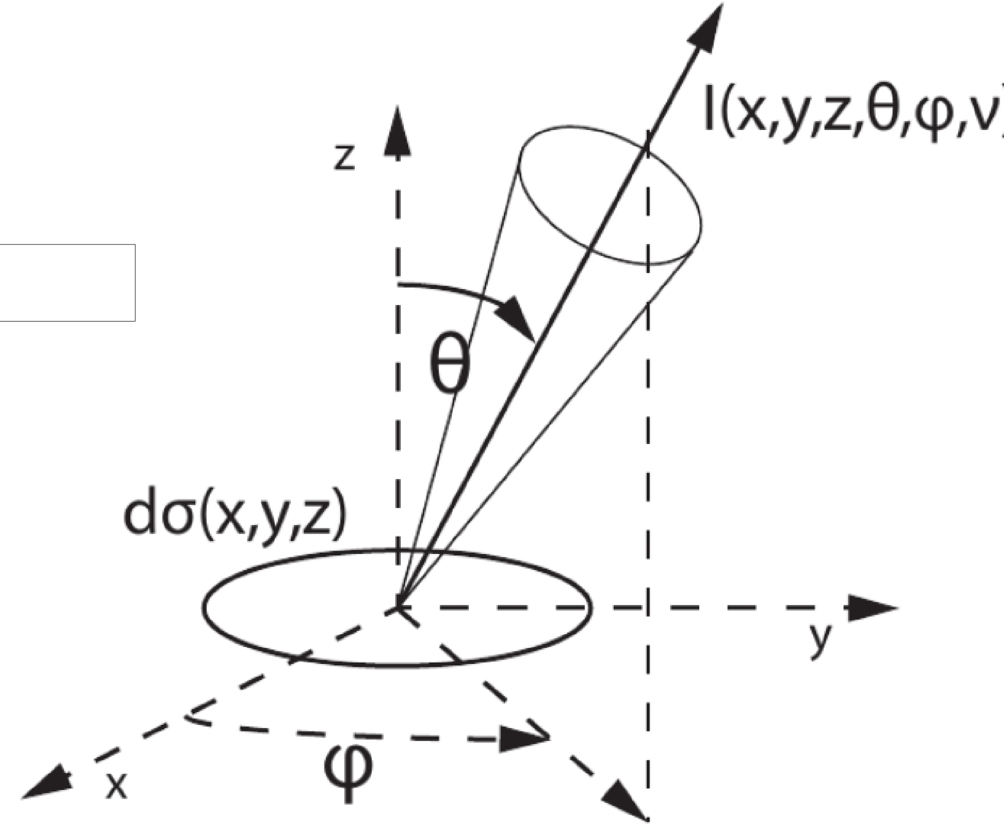


When? Well, we choose

Where, what is the meaning of x,y,z? Discuss, this is important.

At what wavelength? We choose this too.

In which direction? This is specified by the orienation of observed patch w.r.t us

$$I(x, y, z, t, \lambda, \theta, \phi) = \frac{d^4 E}{d\sigma(x, y, z) \cos\theta \, d\Omega(\theta, \phi) \, d\lambda \, dt}$$

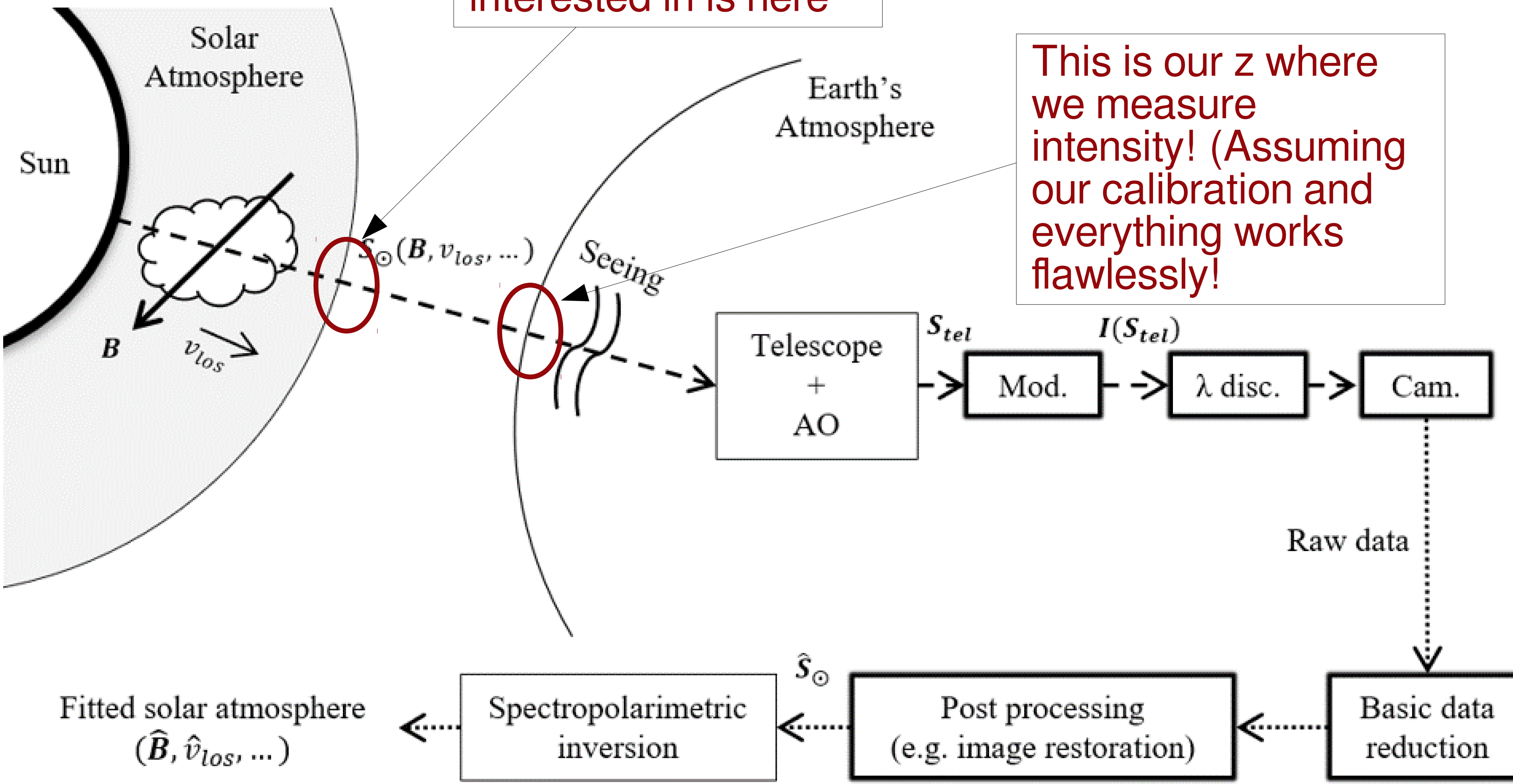
So to summarize

- For simplicity look at the Sun “from above” (heliocentric angle equal to zero)
- x,y - determine where we are on the solar disk
- θ,φ -determined by the position of the pixel, and our reference for azimuth (here technically azimuth is meaningless)
- Wavelength – we choose sampling (hopefully ;))
- Time – the same, we might be limited by our instruments though
- What about z?

Well, technically

But since heliosphere is empty (:P), we can assume z we are interested in is here

This is our z where we measure intensity! (Assuming our calibration and everything works flawlessly!

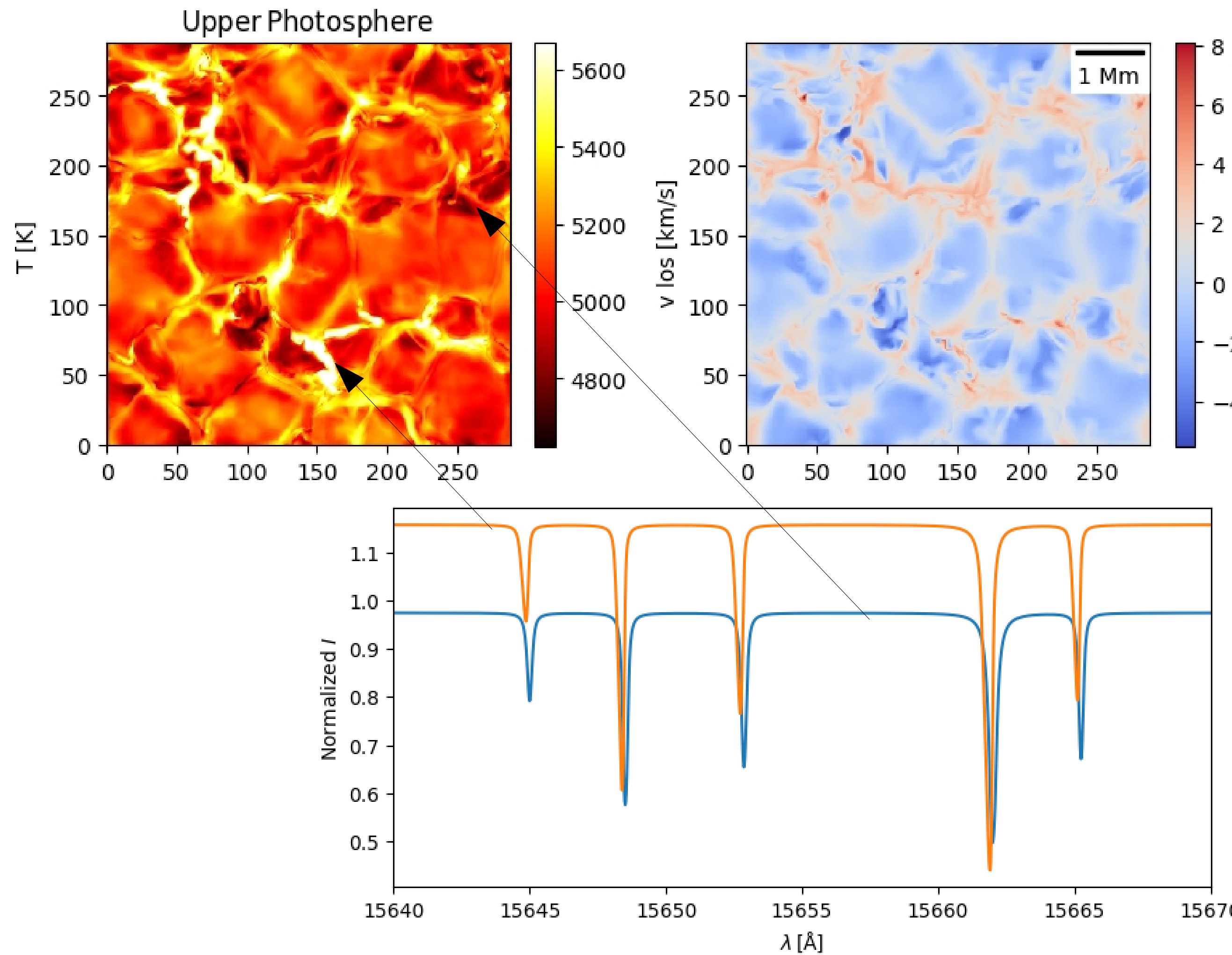


Does it mean we can only probe "top" of the atmosphere?

- Certainly not!
- "Different wavelengths probe different depths!"
- z (height, depth), is a very special coordinate we will spend the whole day today and next week to study what happens with depth
- We need to delve in to the details of **radiative transfer**
- We need to understand how the intensity is changed as it travels through a medium. It's magnitude, spectral distribution and polarization changes.
- Why? How? Can we calculate (model) it? Can we reverse engineer it?

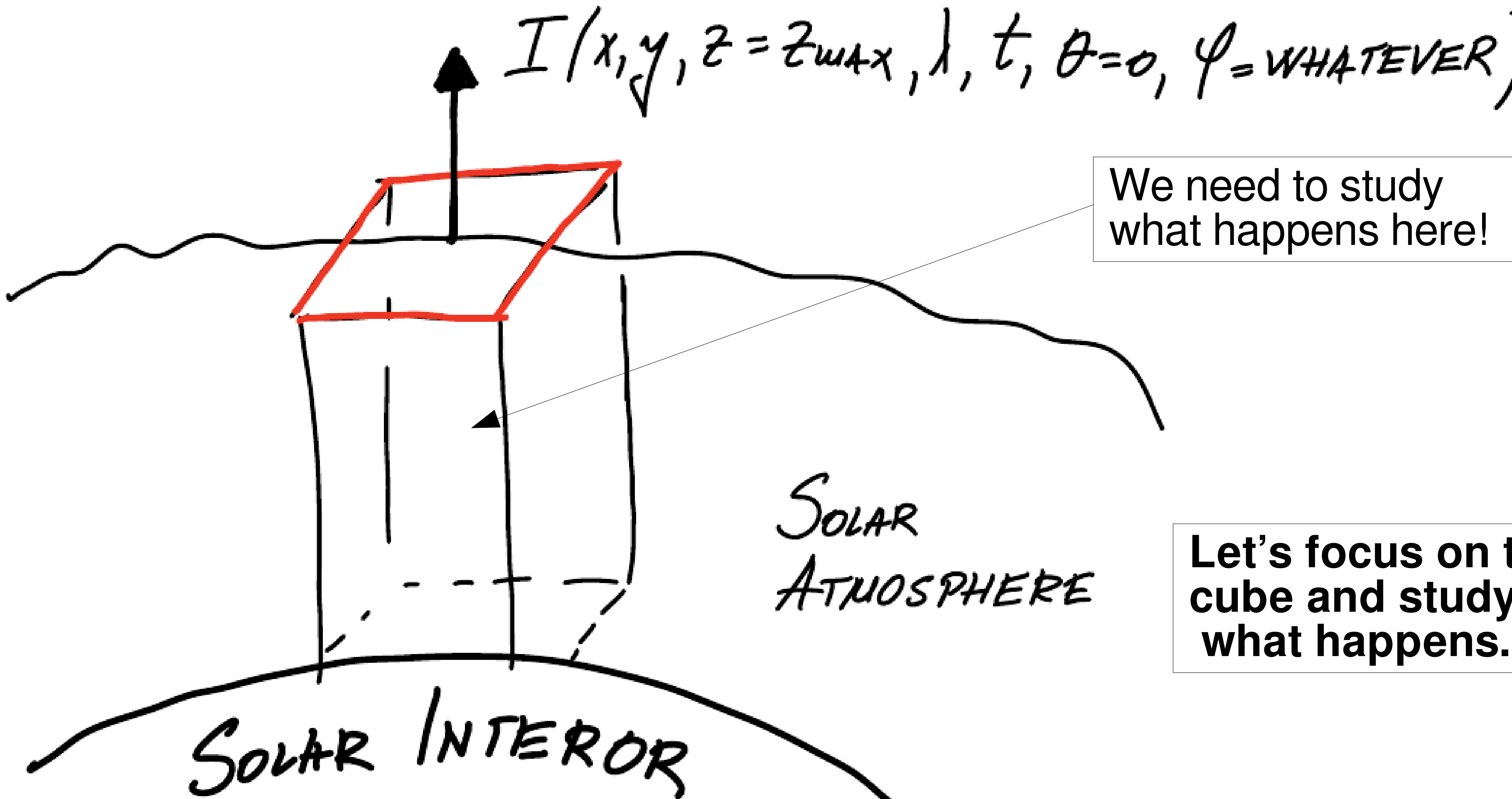
Let’s look at a finished product and unpack

- What is going on here?
- Different points produce different spectra
- Why, how is this spectra generated?
- What influences how will each spectral line look like?
- **What depth is probed by what wavelength? That is something we all would like to know!**



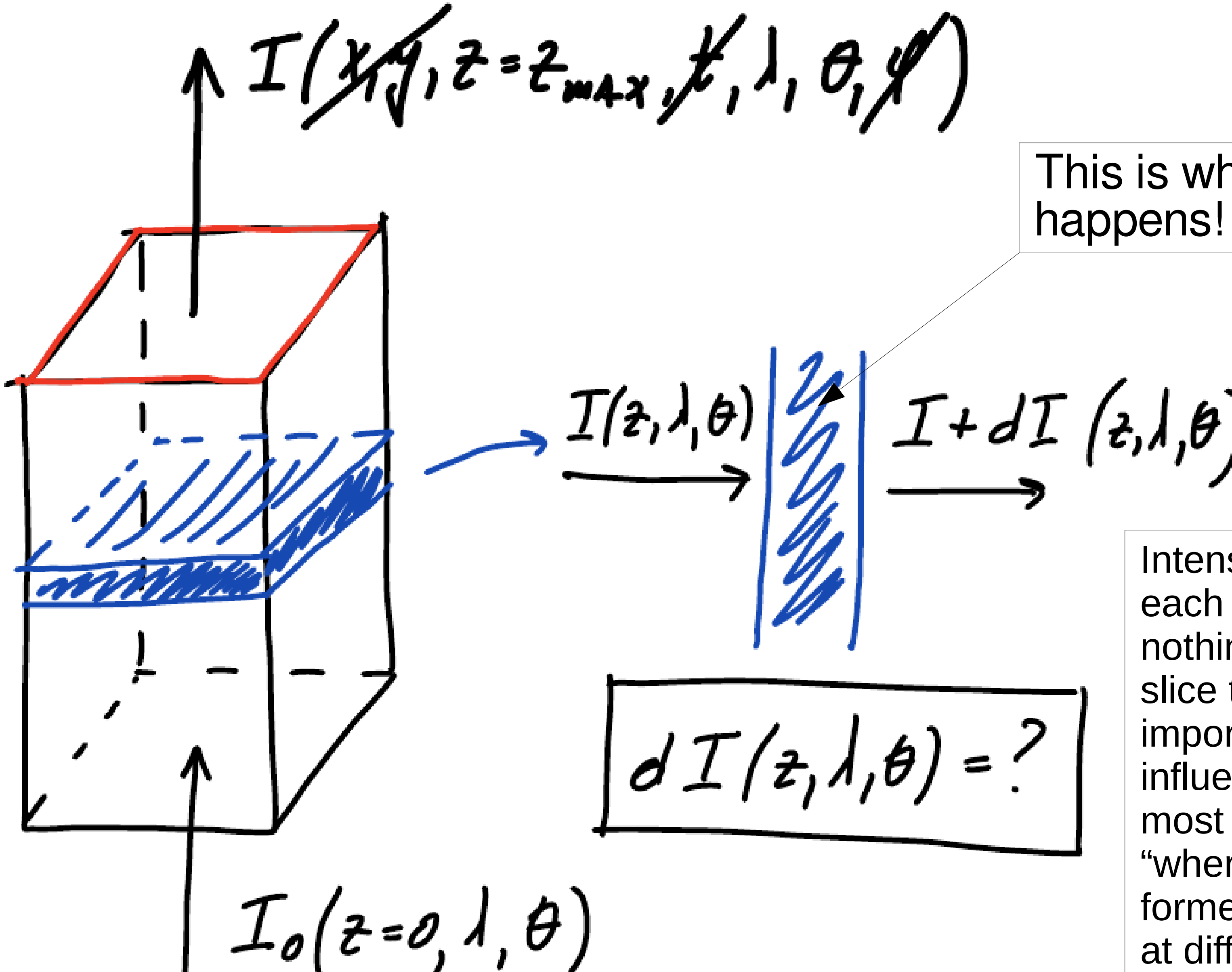
MURAM quiet Sun simulation, courtesy of T. Riethmüller

Radiative transfer equation in the solar atmosphere



We need to study what happens here!

Let's focus on this cube and study what happens.



This is where the magic happens!

Intensity is changed in each of these "slices", if nothing happens in the slice then it is not important. Slices that influence the spectra most are the ones "where the intensity is formed". This will happen at different heights for different wavelengths.

Radiative transfer equation

$$\frac{dI}{c\,dt} + \vec{n} \cdot \nabla I = -\chi I + j$$

- Don't worry it is much simpler in our case.
- Time independent (stationary).
- Only changes with one coordinate (but even 3D case can be reduced to 1D)

Opacity

Emissivity

$$\cos\theta \frac{dI(z, \theta, \lambda)}{dz} = -\chi(z, \theta, \lambda) I(z\, \theta, \lambda) + j(z, \theta, \lambda)$$

This is important for "inclined" rays

Radiative transfer equation

$$\cos\theta \frac{dI(z,\theta,\lambda)}{dz} = -\chi(z,\theta,\lambda)I(z\,\theta,\lambda) + j(z,\theta,\lambda)$$

- The intensity changes with z, there are sources and sinks and they don't have to balance out.
- The derivative is w.r.t. z, so we can solve this for one direction and wavelength at the time.
- Emissivity (emission coefficient) stands alone.
- Opacity (absorption) coefficient, stands with the intensity, why?
- This is a boundary value problem, if I provide an incident intensity on one "side", and all opacities and emissivities, we can solve this equation.

Solving radiative transfer equation

$$\cos\theta \frac{dI(z,\theta,\lambda)}{dz} = -\chi(z,\theta,\lambda)I(z\,\theta,\lambda) + j(z,\theta,\lambda)$$

- We will, for simplicity, assume θ=0, and for brevity, omit dependence on angle (or, if you want, assume we fixed it)
- Then we will divide both sides with - χ

Source function

$$\frac{I_\lambda}{\tau_\lambda} = I_\lambda - S_\lambda$$

Optical depth – our new spatial coordinate.

Let's discuss units a bit

$$\frac{dI(z,\theta,\lambda)}{dz\ \cos\theta} = -\chi(z,\theta,\lambda)I(z,\theta,\lambda) + j(z,\theta,\lambda)$$

$$\frac{dI_\lambda}{d\tau_\lambda} = S_\lambda - I_\lambda$$

Everything changes with wavelength!

We are familiar with this one :-)

$$I = \left[\frac{\mathrm{J}}{\mathrm{m^2\,s\,\mathring{A}\,srad}}\right]$$

$$\chi = \left[\frac{1}{\mathrm{m}}\right]$$

Opacity, "one per meter", or, inverse free path: More opacity, smaller distances the photons travel.

$$j = \left[\frac{\mathrm{J}}{\mathrm{m^3\,s\,\mathring{A}\,srad}}\right]$$

Emissivity: Intensity per meter. We can already feel it is proportional with number density.

Solving RTE

$$\frac{I_\lambda}{\tau_\lambda} = I_\lambda - S_\lambda$$

- If we know opacity and emissivity everywhere, we can go from z to optical depth and we can calculate source function. Then this differential equation is a textbook example. **Formal solution:**

Total optical depth of the atmosphere at chosen wavelength

Incident intensity, boundary condition

t – dummy variable

$$I_\lambda^+ = I_\lambda^0 e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t) e^{-t} dt$$

Emergent intensity, what we need

Source function

Solving RTE

- The emergent intensity is the weighted contribution of: a) Incident intensity b) Source function at various depths/heights in the atmosphere.
- The weigthing factors depend on the wavelength (so does the source function)
- This dependency dictates what heights are important in the formation of which wavelength.
- “H alpha is formed in the chromosphere”
- “Neutral iron lines probe photospheric conditions”
- “ALMA is a good chromospheric thermometer”
- “Coronal lines are formed in optically thin regime, so it is hard. “
- There is a lot to unpack in these statements. Hopefully we can make sense of them.

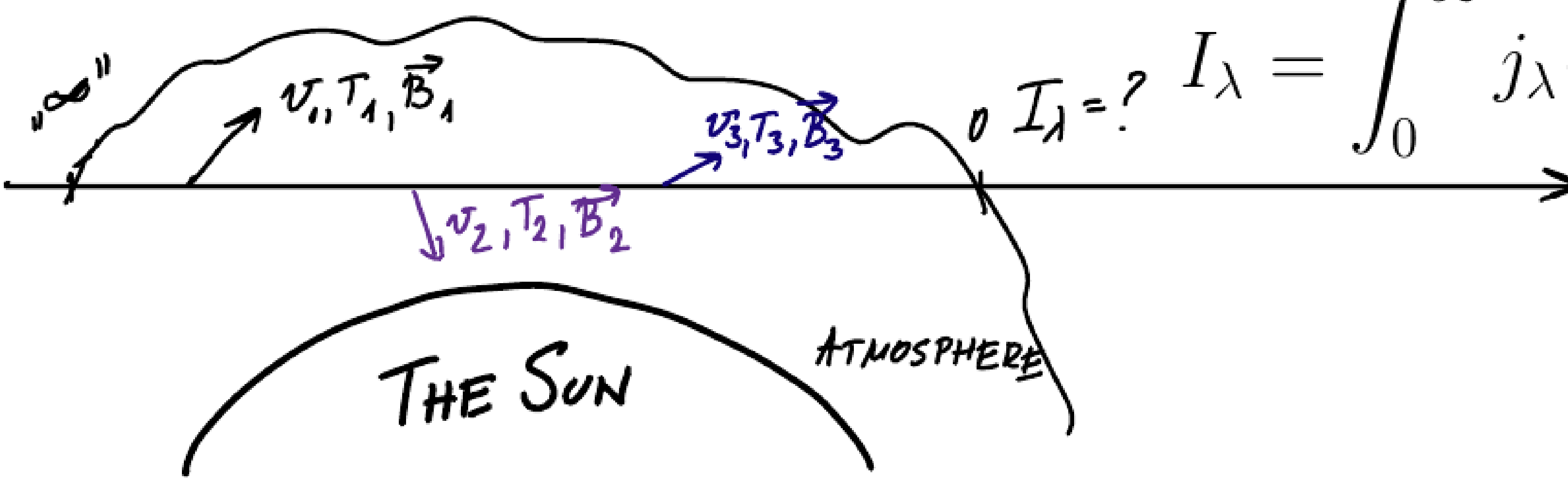
Simplest possible solutions:

- No emissivity, cold, absorbing slab of gas (this is how people modeled spectral lines in the super early days)

$$I_\lambda = I_\lambda^0 e^{-\tau_\lambda}$$

- Simple attenuation.

- No absorption, just “optically thin” emission (obviously we can’t use opacity).



$$I_\lambda = \int_0^\infty j_\lambda(s)ds$$

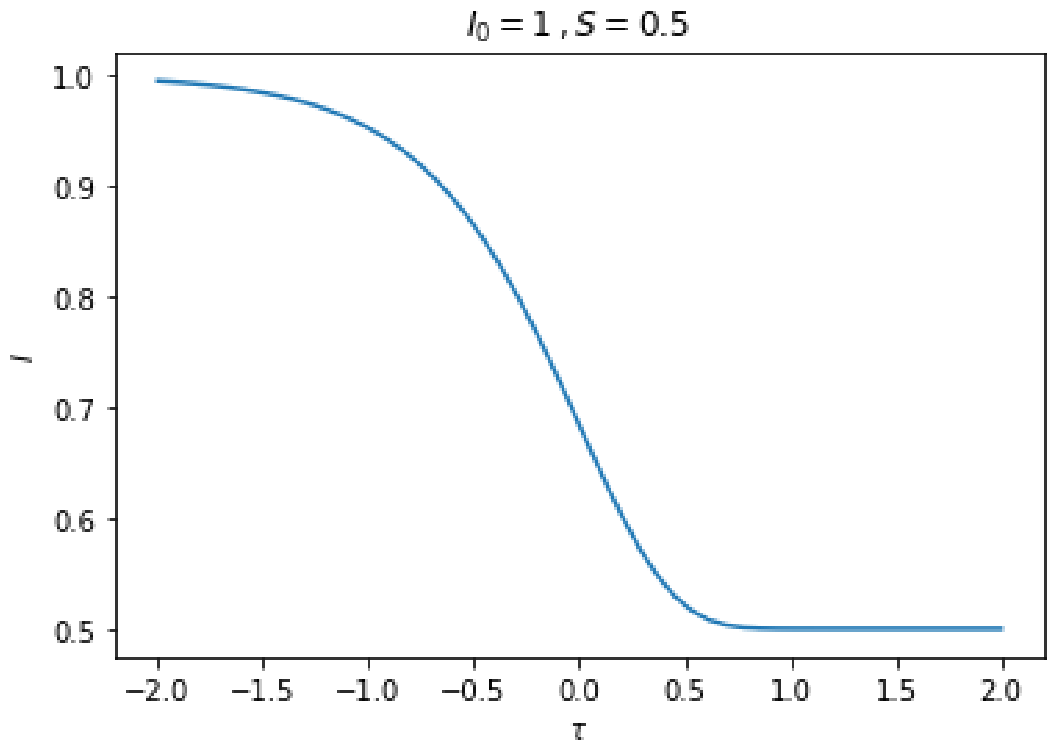
Constant source function

$$I = I_0 e^{-\tau} + \int_0^{\tau} S(t) e^{-t} dt$$

$$I = I_0 e^{-\tau} + S(1 - e^{-\tau})$$

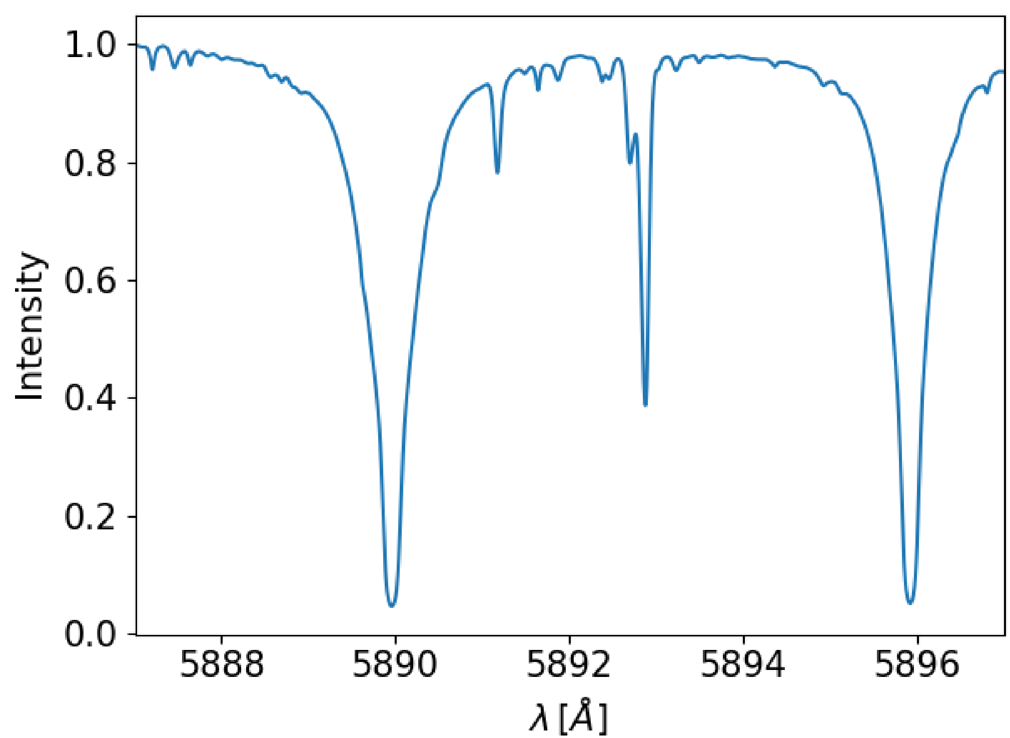
Emergent intensity for a span of optical depth ranges. (From 0.01, to 100, note the log scale).

At first, source function is not important, later, it becomes the sole contributor to the emergent intensity.



Notes

- Opacity changes over wavelengths much more dramatically than the source function.
- Most wavelength dependency comes from **the spectral lines.** When we do spectropolarimetry we focus on few Angstroms at the time. The only processes that change dramatically with the wavelength on such small scales are spectral lines.



Let's try and model a spectral line.

- We now need to focus on a few wavelengths at a time
- We will Assume that wavelength dependent optical depth is some **referent optical depth** times some factor that describes spectral line.

Referent optical depth

$$I_\lambda^+ = I_\lambda^0 e^{-\tau\phi_\lambda} + S(1 - e^{-\tau\phi_\lambda})$$

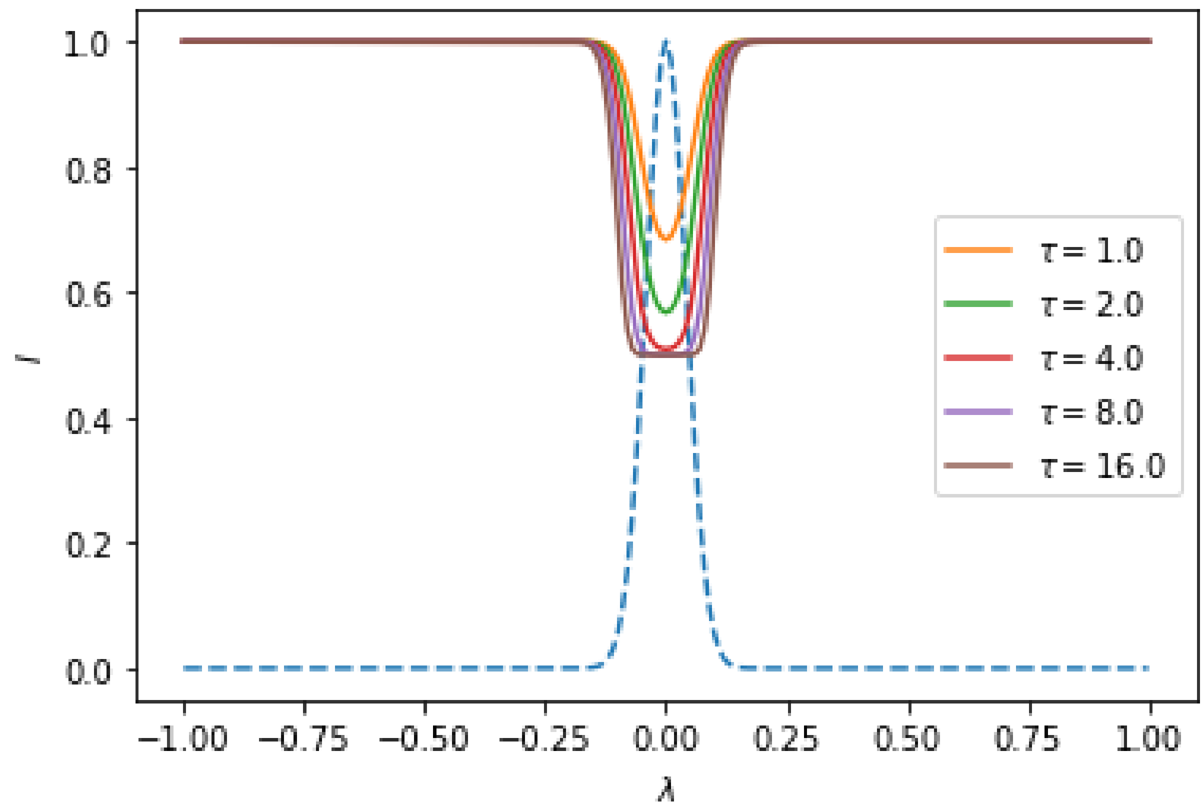
"Line profile"

Now we are interested in wavelength distribution of the emergent intensity!

Let's try and model a spectral line

$$I_\lambda^+ = I_\lambda^0 e^{-\tau\phi_\lambda} + S(1 - e^{-\tau\phi_\lambda})$$

- The line core intensity is set by the value of S
- The continuum is set by incident intensity
- Line never reaches zero!
- Line "saturates"



Source function linear with optical depth

- Assume that the source function increases linearly with the wavelength dependent optical depth and that lower boundary is in infinity:

$$S = a\tau_\lambda + b$$

$$I_\lambda^+ = \int_0^\infty (a + b\tau_\lambda) e^{-\tau_\lambda} d\tau_\lambda$$

$$I_\lambda^+ = a + b = S(\tau_\lambda = 1)$$

- **We “see the source function” coming from optical depth unity. Optical depth unity is different at different wavelengths, so we see different stuff.**

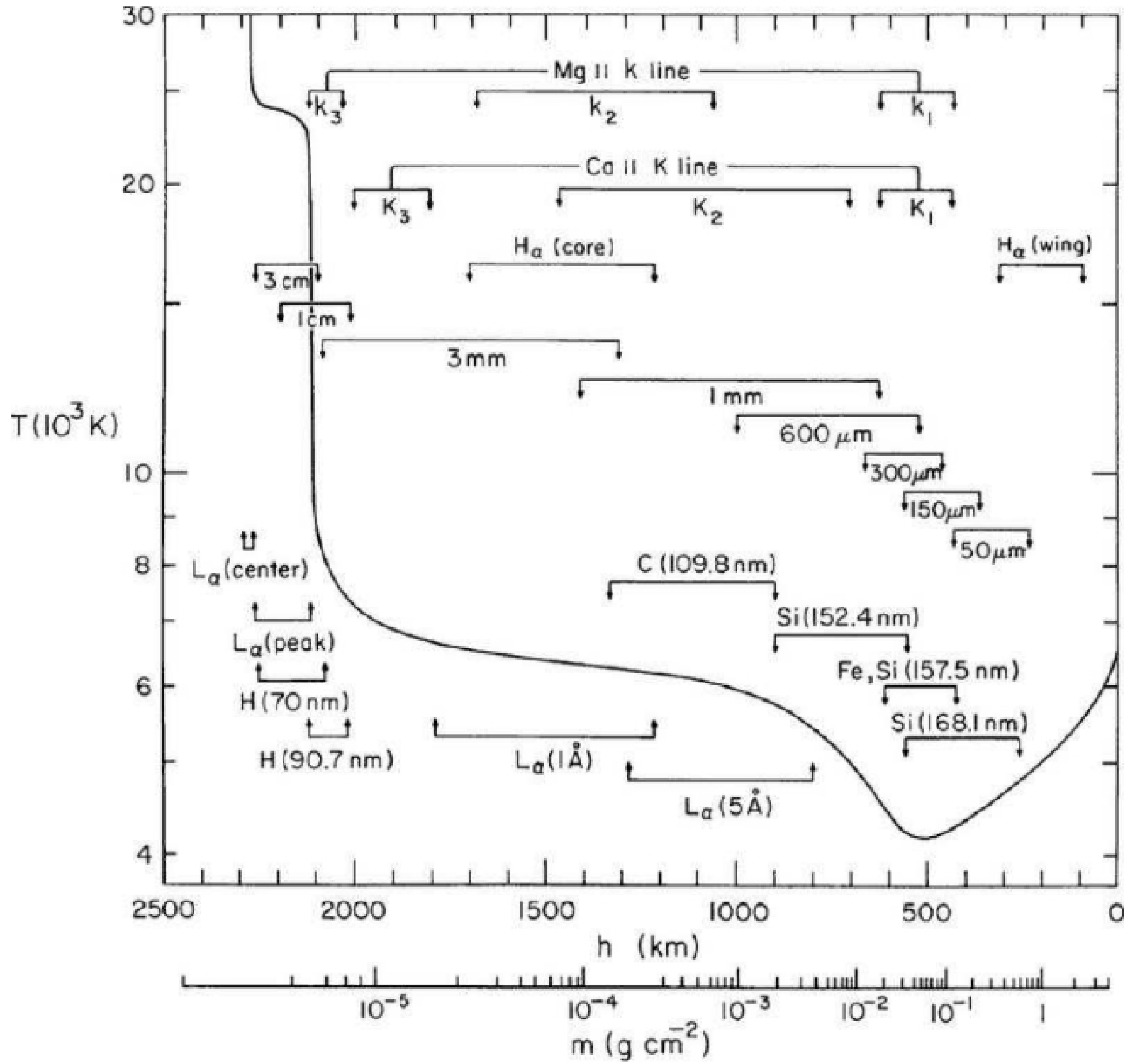
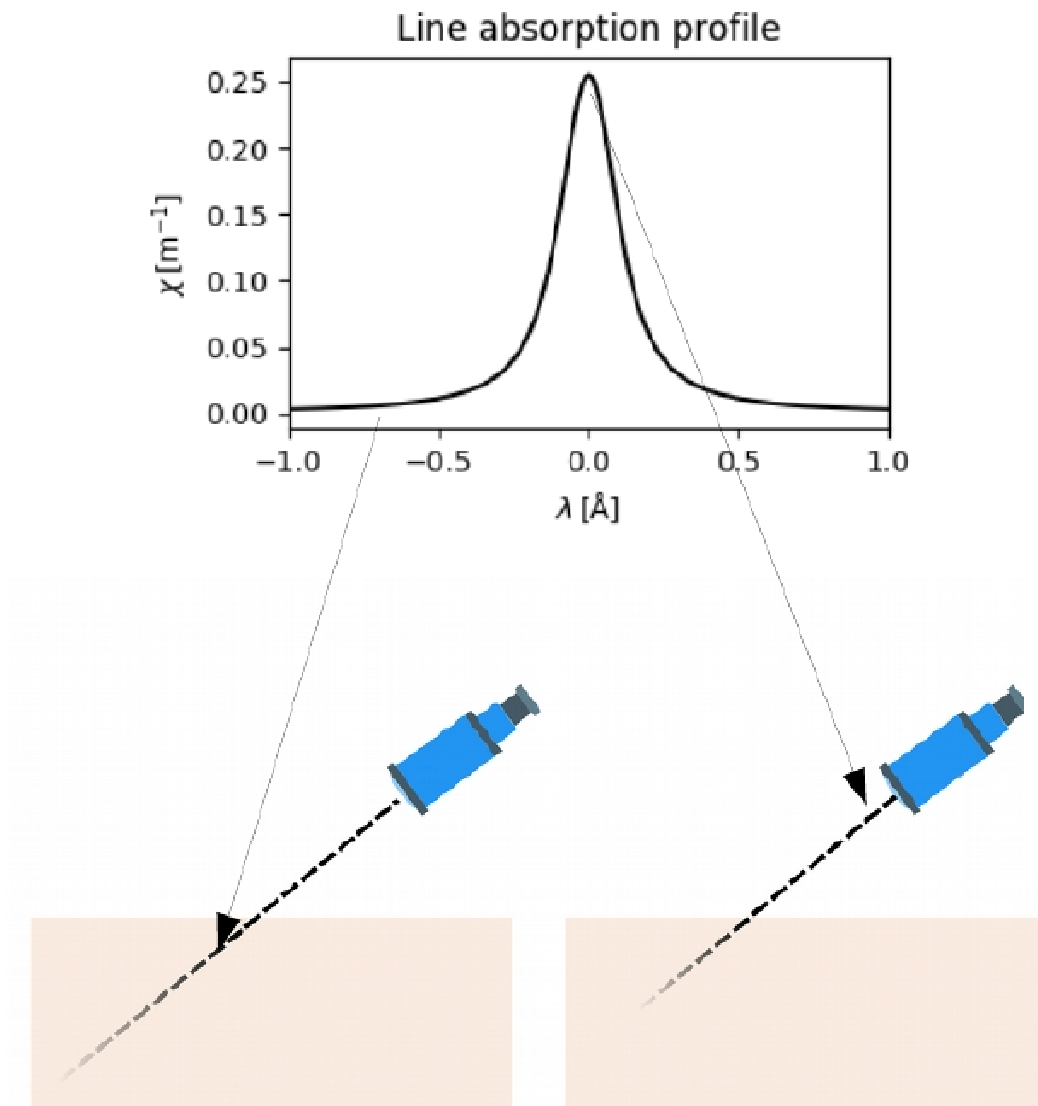
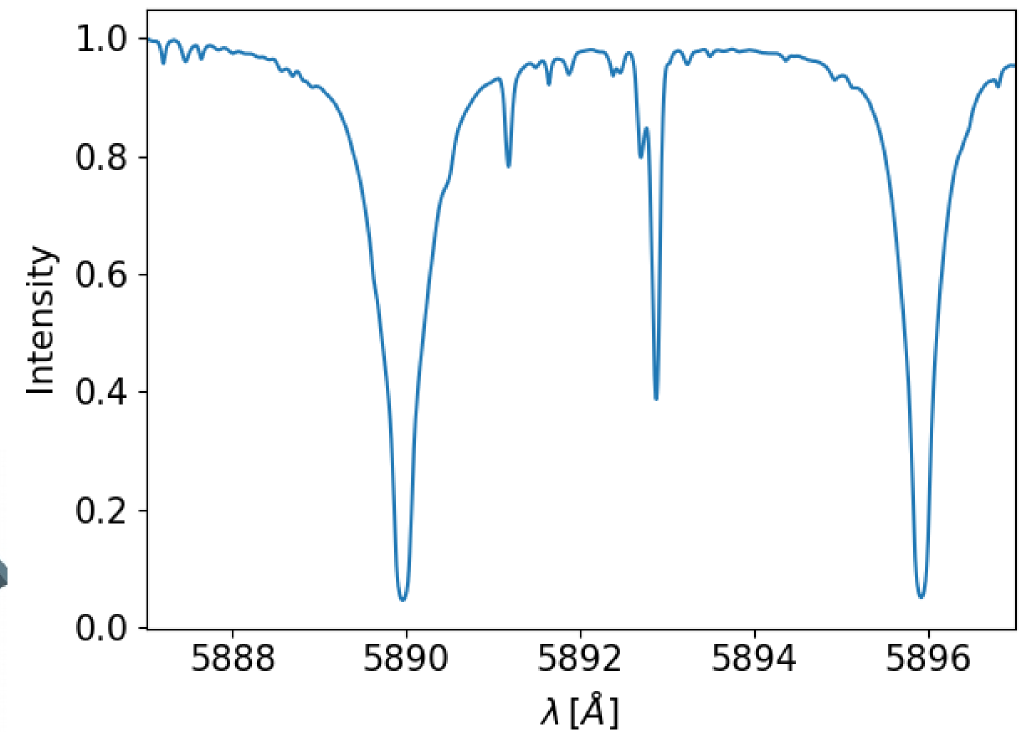


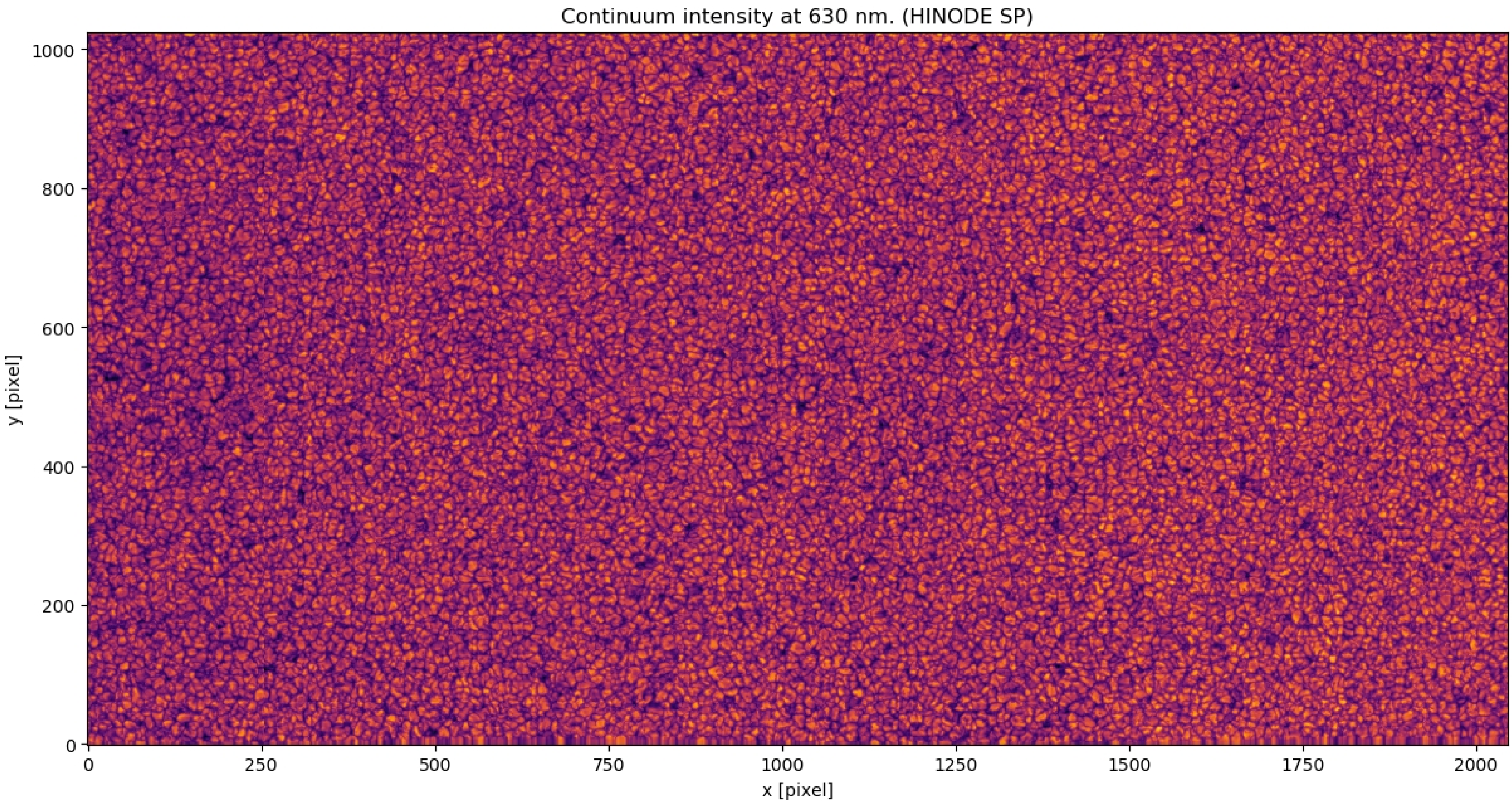
Figure 2: Semi-empirical VALC model atmosphere. From Vernazza et al. 1981

Spectral lines probe a range of heights

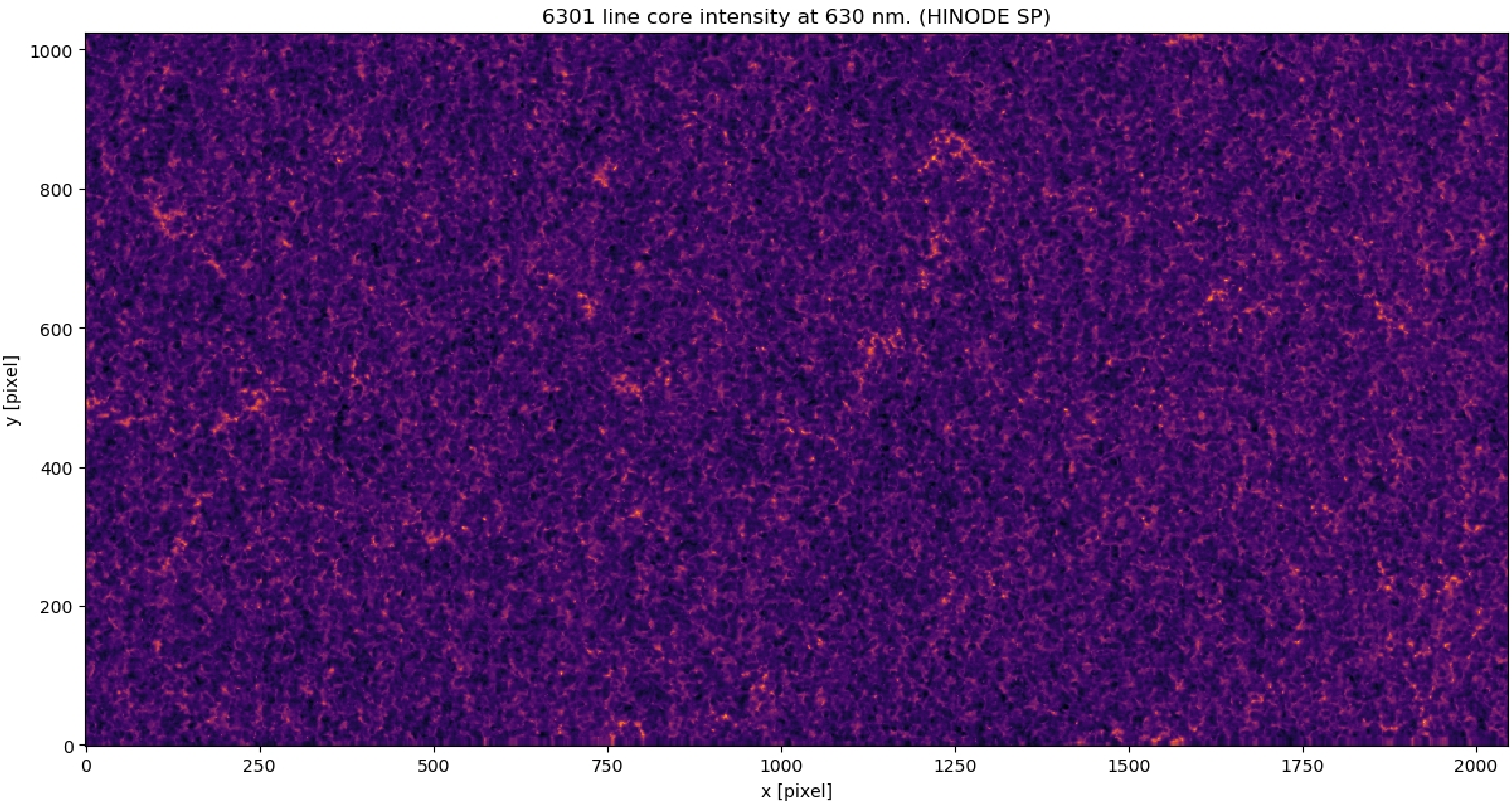




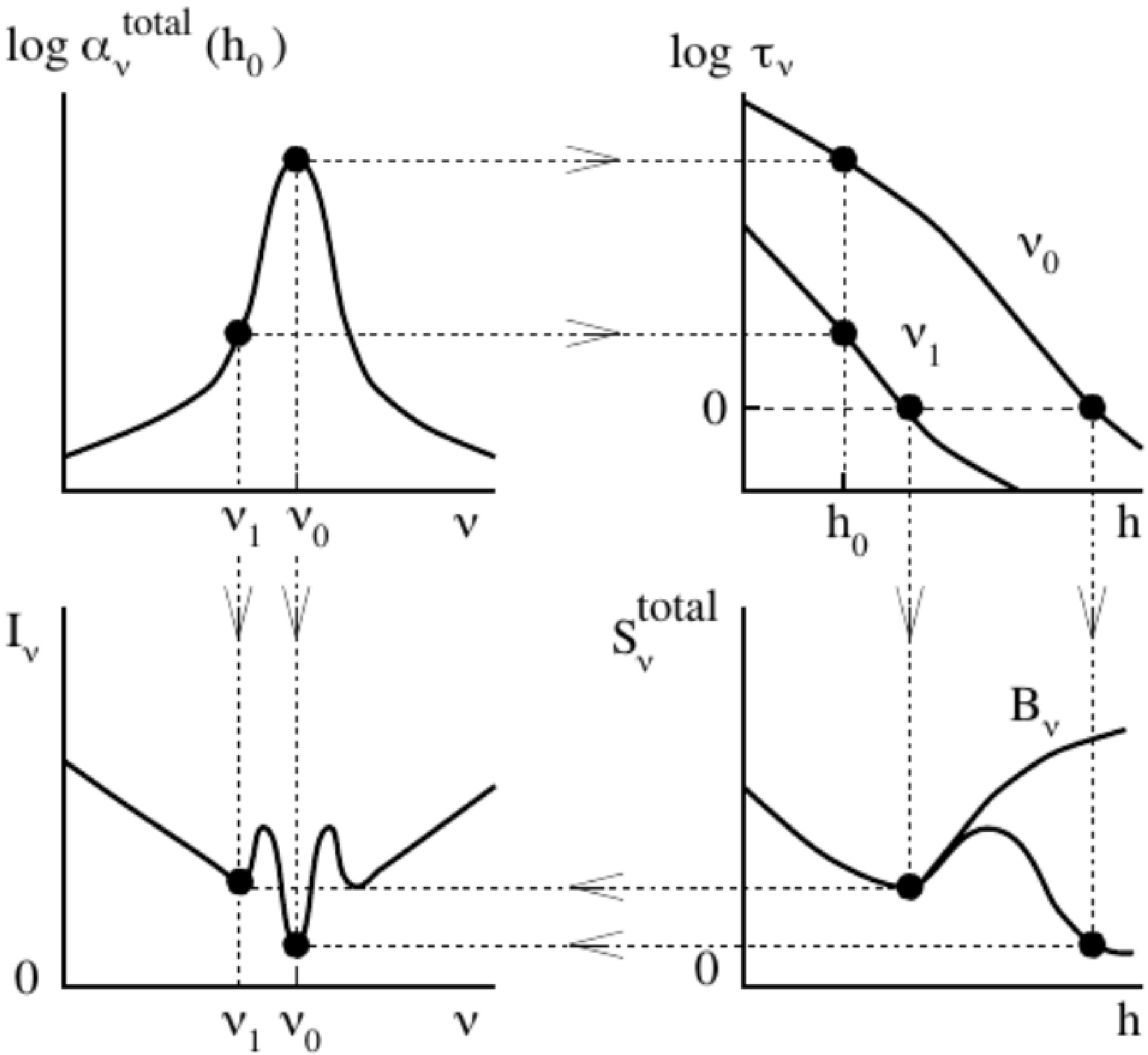
Different wavelengths – different depths



Different wavelengths – different depths

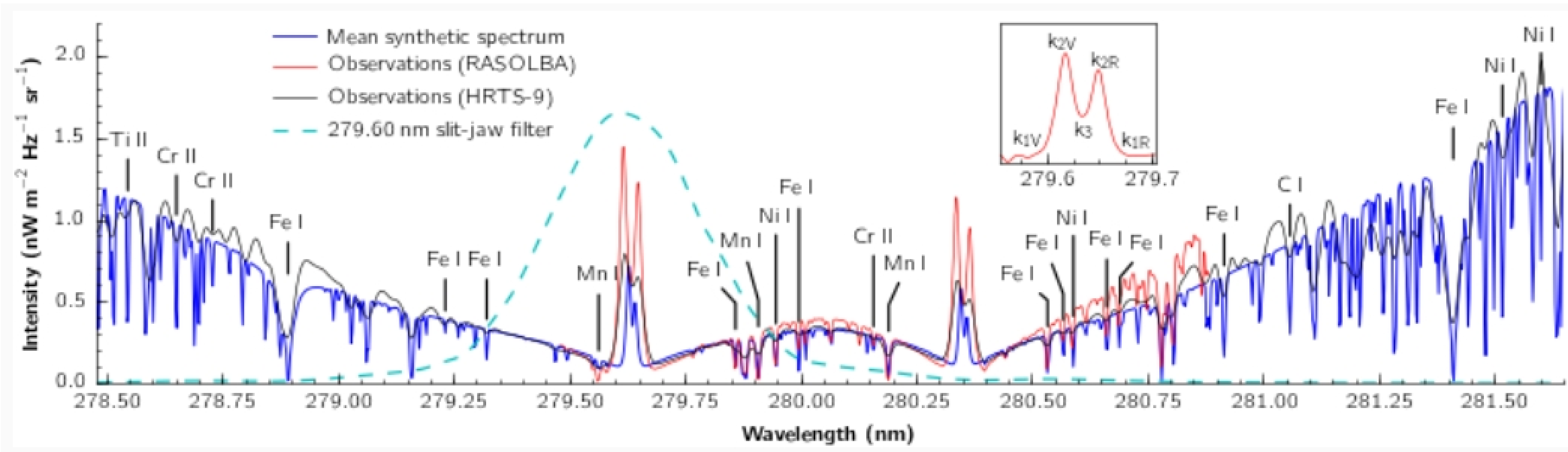


Sometimes the situation gets really wild:



- From “Radiative Transfer in Stellar Atmospheres” by Rob Rutten (freely available pdf!)
- Don’t pay attention to this notation, pay attention to what I am saying!

This does happen in the real Sun:



Summary

- Emergent intensity is the solution of the radiative transfer equation.
- To solve the radiative transfer equation we integrate the source function on a optical depth scale, with appropriate weighting. From the top of the atmosphere to the infinity.
- This all changes with wavelength.
- One simple rule: At given wavelength, intensity is equal to the source function at optical depth unity **at that wavelength.**
- Different spectral lines (different parts of a spectral line!), different opacity $\rightarrow$ different optical depth unity $\rightarrow$ different temperature $\rightarrow$ different source function $\rightarrow$ different emergent intensity $\rightarrow$ Sun looks differently.