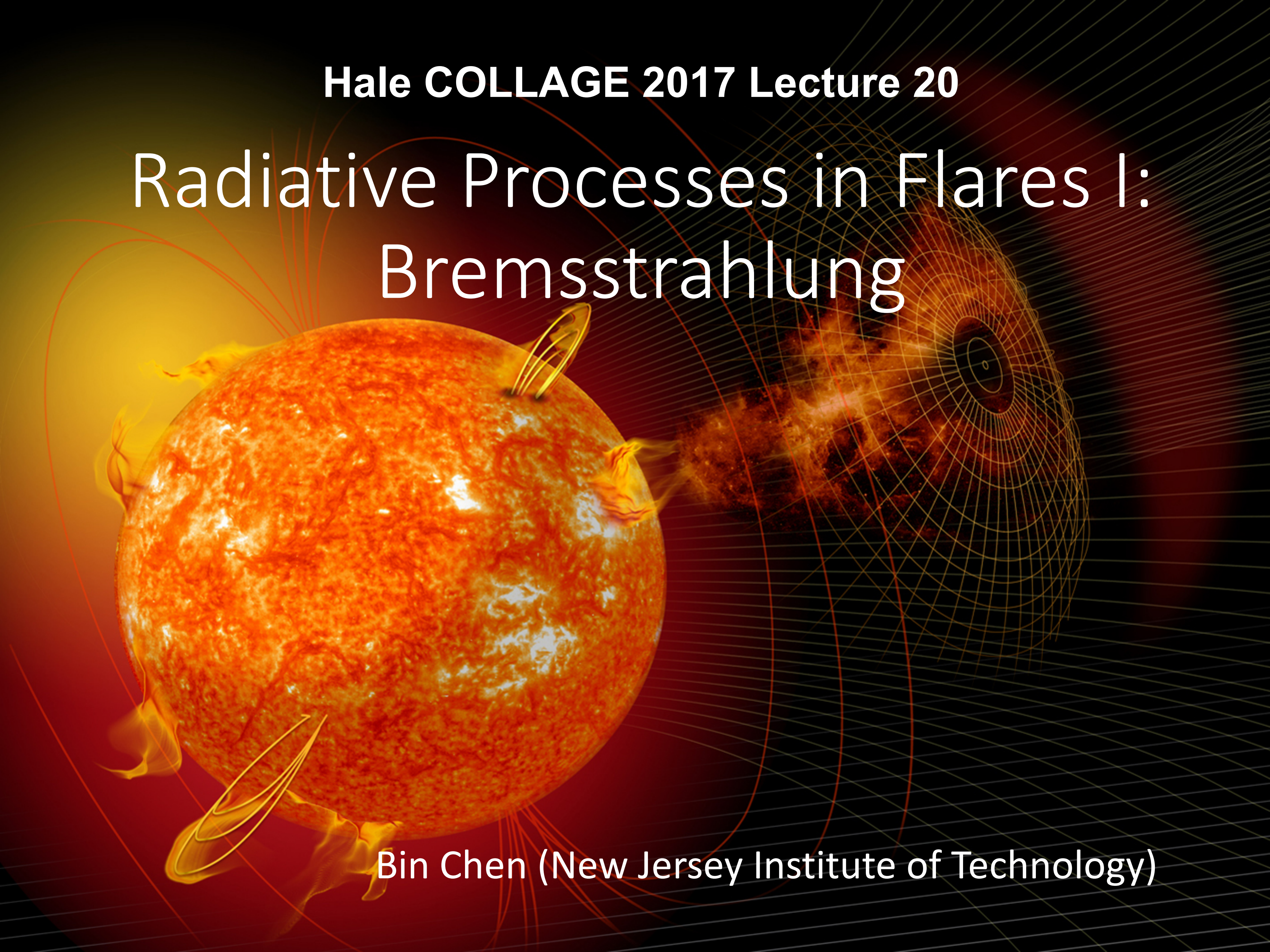
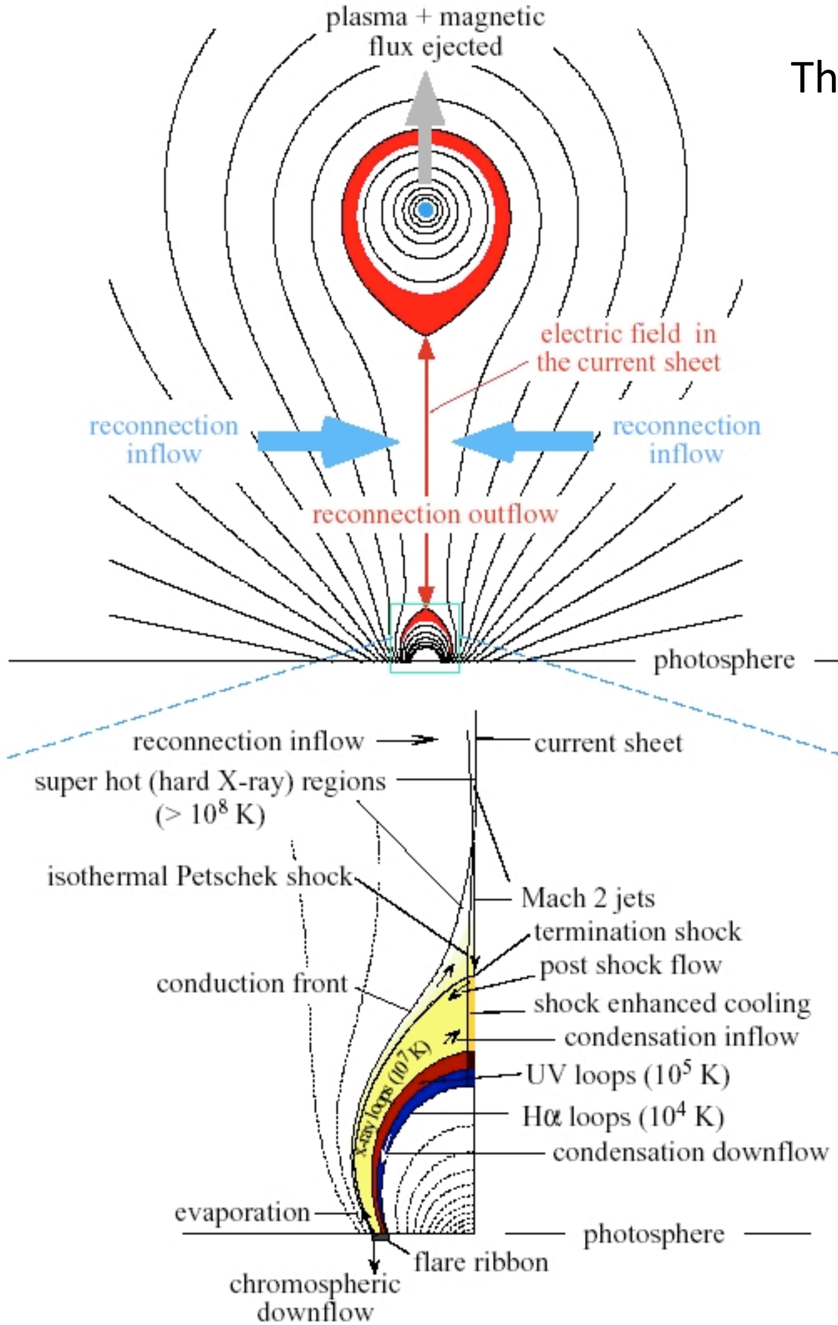
Hale COLLAGE 2017 Lecture 20

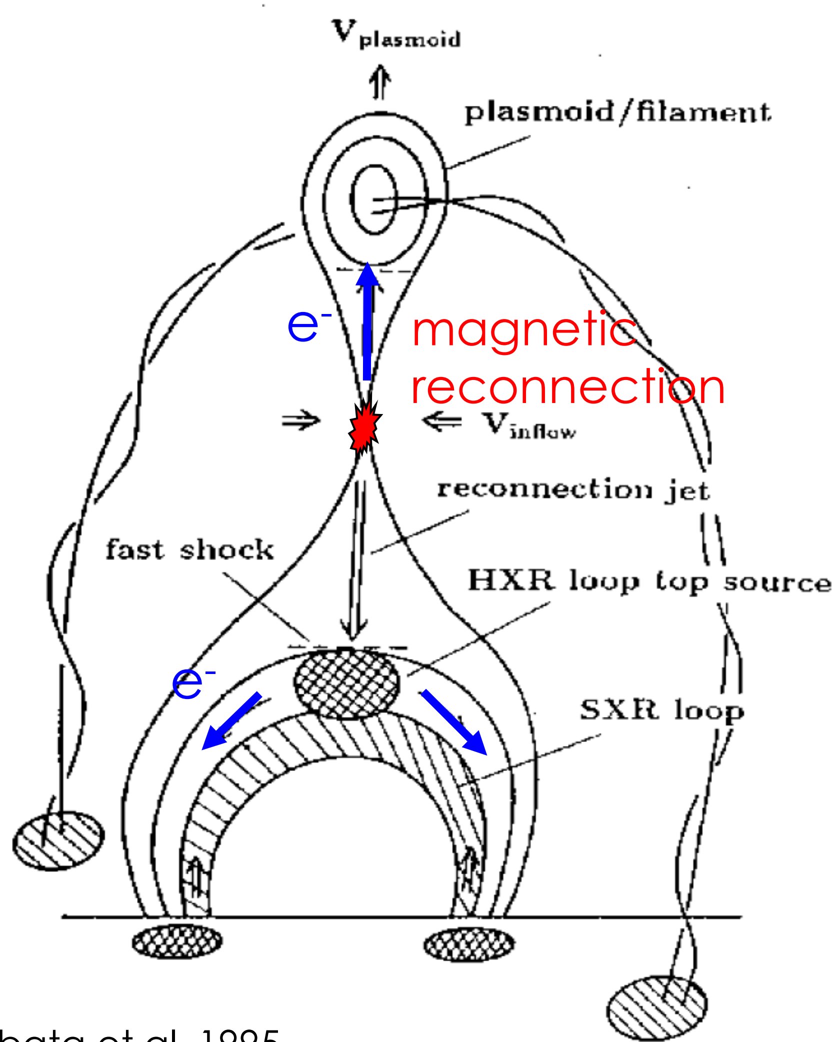
Radiative Processes in Flares I: Bremsstrahlung



Bin Chen (New Jersey Institute of Technology)

The standard flare model





Shibata et al. 1995

1) Magnetic reconnection and energy release
2) Particle acceleration and heating
3) Chromospheric evaporation and loop heating

Previous lectures

Following lectures:
How to diagnose the accelerated particles?

- What?
- Where? ➡ How?
- When?

Outline

- Introduction
- Radiation from energetic particles
    - Bremsstrahlung → this lecture
    - Gyrosynchrotron
    - Other radiative processes (time permitting)
        - Coherent emission
        - Inverse Compton
        - Nuclear processes
- Suggested reading: Ch. 5 of Tandberg-Hanssen & Emslie

Thermal and non-thermal radiation

- Refer to the distribution function of source particles $f(E)$ (# of particles per unit energy per unit volume)
- Radiation from a Maxwellian particle distribution is referred to as **thermal** radiation
- Radiation from a non-Maxwellian particle distribution is referred to as **nonthermal** radiation
  - In flare physics, the nonthermal population we consider usually has much larger energies than that of the thermal "background"
- Example of a nonthermal distribution function:

$f(E)dE = Cn_eE^{-\delta}dE$, where C is the normalization factor to make $\int_0^\infty f(E)dE = n_e$ → "power law"

*See Lecture 17 (by Prof. Longcope) for details on the distribution function

Radiation from an accelerated charge

Larmor formula: $$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\mathbf{a}^2 \sin^2\theta \qquad P = \frac{2q^2}{3c^3}\mathbf{a}^2$$

Relativistic Larmor formula:

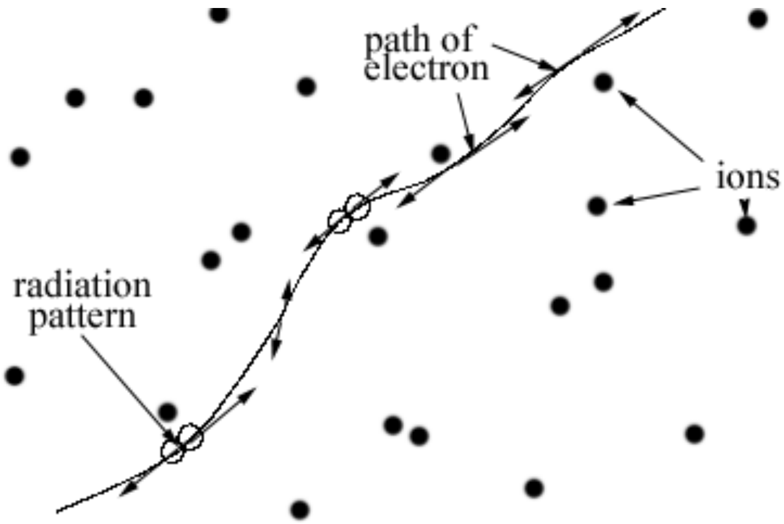
$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\frac{(a_\perp^2 + \gamma^2 a_\parallel^2)}{(1-\beta\cos\theta)^4}\sin^2\theta$$

$$P = \frac{2q^2}{3c^3}\gamma^4(a_\perp{}^2 + \gamma^2 a_\parallel^2)$$

Radio and HXR/gammy-ray emission in flares:

- Acceleration experienced in the Coulomb field: **bremsstrahlung**
- Acceleration experienced in a magnetic field: **gyromagnetic radiation**

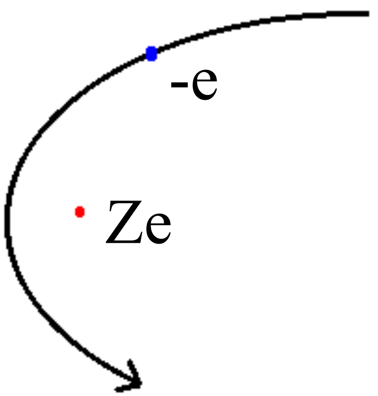
Electron-ion bremsstrahlung



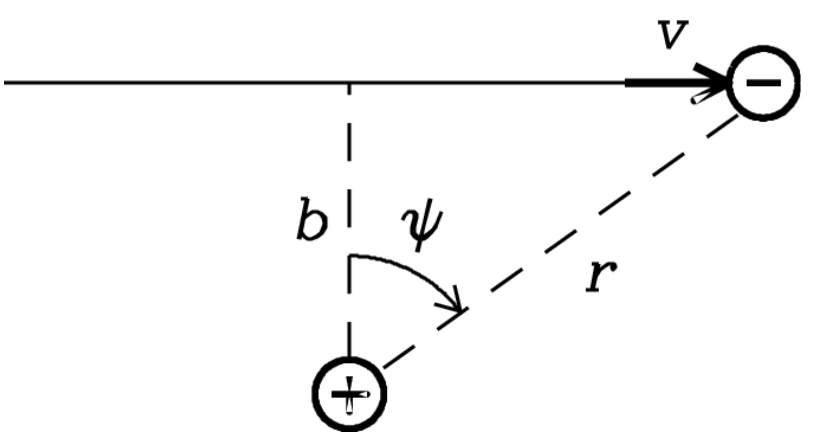
- e-i bremsstrahlung is relevant to quiet Sun, flares, and CMEs

Bremsstrahlung

- Each electron-ion interaction generates a single pulse of radiation

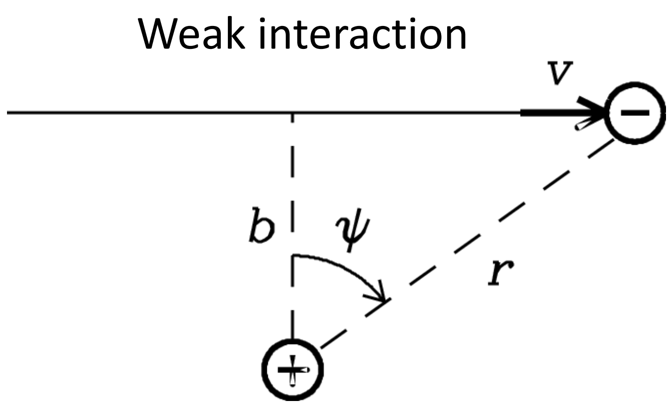


Strong interaction



Weak interaction

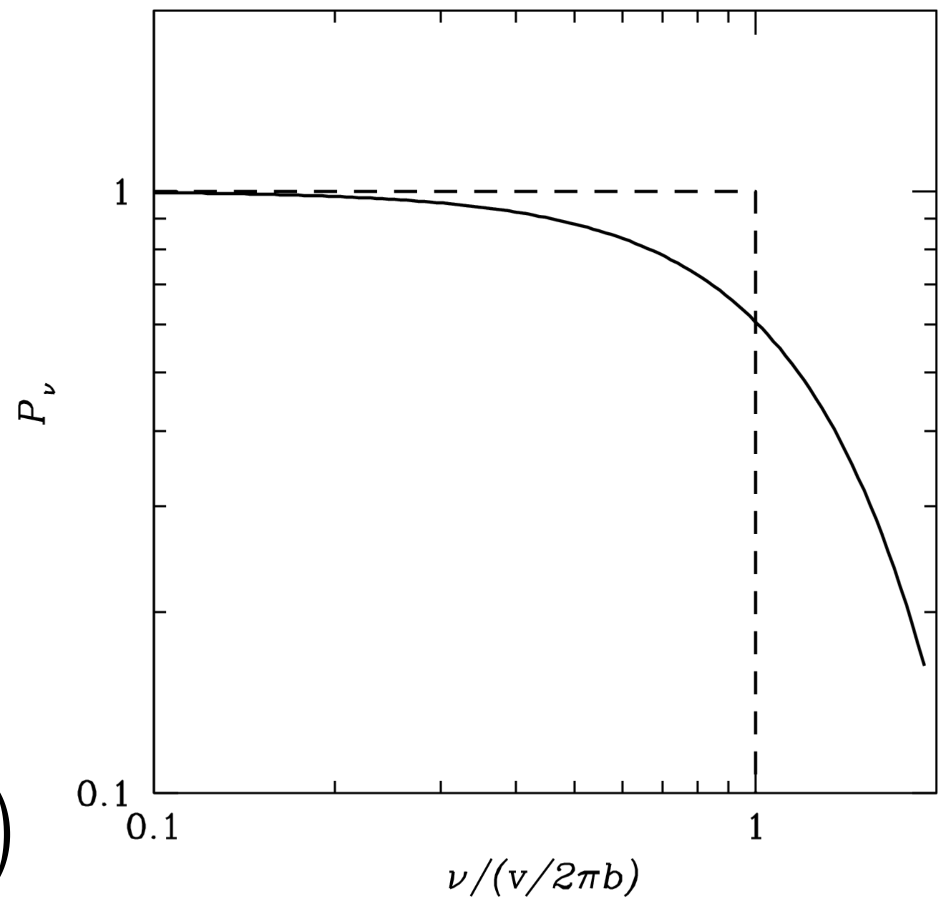
Power spectrum of a single interaction



Single pulse duration $\tau \sim b/v$

Emitted energy

$$W_\nu \approx \frac{\pi^2}{2}\frac{Z^2 e^6}{c^3 m_e^2}\left(\frac{1}{b^2 v^2}\right)$$



A note on the impact parameter

- Maximum value b_{max}
  - ❖ Debye Length $\lambda_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = v_{th}/\omega_{pe}$, Where v_{th} is the thermal speed and $\omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}}$ is the plasma frequency

    Why this length scale sets b_{max}?
  - ❖ $b_{max} \approx \frac{v}{\omega}$, where ω is the observing frequency
- Minimum value b_{min}
  - ❖ $b_{min} \approx \frac{Ze^2}{m_e v^2}$, given by maximum possible momentum change of the electron $\Delta p_e = 2p_{e0}$
  - ❖ $b_{min} = \frac{\hbar}{m_e v}$, from uncertainty principle

Bremsstrahlung emissivity

- # of electrons passing by any ion within $(b, b+db)$ and $(v, v+dv)$

$$n_{\rm e}\ (2\pi b\, db)\ \ v f(v)\ \ dv$$

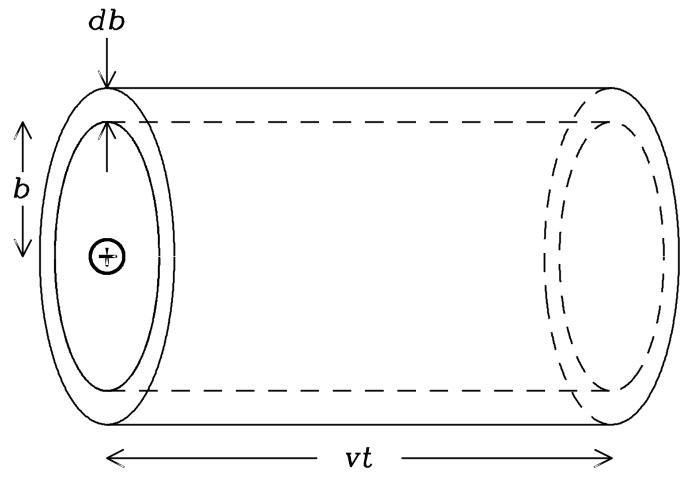
- # of collisions per unit volume per unit time

$$\dot{n}_{\rm c}(v,b) = (2\pi b)\, v f(v)\ \ n_{\rm e} n_{\rm i}$$

- Spectral power emitted at ν

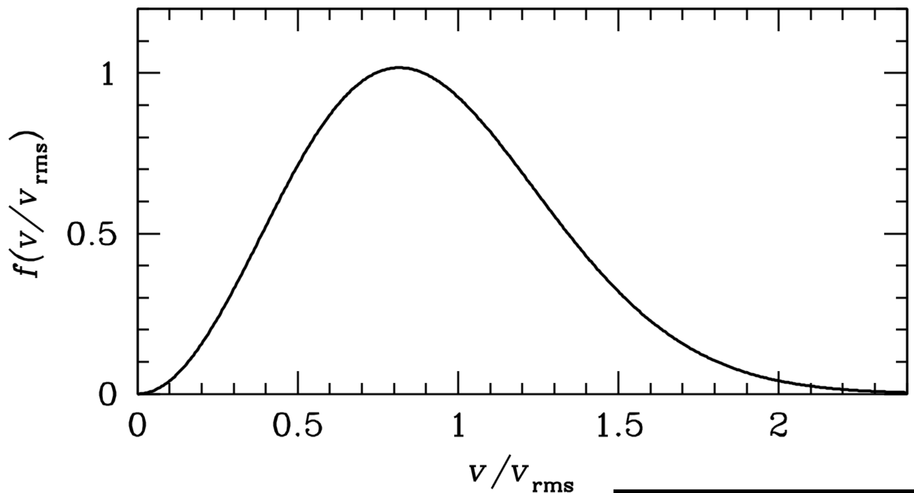
$$4\pi j_\nu = \int_{b=0}^{\infty}\int_{v=0}^{\infty} W_\nu(v,b)\,\dot{n}_{\rm c}(v,b)\,dv\,db.$$

$$4\pi j_\nu = \frac{\pi^3 Z^2 e^6 n_{\rm e} n_{\rm i}}{c^3 m_{\rm e}^2}\int_{v=0}^{\infty}\frac{f(v)}{v}dv\int_{b_{\min}}^{b_{\max}}\frac{db}{b}$$



Thermal bremsstrahlung

- Distribution function of the electron $f(v)$ is Maxwellian



$$f(v) = \frac{4v^2}{\sqrt{\pi}} \left(\frac{m_e}{2kT} \right)^{3/2} \exp\left(-\frac{m_e v^2}{2kT} \right).$$

Emission coefficient:

$$j_\nu = \frac{\pi^2 Z^2 e^6 n_e n_i}{4c^3 m_e^2} \left(\frac{2m_e}{\pi kT} \right)^{1/2} \ln\left(\frac{b_{max}}{b_{min}} \right).$$

Thermal bremsstrahlung

- Absorption coefficient:

$$\kappa = \frac{j_\nu}{B_\nu\,(T)}$$

- Let's first consider radio wavelengths. In the Rayleigh-Jeans regime $B_\nu(T) = \frac{2kT\nu^2}{c^2}$, so

$$\kappa = \frac{1}{\nu^2 T^{3/2}}\left[\frac{Z^2 e^6}{c} n_\mathrm{e} n_\mathrm{i} \frac{1}{\sqrt{2\pi (m_\mathrm{e} k)^3}}\right] \frac{\pi^2}{4} \ln\left(\frac{b_\mathrm{max}}{b_\mathrm{min}}\right).$$

$$\kappa\,(\nu) \propto \nu^{-2.1}$$

Thermal bremsstrahlung opacity

- Considering a spherical cow…

$$\tau = -\int_{\mathrm{los}} \kappa\, ds \propto \int \frac{n_{\mathrm{e}} n_{\mathrm{i}}}{\nu^{2.1} T^{3/2}} ds \approx \int \frac{n_{\mathrm{e}}^2}{\nu^{2.1} T^{3/2}} ds.$$

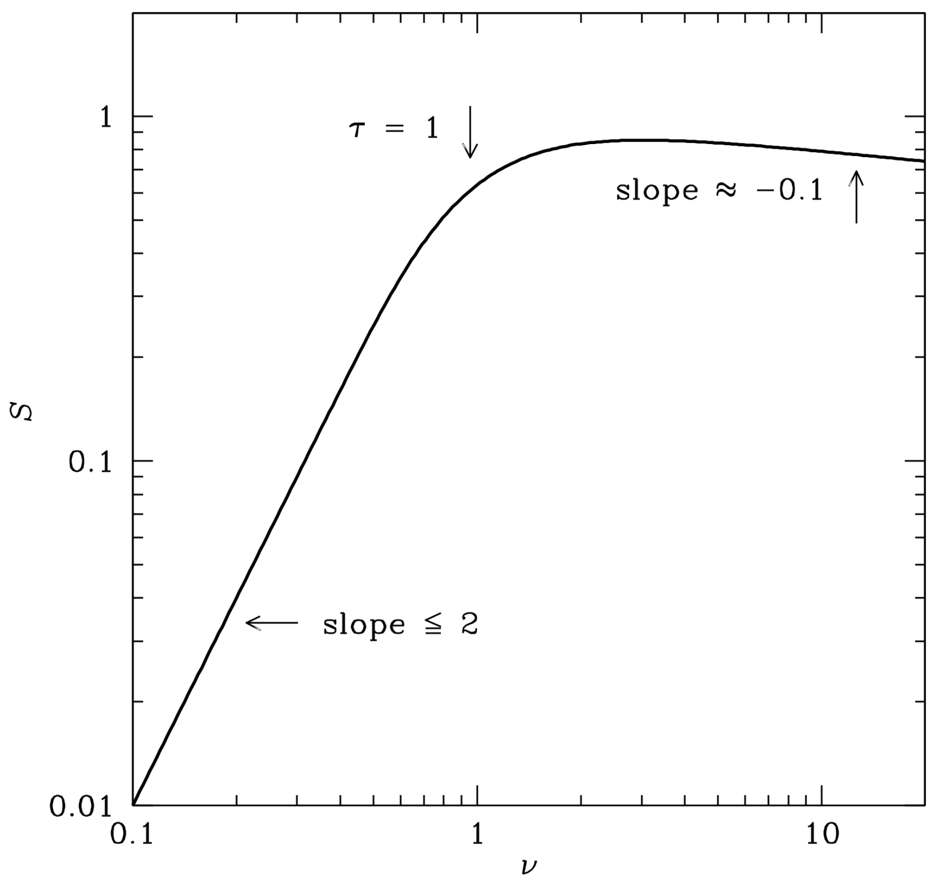
- At low frequencies, $\tau \gg 1$, optically thick:

$$S \approx B_\nu(T) = \frac{2kT\nu^2}{c^2} \propto \nu^2$$

- At high frequencies, $\tau \ll 1$, optically thin:

$$S \propto \frac{2kT\nu^2}{c^2} \tau(\nu) \propto \nu^{-0.1}$$

Thermal bremsstrahlung spectrum in radio



Emission measure

- Optical depth is essential in thermal bremsstrahlung

$$\tau = -\int_{\mathrm{los}} \kappa \, ds \propto \int \frac{n_{\mathrm{e}} n_{\mathrm{i}}}{\nu^{2.1} T^{3/2}} ds \approx \int \frac{n_{\mathrm{e}}^2}{\nu^{2.1} T^{3/2}} ds.$$

- Assuming constant T

$$\tau \approx \nu^{-2.1} T^{-1.5} (n_e^2 L)$$

Column emission measure EM_c

Recap from Lecture 7: Radiative Transfer Equation

- Brightness temperature

$$I_\nu = B_\nu(T_B) = \frac{2\nu^2}{c^2} kT_B$$

- Effective temperature

$$S_\nu = \frac{2\nu^2}{c^2} kT_{eff}$$

- Using our definitions of brightness temperature and effective temperature, the transfer equation can be rewritten

$$\frac{dT_B}{d\tau_\nu} = -T_B + T_{eff}$$

  - Optically thick source, $\tau_\nu \gg 1, T_B \approx T_{eff}$
  - Optically thin source, $\tau_\nu \ll 1, T_B \approx \tau_\nu T_{eff}$

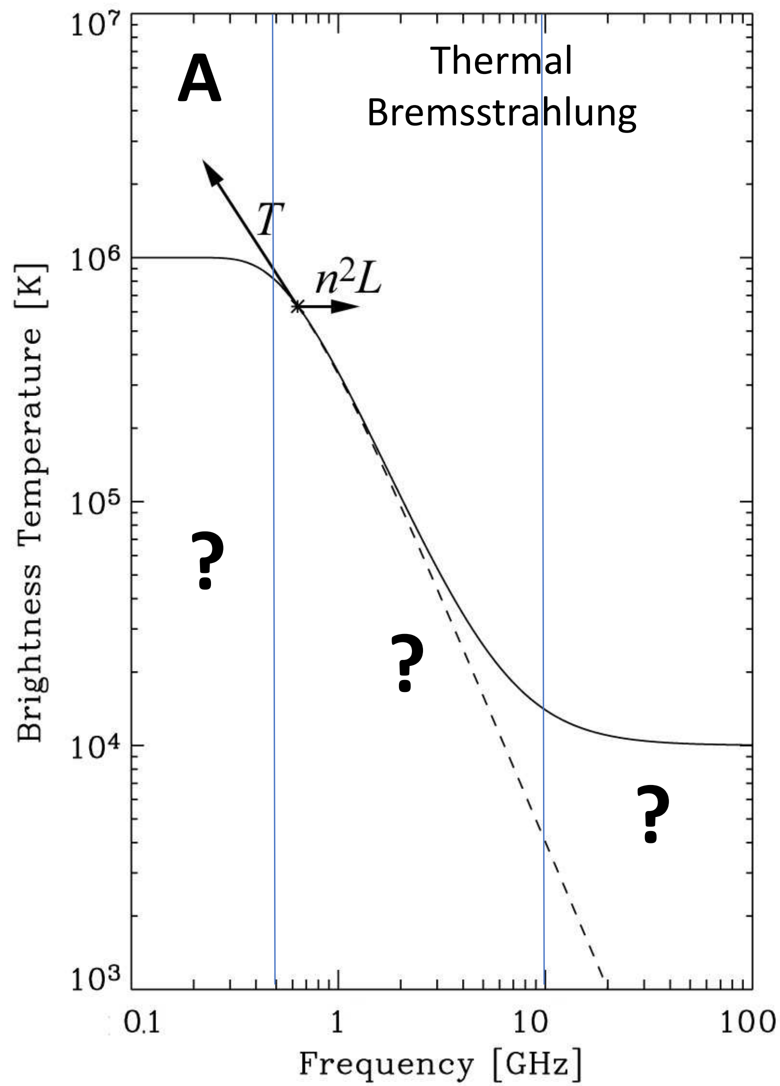
T_b spectrum

- Optically thin regime

$$T_b = T(1 - e^{-\tau}) \approx T\tau$$
$$\approx \nu^{-2.1} T^{-0.5} EM_c$$

- Optically thick

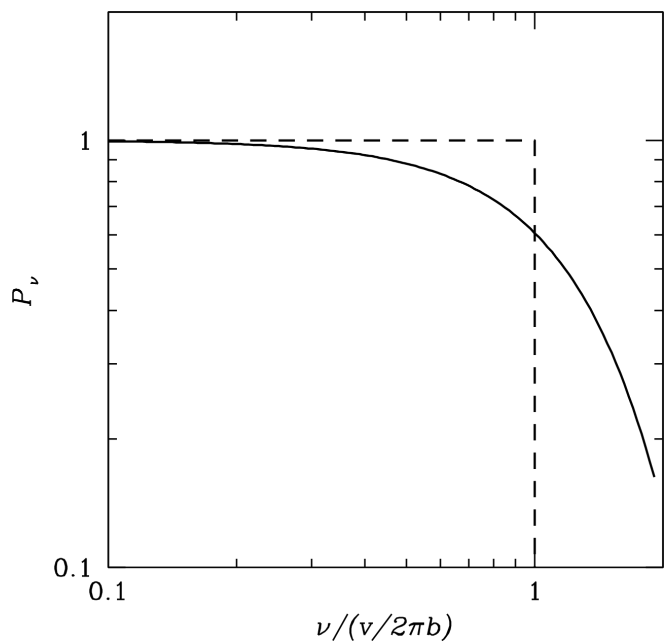
$$T_b = T(1 - e^{-\tau}) \approx \mathrm{T}$$

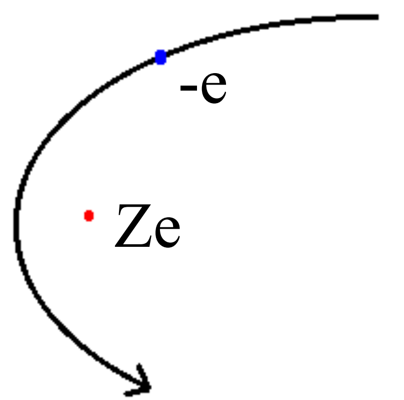


From Problem Set #2

Bremsstrahlung for X-rays

Single pulse duration $\tau \sim b/v$





Strong interaction

- Higher electron energy → larger v
- Closer encounters → smaller b

Introducing the *Cross Section*

- The **cross section** σ_B is defined as if the radiation all comes from the impact within an area around a target ion
- # of electrons that encounter a single target in dt (assume all e have the same speed and emit a photon at the same wavelength):

$$n_e \sigma_B v dt$$

- # of photons produced per unit volume per unit time:

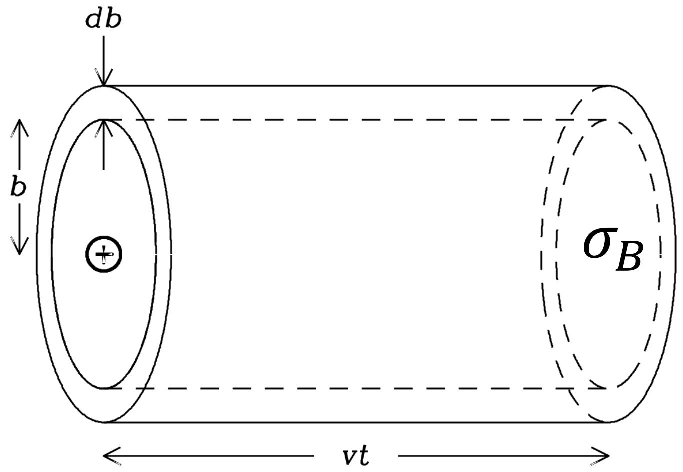
$$n_e n_i \sigma_B v$$

- Photon flux at Earth (cm^{-2} s^{-1} per unit energy) if the incident electron population remains roughly unchanged, or a **"thin target"** scenario:

$$n_e n_i \sigma_B v V_s / (4\pi R^2) \qquad R = 1\ AU$$

$$\approx \frac{\sigma_B v S_s \int n_e n_i dl}{4\pi R^2}$$

$$\approx \sigma_B v S_s / (4\pi R^2) EM_c$$



Differential Cross Section

In fact, σ_B depends on:

- Incident electron energy E
- Outgoing photon energy ϵ,
- Outgoing photon direction Ω

We need a ***differential cross section:***
$d^2\sigma_B/dEd\Omega$, written as $\sigma_B(\epsilon, E, \Omega)$

Bremsstrahlung cross section

- Bremsstrahlung from weak interactions

$$4\pi j_\nu = \frac{\pi^3 Z^2 e^6 n_e n_i}{c^3 m_e^2} \int_{v=0}^{\infty} \frac{f(v)}{v} dv \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

- For close encounters, $\sigma_B(\varepsilon, E, \Omega)$ is much more complicated (quantum physics). In the non-relativistic case, a direction-integrated cross section

$$\sigma_{\rm NRBH}(\epsilon, E) = \frac{\sigma_0 \overline{Z^2}}{\epsilon E} \ln \frac{1 + (1 - \epsilon/E)^{1/2}}{1 - (1 - \epsilon/E)^{1/2}} \quad {\rm cm}^2\ {\rm keV}^{-1}$$

where $\sigma_0 = 7.90 \times 10^{-25}\ {\rm cm}^2\ {\rm keV}$ ($\sigma_{NRBH} = 0$ for $\epsilon > E$)

Known as the Bethe-Heitler cross section

*See Koch & Motz 1959 for full relativistic, angle and polarization dependent cross section

Thin target bremsstrahlung

- With a differential cross section $\sigma_B(\varepsilon, E)$
- Taking into account electron distribution $f(E)$
- The directional integrated ***thin-target bremsstrahlung flux*** $F(\epsilon)$ (photons cm^{-2} s^{-1} per unit energy) becomes

$$F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E=\epsilon}^{\infty} f(E) v(E) \sigma_B(\epsilon, E) dE$$

Where $N_i = \int n_i dl$ is the column density of the target

Thick target bremsstrahlung

- Incident electrons are completely stopped, or thermalized in the source → requires high density.
  - ❖ Usually occurs when energetic electrons precipitating onto the chromosphere
- Much quicker energy loss from electrons → lots of X-ray photons emitted → Produces intense X-ray emission
- Usually dominates the hard X-ray (~10 keV – 300 keV) spectrum

Thick target bremsstrahlung

- Electrons change their energy in time (quickly)

$$F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int f(E) v(E) \sigma_B(\epsilon, E) dE$$

Time and space dependent

- Instead we need to have

$$F(\epsilon) = \frac{S_s}{4\pi R^2} \int_{E_0=\epsilon}^{\infty} f(E_0) v(E_0) m(\epsilon, E_0) dE_0$$

where

$$m(\epsilon, E_0) = \int_{t_1(E=E_0)}^{t_2(E=\epsilon)} n_i\big(l(t)\big) \sigma_B\big(\epsilon, E(t)\big) v\big(E(t)\big) dt$$

is the number of photons at energy ϵ emitted per unit energy by an electron of initial energy E_0

Thick target bremsstrahlung

- We need something to describe $E(t)$
- Q: what is the main mechanism for electron energy loss?
- Energy loss mainly due to e-e Coulomb collisions. We need another cross section to describe dE/dt

→ the Rutherford cross section:

$$\sigma_e = \frac{C}{E^2} \approx 10^{-17}\mathrm{cm}^2 \times \left(\frac{E}{\mathrm{keV}}\right)^{-2}, \text{ where } C = 2\pi e^4 \ln \Lambda$$

So $$\frac{dE}{dt} = -\sigma_e(E) n_i v(E) E$$

Thick target bremsstrahlung

- Photon flux

$$F(\epsilon) = \frac{S_s}{4\pi R^2 C} \int_{E_0=\epsilon}^{\infty} f(E_0) v(E_0) \left(\int_{\epsilon}^{E_0} E \sigma_B(\epsilon, E) dE \right) dE_0$$

- Comparing to the thin-target case

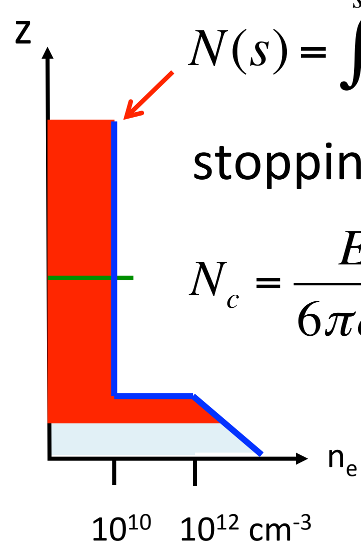
$$F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E=\epsilon}^{\infty} f(E) v(E) \sigma_B(\epsilon, E) dE$$

- Effective column density

$$N_{eff}(\epsilon, E_0) = \frac{1}{C\sigma_B(\epsilon, E_0)} \int_{\epsilon}^{E_0} E\, \sigma_B(\epsilon, E) dE$$

$$\propto E^2$$

Similar to the stopping column N_c!



$$N(s) = \int^{s} n_e(s') ds' \quad [\text{cm}^{-2}]$$

stopping column: $N \sigma_e = 1$

$$N_c = \frac{E_c^2}{6\pi e^4 \Lambda} = 1.4 \times 10^{17} \text{cm}^{-2} E_{c,keV}^2$$

stop cut-off

From Lecture 10 by Prof. Longcope

Thin target vs. Thick target

- The effective column depth $N_{eff}(\epsilon, E_0)$ is independent of the target
- $N_i \ll N_{eff}(\epsilon, E_0)$: incident electron distribution is nearly unchanged → thin target
- $N_i \geq N_{eff}(\epsilon, E_0)$: substantial change in incident electron distribution → we must use the thick target expression

From X-ray spectrum to electron distribution

- $F(\epsilon)$ is what we observe (after taking out instrument response)

Thin target

$$F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E=\epsilon}^{\infty} f(E) v(E) \sigma_B(\epsilon, E) dE$$

Thick target

$$F(\epsilon) = \frac{S_s}{4\pi R^2 C} \int_{E_0=\epsilon}^{\infty} f(E_0) v(E_0) \left(\int_{\epsilon}^{E_0} E \sigma_B(\epsilon, E) dE \right) dE_0$$

- Obtaining $f(E)$ becomes an inversion problem
- Many approaches (Brown 1971 and after), but difficult to obtain an accurate $f(E)$ due to the “smoothing” effect of the integral (e.g., Craig & Brown 1985)

If we pretend to know the form of $f(E)$…

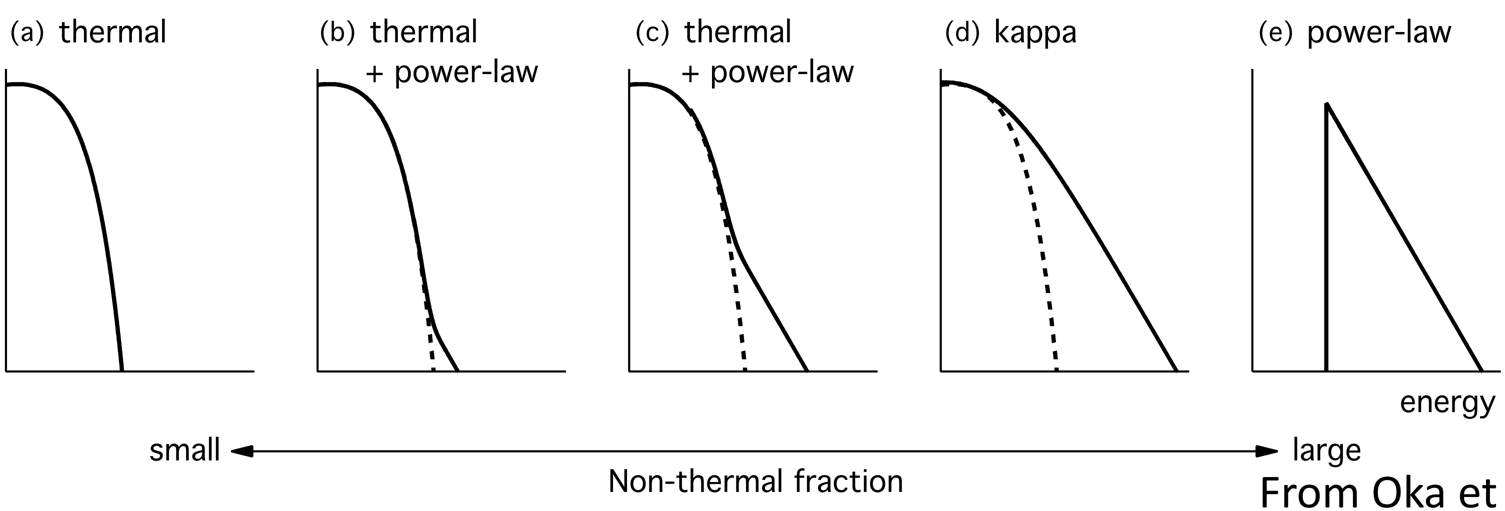
- Thermal:

Maxwellian: $f(E) = \frac{2n_e}{\pi^{1/2}(KT)^{3/2}} E^{1/2} \exp(-E/KT)$

- Nonthermal:

Power law: $f(E) = Cn_e E^{-\delta}$

Kappa: $f(E) \propto \left[1 + \frac{E}{kT(\kappa - 3/2)}\right]^{-(\kappa+1)}$

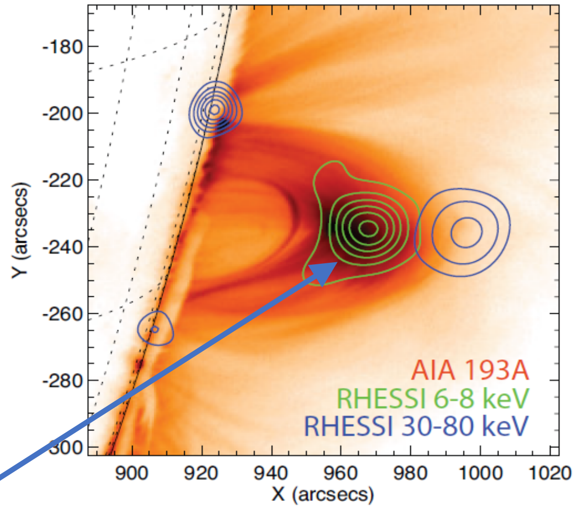


From Oka et al. 2015

Thermal bremsstrahlung

- Resulted from $f(E)$ with a Maxwellian distribution
  - Radio thermal bremsstrahlung do not require high speed electrons
  - X-ray thermal bremsstrahlung does require high speed electrons $E > \epsilon$. For 3 keV X-ray, $T_e \sim 3.5\times10^7\ K$

→ from flaring loops



From Krucker & Battaglia 2014

Question:

- Why is thermal bremsstrahlung not so important in optical and UV?

Nonthermal thin target bremsstrahlung

- Assume electron distribution is a power law:

$$f(E) = C n_e E^{-\delta}$$

- Plug in $F(\epsilon)$ for thin target, we have

$$F(\epsilon) = 2\beta K \epsilon^{-(\delta+1/2)} \int_1^\infty u^{-(\delta+1/2)} \log\left(\sqrt{u} + \sqrt{1-u}\right) du$$

$$F(\epsilon) \propto \epsilon^{-(\delta+1/2)}$$

$$u = E/\epsilon$$

- $F(\epsilon)$ usually have a power-law shape in HXRs:

$$F(\epsilon) = K\epsilon^{-\gamma}$$

If thin-target, the electron energy spectrum is

$$f(E) \propto E^{-(\gamma-1/2)}$$

* See J. Brown 1971 for details

Nonthermal thick target bremsstrahlung

- Assume electron distribution is a power law:

$$f(E) = Cn_e E^{-\delta}$$

- Plug in $F(\epsilon)$ for thick target, we have

$$F(\epsilon) \propto \epsilon^{-(\delta-1)}$$

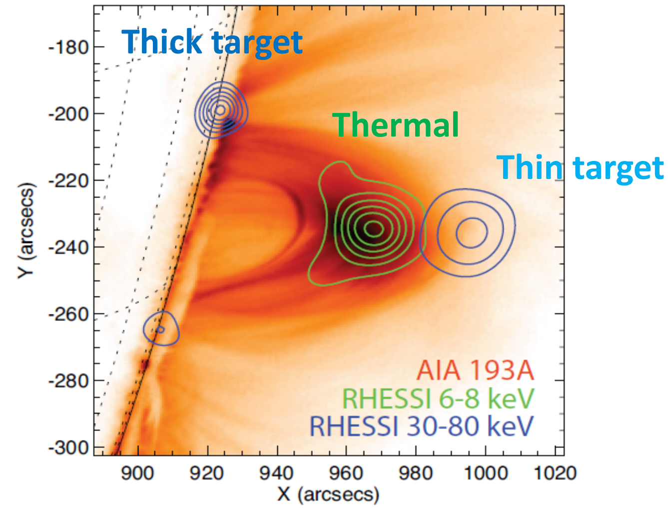
Much flatter than thin-target!

- Inverting from a power-law photon spectrum

$$f(E) \propto E^{-(\gamma+3/2)}$$

- Inferred spectral index is steeper by 2 for thick target than thin target

X-ray emission in flares

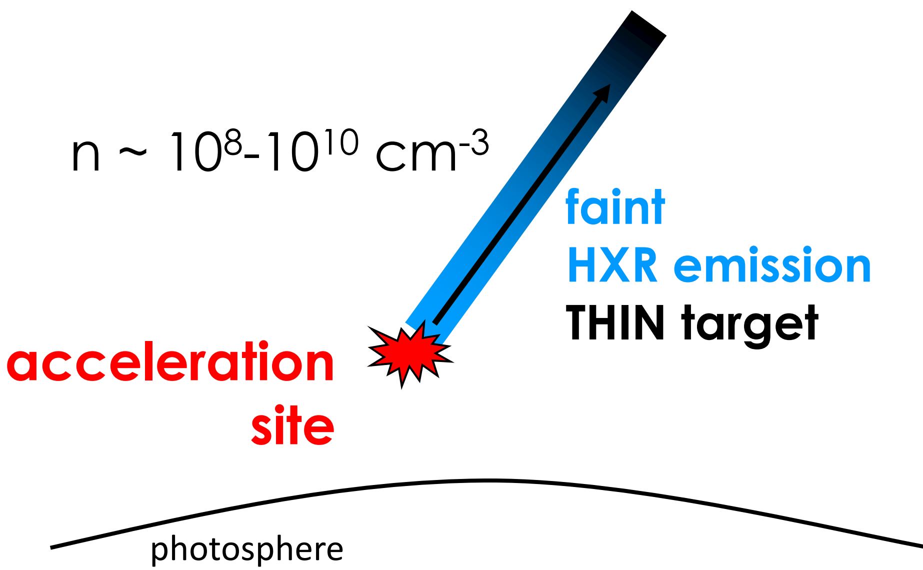


In solar corona:

low density → very few collisions

→ energy loss small (dE<<E)

→ faint X-ray emission

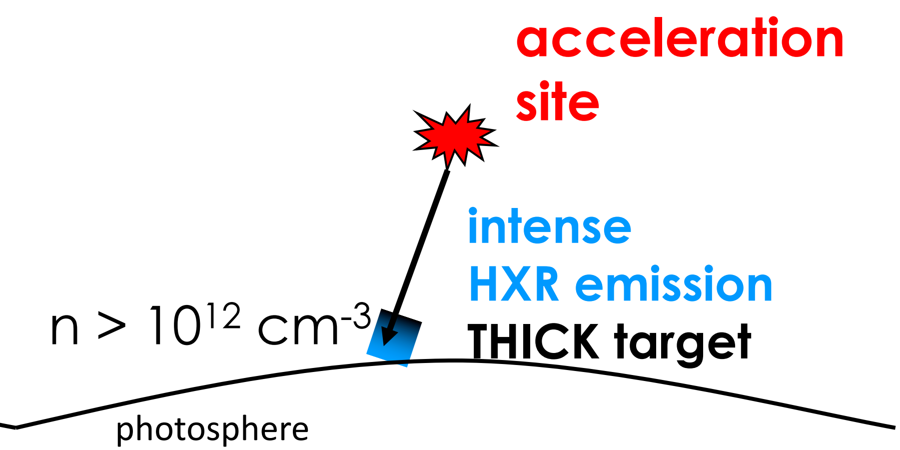


Below transition region:

high density → many collisions

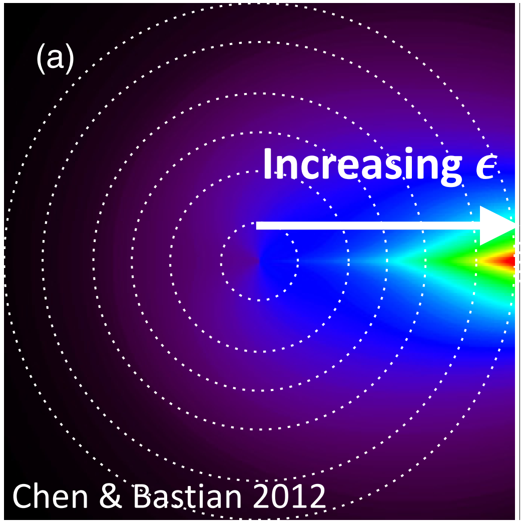
→ energy loss very fast

→ strong X-ray emission



Other bremsstrahlung contributions

Bremsstrahlung emissivity from an electron beam



- At higher (relativistic) energies, corrections for the e-i cross section must be included. Moreover, the radiation pattern becomes highly beamed
- Additional contributions need to be included
  - e-e bremsstrahlung → important at $\epsilon > 300$ keV with a flatter spectrum (Haug 1975; Kontar 2007)
  - e^+-e bremsstrahlung (Haug 1985)
  - i-e bremsstrahlung is usually insignificant (Emslie & Brown 1985; Haug 2003)

Summary

- Bremsstrahlung emission is one of the most important diagnostics for energetic electrons in flares
- Thermal bremsstrahlung: radio, X-ray
- Nonthermal bremsstrahlung: X-ray
  - Thin target → corona
  - Thick target → chromosphere (sometimes corona)
- To obtain $f(E)$, we need
  - Observation of X-ray spectrum $F(\epsilon)$ with high resolution
  - Application of the correct emission mechanism(s)
  - Appropriate inversion