Non-thermal particles

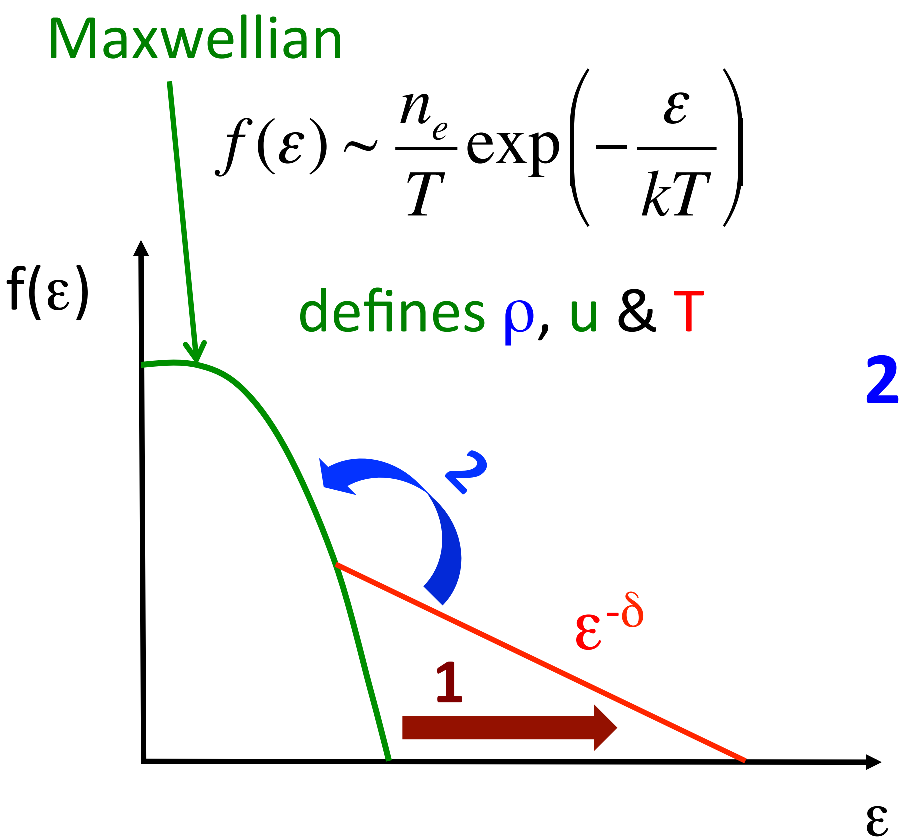
Motions of charged particles

Lecture 16

March 22, 2017

Non-thermal electrons

From Lecture 10



Electron distribution function
(complete details next lecture)

1. **Some process*** adds energy to subset of e^-s from Maxwellian — creates **NT tail**. Often a power law:

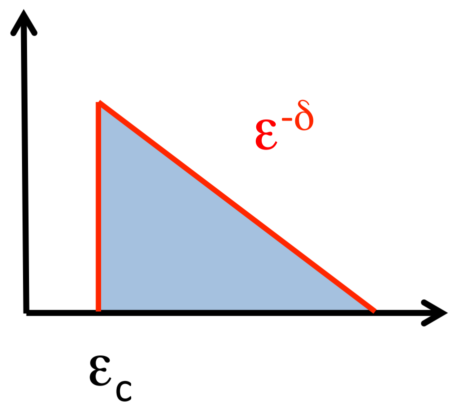
   $$f(\varepsilon) \sim \varepsilon^{-\delta}$$

2. **Collisions** return NT e^-s to Maxwellian: **thermalization.** Adds energy to Maxwellian: heating **h**

* This will be our focus **NOW**

What would we like to know?

- What is the flare acceleration process?
  What gives 10 – 1000 keV to e^-s initially w/ ε ~ 0.3 keV (T_e ~ 3 MK)?
- Why does it produce a power-law distribution of e^- energies?
  - What determines δ? ε_c?
  - How are of pitch angles distributed?
- What fraction of e^-s are accelerated?
- What fraction of released energy is given to NT e^-s?
- Are ions also accelerated?



Formulating an answer:

Motion of single charged particle

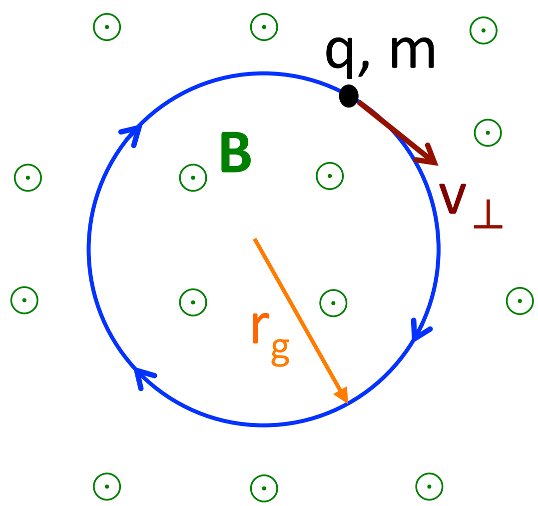
$$m\frac{d\mathbf{v}}{dt} = q\mathbf{E} + \frac{q}{c}\mathbf{v}\times\mathbf{B}$$

$$\Rightarrow \quad \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}m|\mathbf{v}|^2}_{\varepsilon}\Big) = \frac{d\varepsilon}{dt} = q\mathbf{v}\cdot\mathbf{E}$$

Use non-relativistic form –

- valid for e^-s w/ $\varepsilon < 200$ keV
- valid for protons w/ $\varepsilon < 400$ MeV

One charged particle in B



gyrofrequency:

$$\Omega_c = \frac{|q\mathbf{B}|}{mc} = 2\pi f_c$$

e⁻s: $\Omega_{ce} = 2\times10^9\,\text{rad/s}\left(\frac{|\mathbf{B}|}{100\,\text{G}}\right)$

really fast

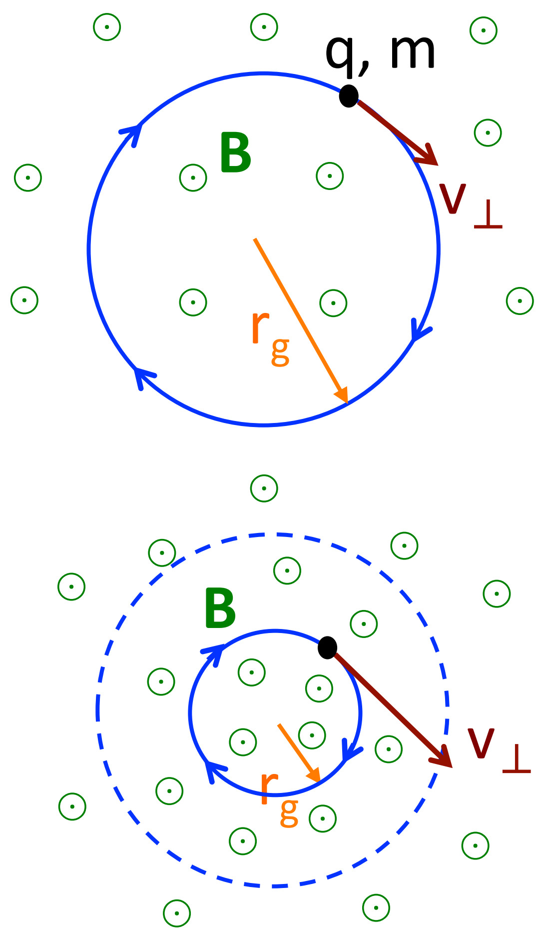
gyroradius:

$$r_g = \frac{v_\perp}{\Omega_c}$$

really small

e⁻s: $r_{ge} = 1.0\,\text{cm}\left(\frac{|\mathbf{B}|}{100\,\text{G}}\right)^{-1}\left(\frac{\varepsilon}{1\,\text{keV}}\right)^{1/2}$

One charged particle in B



Increase |**B**| slowly:

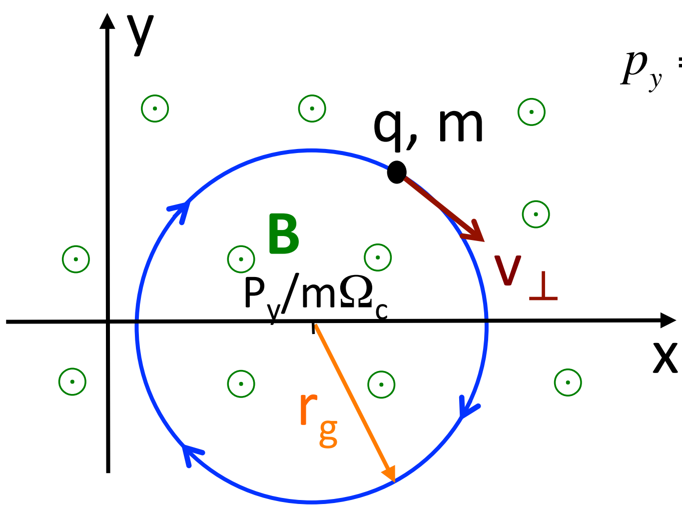
$$\tau >> \Omega_c^{-1} \sim 10^{-9}\ \mathrm{s}$$

Q: what remains fixed?

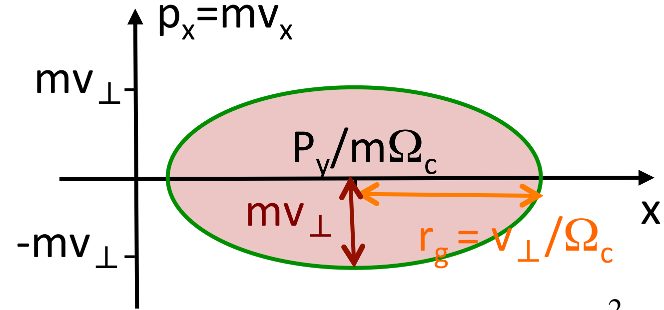
~~energy?~~

$$\frac{\partial \mathbf{B}}{\partial t} \neq 0 \Rightarrow \mathbf{E} \neq 0$$

One charged particle in B

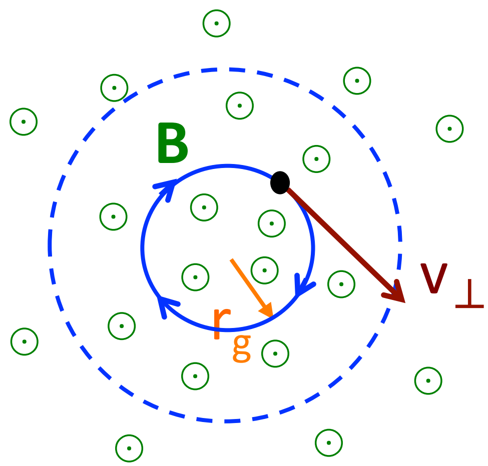


$$p_y = m\mathrm{v}_y + \frac{qB}{c}x = \text{const.} \;\Rightarrow\; x = \frac{p_y}{m\Omega_c} - \frac{\mathrm{v}_y}{\Omega_c}$$



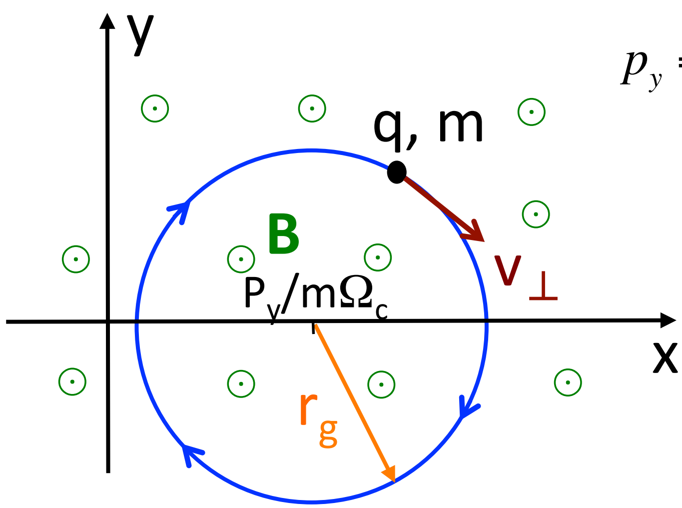
action: $J_1 = \oint p\,dq = \pi \frac{m\mathrm{v}_\perp^2}{\Omega_c}$ =area

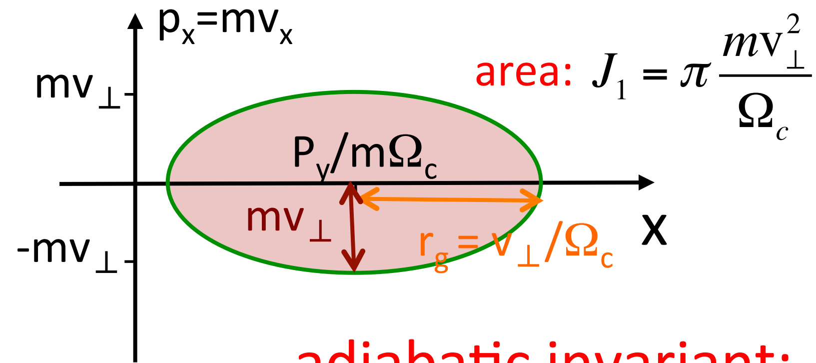
phase space: p_x *vs.* x



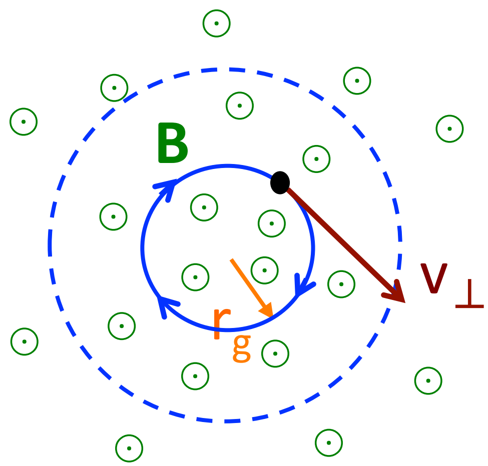
One charged particle in B

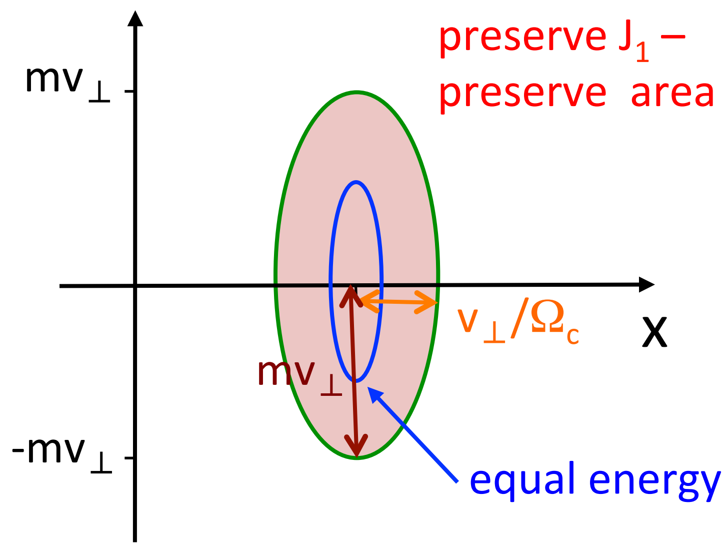
$$p_y = m\mathrm{v}_y + \frac{qB}{c}x = \text{const.} \;\; \Rightarrow \;\; x = \frac{p_y}{m\Omega_c} - \frac{\mathrm{v}_y}{\Omega_c}$$



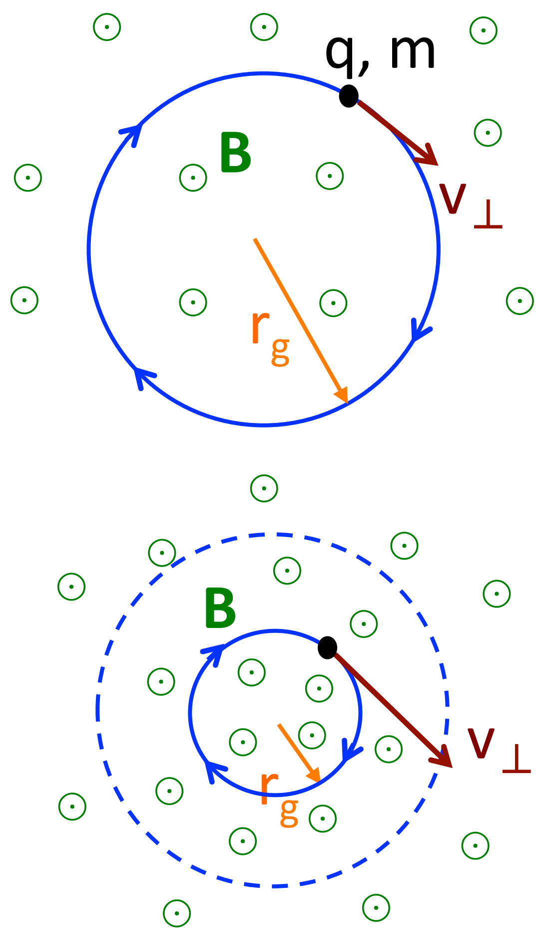


adiabatic invariant:





One charged particle in B



Increase |**B**| slowly:

$$\tau >> \Omega_c^{-1} \sim 10^{-9}\ \mathrm{s}$$

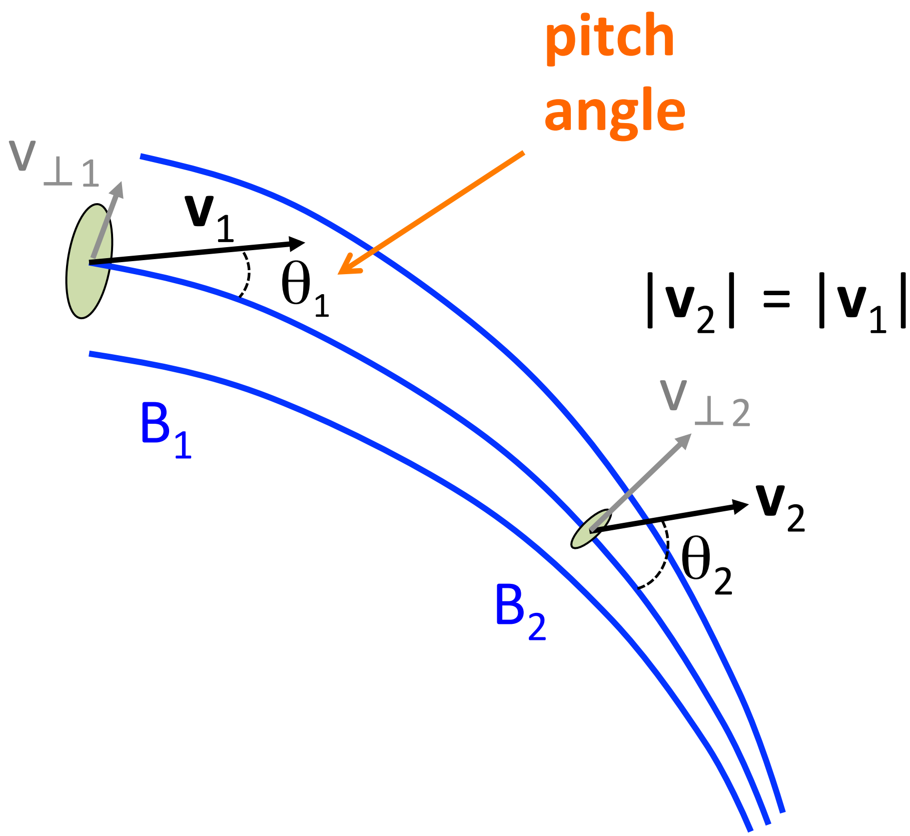
Q: what remains fixed?

~~energy?~~

adiabatic invariant

$$J_1 = \pi \frac{m v_\perp^2}{\Omega_c} \propto \frac{\varepsilon_\perp}{B} \propto B r_g^2 \propto \Phi$$

double |**B**| ➔ double $\varepsilon_\perp$



Static 3d field

- particle follows helix
- r_g from center
- center follows field line

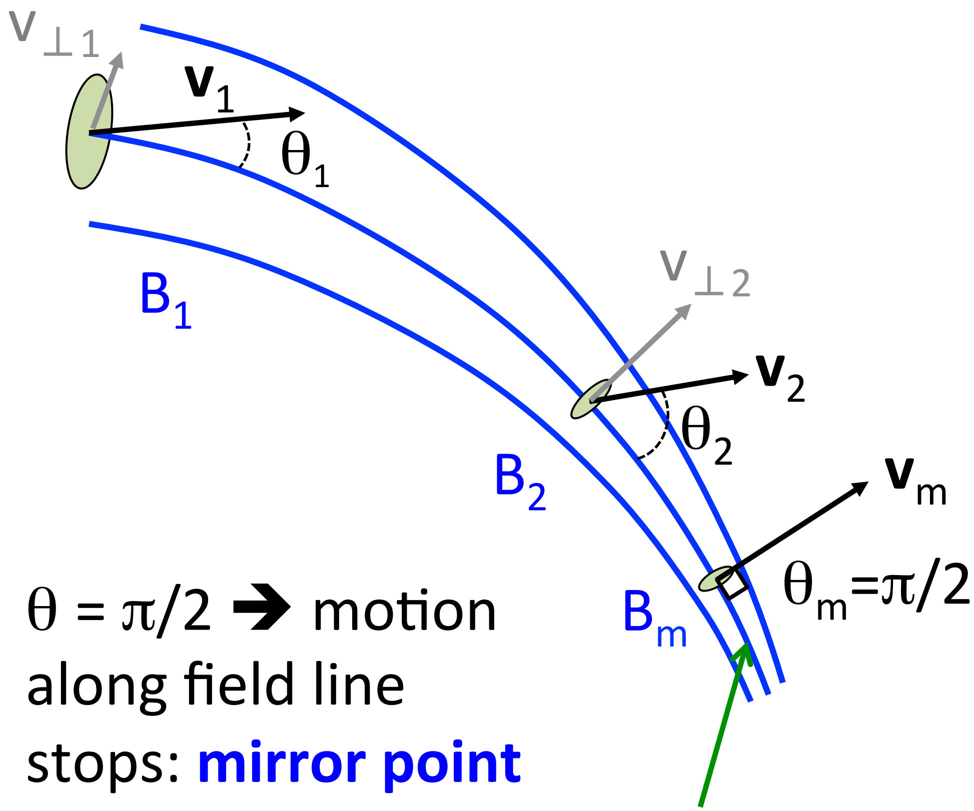
$$\frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\Rightarrow \quad \varepsilon = \tfrac{1}{2} m \mathrm{v}^2 = \text{const.}$$

- also conserved: adiabatic invariant for gyromotion

$$\frac{\mathrm{v}_\perp^2}{B} = \mathrm{v}^2 \boxed{\frac{\sin^2\theta}{B}}$$

const.



$\theta = \pi/2$ ➔ motion along field line stops: **mirror point**

$$B_m = \frac{B_1}{\sin^2\theta_1}$$

$B > B_m$: inaccessible

Static 3d field

- particle follows helix
- r_g from center
- center follows field line

$$\frac{\partial \mathbf{B}}{\partial t} = 0$$

➔ $\varepsilon = \frac{1}{2} m \mathrm{v}^2 = \text{const.}$

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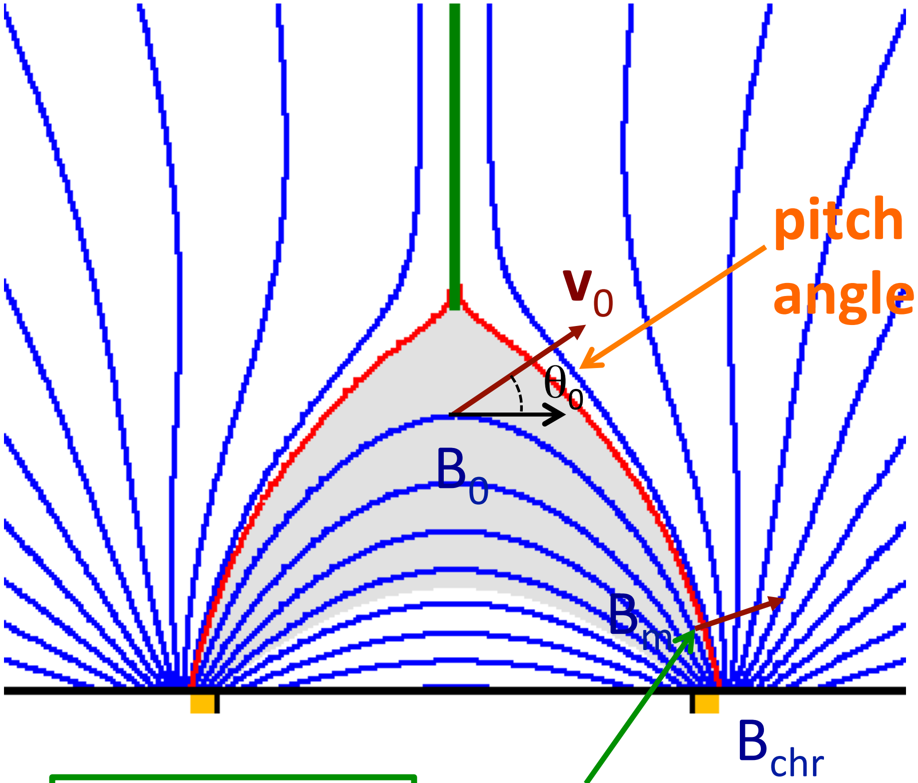
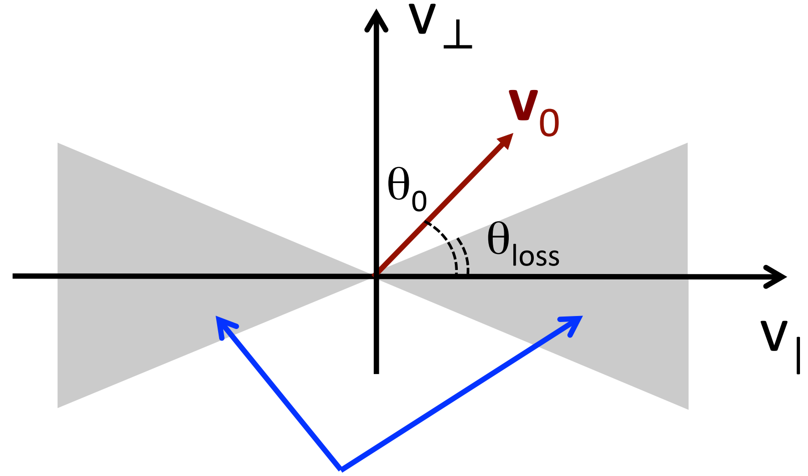
const.



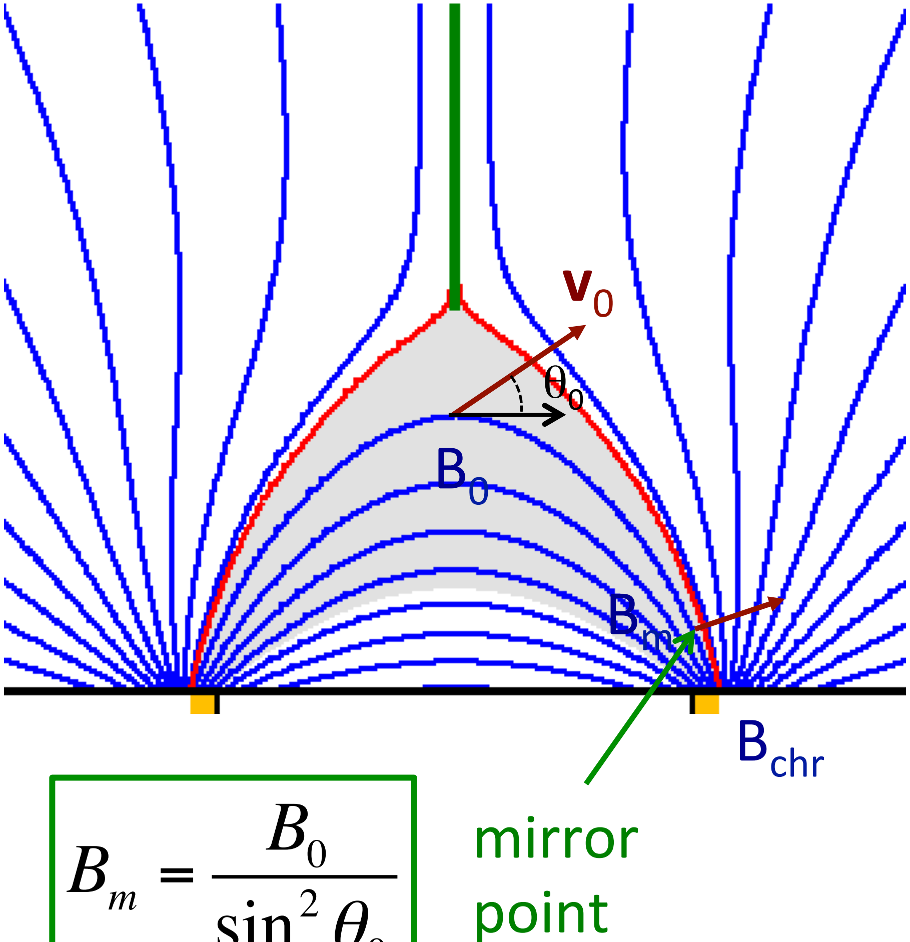
$$B_m = \frac{B_0}{\sin^2 \theta_0}$$



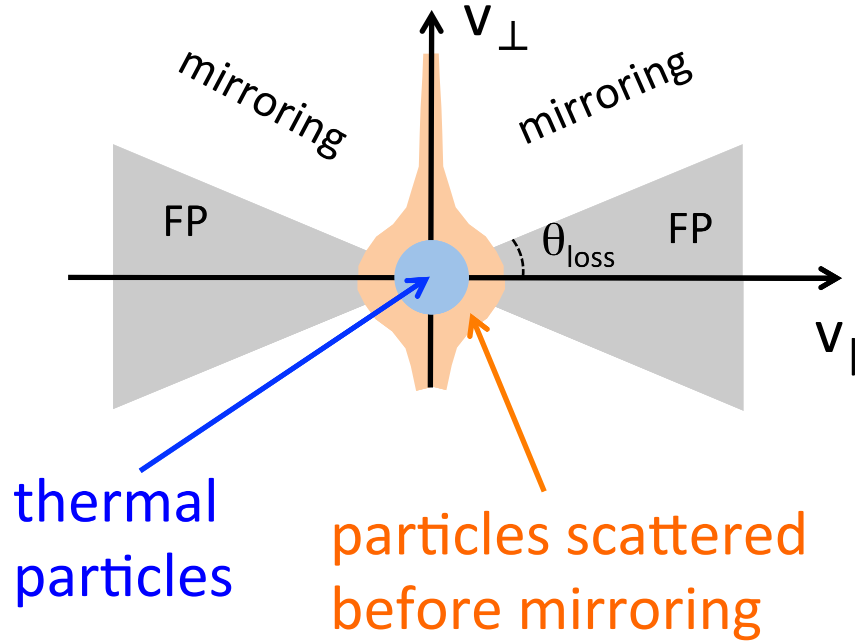
Loss cone: particles reach chromosphere before mirroring

$$\theta_0 < \theta_{\mathrm{loss}}$$

$$\theta_{\mathrm{loss}} = \sin^{-1}\left(\sqrt{\frac{B_0}{B_{\mathrm{chr}}}}\right)$$



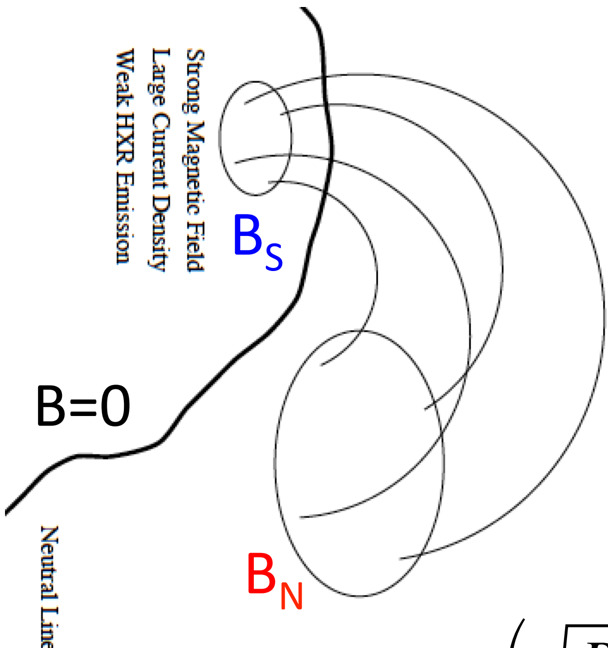
$$B_m = \frac{B_0}{\sin^2\theta_0}$$



$$E_{keV}^2 \cos\theta_0 < \left(\frac{N_{\rm chr}}{1.4\times10^{17}\,{\rm cm}^{-2}}\right) \approx 30$$

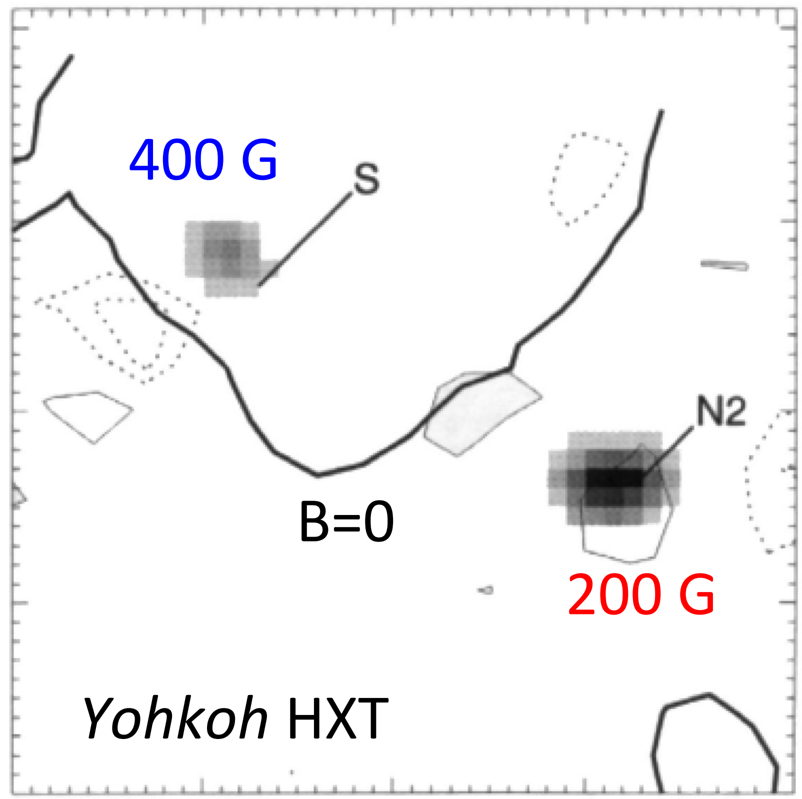
Consequence: Cornucopia

$$\theta_S = \sin^{-1}\left(\sqrt{\frac{B_0}{B_S}}\right)$$



$$\theta_N = \sin^{-1}\left(\sqrt{\frac{B_0}{B_N}}\right)$$

asymmetric mirrors ➔
asymmetric footpoints



Li et al. 1997

$B_N < B_S$ ➔ $\theta_N > \theta_S$ ➔ more precip @ N

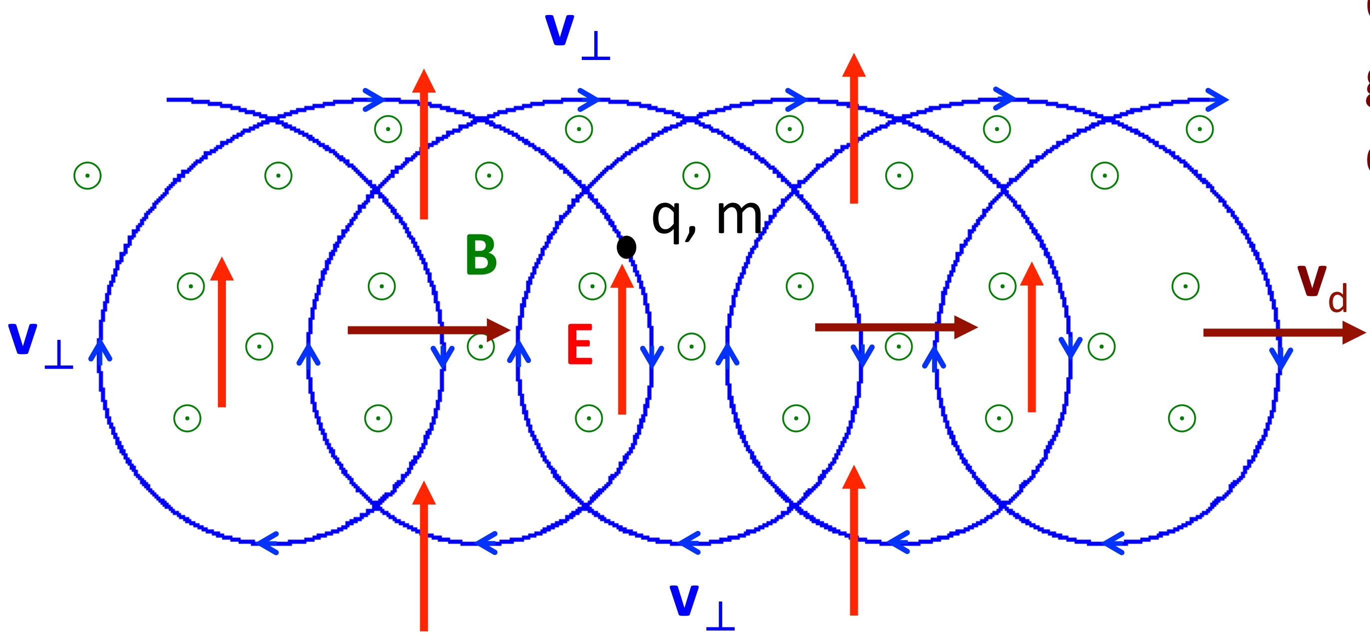
What can accelerate e⁻s?

Must ultimately be **E** …

$$\frac{d}{dt}\left(\tfrac{1}{2}m|\mathbf{v}|^2\right) = \frac{d\varepsilon}{dt} = q\mathbf{v}\cdot\mathbf{E}$$

… but what creates/sustains **E**?

DC **E** field **cannot** increase $\varepsilon_\perp$



Mean motion:
drift of
gyrocenter @
constant speed

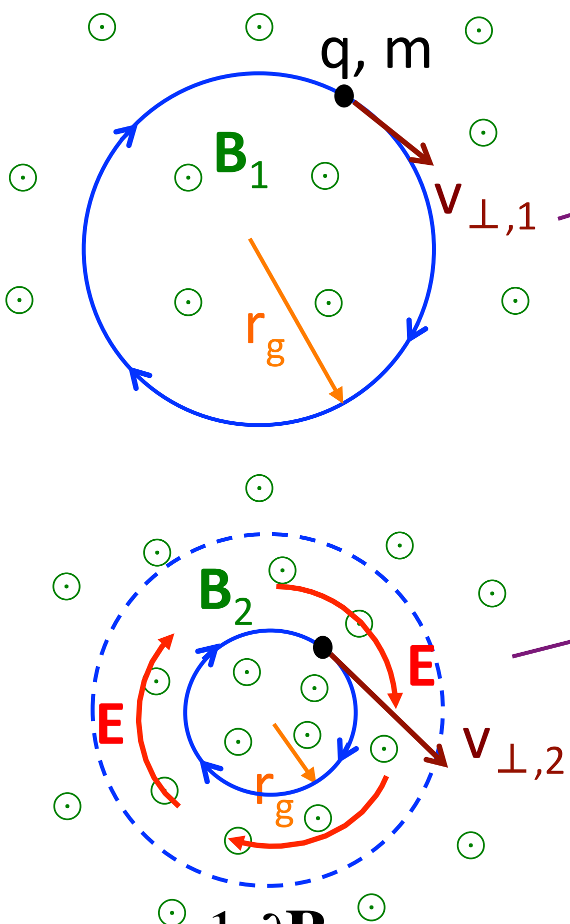
$$\langle \mathbf{v}_\perp \rangle = \mathbf{v}_d = c\frac{\mathbf{E}\times\mathbf{B}}{|\mathbf{B}|^2}$$

NB: x-form to drift frame:

$$\mathbf{E}' = \mathbf{E} + \frac{1}{c}\mathbf{v}_d \times \mathbf{B} = 0$$ ➔ simple gyromotion

Changing B **can** increase ε ⊥

Betatron acceleration



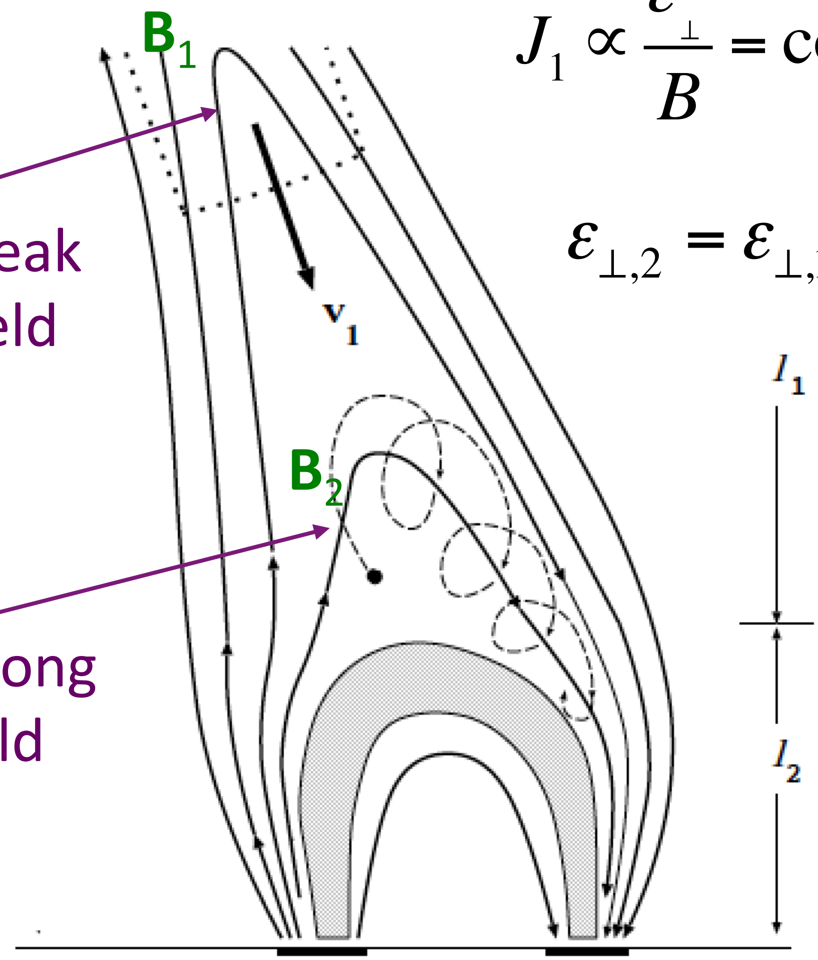
$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

weak field

strong field

$$J_1 \propto \frac{\varepsilon_\perp}{B} = \text{const.}$$

$$\varepsilon_{\perp,2} = \varepsilon_{\perp,1}\frac{B_2}{B_1}$$



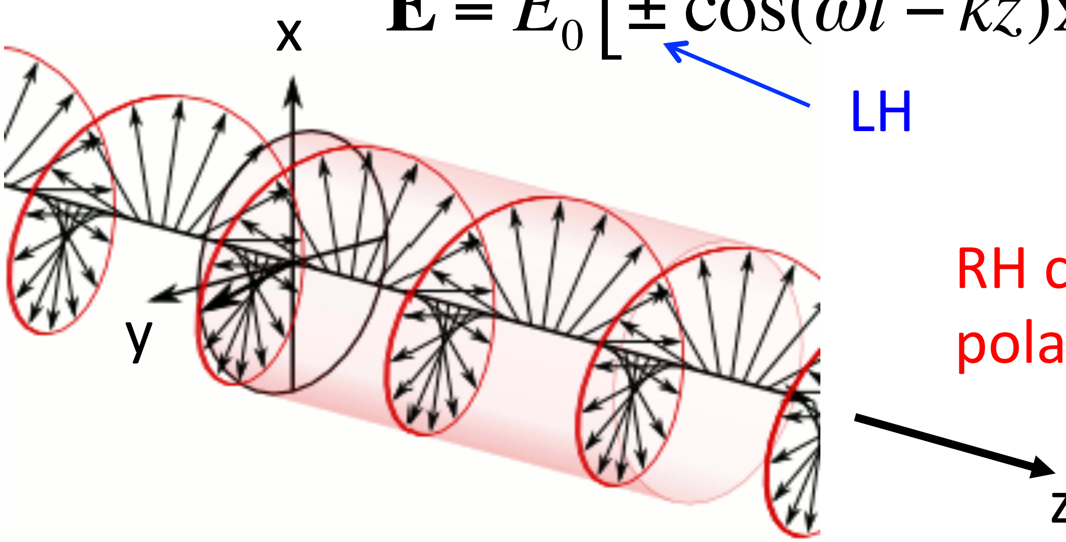
Somov & Kosugi 1997

Electromagnetic wave can too

$$\mathbf{E} = E_0\left[\pm\cos(\omega t - kz)\hat{\mathbf{x}} + \sin(\omega t - kz)\hat{\mathbf{y}}\right]$$

RH

LH



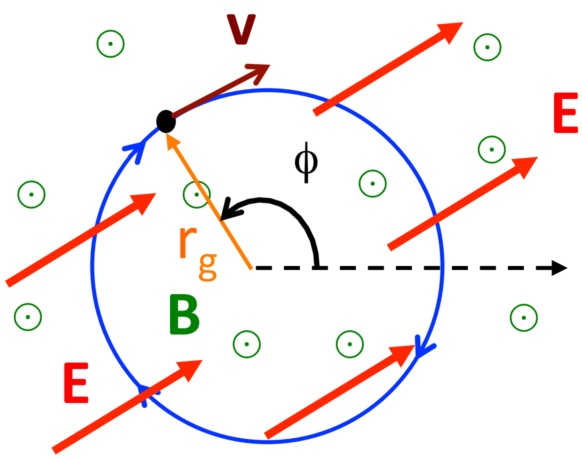
RH circularly polarized wave

$$\hat{\phi} = \cos\varphi\;\hat{\mathbf{y}} - \sin\varphi\;\hat{\mathbf{x}}$$

$$E_\phi = \mathbf{E}\cdot\hat{\phi} = E_0\sin\left[(\omega t - kz)\mp\varphi\right]$$

Electromagnetic wave can too

$$E_\phi = \mathbf{E}\cdot\hat{\phi} = E_0 \sin\left[(\omega t - kz) \mp \varphi\right]$$



Particle position:

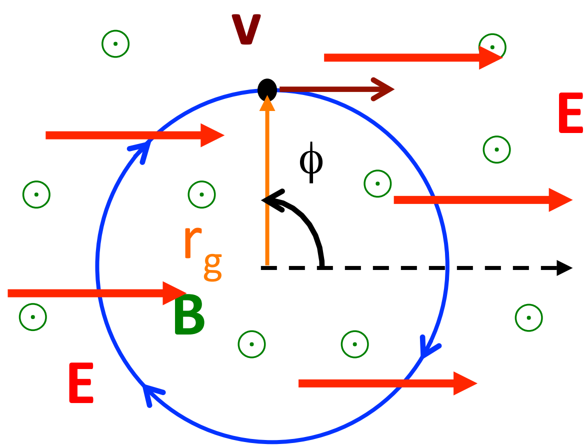
$$z(t) = \mathrm{v}_z t$$

$$\varphi(t) = -s\,\Omega_c t + \varphi_0$$

$$s = \frac{q}{|q|} = \pm 1$$ sign of charge

Electromagnetic wave can too

$$E_\phi = \mathbf{E} \cdot \hat{\phi} = E_0 \sin\left[(\omega t - kz) \mp \varphi\right]$$



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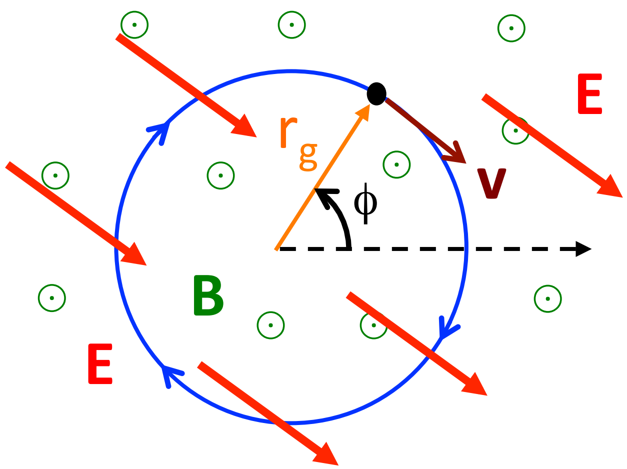


Particle position:

$$z(t) = \mathrm{v}_z t$$

$$\varphi(t) = -s\,\Omega_c t + \varphi_0$$

$$s = \frac{q}{|q|} = \pm 1$$ sign of charge

$$\frac{d\mathrm{v}_\perp}{dt} = qE_\phi = qE_0 \sin\left[(\omega - k\mathrm{v}_z \pm s\,\Omega_c)t \mp \varphi_0\right]$$

RH (upper signs), LH (lower signs)

resonance: $\omega - k\mathrm{v}_z = \mp s\,\Omega_c$ (RH: upper sign)

electrons (s=-1) resonate w/ **RH** wave

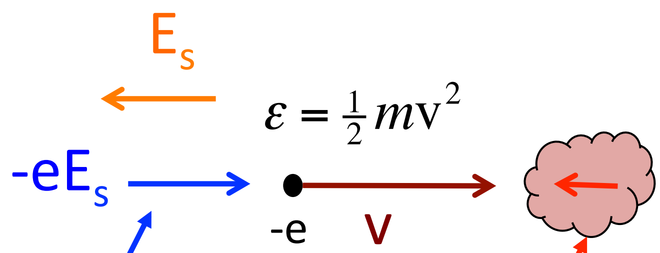
Electromagnetic wave can too

resonance: $$\omega - k\mathrm{v}_z = \mp s\,\Omega_c$$

Resonant particles gain energy
– at expense of the wave
➔ wave loses energy (amplitude diminishes)
a.k.a. **wave damping**
a.k.a. **wave-particle interaction**

DC **E** field **can** increase $\varepsilon_{||}$

v || B ➔ no effect – consider only electric field



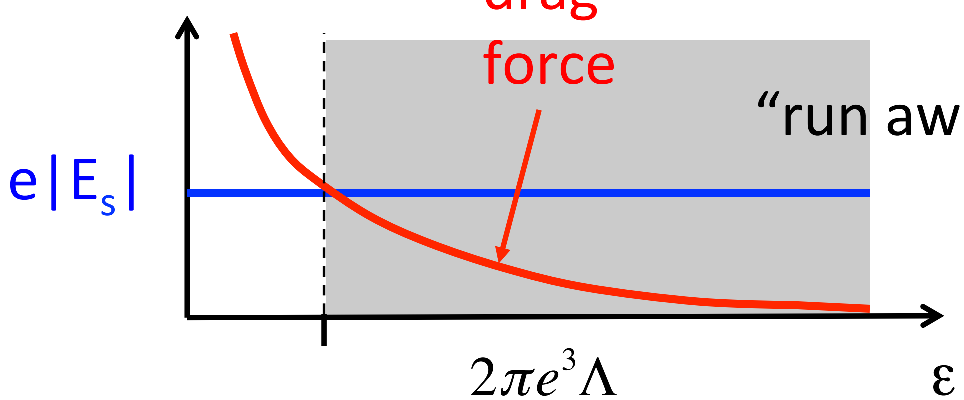
Rutherford x-section

$$\sigma_{ei} = \frac{2\pi e^4 \Lambda}{\varepsilon^2}$$

acceleration

$$\frac{d\varepsilon}{ds} = -eE_s - n_i \sigma_{ei} \varepsilon = e|E_s| - n_i \frac{2\pi e^4 \Lambda}{\varepsilon} = e|E_s|\left(1 - \frac{\varepsilon_{ra}}{\varepsilon}\right)$$

drag force



$$\varepsilon_{ra} = n_i \frac{2\pi e^3 \Lambda}{|E_s|}$$

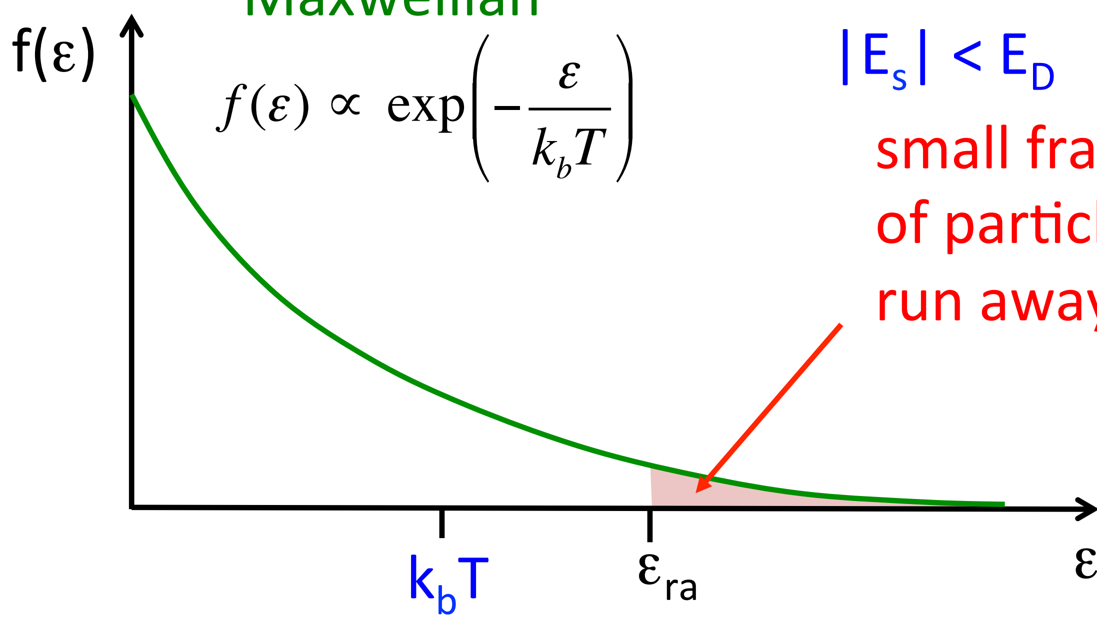
acceleration:
$d\varepsilon/ds > 0$
$\varepsilon > \varepsilon_{ra}$

DC **E** field **can** increase $\varepsilon_{||}$

$$\varepsilon_{ra} = n_i \frac{2\pi e^3 \Lambda}{|E_s|} = k_b T \frac{E_D}{|E_s|}$$

Dreicer field

$$E_D = n_i \frac{2\pi e^3 \Lambda}{k_b T} = 6\,\mathrm{mV/m}\ \frac{n_9}{T_6}$$



$$f(\varepsilon) \propto \exp\left(-\frac{\varepsilon}{k_b T}\right)$$

Sub-Dreicer

$|E_s| < E_D$

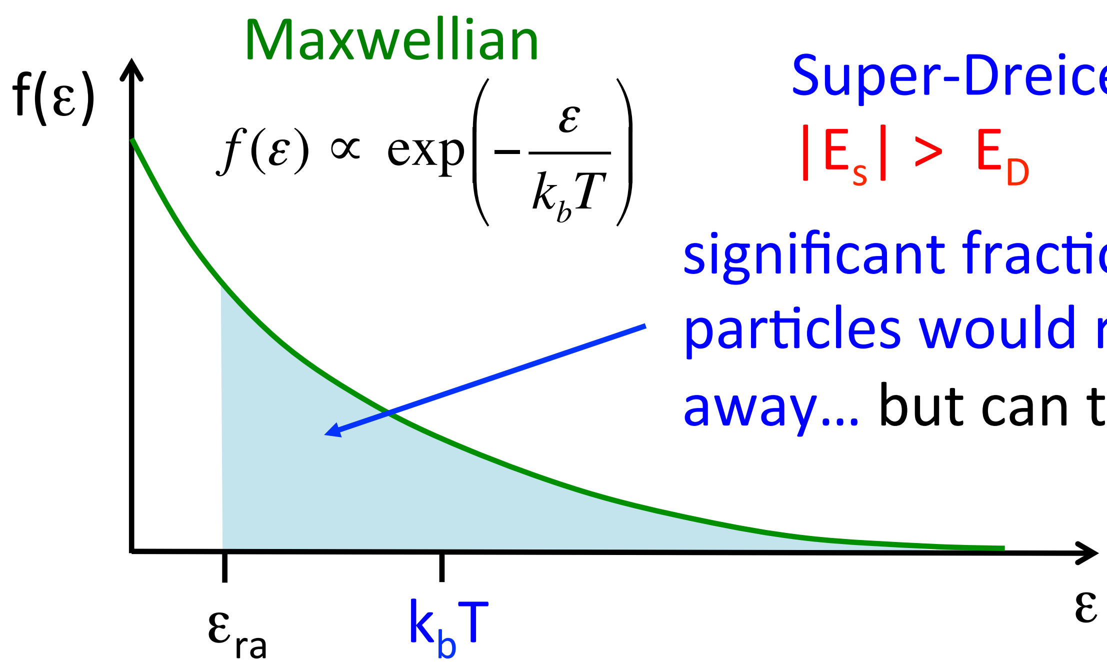
small fraction of particles run away

DC **E** field **can** increase $\varepsilon_{||}$

$$\varepsilon_{ra} = n_i \frac{2\pi e^3 \Lambda}{|E_s|} = k_b T \frac{E_D}{|E_s|}$$

Dreicer field

$$E_D = n_i \frac{2\pi e^3 \Lambda}{k_b T} = 6\,\mathrm{mV/m}\ \frac{n_9}{T_6}$$

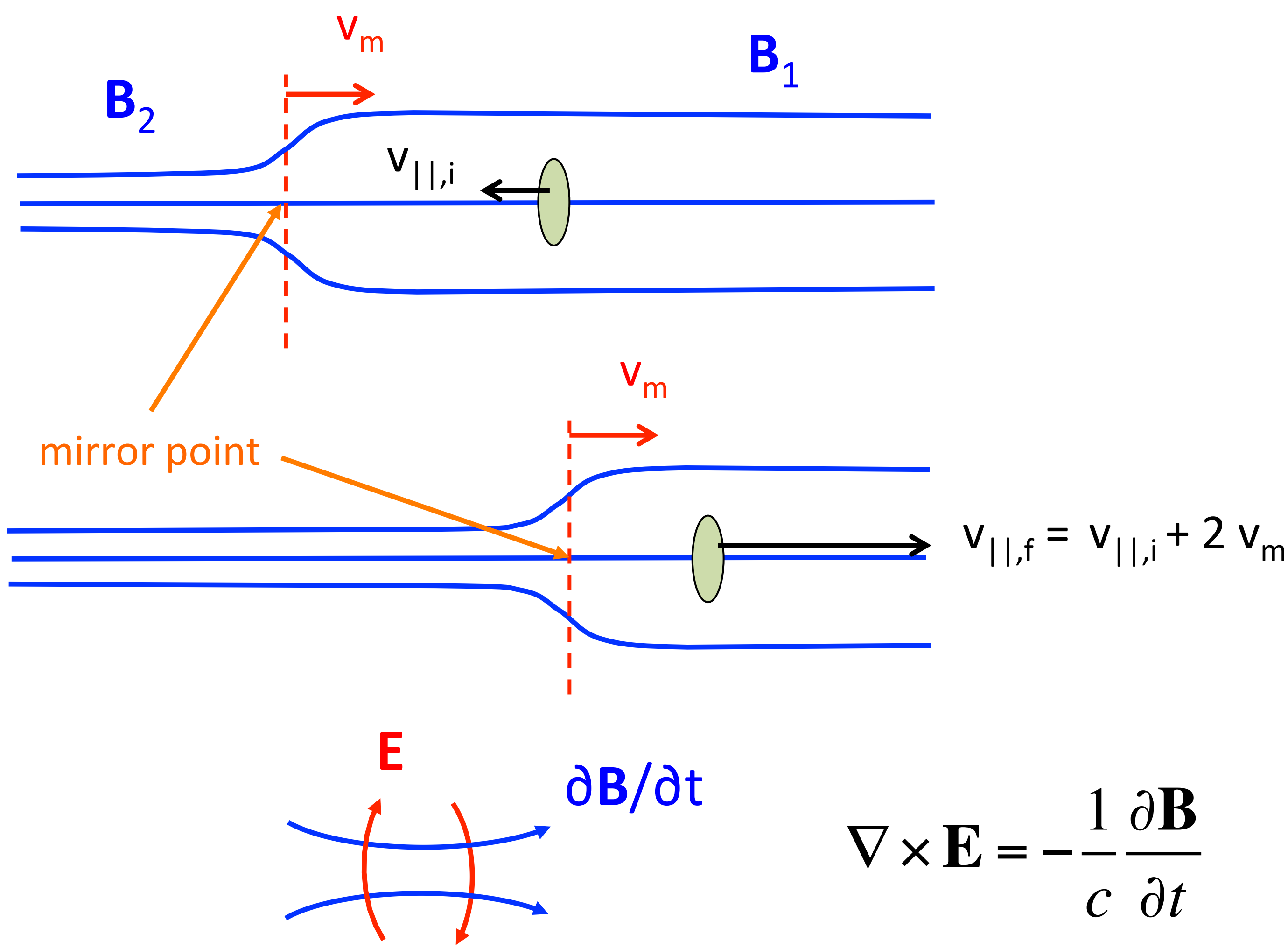


Super-Dreicer

$|E_s| > E_D$

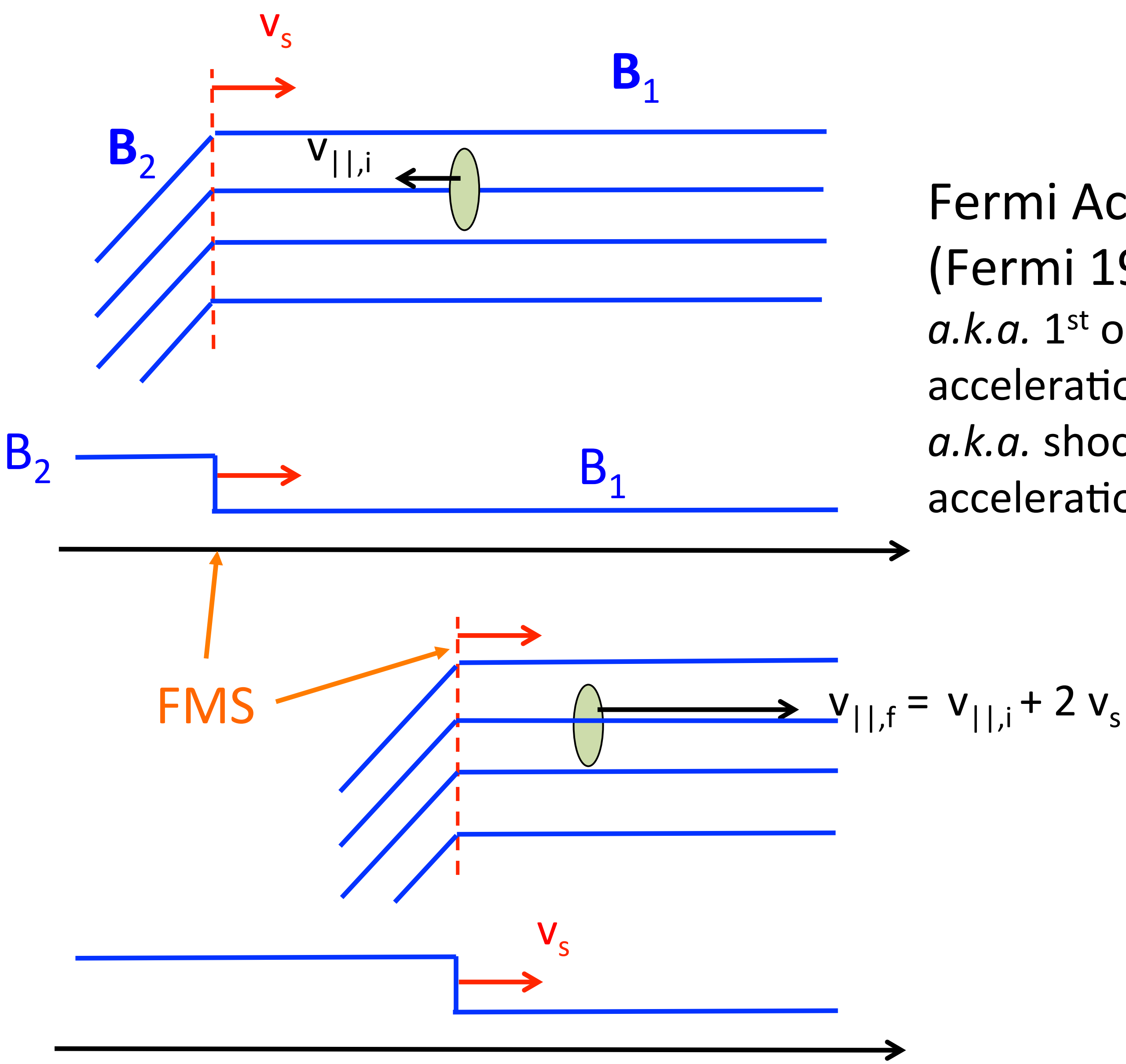
significant fraction of particles would run away… but can they?

Moving mirror can increase $\varepsilon_{||}$



$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Fast MS shock: a moving mirror



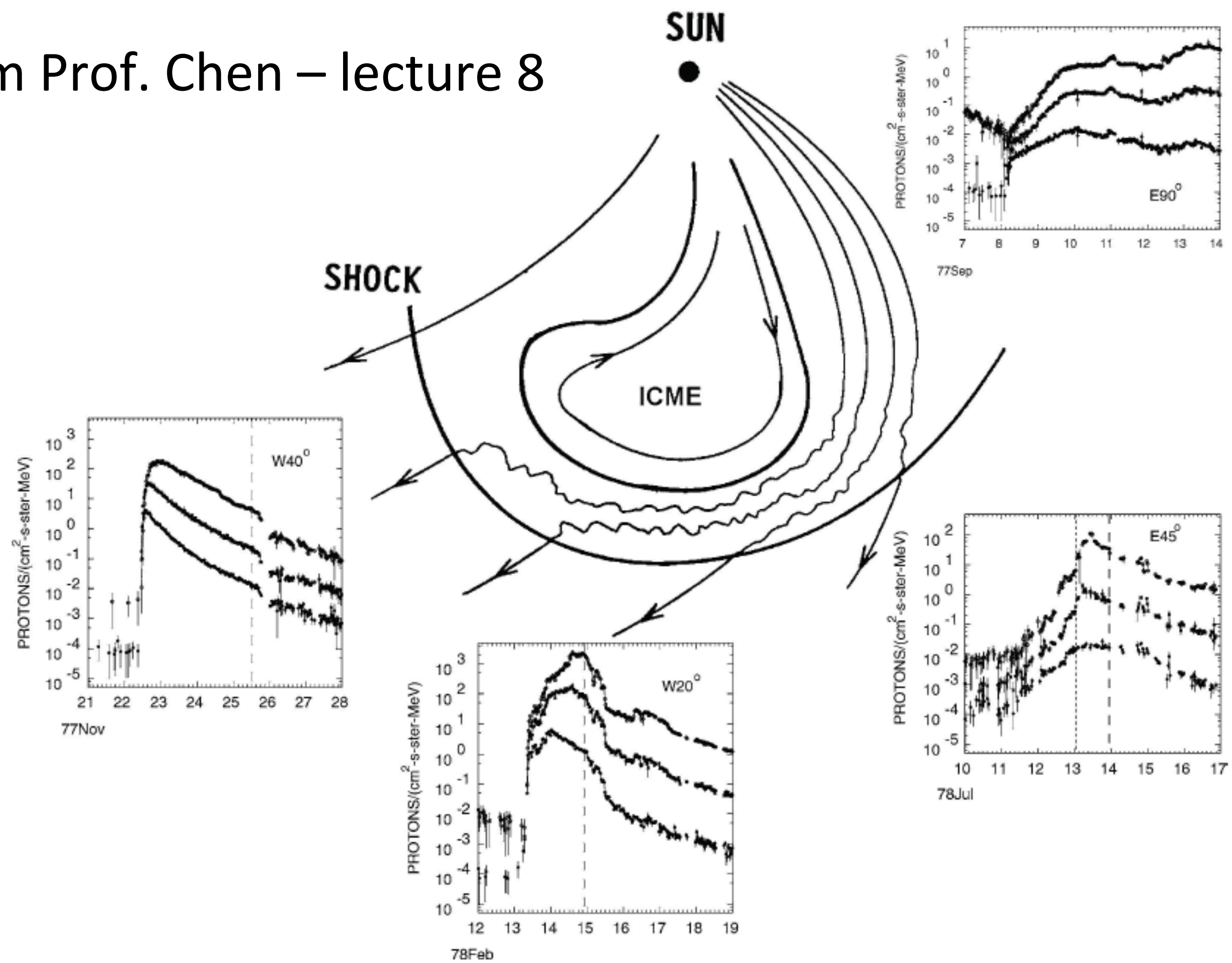
Fermi Acceleration (Fermi 1949)

a.k.a. 1st order Fermi acceleration

a.k.a. shock-drift acceleration

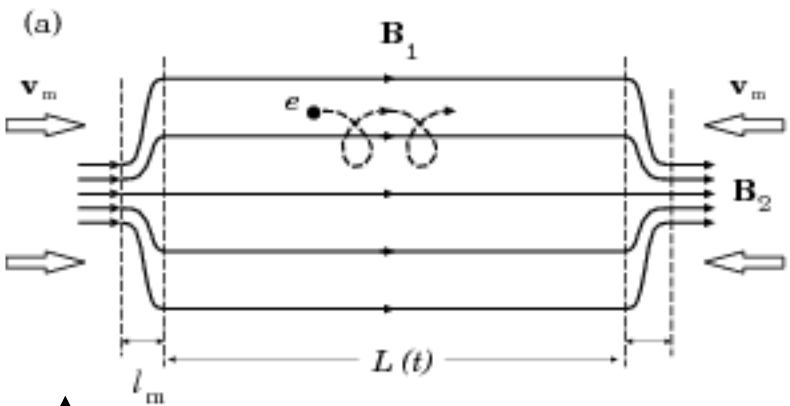
Shocks are good particle accelerators

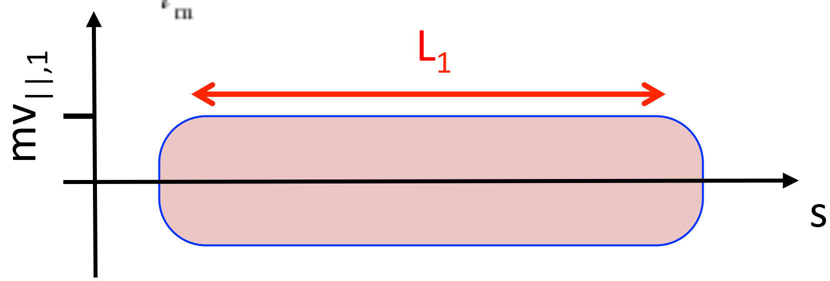
From Prof. Chen – lecture 8



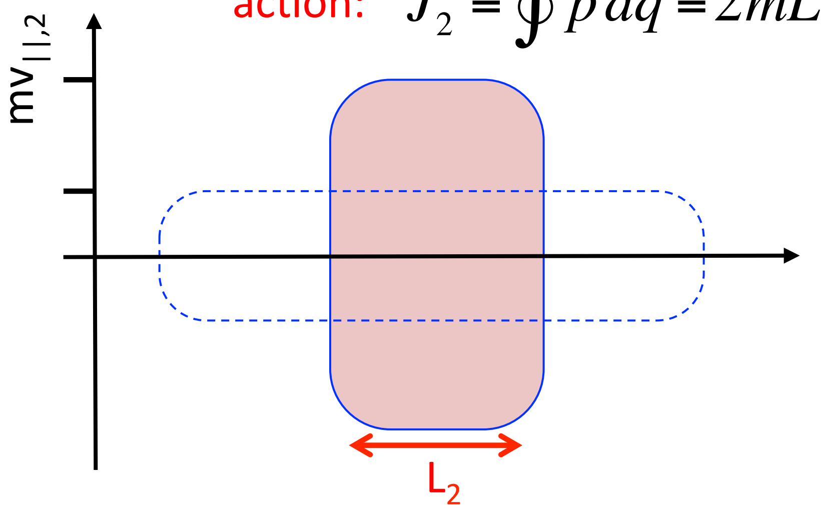
From Cane & Lario 2006

Standing Fast MS shock

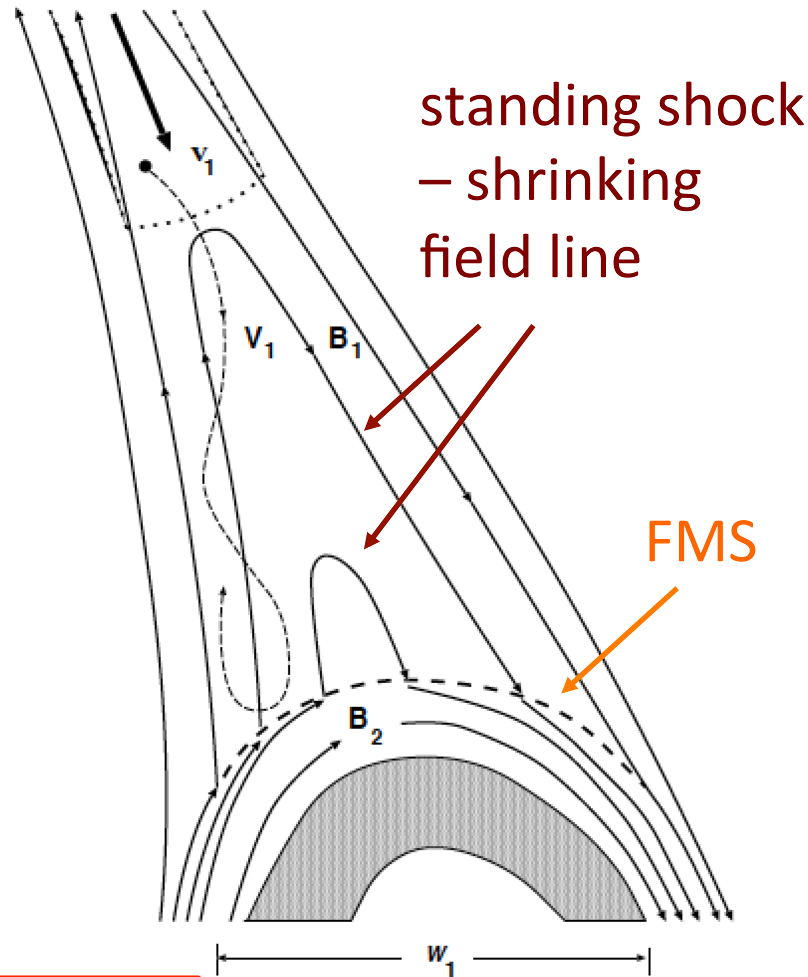




action: $J_2 = \oint p\,dq = 2mL\mathrm{v}_{\parallel}$ = area

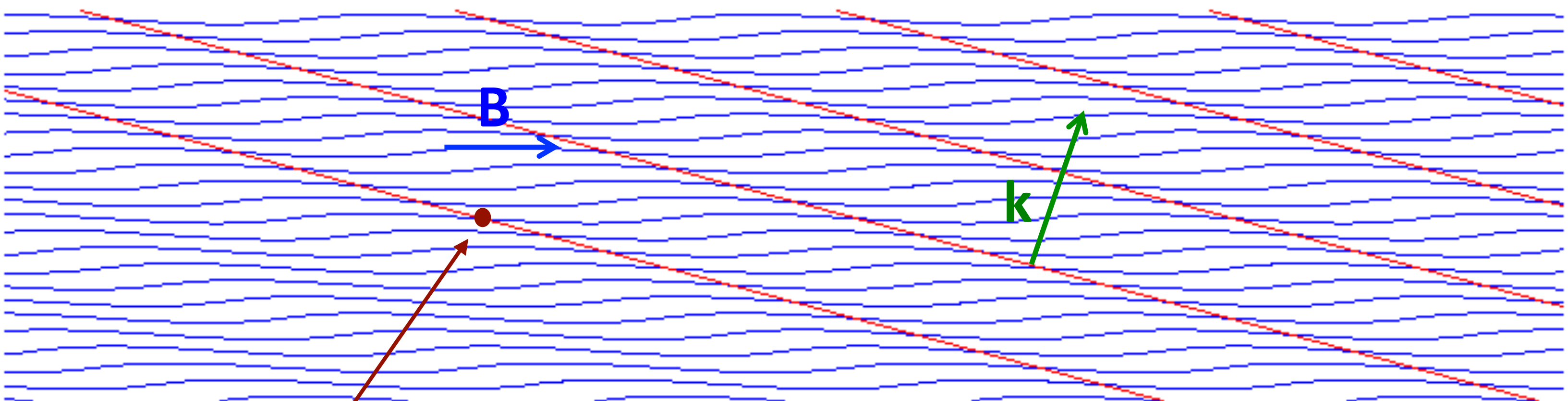


$$\varepsilon_{\parallel,2} = \left(\frac{L_1}{L_2}\right)^2 \varepsilon_{\parallel,1}$$



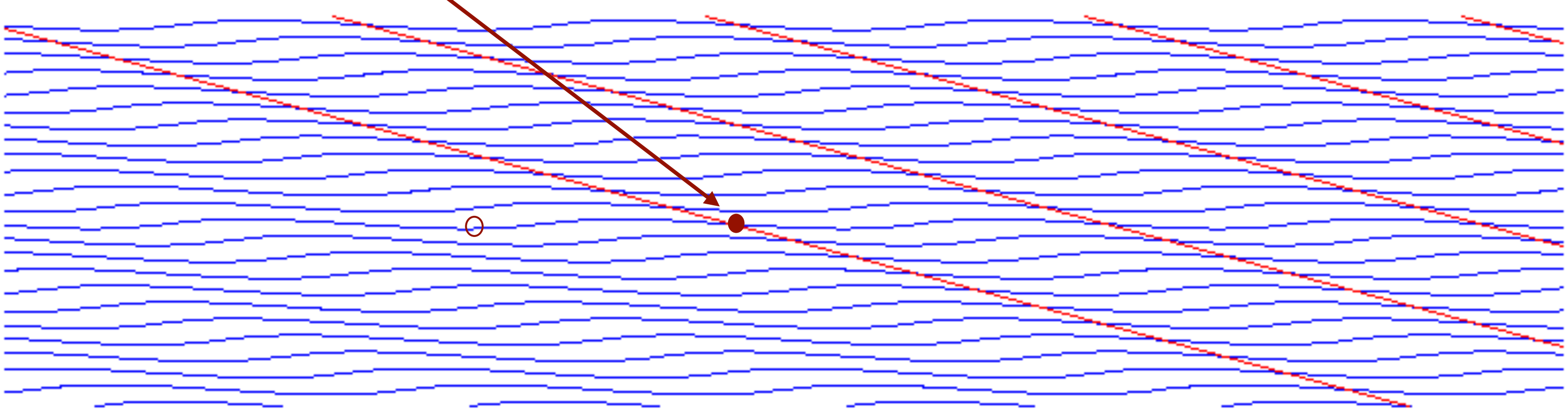
Somov & Kosugi 1997

Fast MS waves are moving mirrors

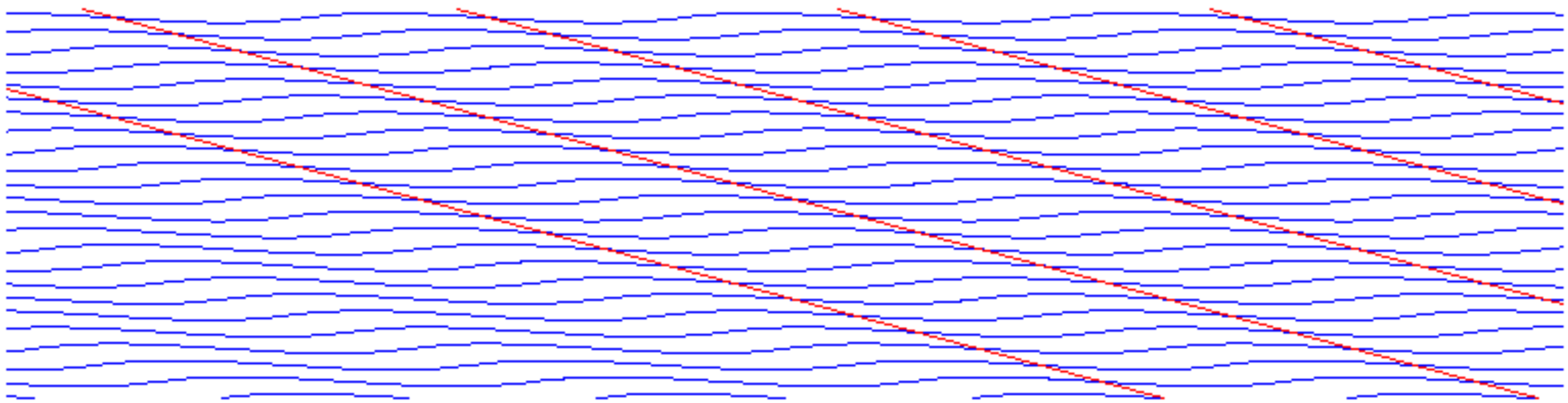


moving mirror point

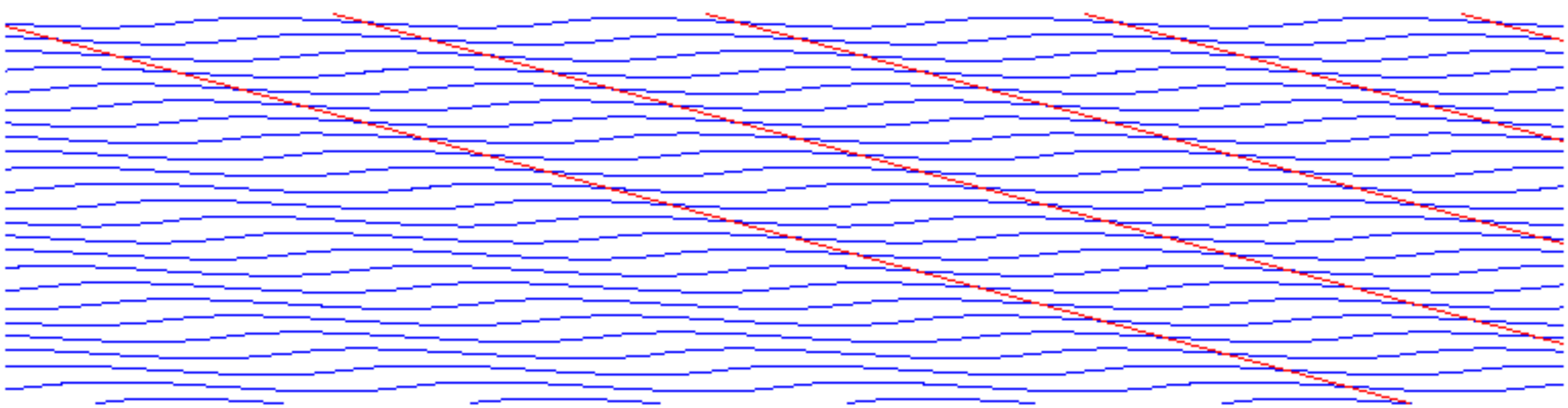
small mirror ratio ➔ only mirrors particles moving near mirror speed (phase speed of wave) i.e. resonant particles



Fast MS waves are moving mirrors



wave-particle interaction



Summary

- Ions and electrons follow magnetic field lines (gyroradii are **very** small)
- Can **mirror** from points of strong field – if their pitch angle is large enough
- Can gain energy in various ways
  - DC parallel **E** field (prob. sub-Dreicer)
  - increasing **B** (betatron acceleration)
  - moving mirrors (Fermi acceleration)
  - wave-particle interaction