

Flare Loops

1d gas dynamics

Lecture 10

Feb. 22, 2017

The 1d flare loop

$\rho(s,t)$, $u(s,t)$ & $T(s,t)$

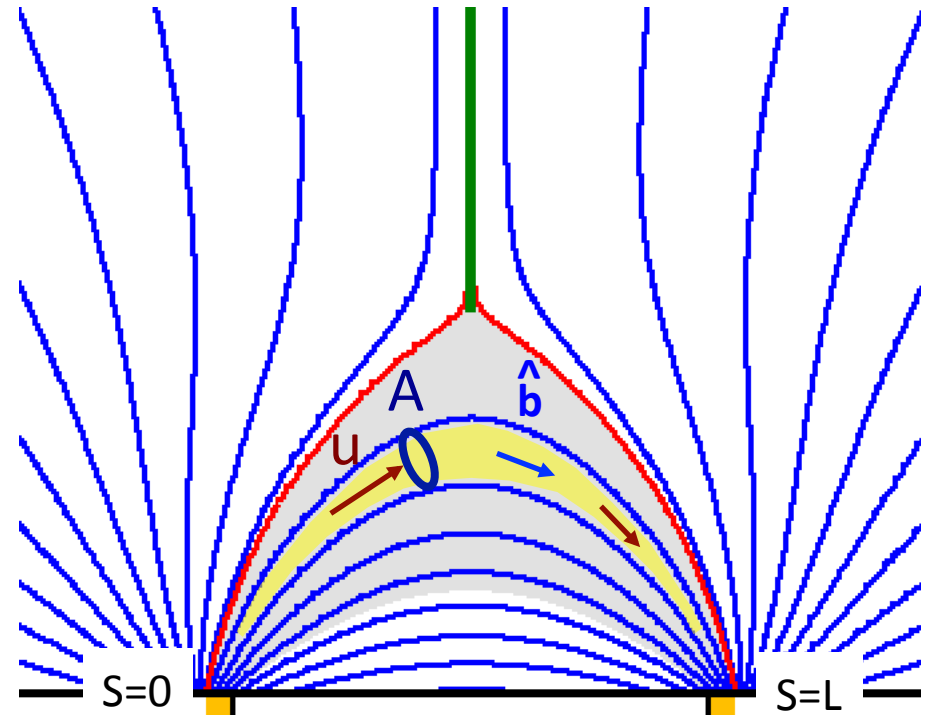
Fixed $A(s)$ & $g_{||}(s)$

$$\frac{\partial \rho}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial s} (A \rho u)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) = -\frac{\partial p}{\partial s} + \rho g_{||} + \frac{\partial}{\partial s} \left(\frac{4}{3} \mu \frac{\partial u}{\partial s} \right)$$

$$\rho c_v \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial s} \right) = -\frac{p}{A} \frac{\partial}{\partial s} (A u) + \frac{4}{3} \mu \left| \frac{\partial u}{\partial s} \right|^2 + \frac{1}{A} \frac{\partial}{\partial s} \left[A \kappa \frac{\partial T}{\partial s} \right] - n_e^2 \Lambda(T) + h$$

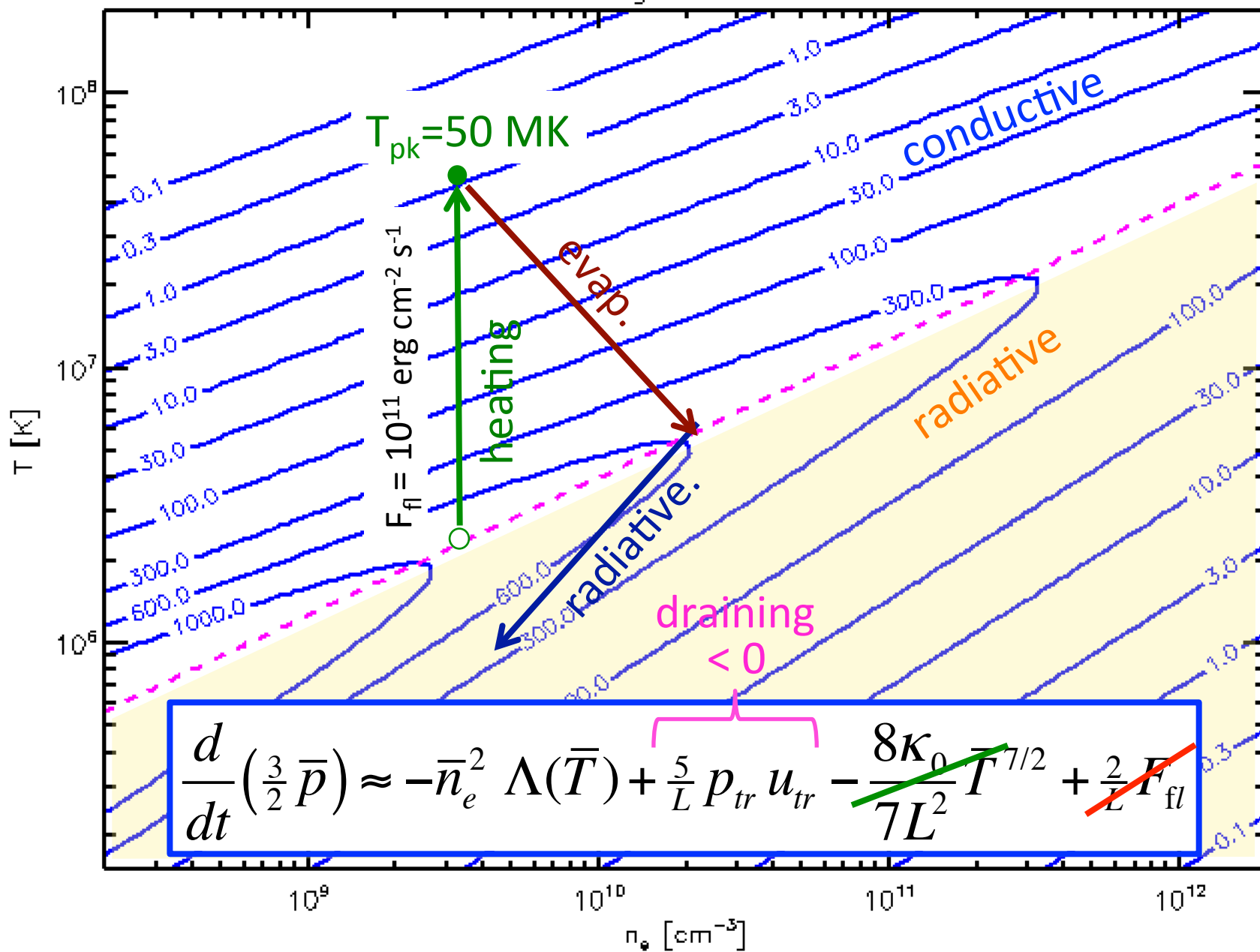
$$p = \frac{k_b}{\bar{m}} \rho T \quad c_v = \frac{3}{2} \frac{k_b}{\bar{m}}$$



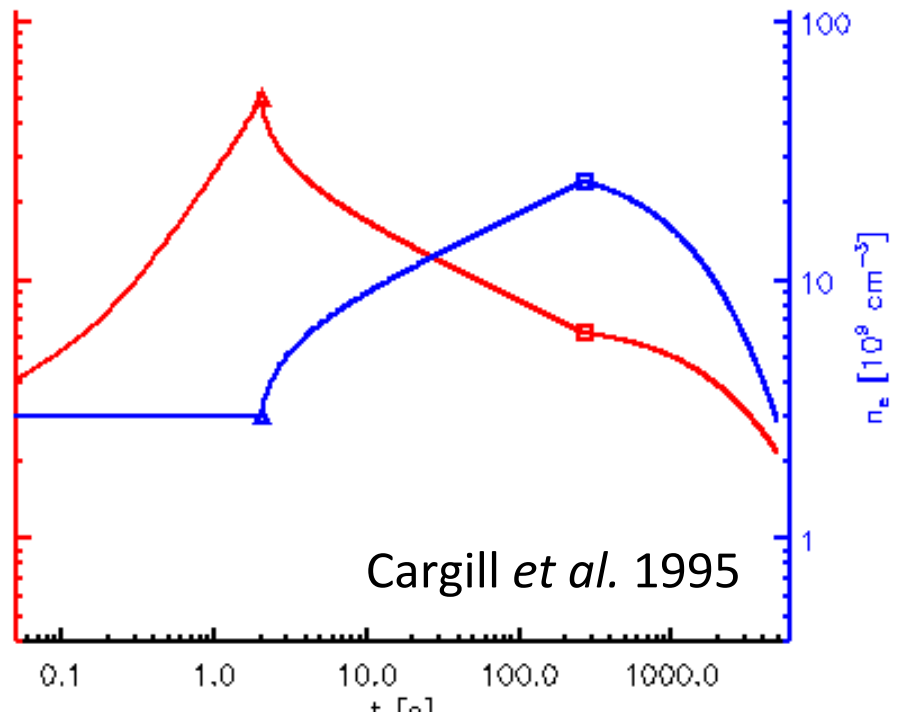
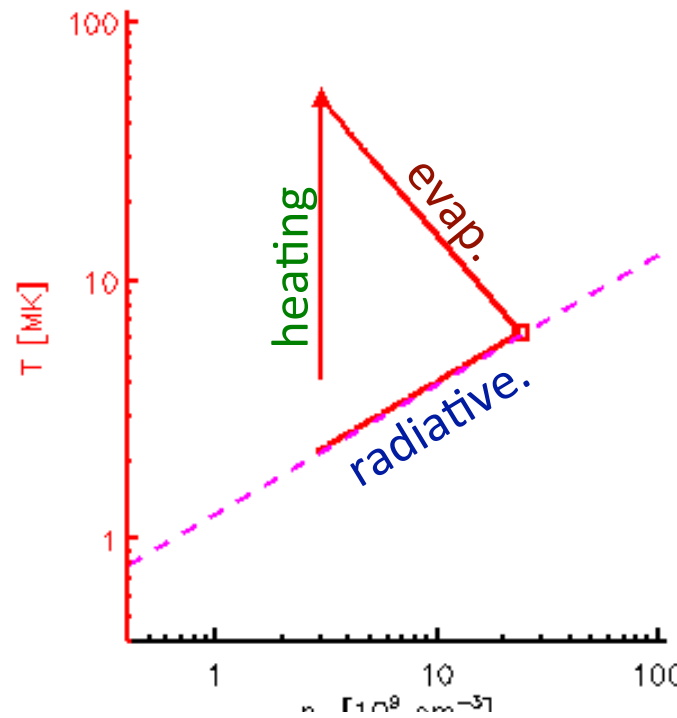
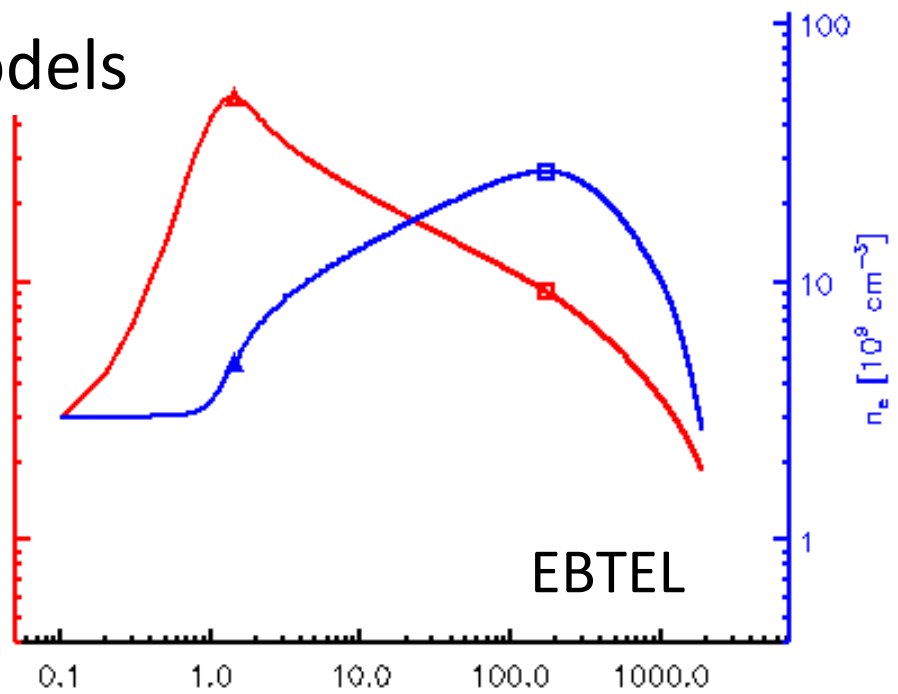
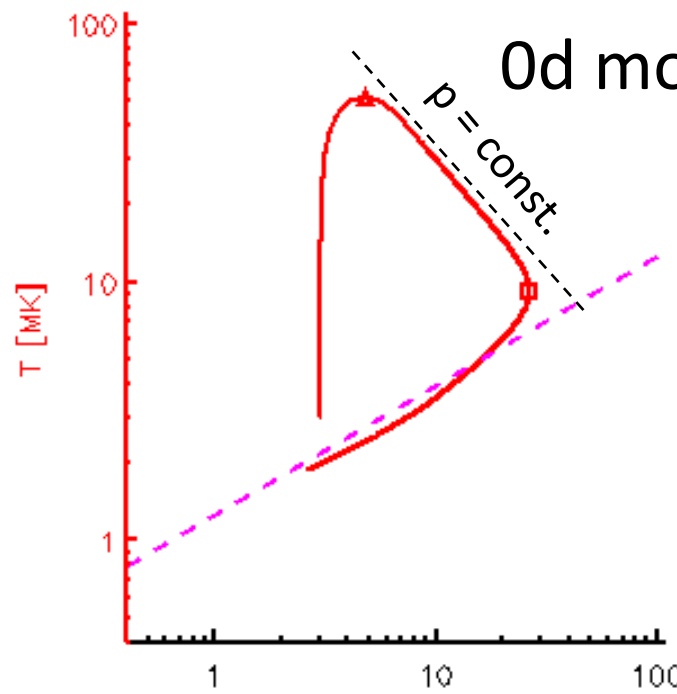
Source of flare energy



full length = 50 Mm



0d models



$$\bar{n}_{e,\max} = 2.6 \times 10^{12} \frac{E^{2/3}}{V^{2/3} L^{1/3}}$$

$$\bar{T}_{em,pk} = 930 \left(\frac{EL}{V} \right)^{1/3}$$

IF flare were a single loop

$$\max(EM) = V \bar{n}_{e,\max}^2 = 7 \times 10^{24} \frac{E^{4/3}}{V^{1/3} L^{2/3}} = 7 \times 10^{49} \text{ cm}^3 \frac{E_{30}^{4/3}}{V_{27}^{1/3} L_9^{2/3}}$$

$$\bar{T}_{em,pk} = 930 \left(\frac{EL}{V} \right)^{1/3} = 9 \times 10^6 \text{ K} \left(\frac{E_{30} L_9}{V_{27}} \right)^{1/3}$$

$$E_{30} = E/10^{30} \text{ ergs}$$

$$\sim E^{7/4}$$

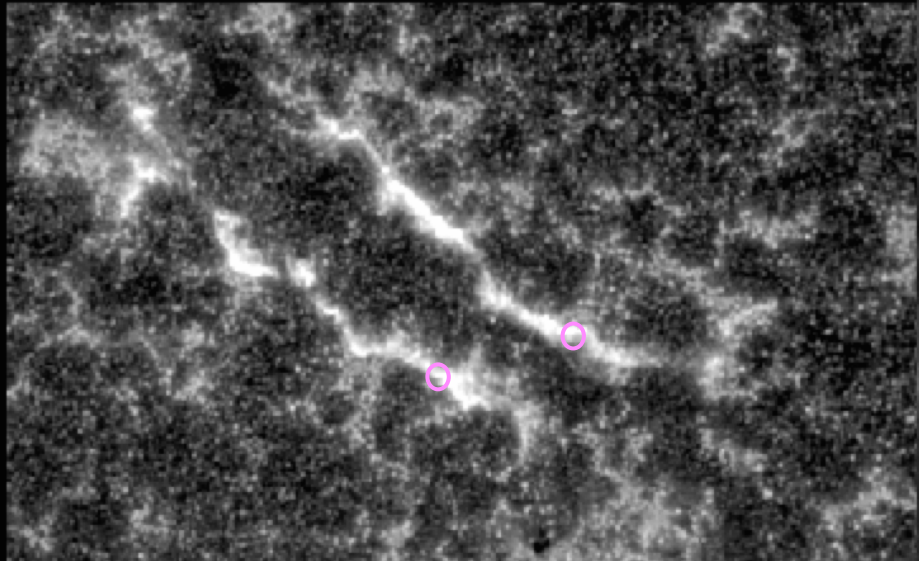
GOES peak:

$$F_{1-8} \approx 10^{-63} \frac{\text{W}}{\text{m}^2} EM T^{5/4} = 4 \times 10^{-5} \frac{\text{W}}{\text{m}^2} \cdot \frac{E_{30}^{7/4}}{L_9^{1/4} V_{27}^{3/4}}$$

Warren & Antiochos 2004

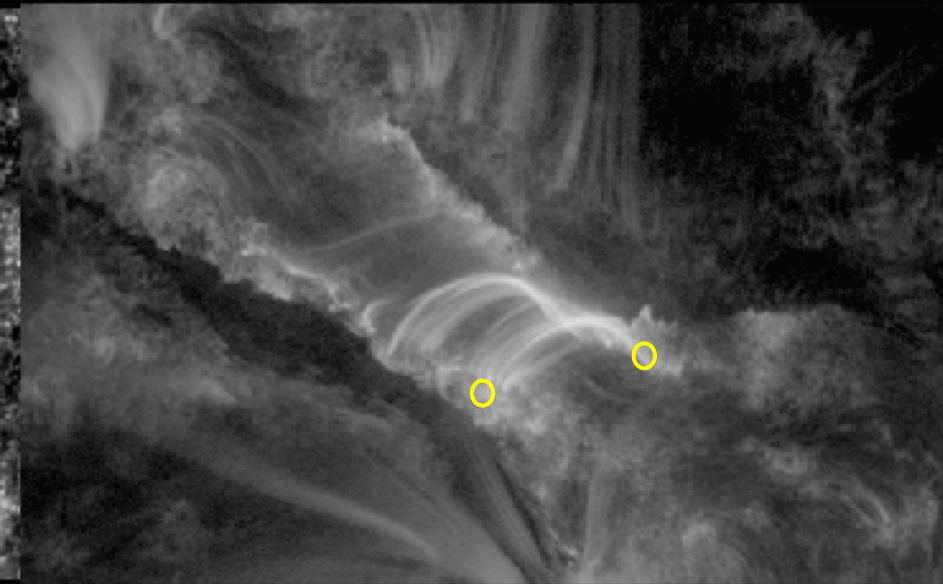
M4 flare

26-Dec-2011 11:31:53.120

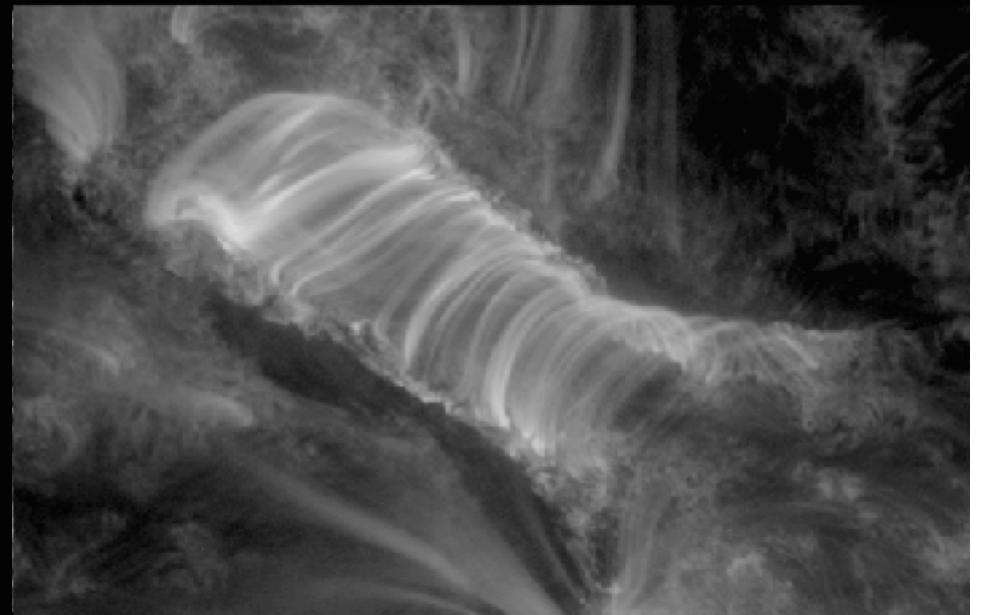


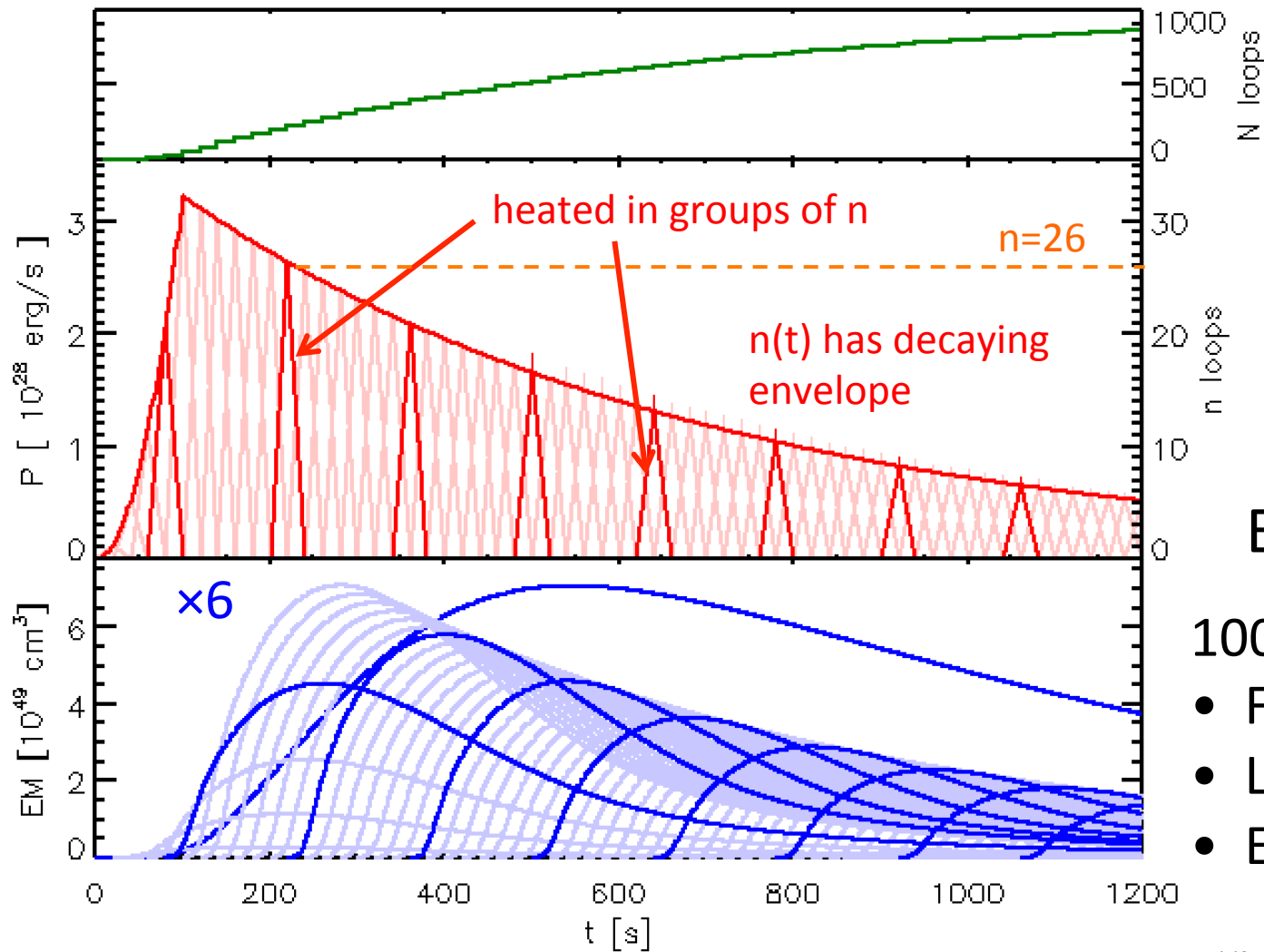
BUT a real flare is built from many diff. loops – each evolving independently

28-Dec-2011 12:08:14.540



28-Dec-2011 12:44:13.450





Example model:

Hori et al. 1997,
 Warren et al. 2002,
 Reeves & Warren
 2002, Qiu et al. 2012,
 2013, ...

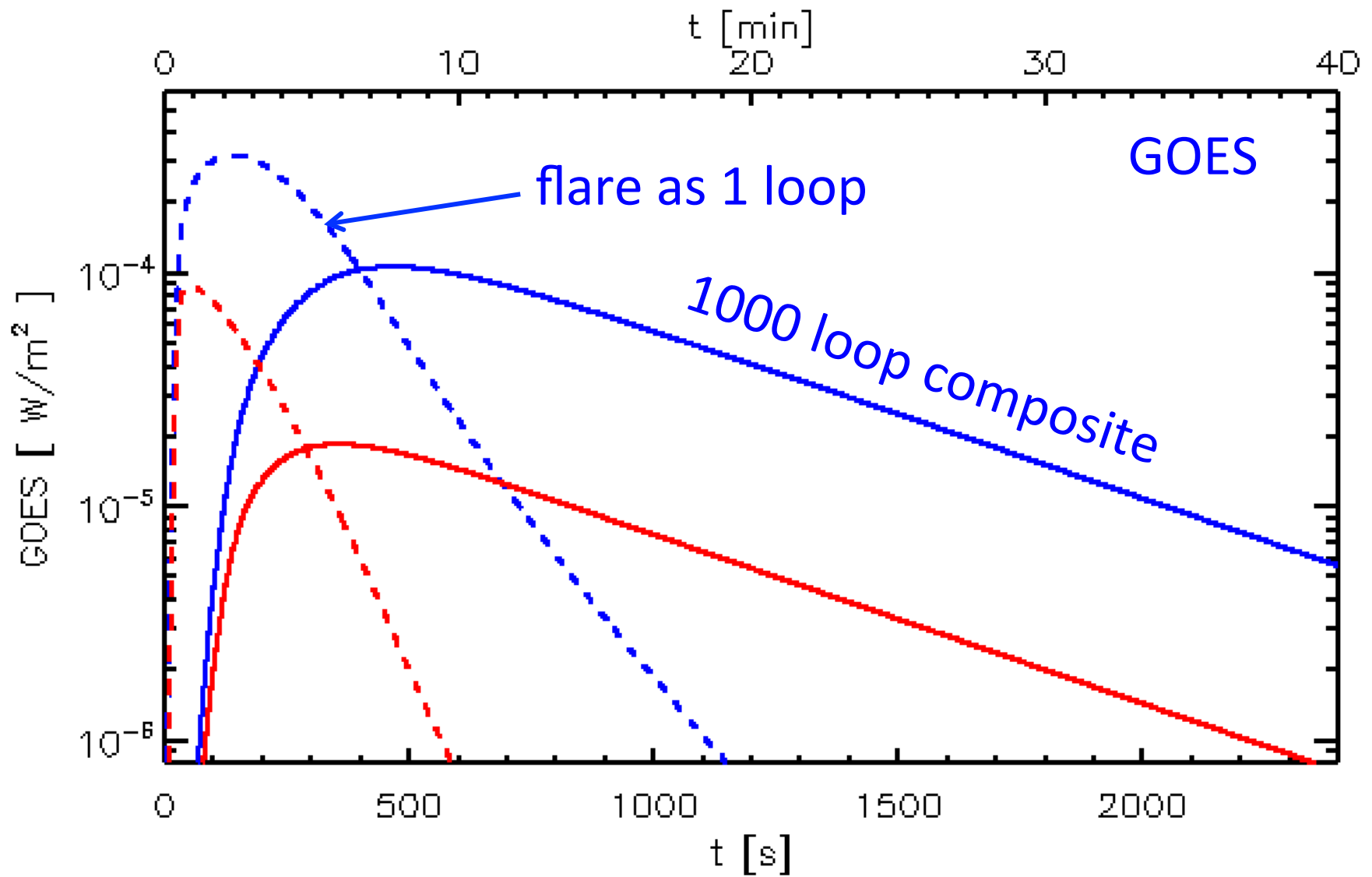
$$E = 2 \times 10^{31} \text{ erg}$$

1000 loops:

- $F_{\text{fl}} = 10^{11} \text{ erg/s/cm}^2$
- $L = 5 \times 10^9 \text{ cm}$
- $E_i = 2 \times 10^{28} \text{ erg}$

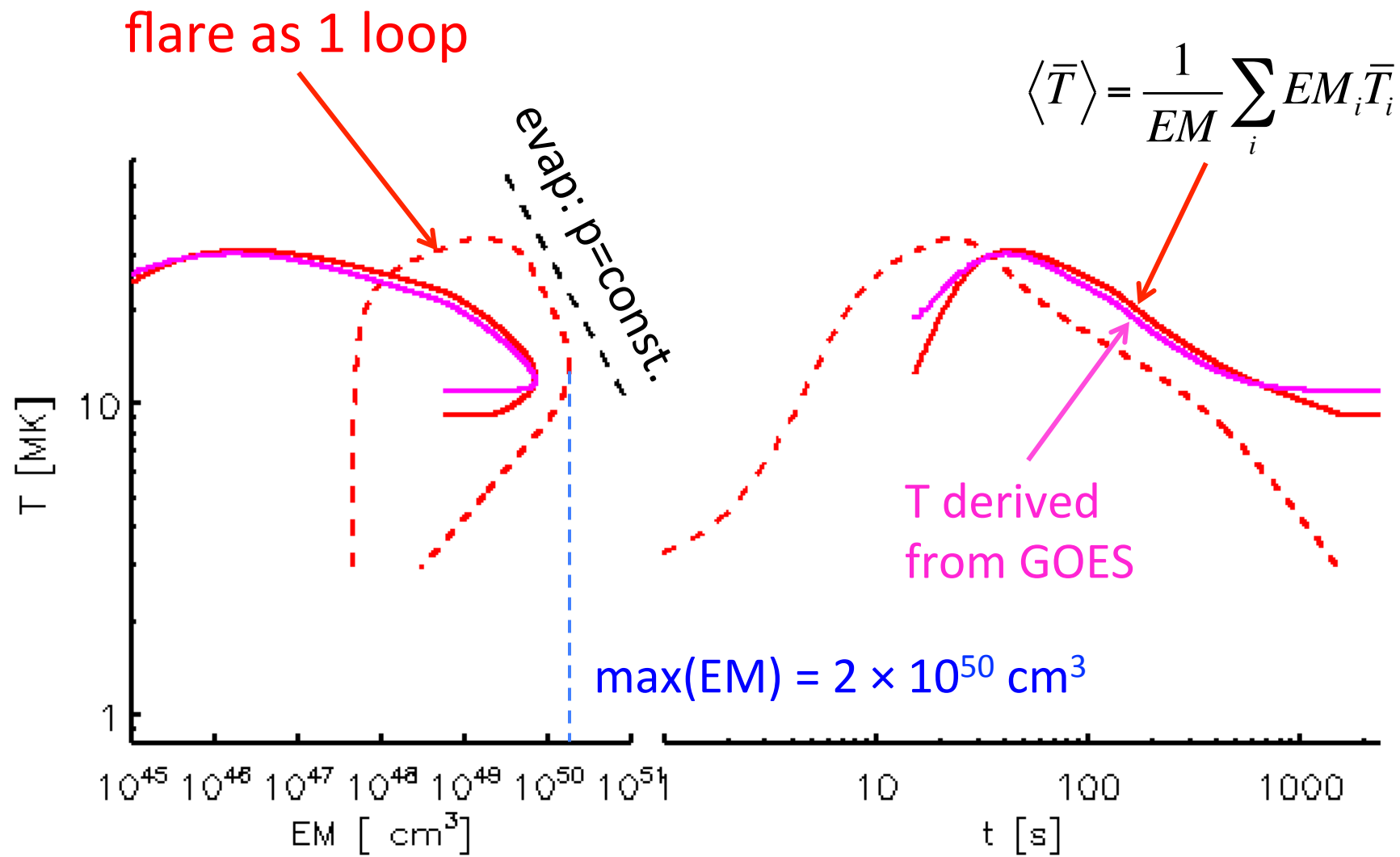
Each loop: $\max(EM) = 7 \times 10^{49} \text{ cm}^3 \frac{E_{30}^{4/3}}{V_{27}^{1/3} L_9^{2/3}} = 0.03 \times 10^{49} \text{ cm}^3$

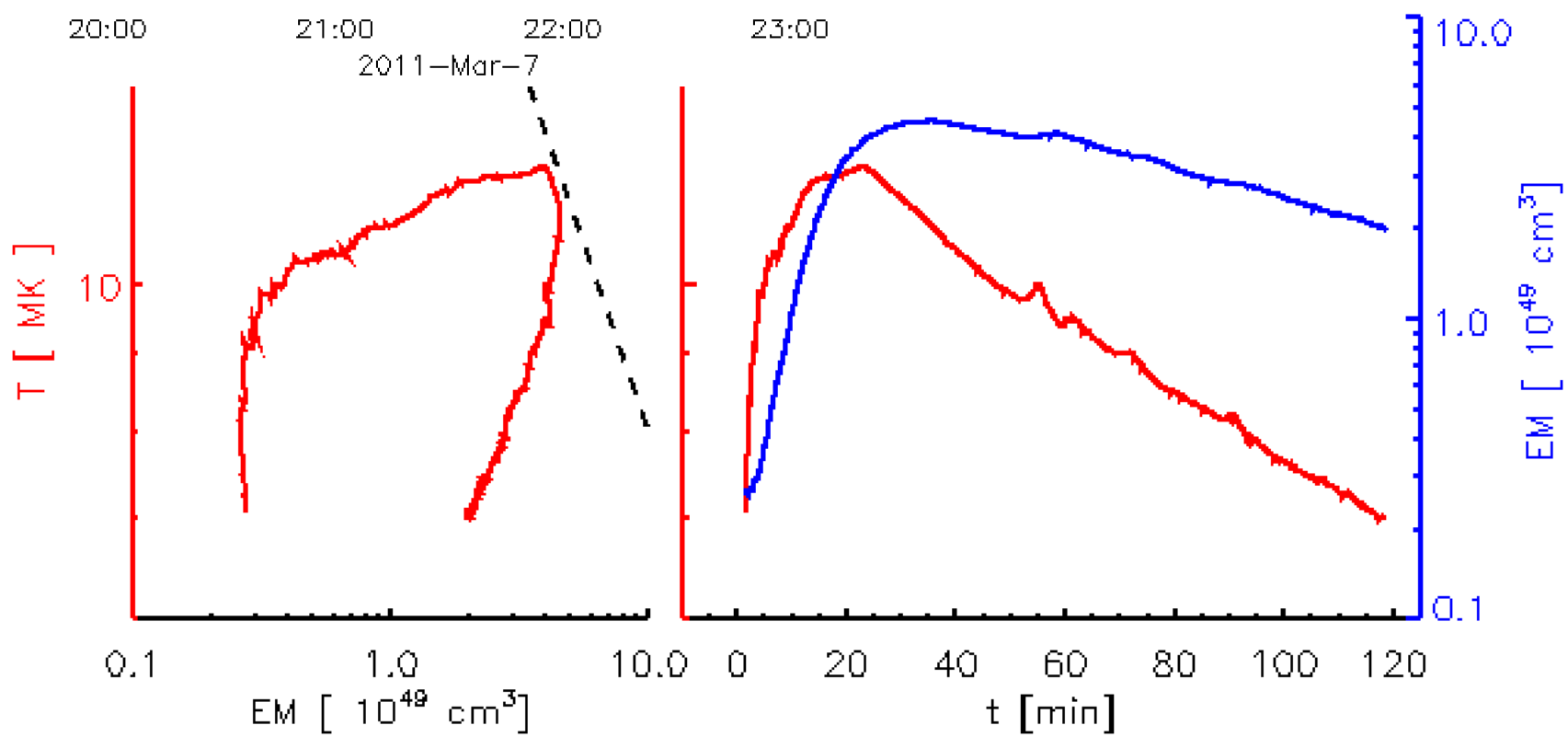
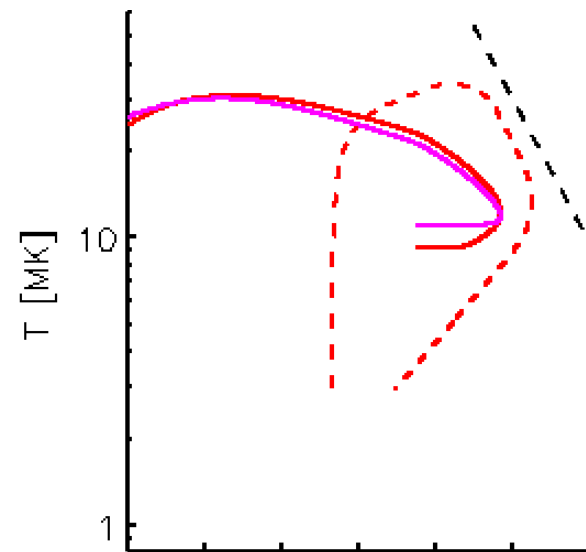
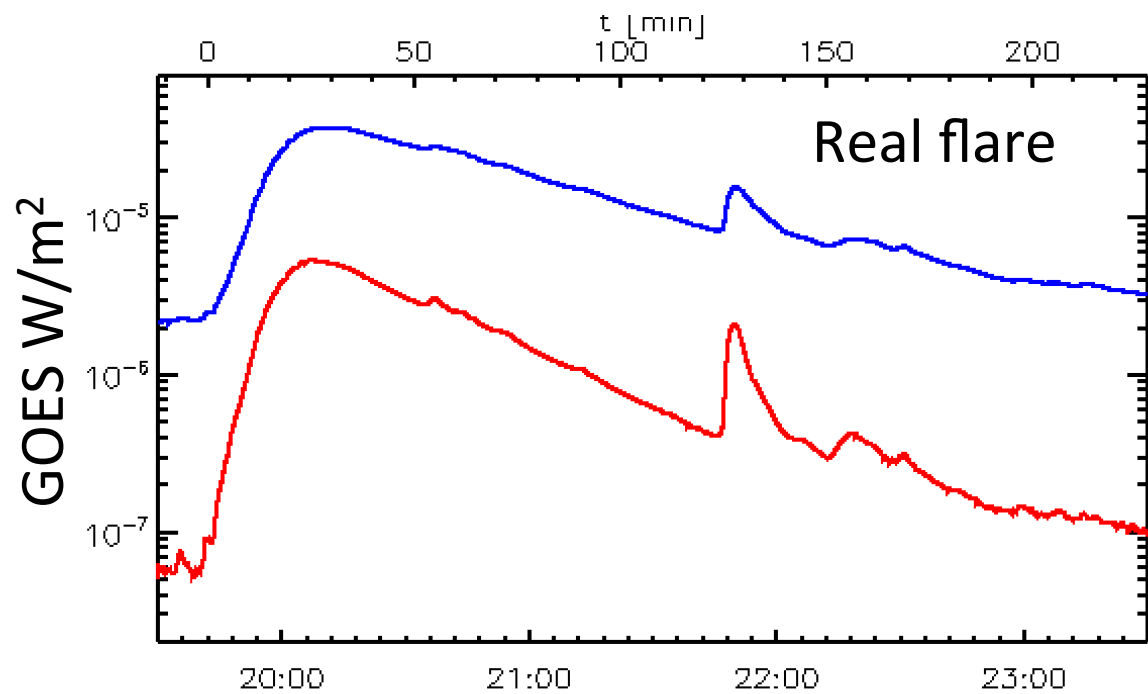
Flare as 1 loop: $\max(EM) = 7 \times 10^{49} \text{ cm}^3 \frac{E_{30}^{4/3}}{V_{27}^{1/3} L_9^{2/3}} = 30 \times 10^{49} \text{ cm}^3$



Flare as 1 loop:

$$F_{1-8} \approx 4 \times 10^{-5} \frac{\text{W}}{\text{m}^2} \cdot \frac{E_{30}^{7/4}}{L_9^{1/4} V_{27}^{3/4}} = 3 \times 10^{-4} \frac{\text{W}}{\text{m}^2}$$



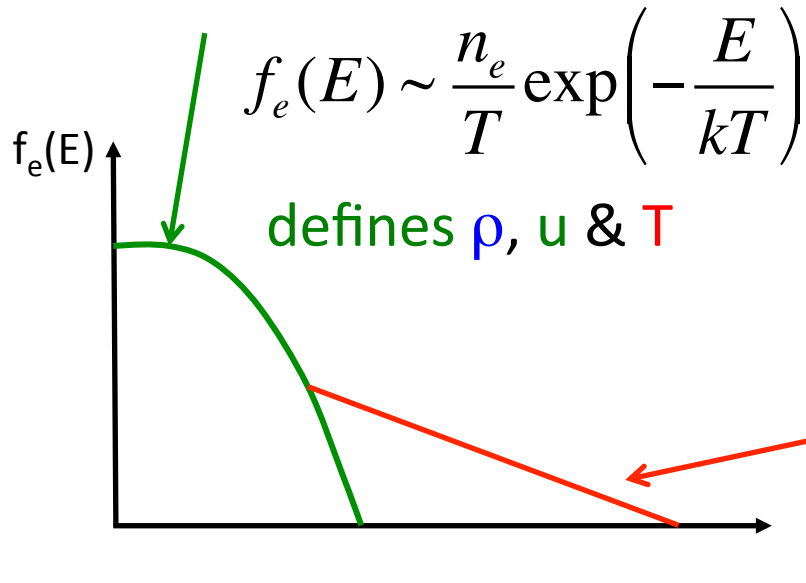


Solving the 1d problem

$$\frac{\partial \rho}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial s} (A \rho u) \quad \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) = -\frac{\partial p}{\partial s} + \rho g_{\parallel} + \frac{\partial}{\partial s} \left(\frac{4}{3} \mu \frac{\partial u}{\partial s} \right)$$

$$\rho c_v \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial s} \right) = -\frac{p}{A} \frac{\partial}{\partial s} (A u) + \frac{4}{3} \mu \left| \frac{\partial u}{\partial s} \right|^2 + \frac{1}{A} \frac{\partial}{\partial s} \left[A \kappa \frac{\partial T}{\partial s} \right] - n_e^2 \Lambda(T) + h(s, t)$$

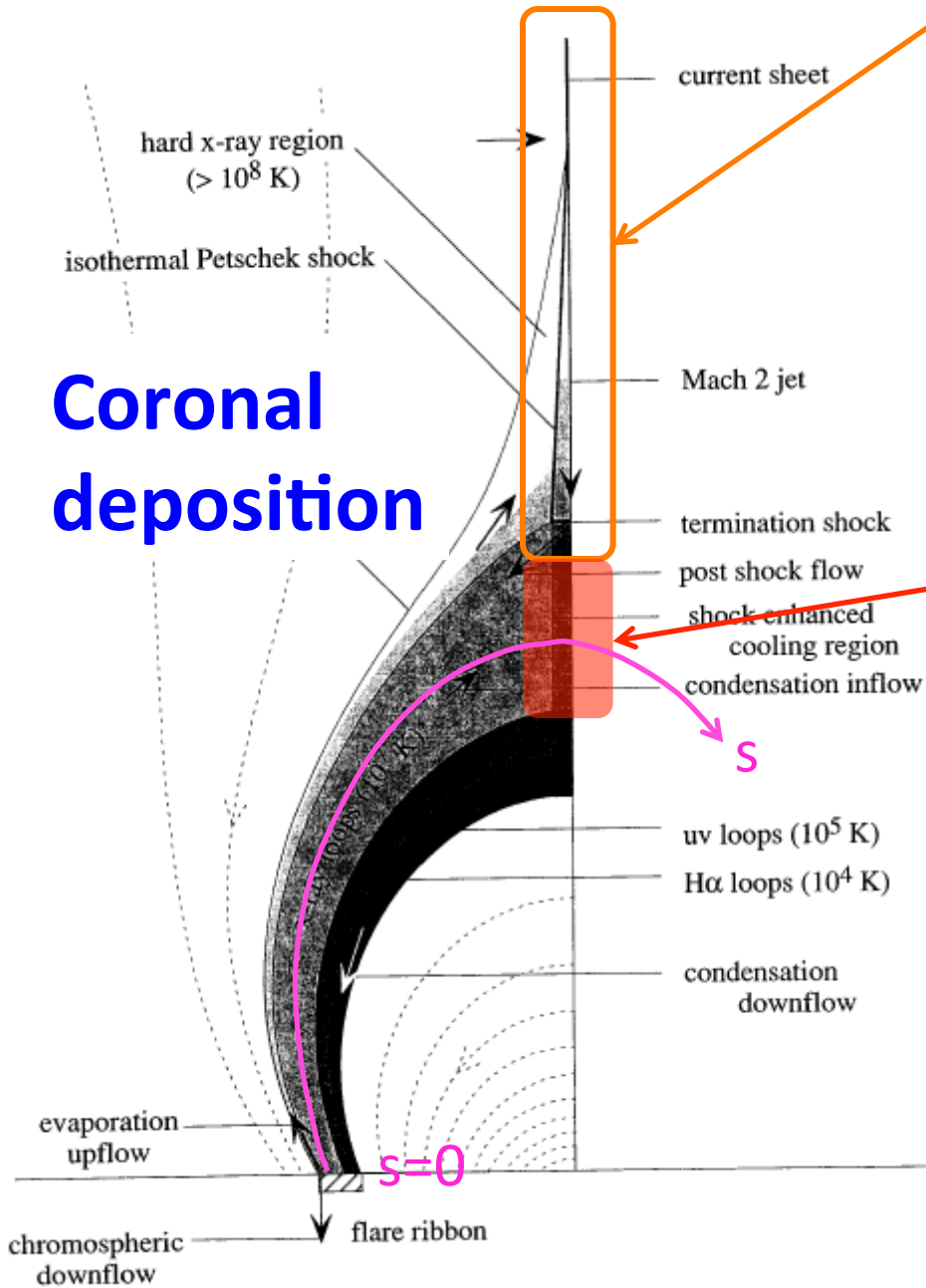
Maxwellian



Source of flare energy:
 heat originating in
 magnetic energy,
 kinetic energy, or
 non-thermal electrons

$$F_{\text{fl}}(t) = \int_0^{L/2} h(s, t) ds$$

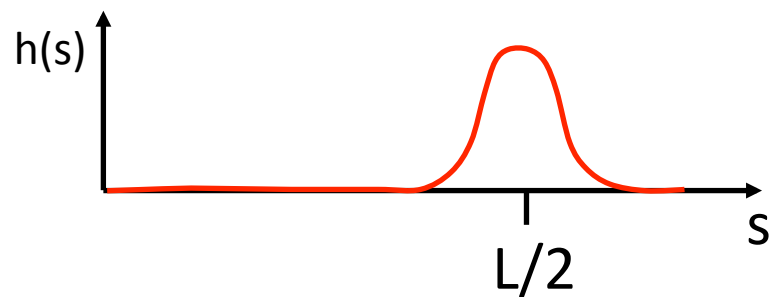
Coronal deposition



Petschek reconnection:

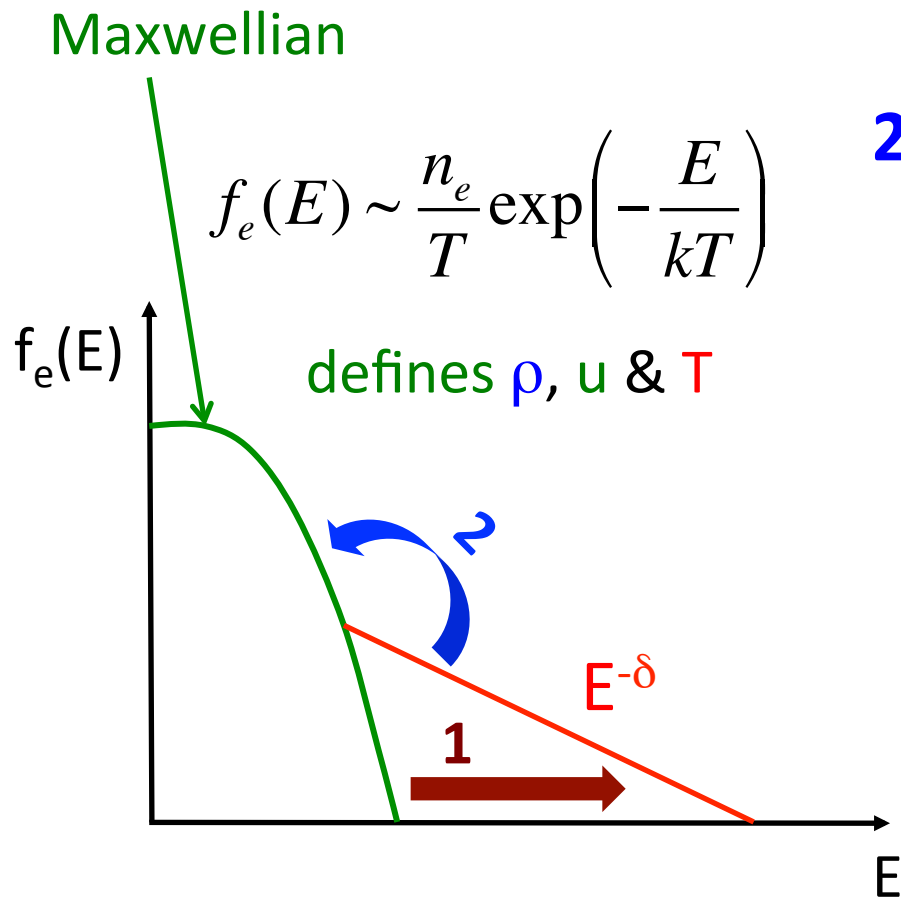
- converts magnetic energy to kinetic energy of outflow
- shocks convert KE to heat

$h(s,t)$ localized to loop apex ($s=L/2$)



$$F_{fl} = \zeta \frac{B^2}{8\pi} v_A = \frac{\zeta}{2\sqrt{\rho}} \left(\frac{B^2}{4\pi} \right)^{3/2}$$

Non-thermal electrons



1. **Some process*** adds energy to subset of e^- s from Maxwellian — creates **NT tail**. Often a power law:

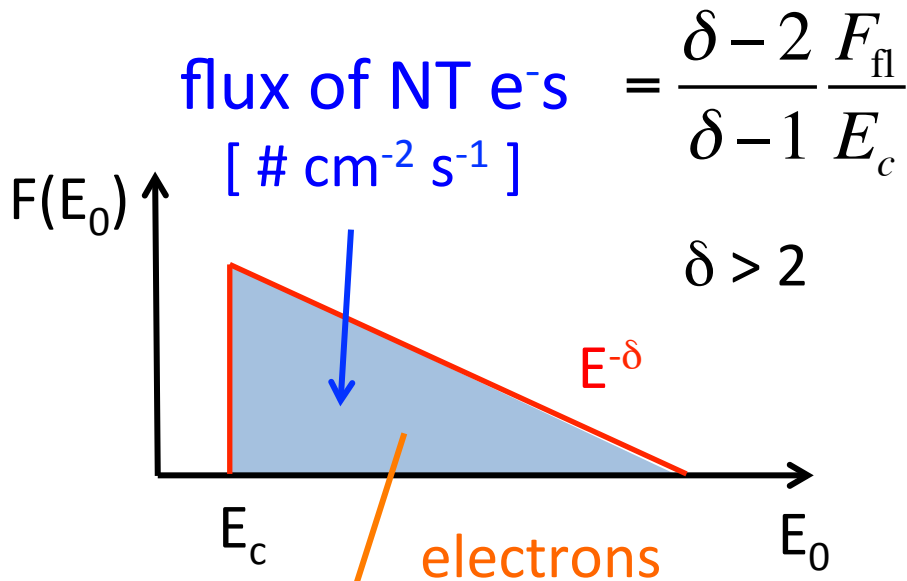
$$f_e(E) \sim E^{-\delta}$$

2. **Collisions** return NT e^- s to Maxwellian: **thermalization**. Adds energy to Maxwellian: heating **h**

Q: where does thermalization occur?
i.e. what is **h(s)**?

*acceleration – more later

Non-thermal e⁻s



collision cross section

$$\sigma_e = 10^{-17} \text{ cm}^2 \times E_{\text{keV}}^{-2}$$

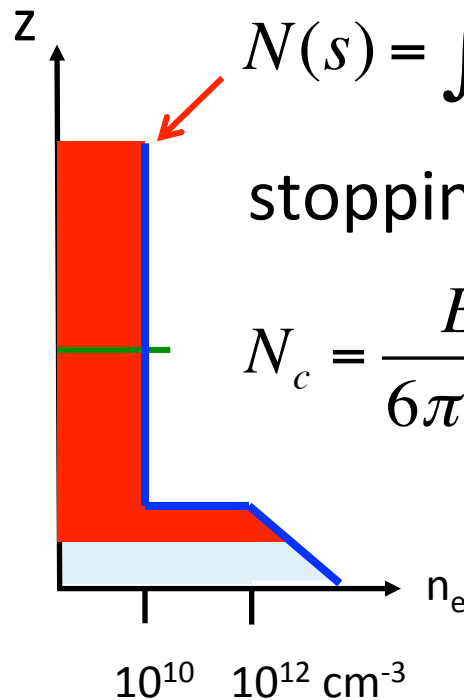
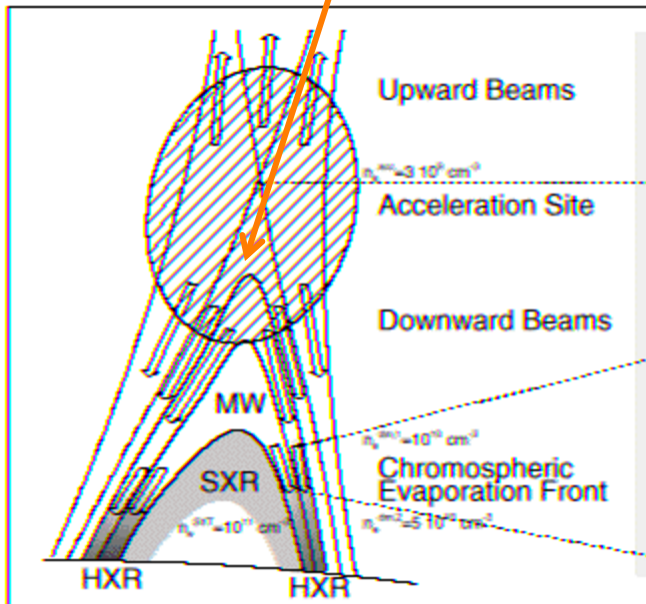
column depth:

$$N(s) = \int^s n_e(s') ds' \quad [\text{cm}^{-2}]$$

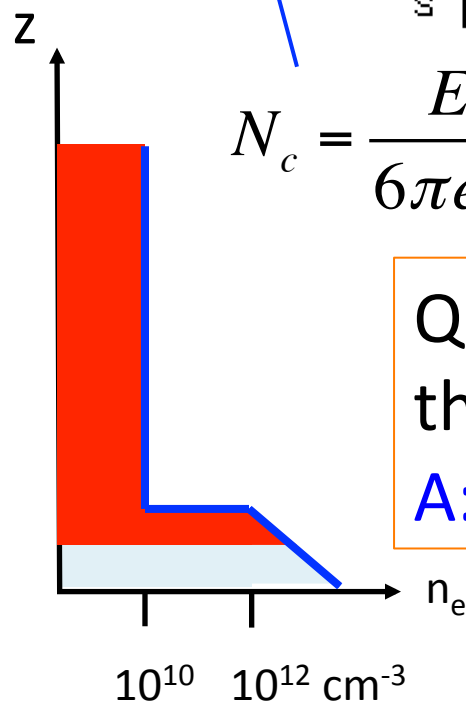
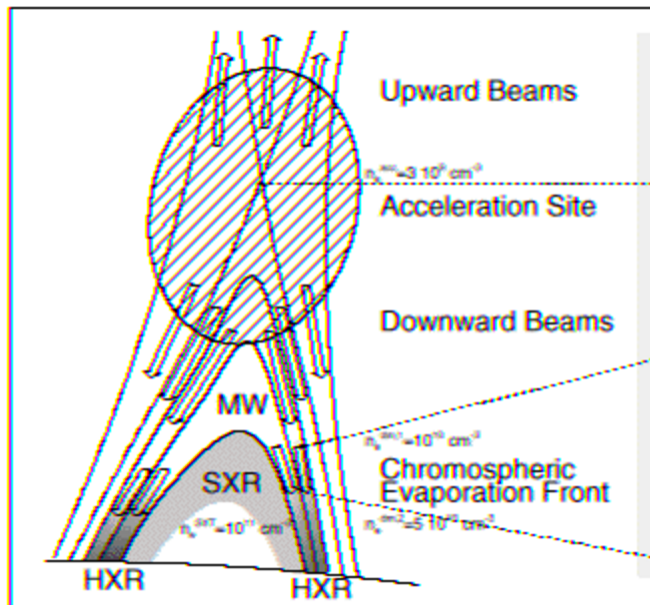
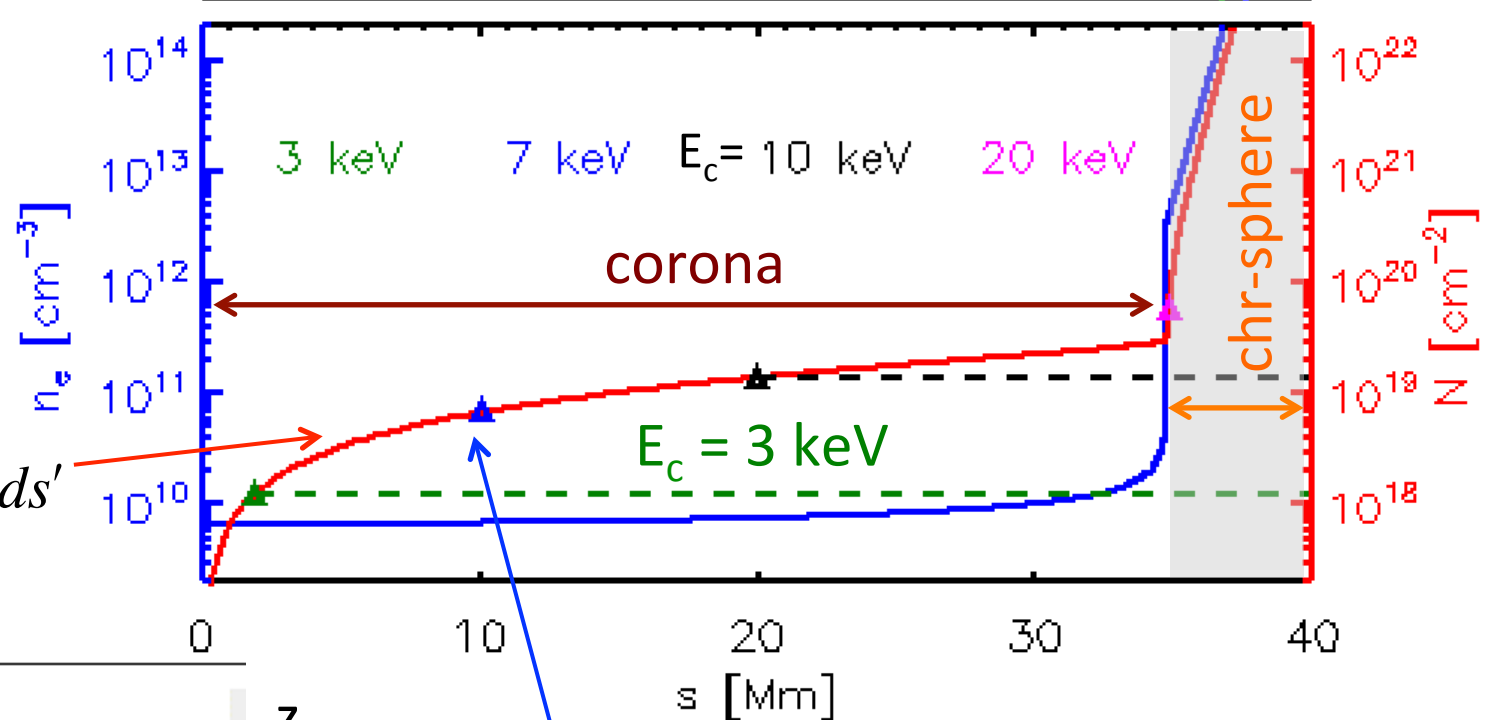
stopping column: $N \sigma_e = 1$

$$N_c = \frac{E_c^2}{6\pi e^4 \Lambda} = 1.4 \times 10^{17} \text{ cm}^{-2} E_{c, \text{keV}}^2$$

stop cut-off



$$N(s) = \int^s n_e(s') ds'$$



$$N_c = \frac{E_c^2}{6\pi e^4 \Lambda} = 1.4 \times 10^{17} \text{ cm}^{-2} E_{c,keV}^2$$

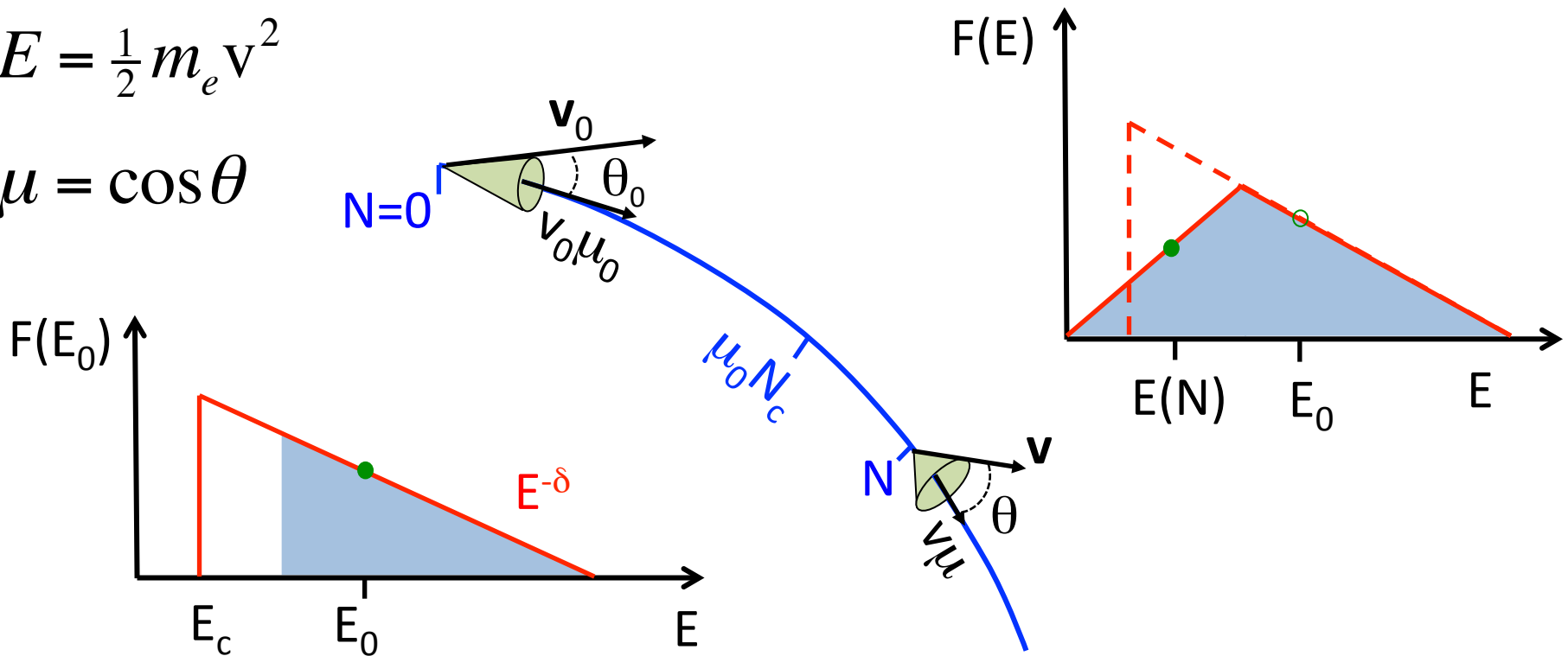
Q: where does thermalization occur?
 A: Where the e⁻s stop

$$\frac{\mu}{\mu_0} = \frac{E}{E_0} \quad \frac{dE}{dN} = -\frac{2\pi e^4 \Lambda}{\mu E} = -\frac{2\pi e^4 \Lambda}{\mu_0 E_0} \left(\frac{E_0}{E}\right)^2$$

$$\frac{E(N)}{E_0} = \left[1 - \frac{6\pi e^4 \Lambda}{\mu_0 E_0^2} N\right]^{1/3} = \left[1 - \frac{E_c^2}{E_0^2} \frac{N}{\mu_0 N_c}\right]^{1/3}$$

$$E = \frac{1}{2} m_e v^2$$

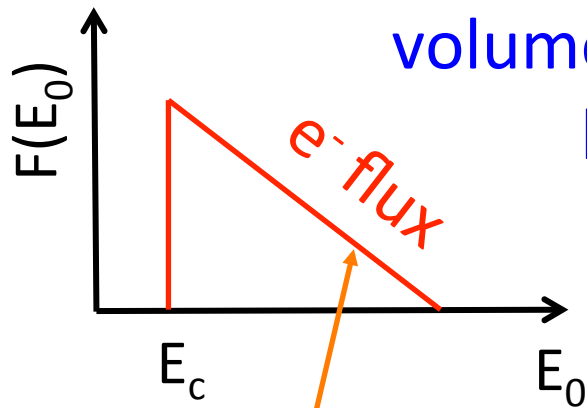
$$\mu = \cos \theta$$



$$E(N) = E_0 \left[1 - \frac{E_c^2}{E_0^2} \frac{N}{\mu_0 N_c} \right]^{1/3} \quad \frac{dE}{dN} = - \frac{E_c^2}{3\mu_0 N_c E_0} \left(\frac{E_0}{E} \right)^2$$

$$\text{loss per NT e}^- \frac{dE}{ds} = \frac{dN}{ds} \frac{dE}{dN} = - \frac{n_e E_c^2}{3\mu_0 N_c E_0} \left[1 - \frac{E_c^2}{E_0^2} \frac{N}{\mu_0 N_c} \right]^{-2/3}$$

gain by plasma:
volumetric
heat



$$h(N, \mu_0) = \int \left(- \frac{dE}{ds} \right) F(E_0) dE_0$$

$$= \frac{n_e E_c^2}{\mu_0 N_c} \int \left[1 - \frac{E_c^2}{\mu_0 E_0^2} \frac{N}{N_c} \right]^{-2/3} F(E_0) \frac{dE_0}{E_0}$$

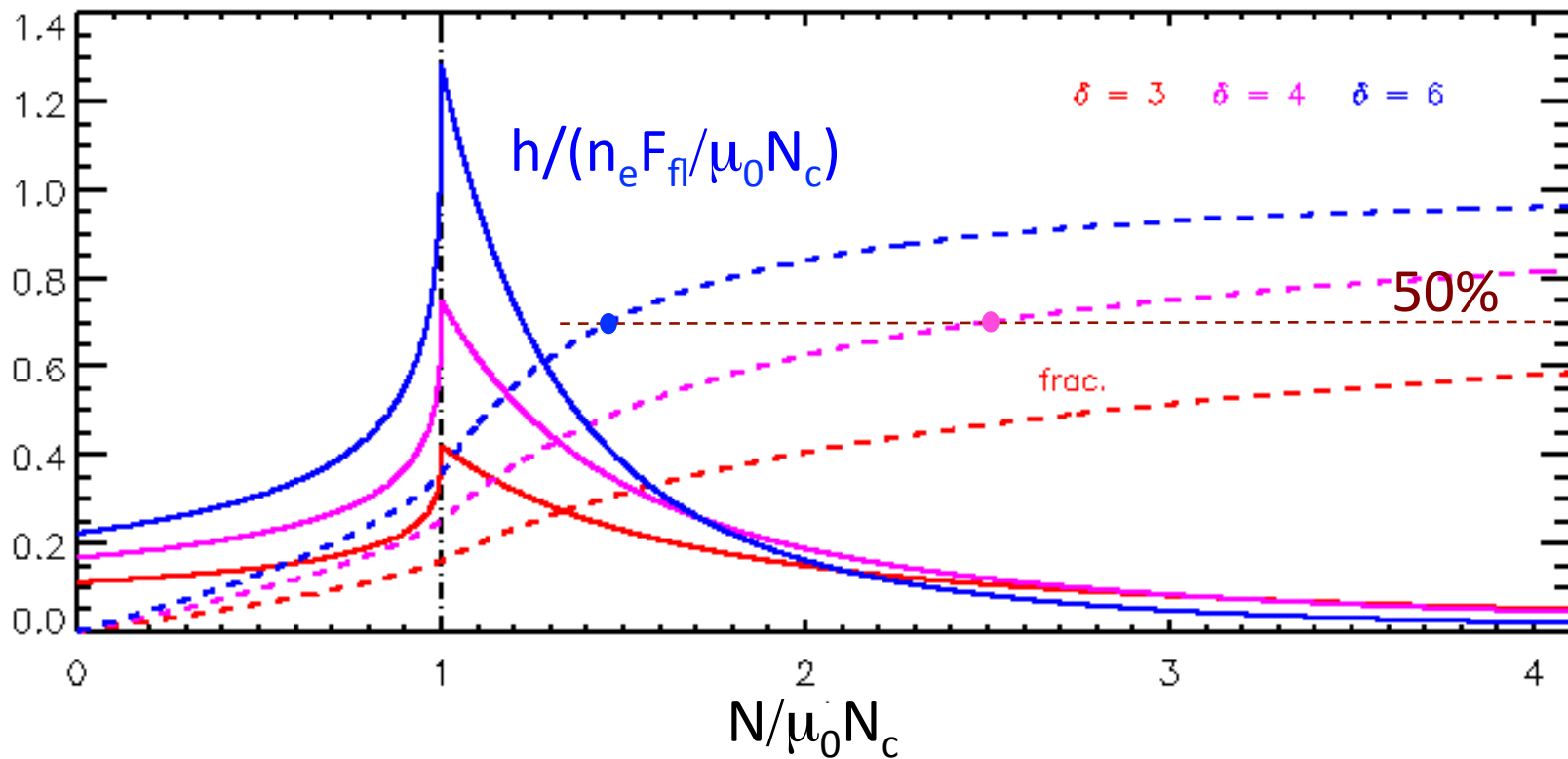
$$F(E_0) = (\delta - 2) \frac{F_{\text{fl}}(t)}{E_c^2} \left(\frac{E_0}{E_c} \right)^{-\delta}$$



$$h \sim F_{\text{fl}}(t) \frac{n_e(s)}{\mu_0 N_c} \left(\frac{N}{N_c \mu_0} \right)^{-\delta/2}$$

NT e⁻ heating

$$h = \frac{\delta - 2}{6} \left(\frac{n_e F_{\text{fl}}}{\mu_0 N_c} \right) \left(\frac{N}{\mu_0 N_c} \right)^{-\delta/2} \begin{cases} B\left(\frac{N}{\mu_0 N_c}; \frac{\delta}{2}, \frac{1}{3}\right) & , N < \mu_0 N_c \\ B\left(\frac{\delta}{2}, \frac{1}{3}\right) = \frac{\Gamma(\frac{1}{2}\delta + \frac{1}{3})}{\Gamma(\frac{1}{2}\delta)\Gamma(\frac{1}{3})} & , N > \mu_0 N_c \end{cases}$$



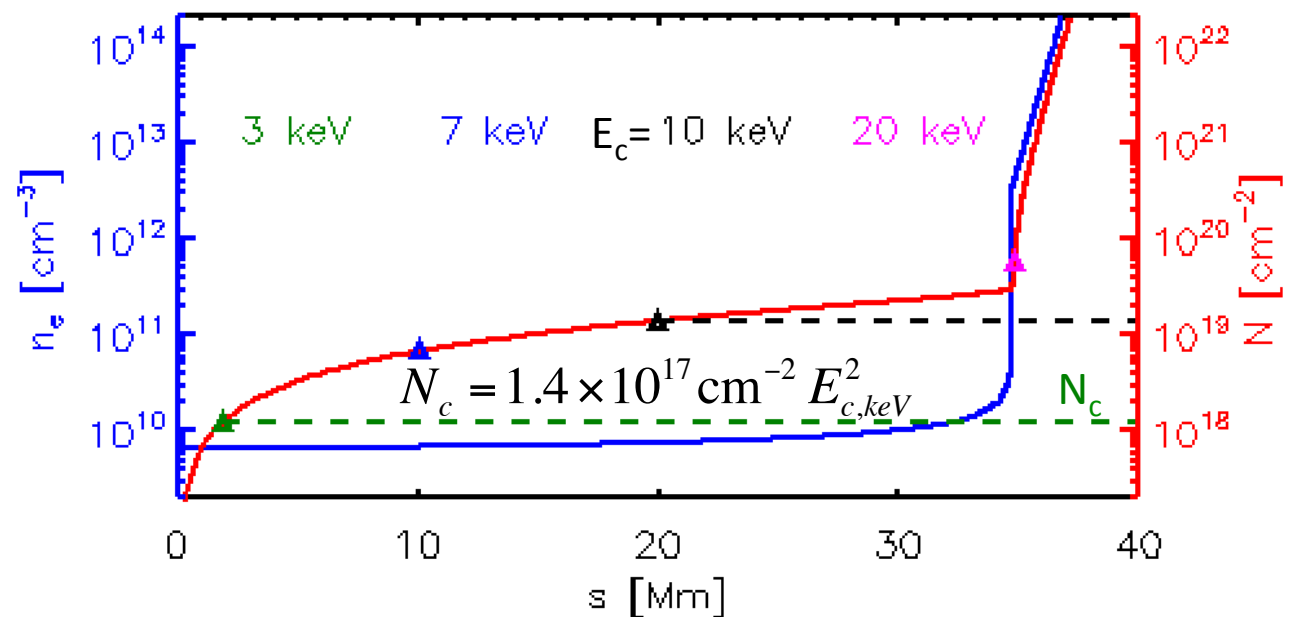
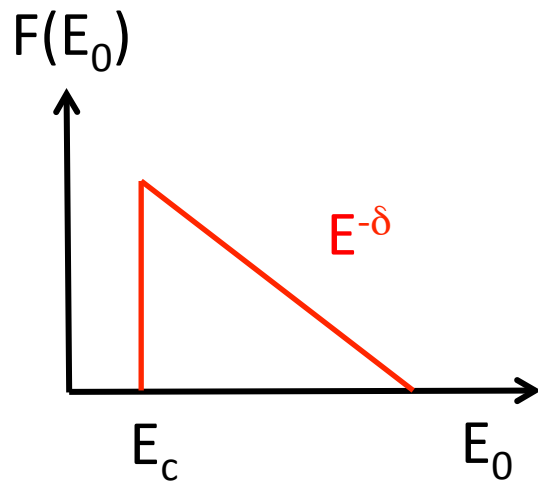
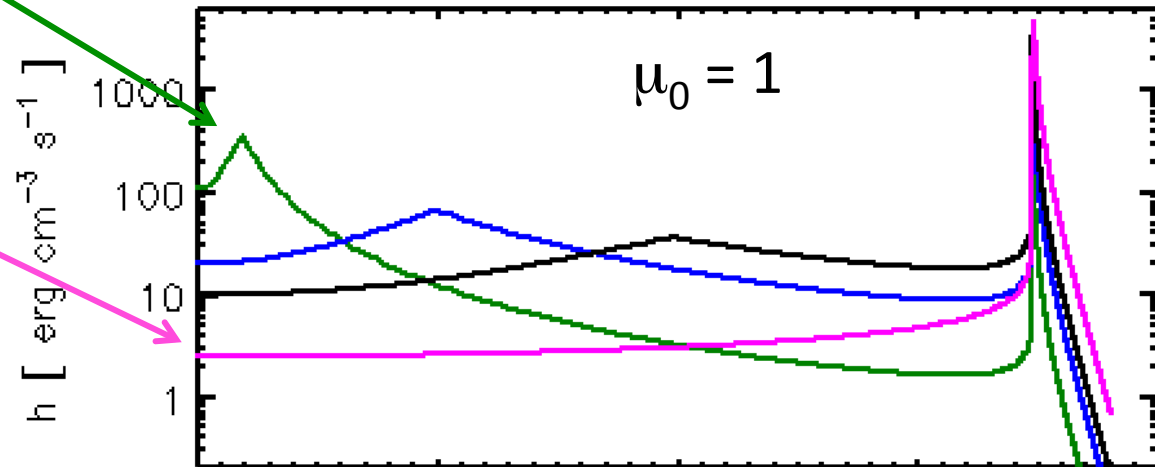
$$h \sim F_{\text{fl}}(t) \frac{n_e(s)}{\mu_0 N_c} \left(\frac{N}{N_c \mu_0} \right)^{-\delta/2}$$

$E_c = 3$ keV:
coronal
deposition

$E_c = 20$ keV:
chr-spheric
deposition

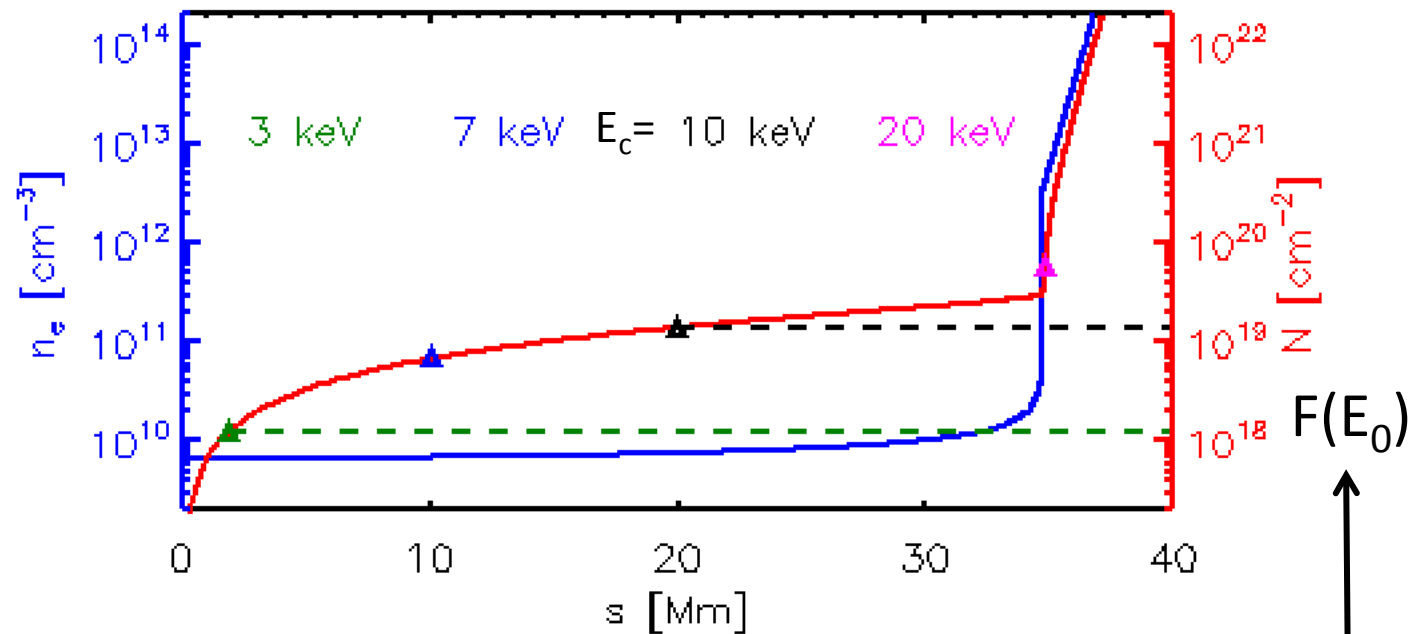
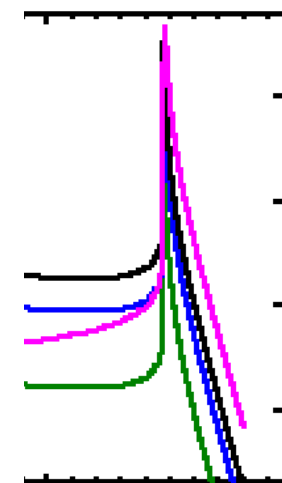
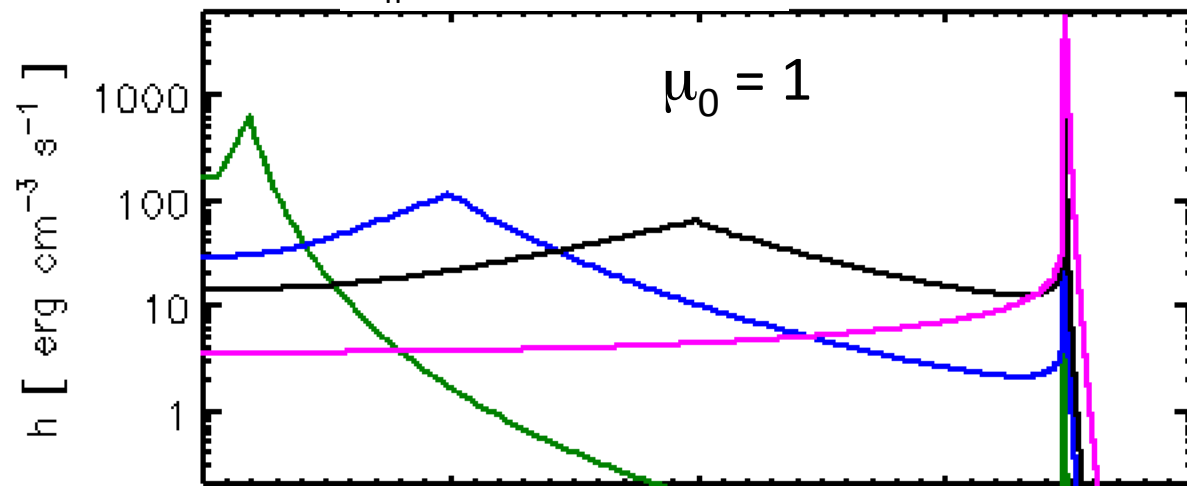
$$F_{\text{fl}} = 10^{11} \text{ erg cm}^{-2} \text{ s}^{-1} \quad \delta = 4.0$$

$$\mu_0 = 1$$

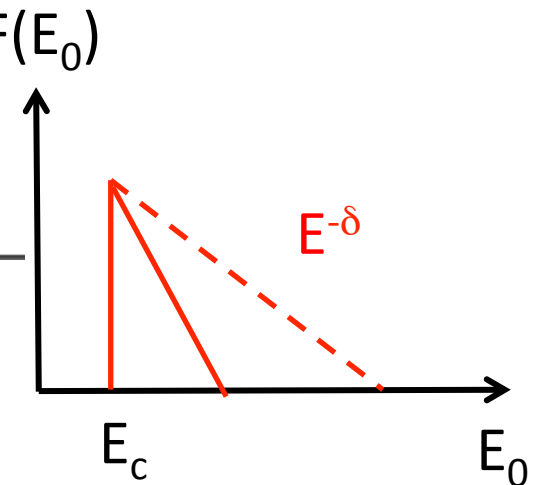


$F_{\text{fl}} = 10^{11} \text{ erg cm}^{-2} \text{ s}^{-1}$ $\delta = 7.0$

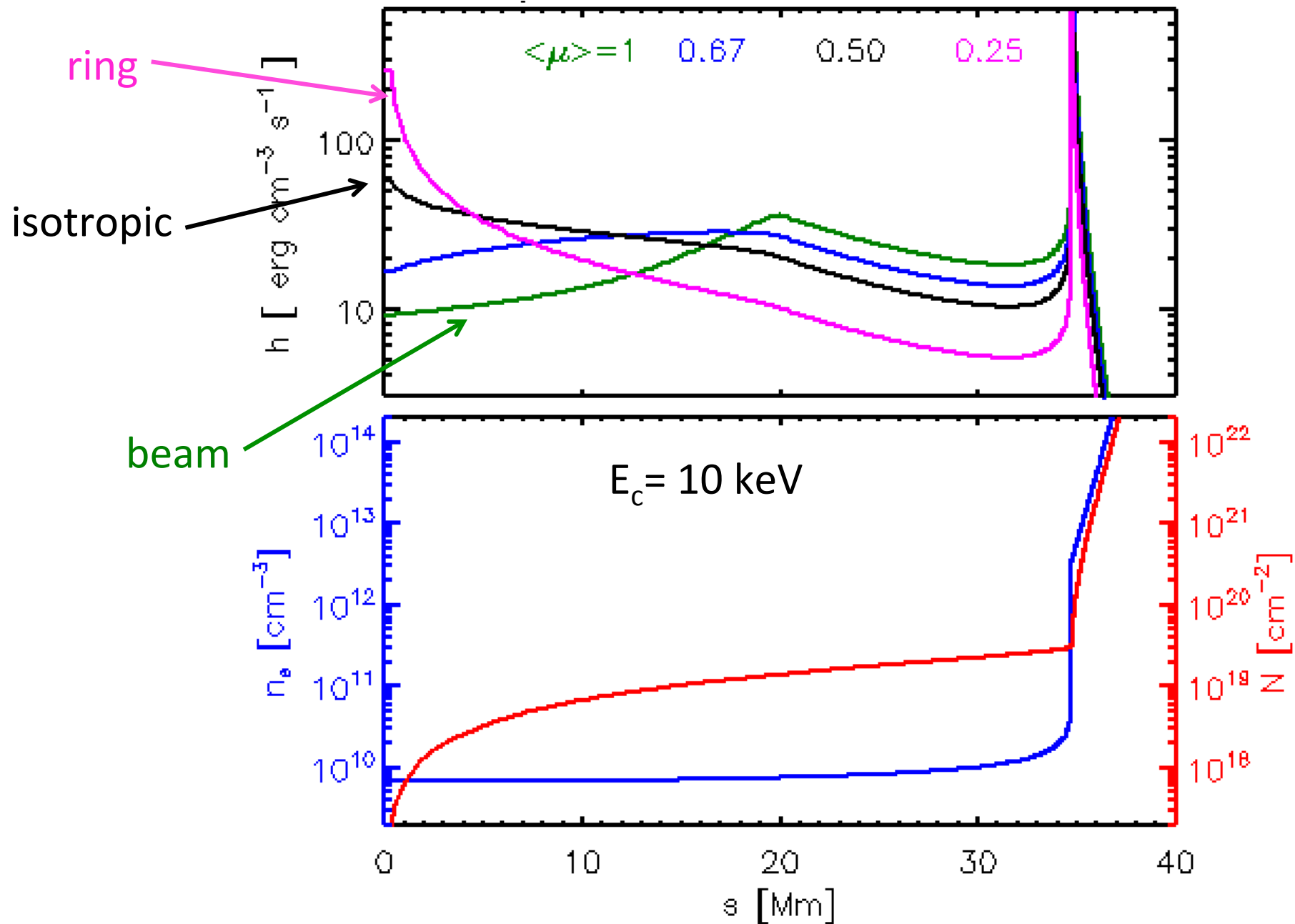
$\delta = 4$



softer
spectrum



$$F_{\text{fl}} = 10^{11} \text{ erg cm}^{-2} \text{ s}^{-1} \quad \delta = 4.0$$



Coronal deposition

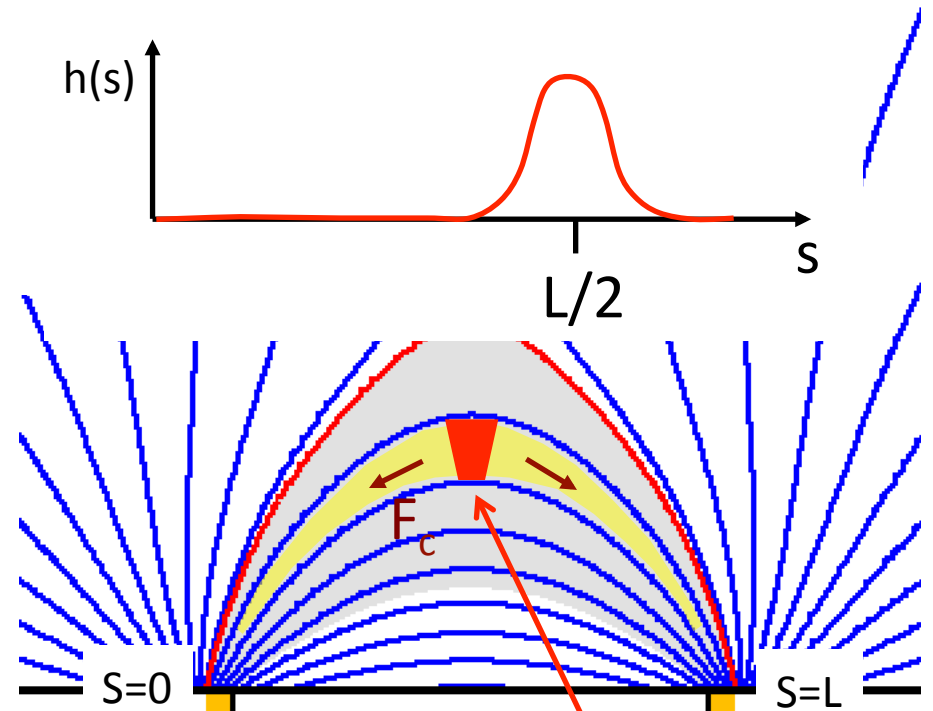
Reconnection, or small E_c ,
or large δ , or small μ_0

$$\frac{\partial \rho}{\partial t} = -\frac{1}{A} \frac{\partial}{\partial s} (A \rho u)$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) = -\frac{\partial p}{\partial s} + \rho g_{\parallel} + \frac{\partial}{\partial s} \left(\frac{4}{3} \mu \frac{\partial u}{\partial s} \right)$$

$$\rho c_v \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial s} \right) = -\frac{p}{A} \frac{\partial}{\partial s} (A u) + \frac{4}{3} \mu \left| \frac{\partial u}{\partial s} \right|^2 + \frac{1}{A} \frac{\partial}{\partial s} \left(A \underbrace{\kappa \frac{\partial T}{\partial s}}_{F_c} \right) - n_e^2 \Lambda(T) + h$$

$$p = \frac{k_b}{\bar{m}} \rho T \quad c_v = \frac{3}{2} \frac{k_b}{\bar{m}}$$



Conductive flux

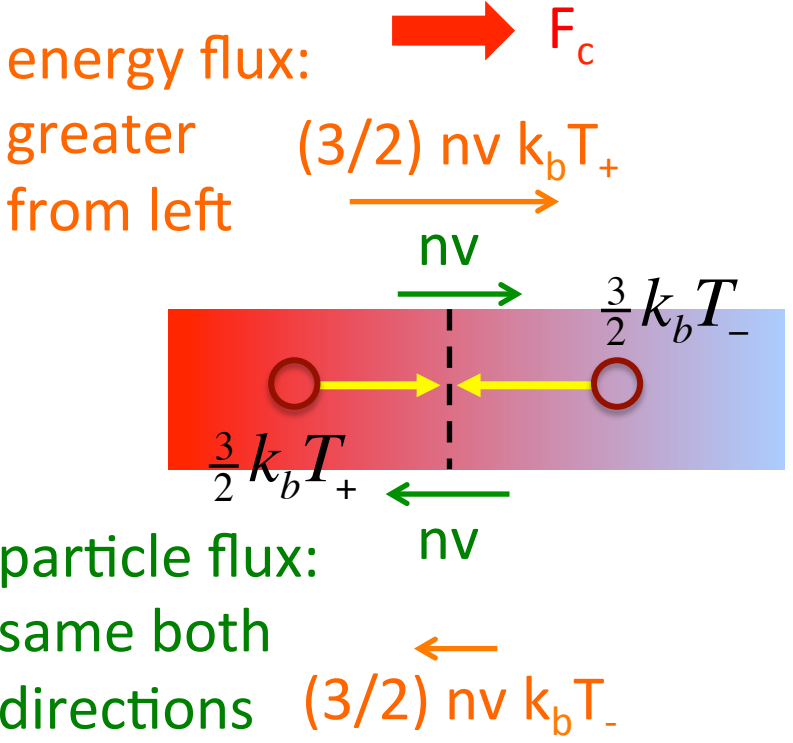
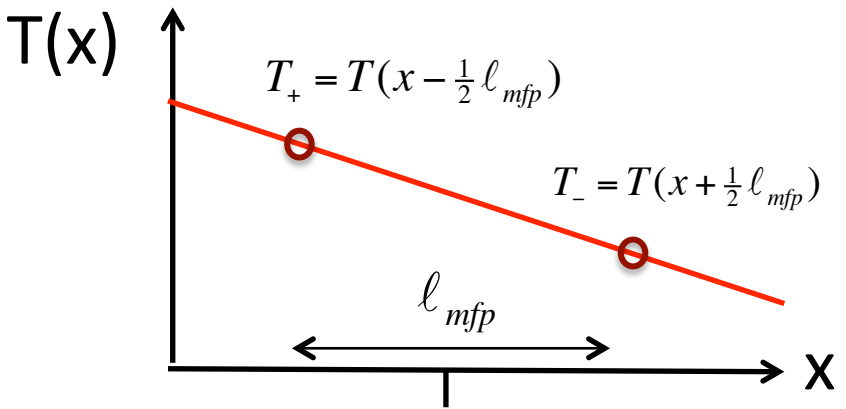
$$F_c = \frac{3}{2} n v k_b (T_+ - T_-)$$

IF gradient is shallow
or m.f.p. is small

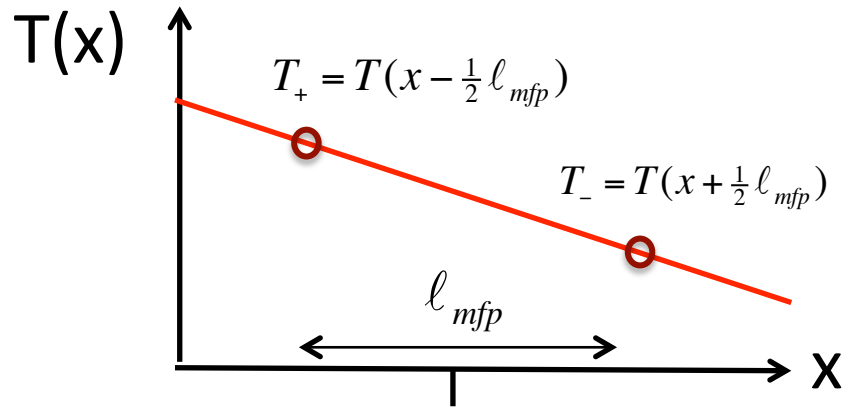
$$F_c \approx - \underbrace{\left(\frac{3}{2} n v k_b \ell_{mfp} \right)}_{= \kappa} \frac{\partial T}{\partial x}$$

$$\vec{F}_c = -\kappa \nabla T$$

Fourier's law –
classical heat flux



Conductive flux



$$F_c = \frac{3}{2} n v k_b (T_+ - T_-)$$

IF gradient is shallow
or m.f.p. is small

$$\kappa = \frac{3}{2} k_b v_{th} n \ell_{mfp} = \frac{3}{2} \frac{k_b v_{th}}{\sigma_{sc}}$$

$$\sigma_{sc} \sim \frac{q^4}{m^2 v_{th}^4} \quad \text{Rutherford scattering}$$

$$\kappa \sim \frac{k_b m^2 v_{th}^5}{q^4} \sim \left(\frac{k_b^{7/2}}{m^{1/2} q^4} \right) T^{5/2}$$

$$F_c \approx - \boxed{\frac{3}{2} n v k_b \ell_{mfp}} \frac{\partial T}{\partial x} = \kappa$$

e⁻s : smallest m →
most heat flux

$$\kappa = \kappa_0 T^{5/2} \quad [\text{erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-1}]$$

$$\kappa_0 \approx 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-7/2}$$

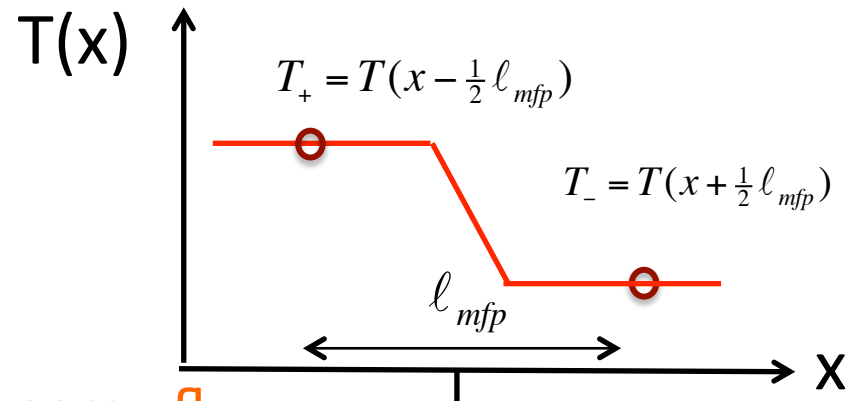
Conductive flux

$$F_c = \frac{3}{2} n v k_b (T_+ - T_-)$$

IF NOT

$$|F_c| < \frac{3}{2} n v_{th} k_b T_+ = \frac{3}{2} n_e \frac{k_b^{3/2}}{m_e^{1/2}} T_e^{3/2} = F_{fs}$$

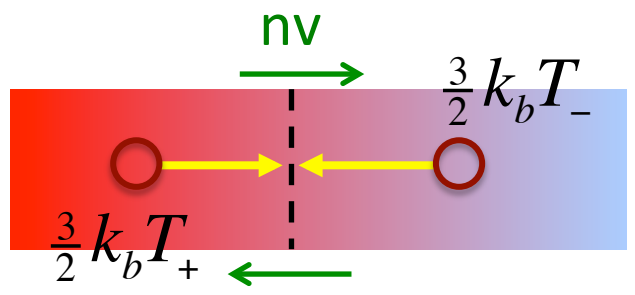
**Free-streaming
heat flux:
upper bound**



energy flux:
greater from
left



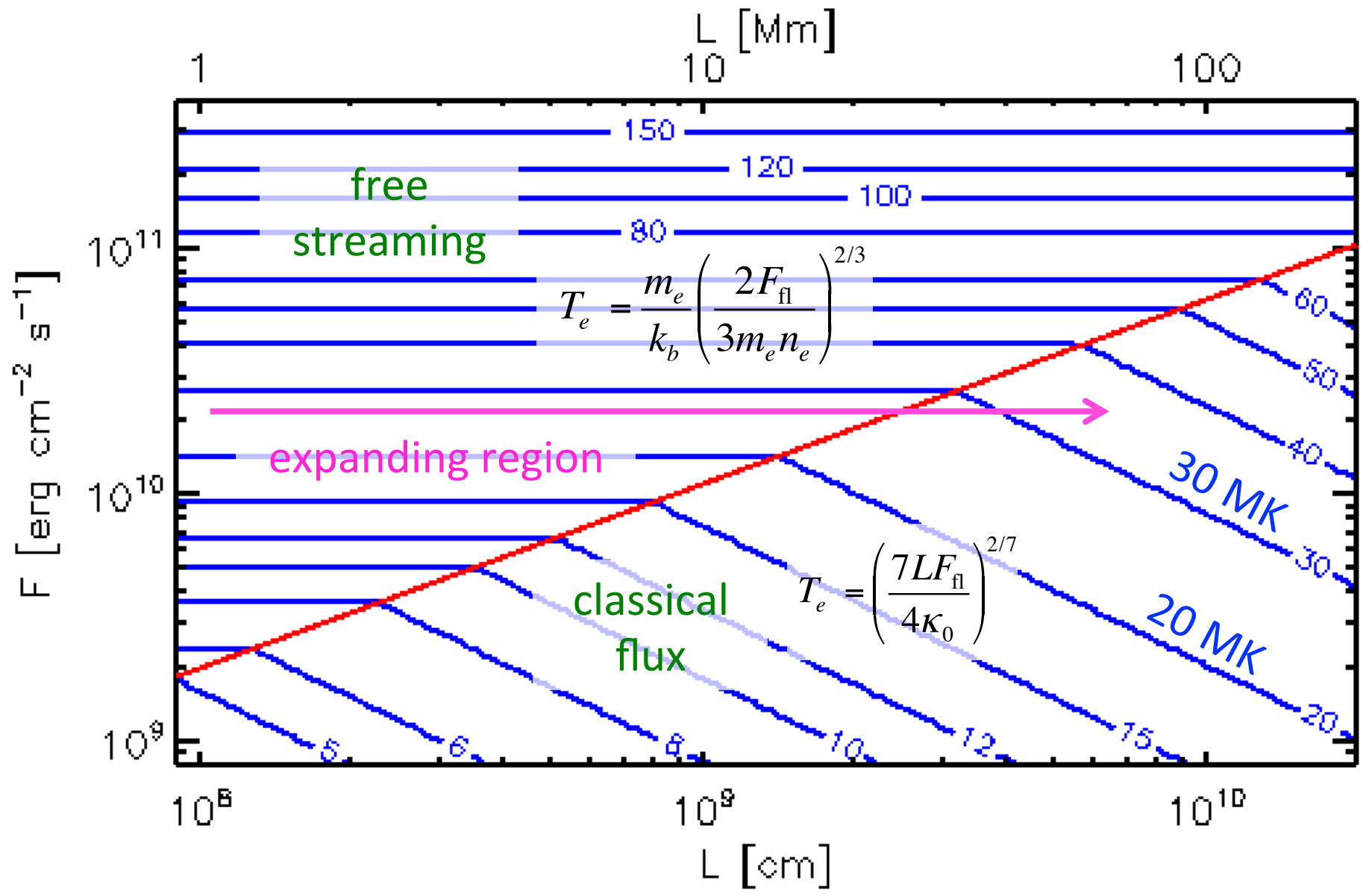
$$\frac{3}{2} n v k_b T_+$$



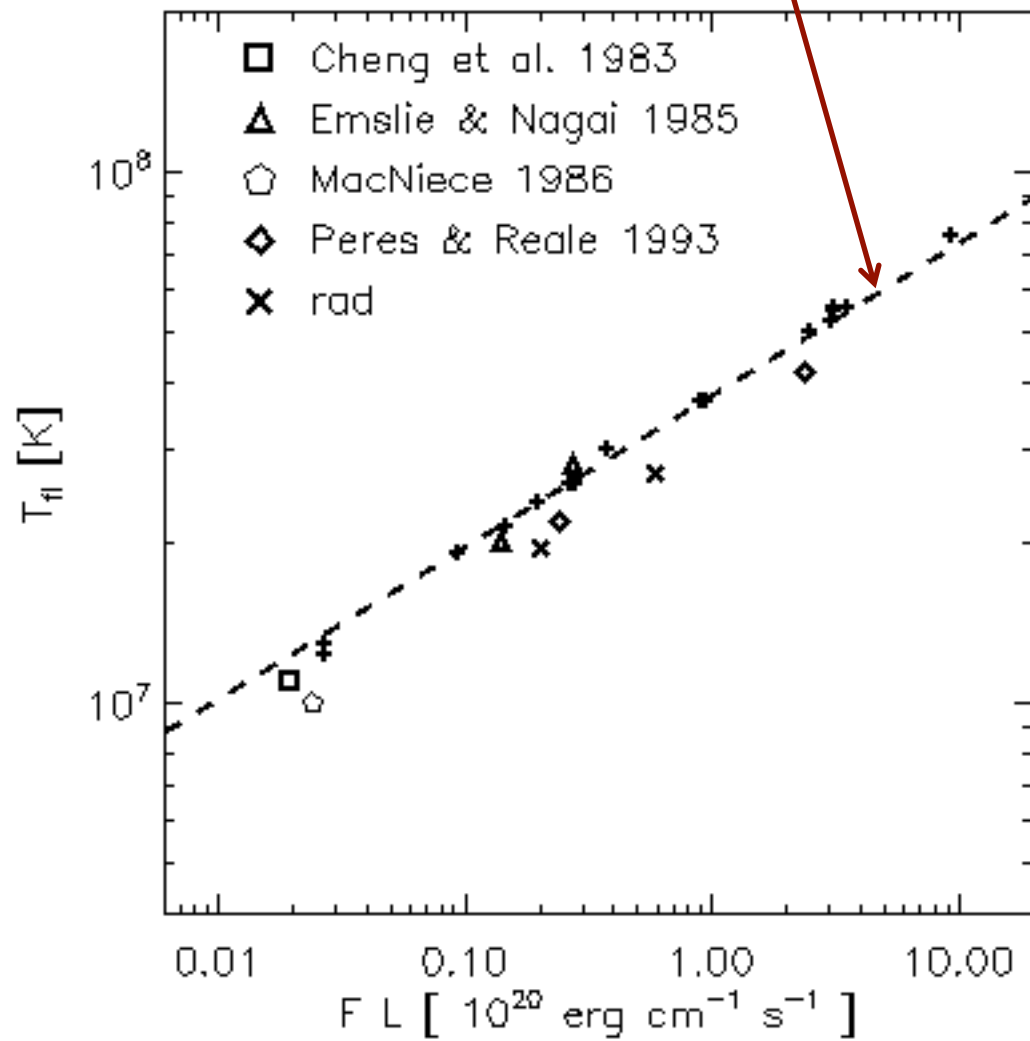
particle flux:
same both
directions

$$\frac{3}{2} n v k_b T_-$$

$$n_e = 3 \times 10^9 \text{ cm}^{-3}$$

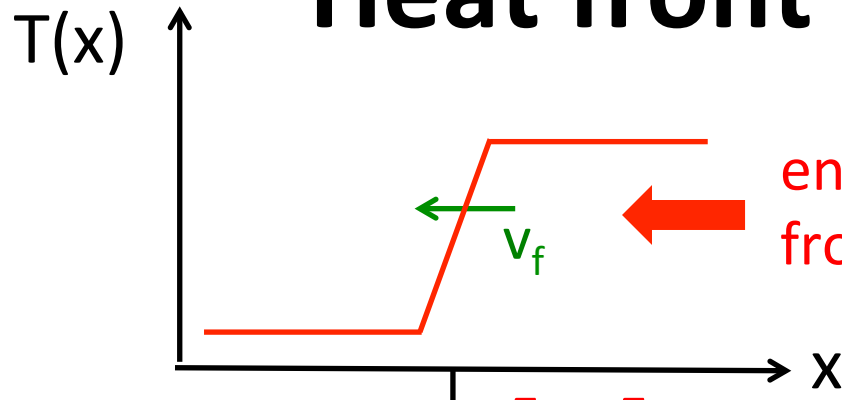


$$T_{fl} = C_T (FL/\kappa_0)^{2/7}, \quad C_T=1.46$$



Longscope 2014

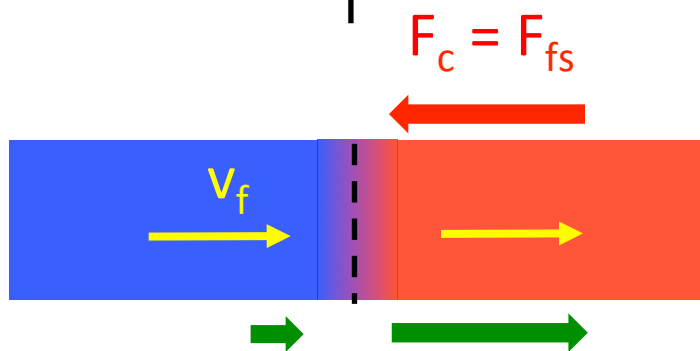
Heat front



$$|F_c| = \frac{3}{2} n_e \frac{k_b^{3/2}}{m_e^{1/2}} T_e^{3/2}$$

free-streaming
heat flux

energy flux
from flare: F_{fl}



$$k_b T_e = m_e \left(\frac{2F_{fl}}{3m_e n_e} \right)^{2/3}$$

Co-moving
frame

enthalpy flux

$$5v_f n_e k_b T_e$$

$$= 5v_f n_e m_e \left(\frac{2F_{fl}}{3m_e n_e} \right)^{2/3} = F_{fl}$$

speed of
heat front

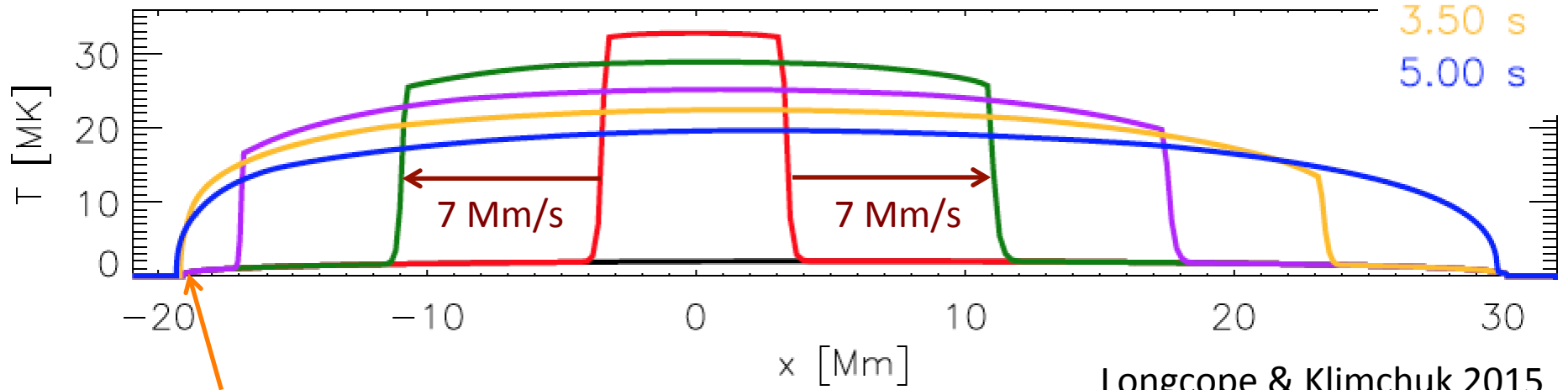
$$v_f = 0.26 \left(\frac{F_{fl}}{m_e n_e} \right)^{1/3}$$

$$F_{\text{fl}} = 1.2 \times 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1}$$

$$n_e = 10^9 \text{ cm}^{-3} \text{ at first}$$

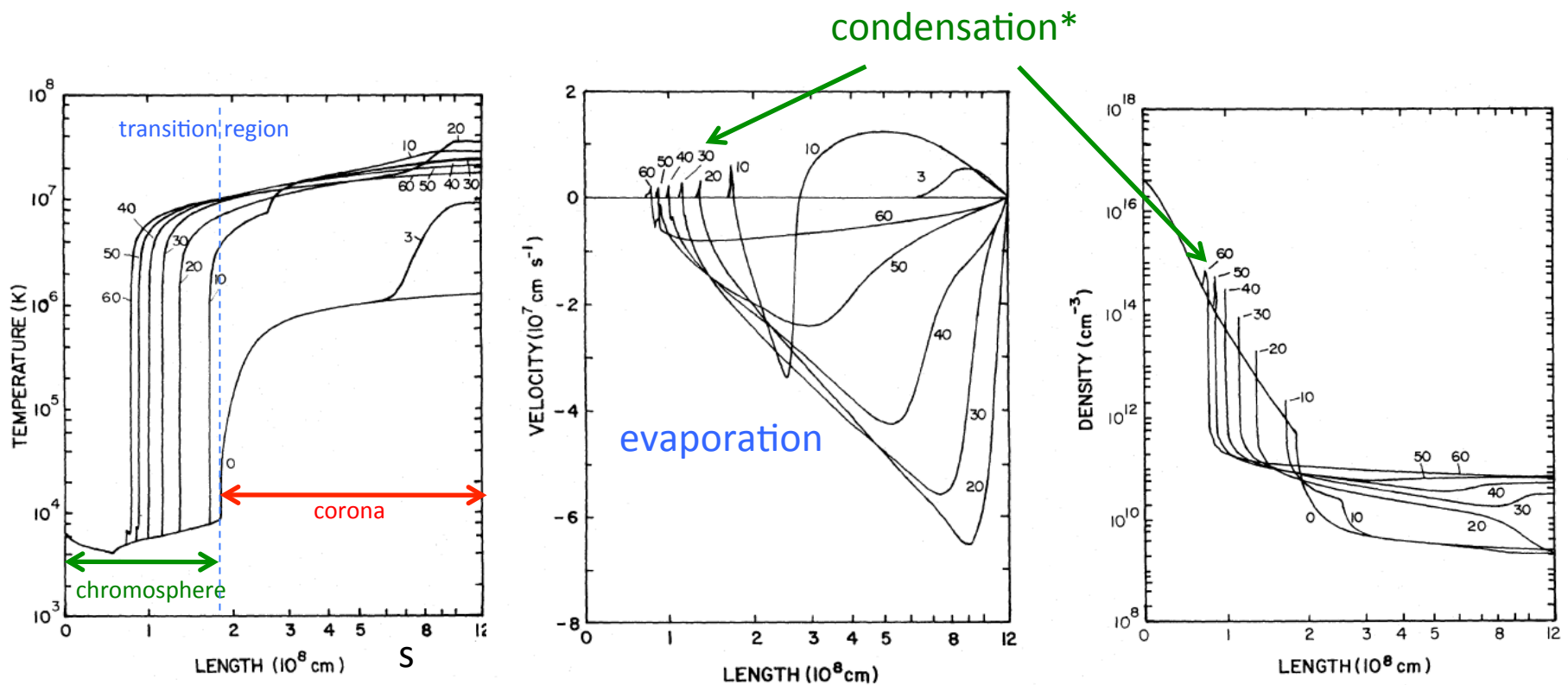
$$T_e = \frac{m_e}{k_b} \left(\frac{2F_{\text{fl}}}{3m_e n_e} \right)^{2/3} = 2.8 \times 10^7 \text{ K}$$

$$v_f = 0.26 \left(\frac{F_{\text{fl}}}{m_e n_e} \right)^{1/3} = 6 \times 10^8 \text{ cm/s}$$



heat front reaches TR in ~ 3 sec.

Conduction-driven evaporation

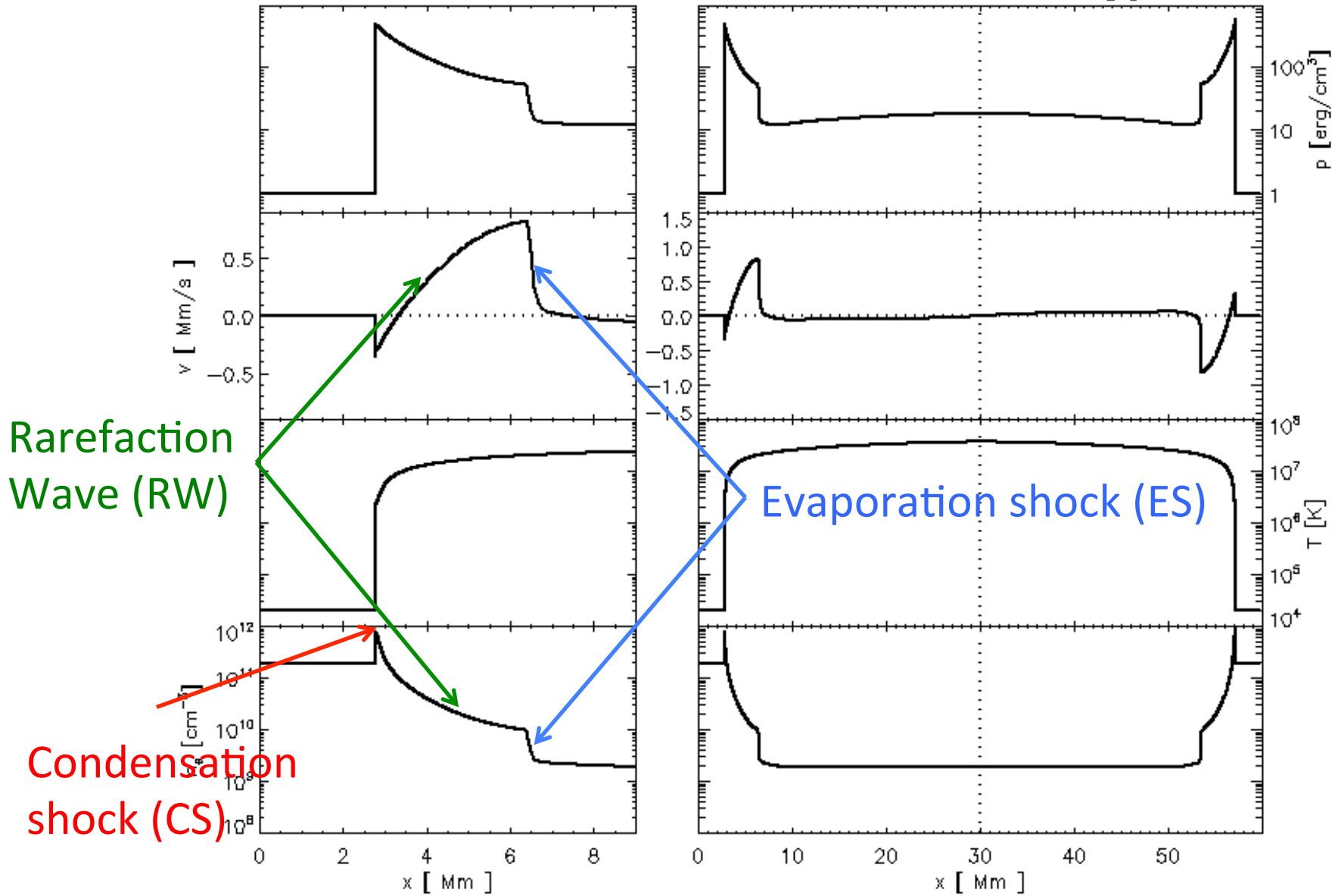


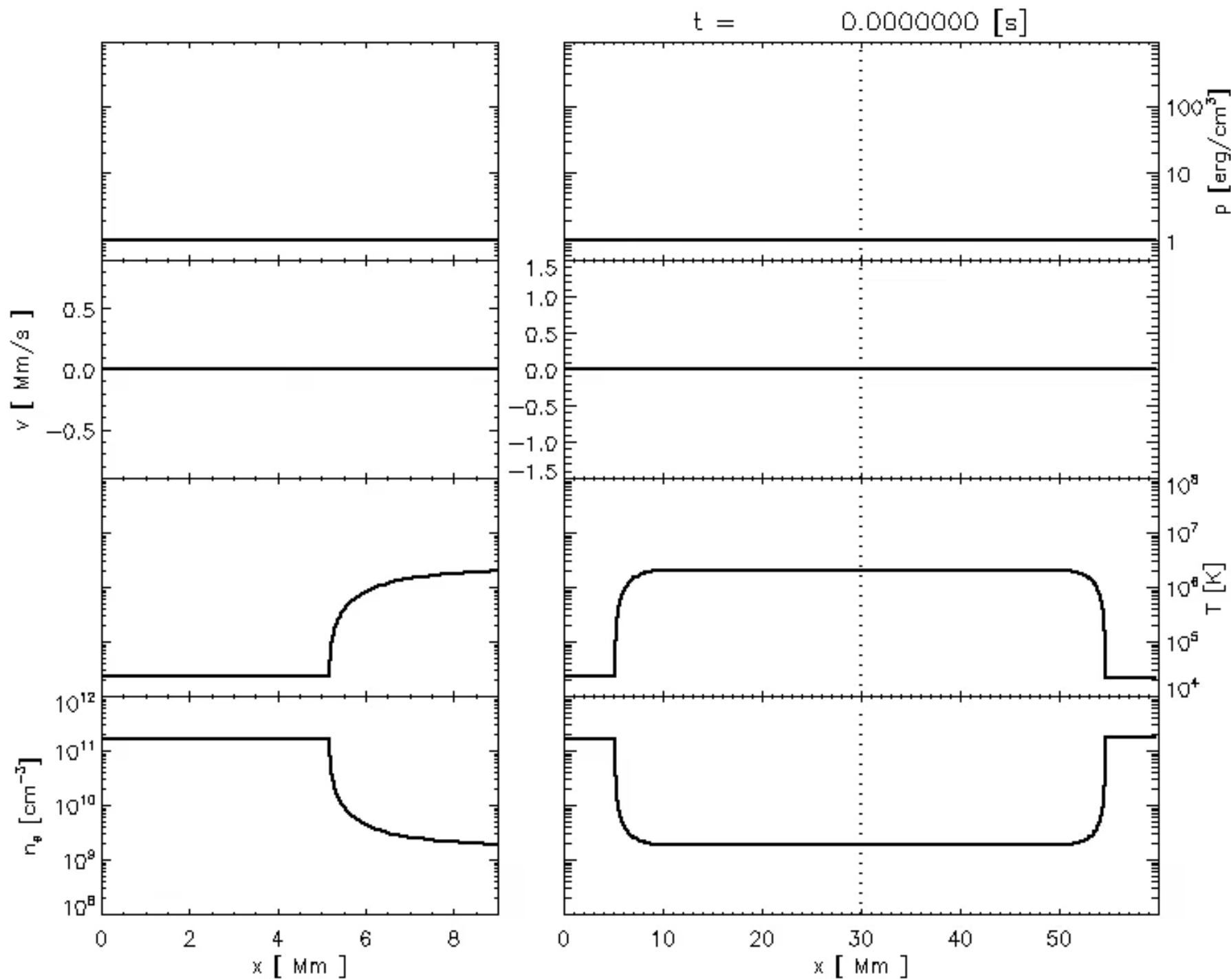
Emslie & Nagai 1985

* Another historical term

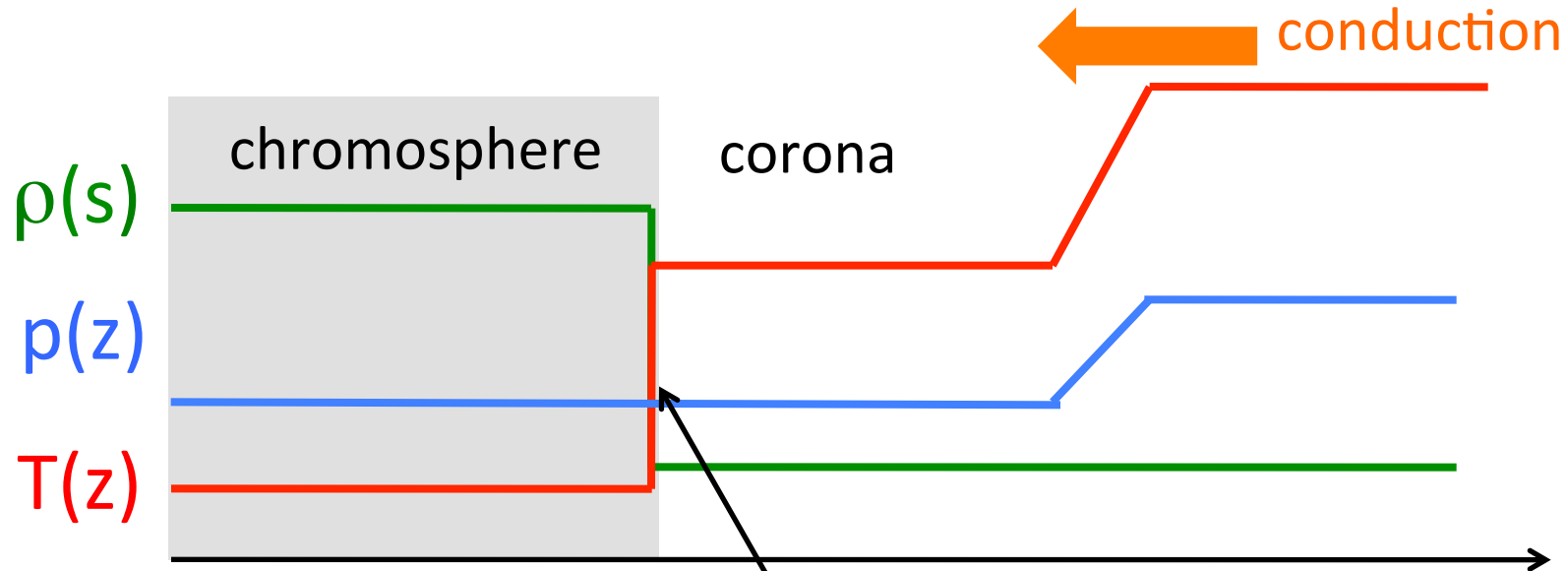
Evaporation

t = 4.2000001 [s]





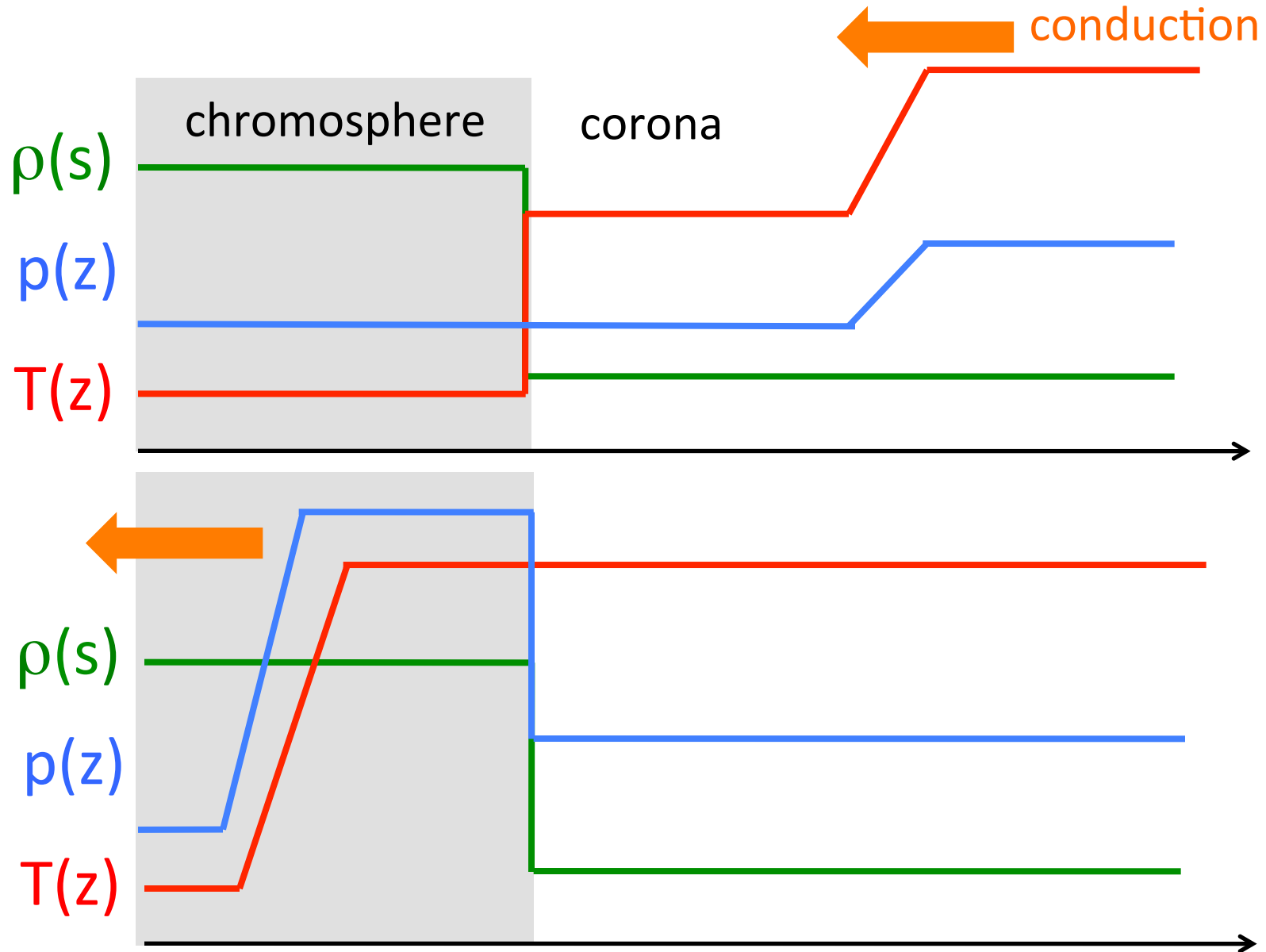
Conduction-driven evaporation



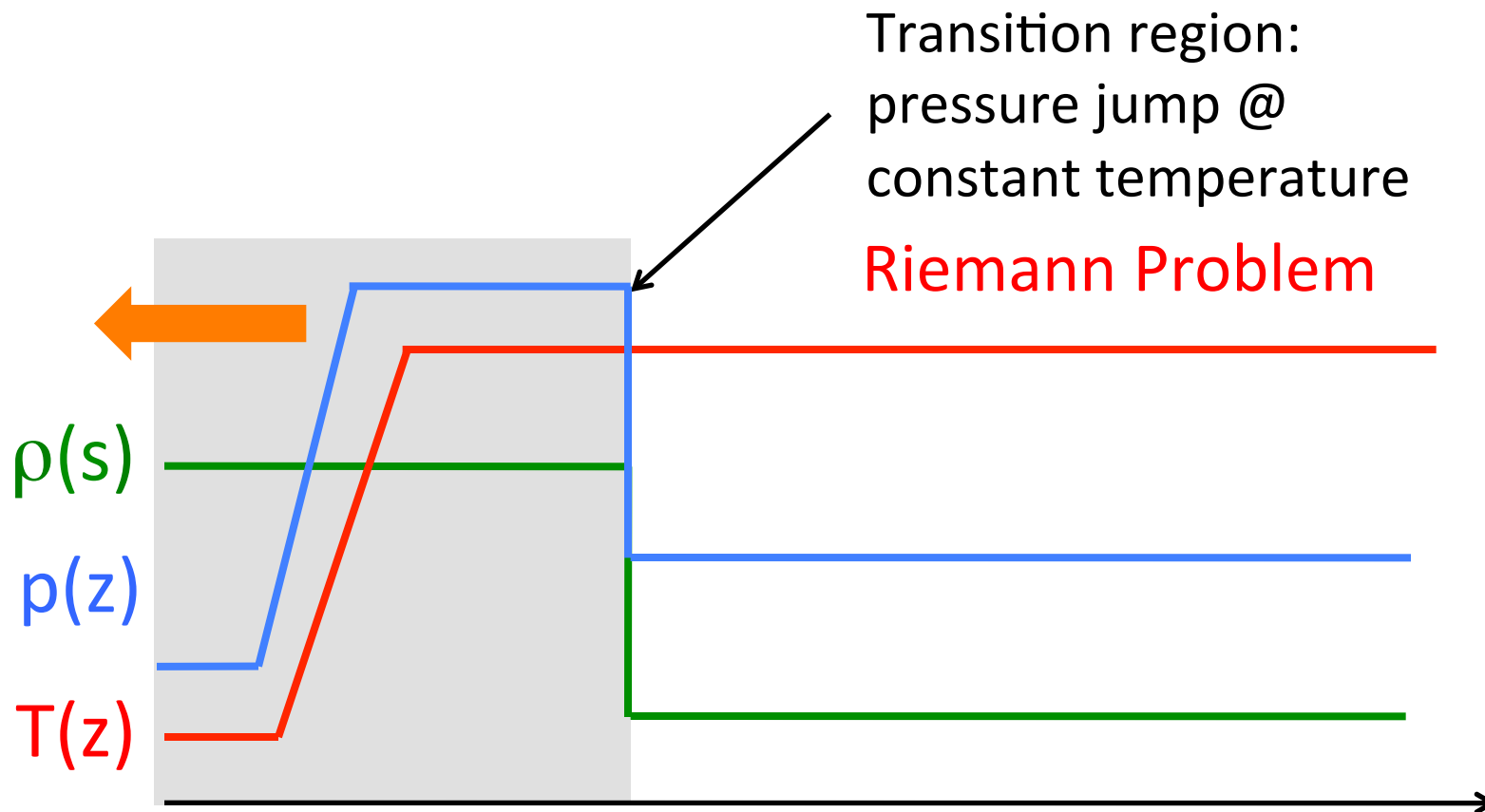
Transition region:
density/temperature
jump @ constant
pressure

$$\frac{\rho_{\text{ch},0}}{\rho_{\text{co},0}} = \frac{T_{\text{co},0}}{T_{\text{ch},0}} = R_{\text{tr}}$$

Conduction-driven evaporation



Conduction-driven evaporation

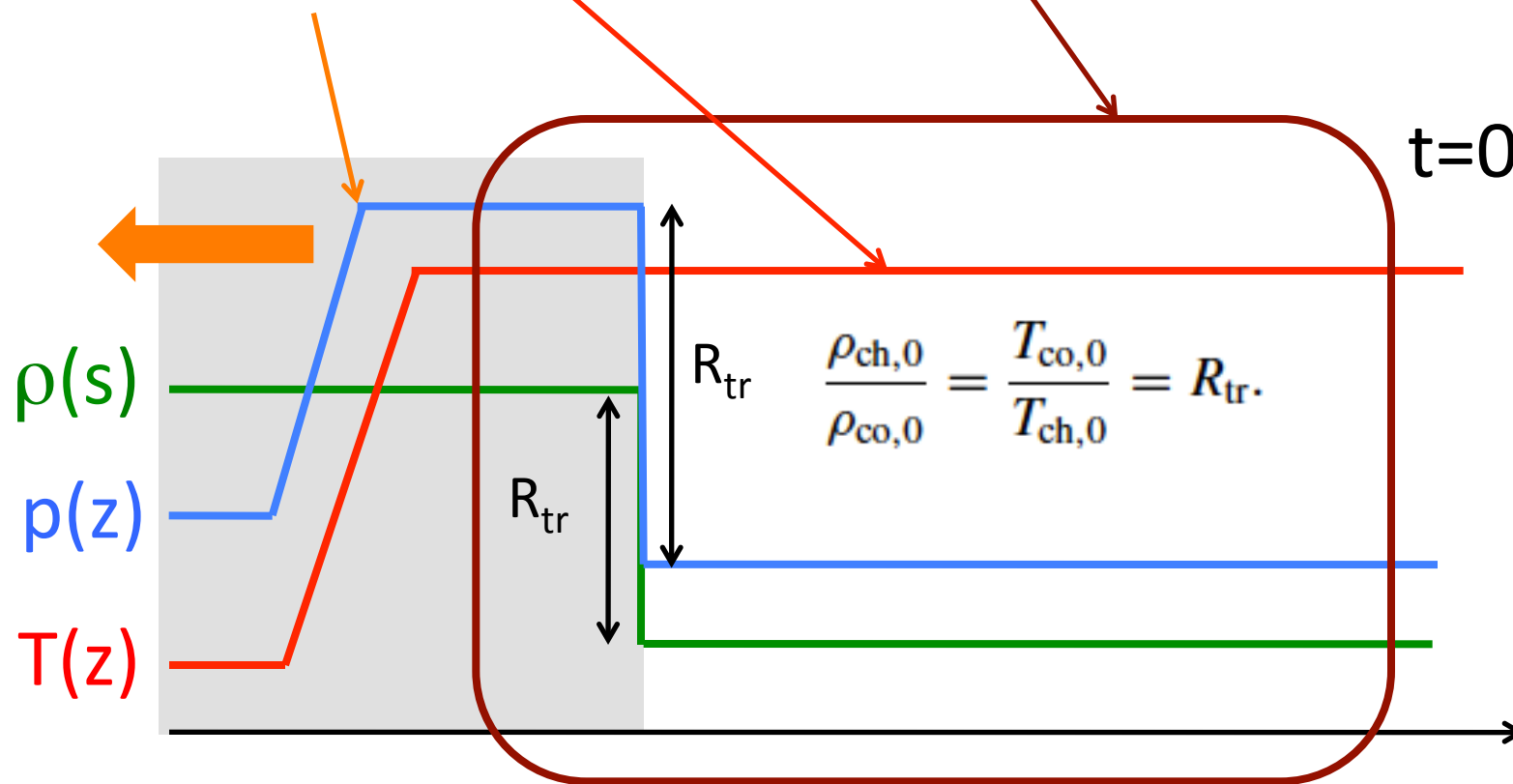


Evaporation as Riemann Problem

- conduction creates uniform high temp. around TR
- evaporation occurs @ constant T for $t > 0$

Isothermal Riemann problem

- isothermal – sound speed a ; iso-T Mach #: $v/a = M^{(it)}$
- condensation shock (hypersonic) propagates down



$$[M_{es}^{(it)}]^2 = 4R_{tr} \exp\left(-\sqrt{3} - M_{es}^{(it)} + \frac{1}{M_{es}^{(it)}}\right),$$

ES:

$$\frac{v_e}{a} = M_{es}^{(it)} - \frac{1}{M_{es}^{(it)}}$$

$$\frac{\rho_e}{\rho_{co,0}} = [M_{es}^{(it)}]^2$$

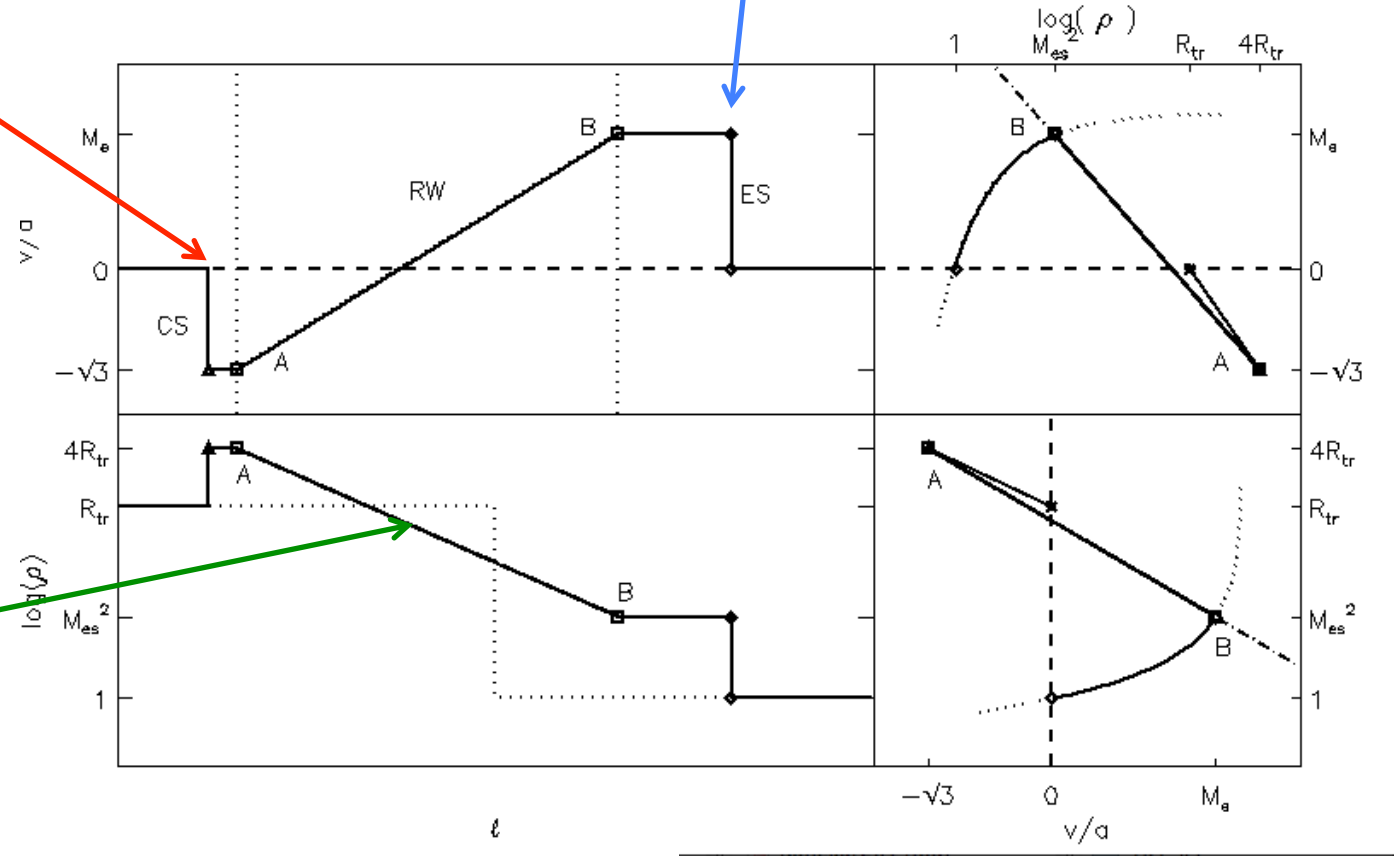
Condensation shock (CS):
(hypersonic)

$$v_c = -\sqrt{3} a$$

$$\rho_c = 4\rho_{ch,0} = 4R_{tr} \rho_{co,0}$$

Rarefaction Wave (RW):

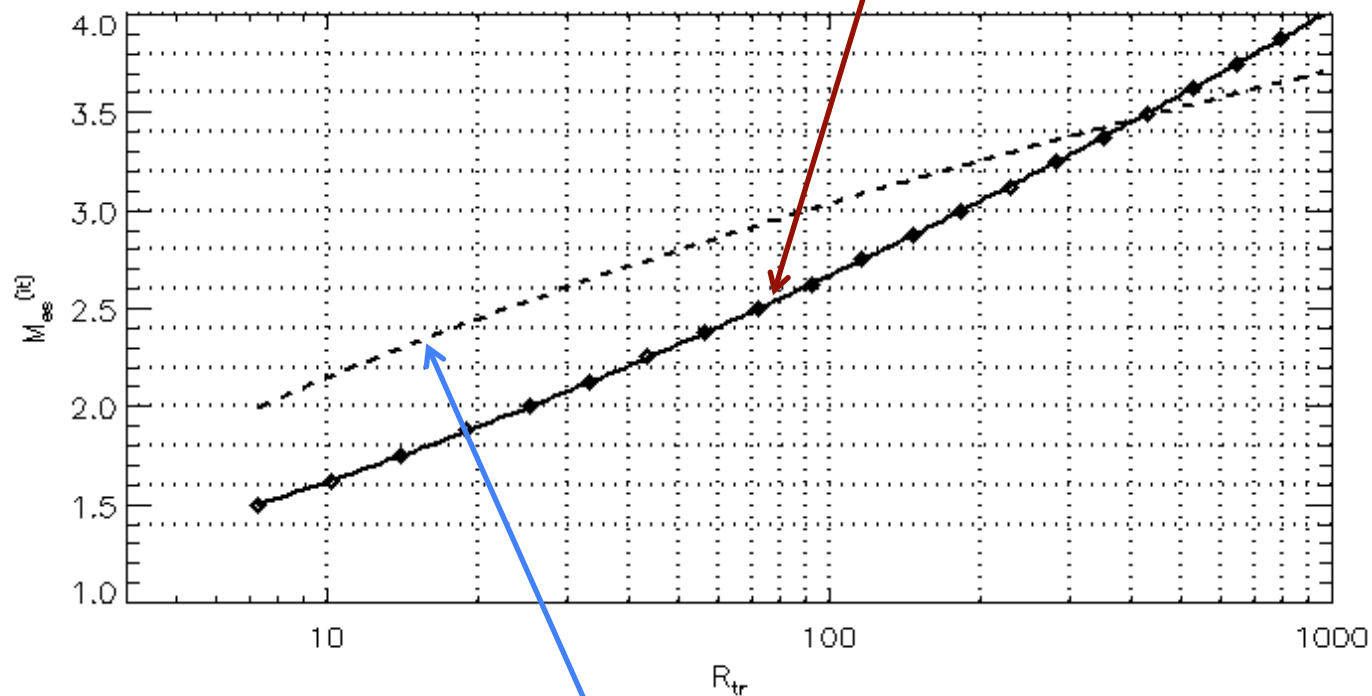
$$v(\ell, t) = \frac{\ell}{t} + a,$$



$$\rho(\ell, t) = \rho_0 \exp\left(-\frac{\ell}{at}\right) = \rho_0 \exp\left(1 - \frac{v}{a}\right) = 4R_{tr} \exp\left(-\sqrt{3} - \frac{v}{a}\right) \rho_{co,0}$$

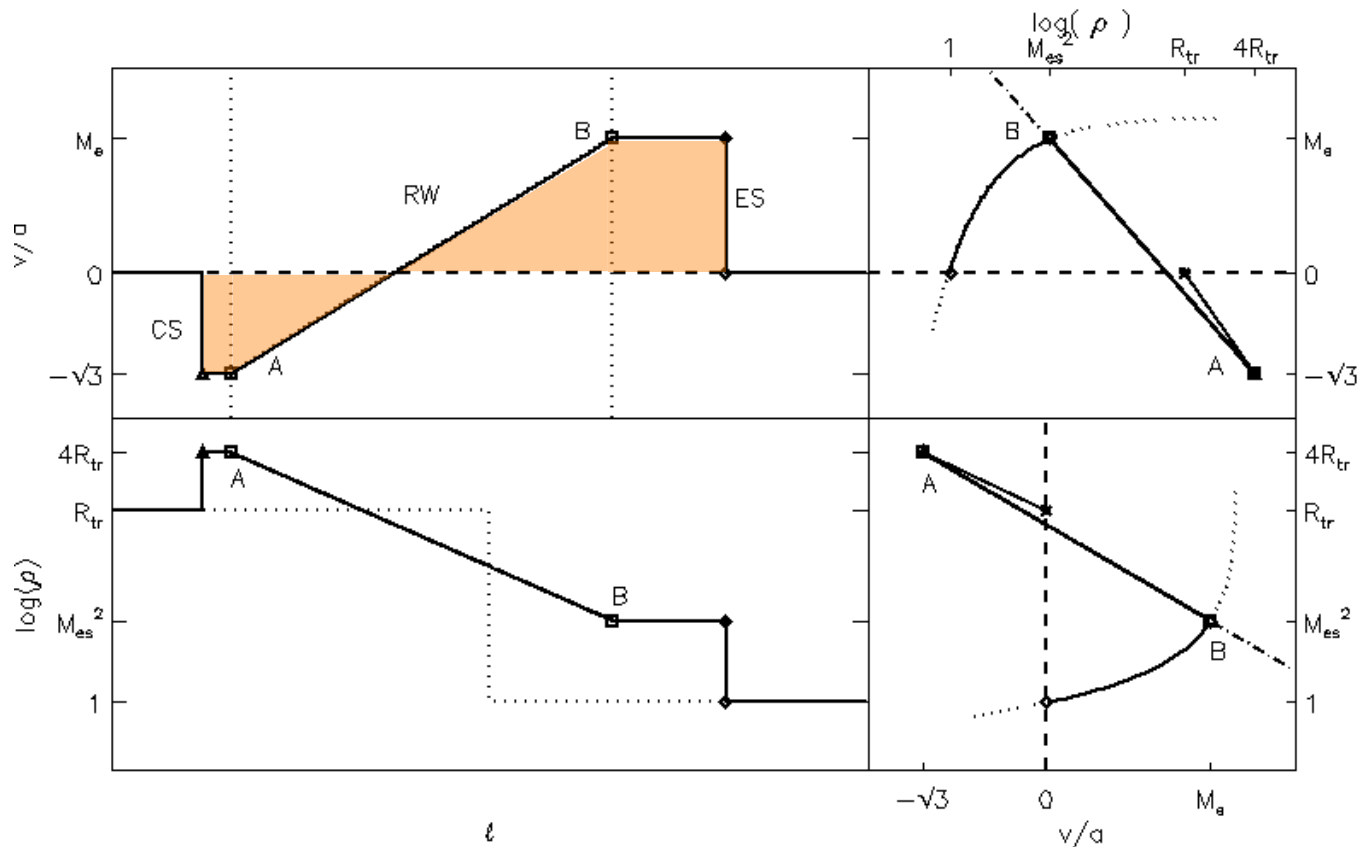
$$[M_{\text{es}}^{(it)}]^2 = 4R_{\text{tr}} \exp\left(-\sqrt{3} - M_{\text{es}}^{(it)} + \frac{1}{M_{\text{es}}^{(it)}}\right),$$

$$M_{\text{es}}^{(it)} \simeq 2.670 + 1.209 \log(R_{\text{tr}}/100) [1 + 0.126 \log(R_{\text{tr}}/100)].$$



$$v_{\text{FCM}} = \sqrt{(6/5) \ln R_{\text{tr}}} c_{s,*} = \sqrt{2 \ln R_{\text{tr}}} a.$$

Fisher, Canfield &
McClymont 1984

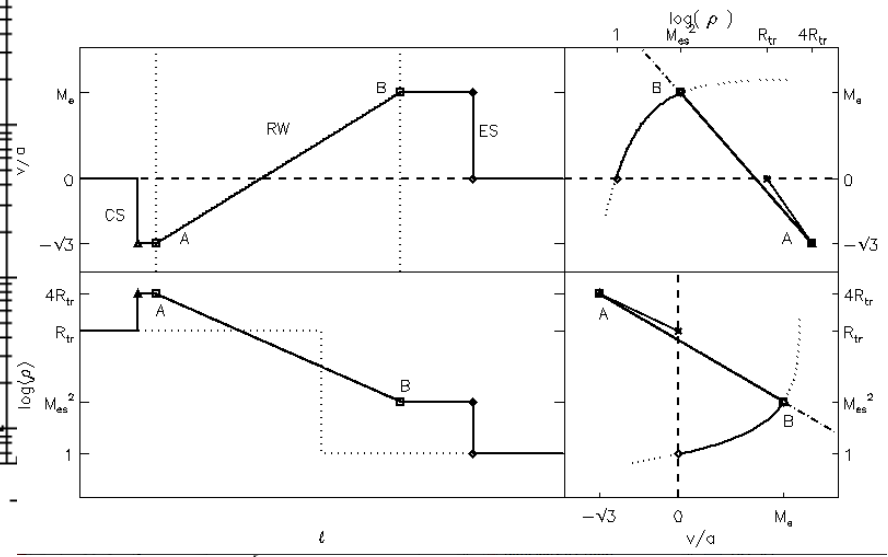
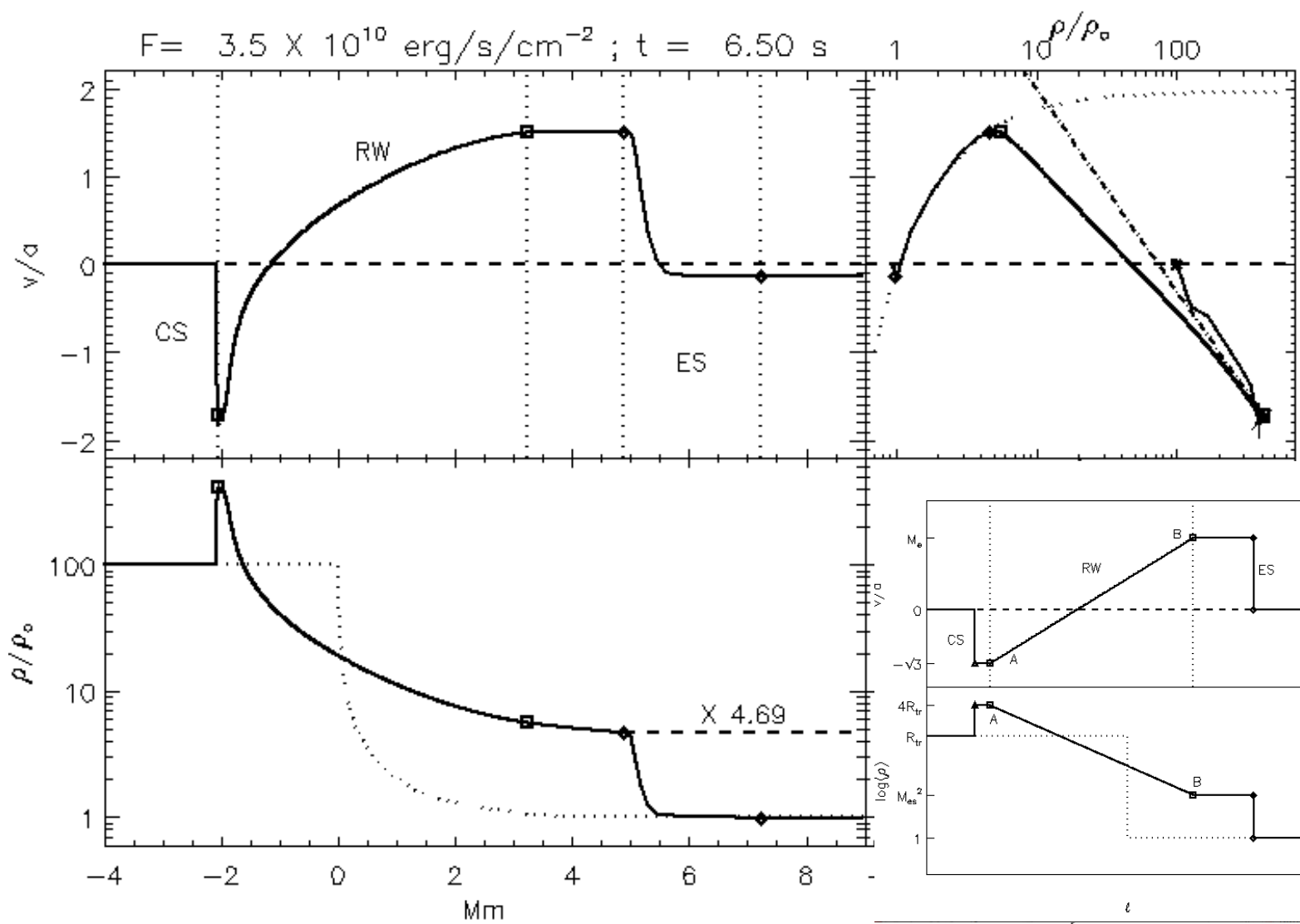


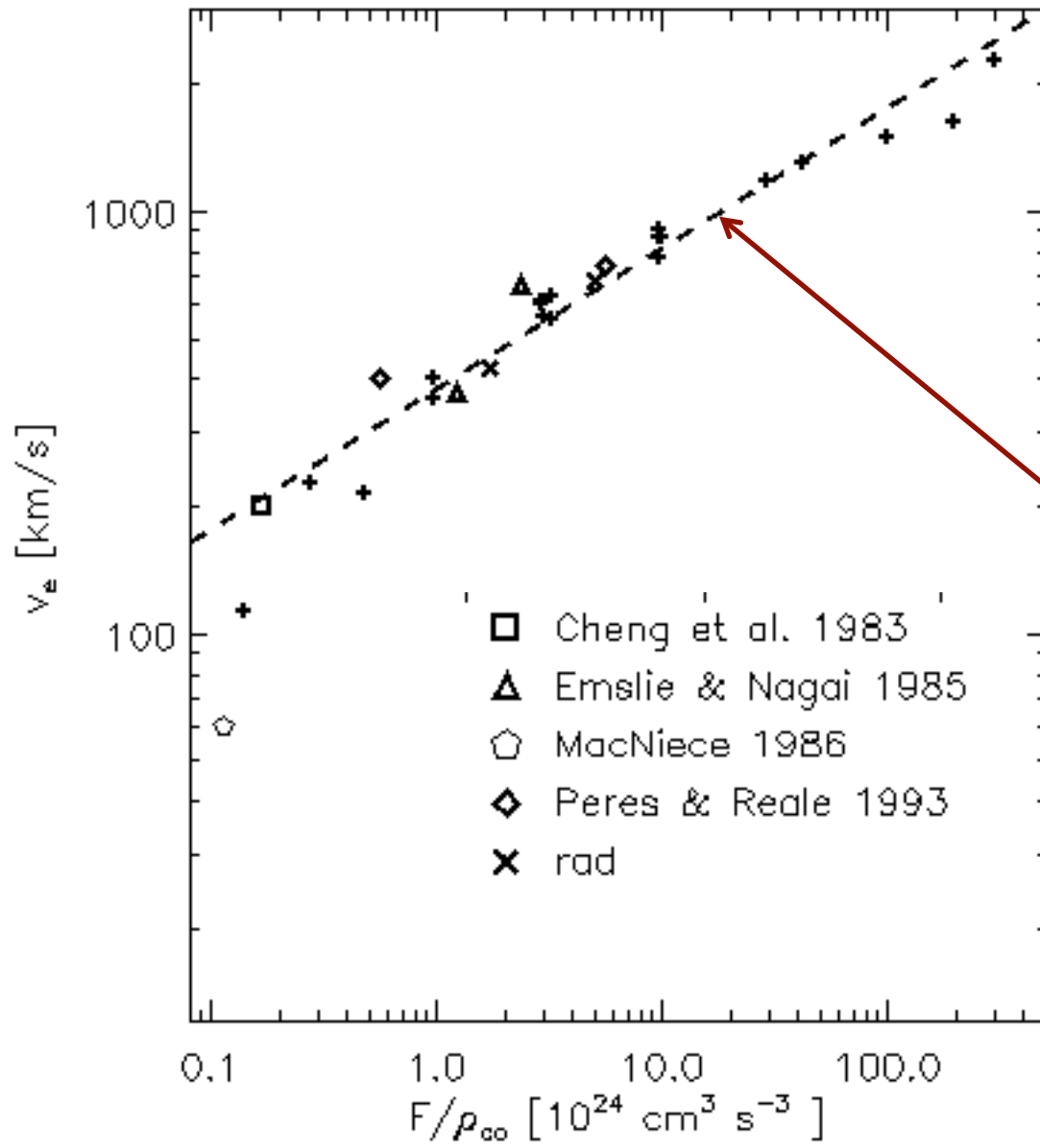
$$E_K = \int \frac{1}{2} \rho v^2 dl = \frac{1}{2} \rho_{co,0} a^3 t \int \frac{\rho(\tilde{l})}{\rho_{co,0}} [M^{(it)}(\tilde{l})]^2 d\tilde{l},$$

Flare
energy
flux

$$F = \frac{dE_K}{dt} \sim \rho_{co,0} a^3$$

$$v_e = a M_e^{(it)} \approx C_e \left(\frac{F}{\rho_{co,0}} \right)^{1/3}$$





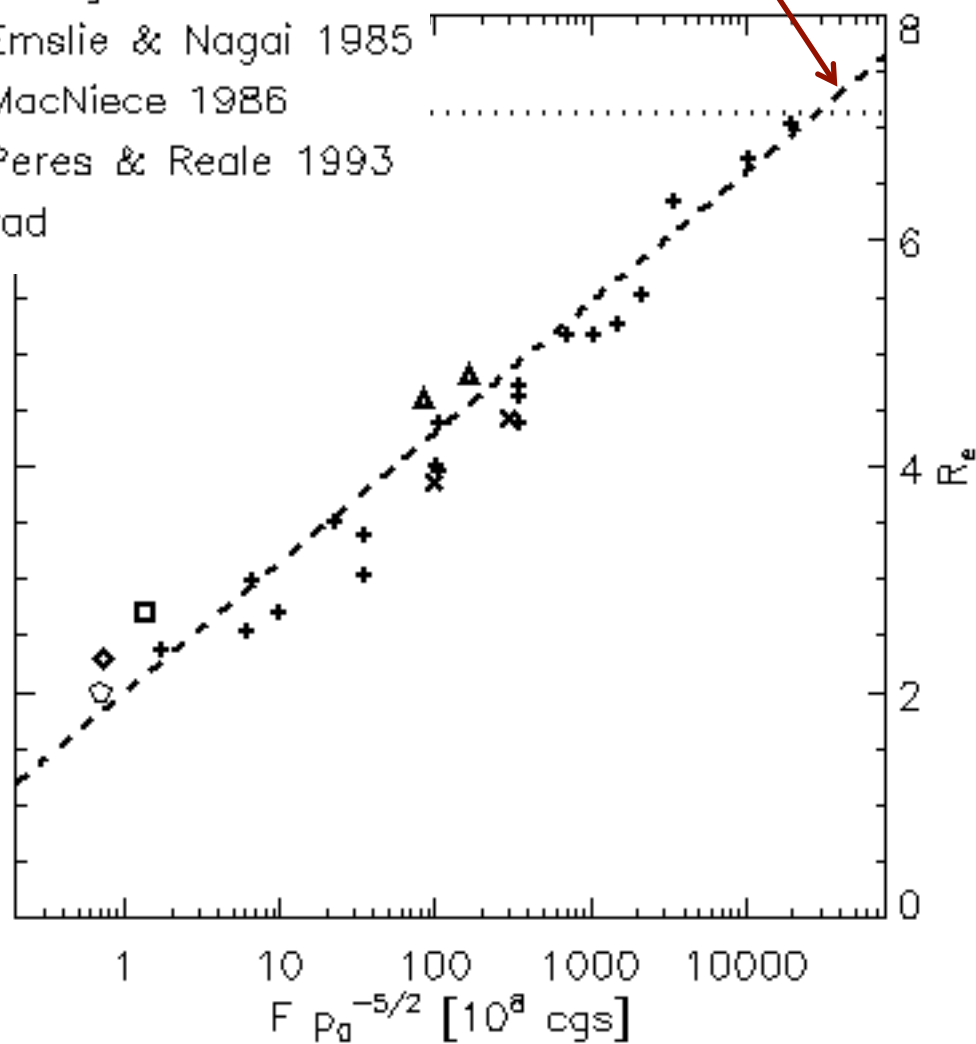
Simulations
from the past

$$v_e = C_e \left(\frac{F}{\rho_{co,0}} \right)^{1/3}$$

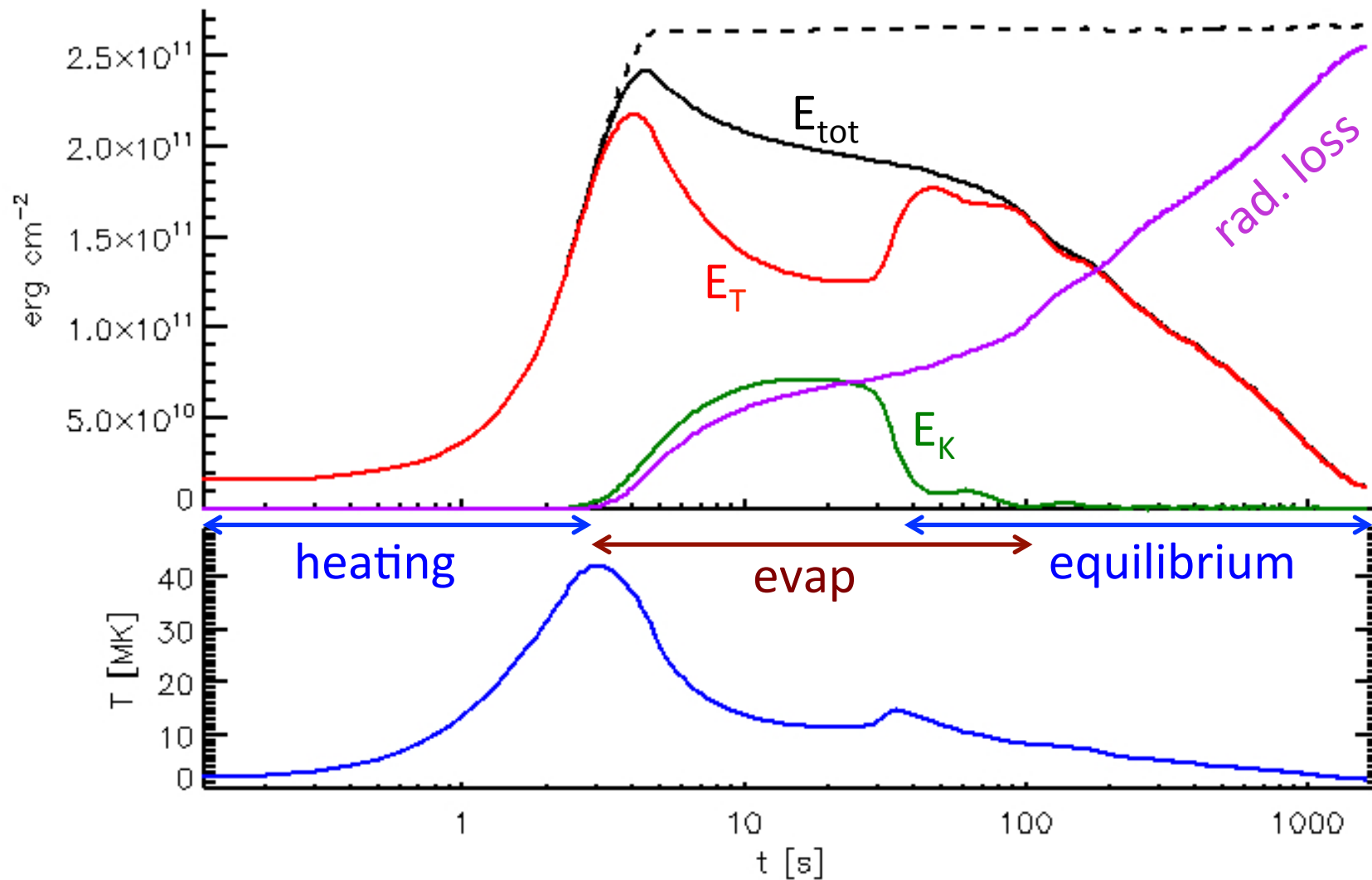
$$C_e = 0.38$$

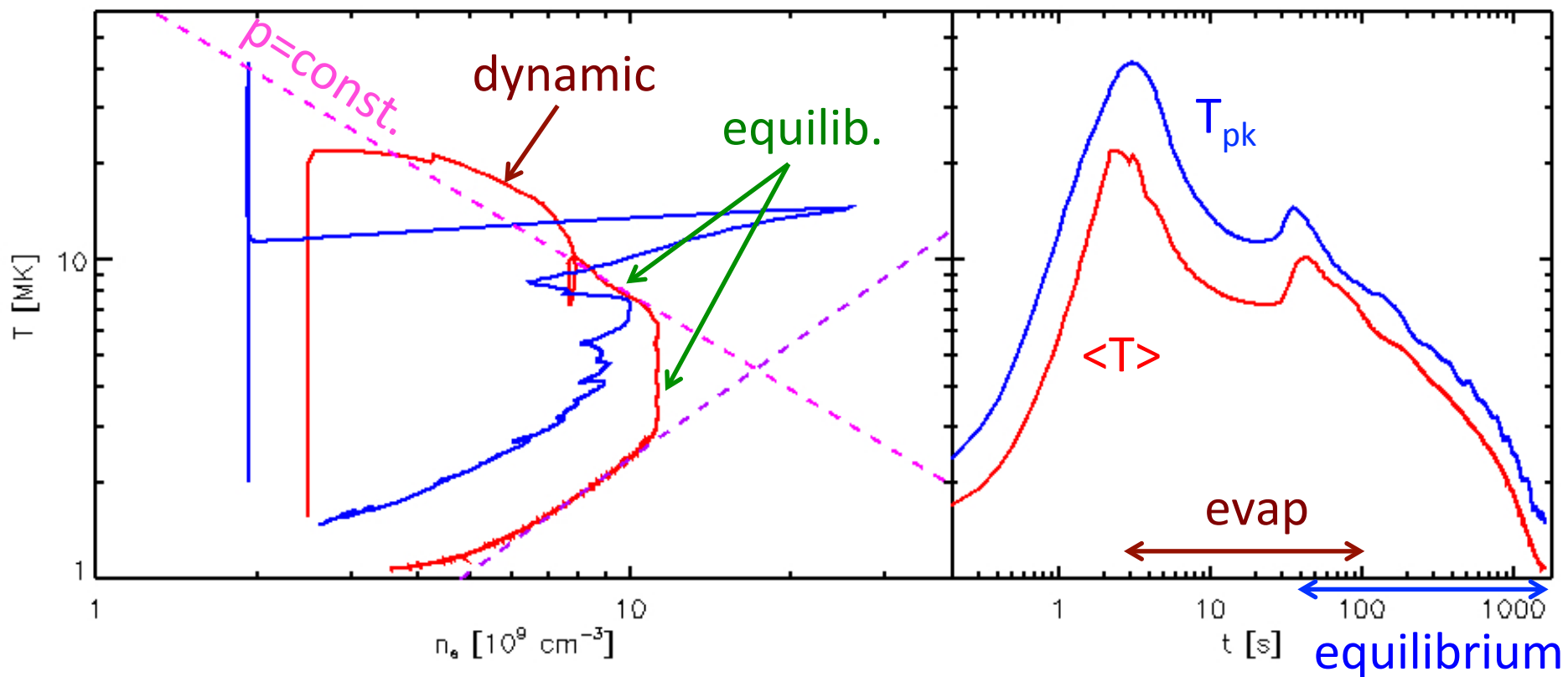
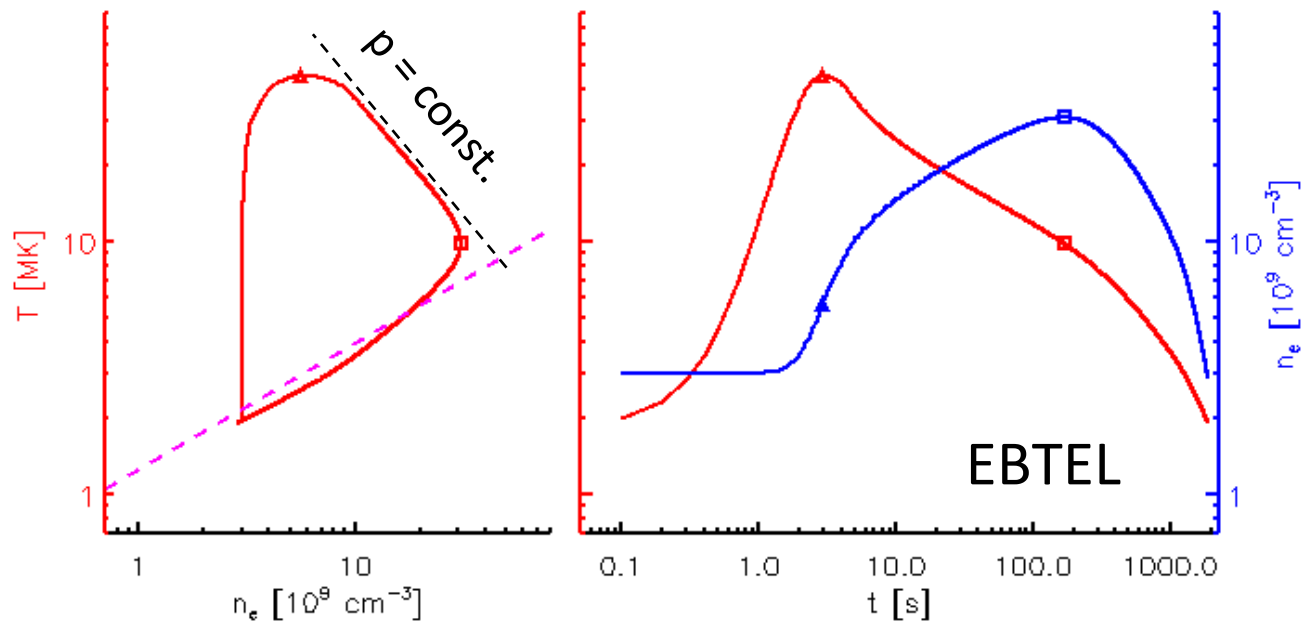
$$R_{\text{es}} = \frac{1}{2} \ln (F p_0^{-5/2} / C_M) + 7.13,$$

- Cheng et al. 1983
- △ Emslie & Nagai 1985
- MacNiece 1986
- ◇ Peres & Reale 1993
- × rad



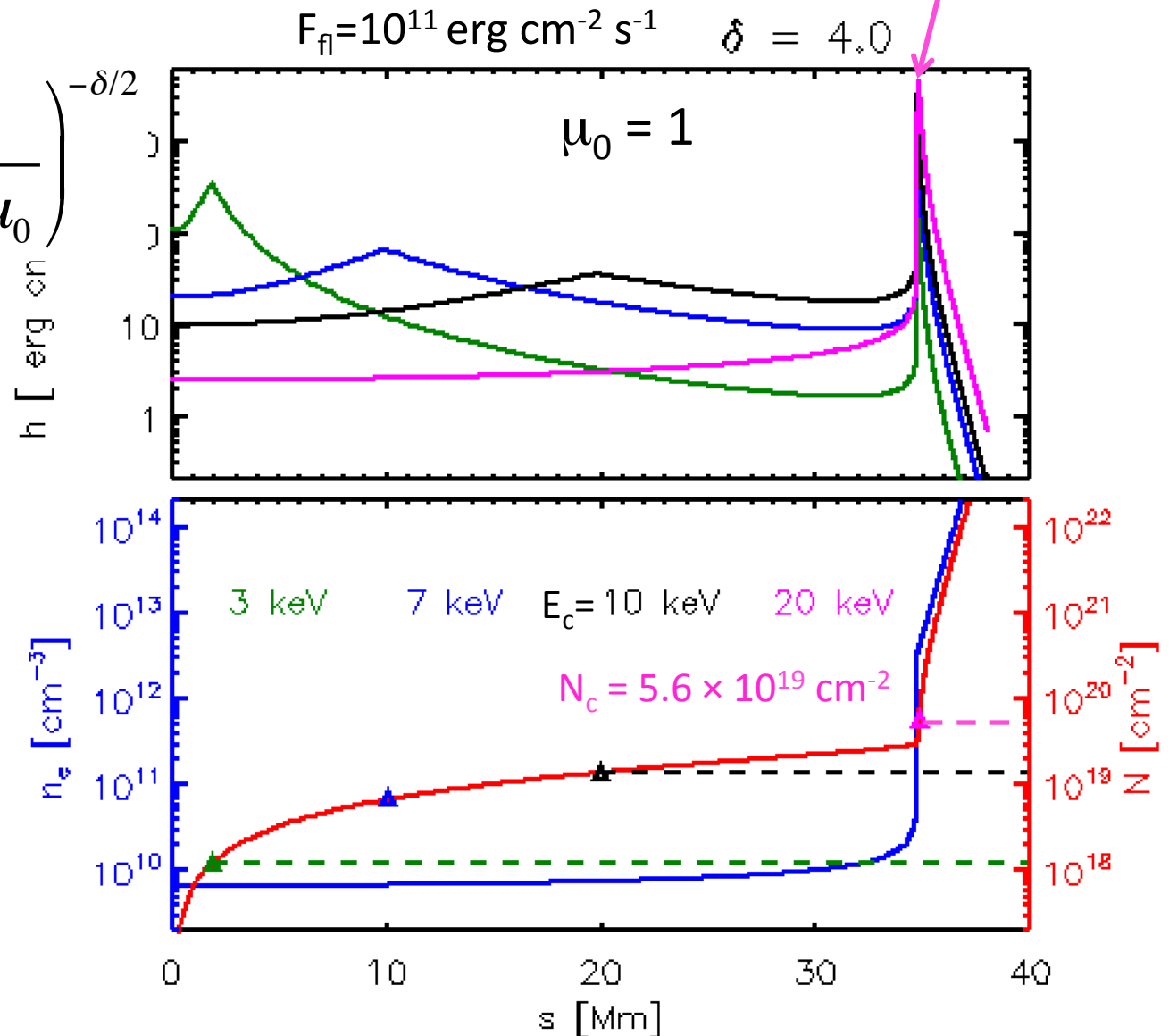
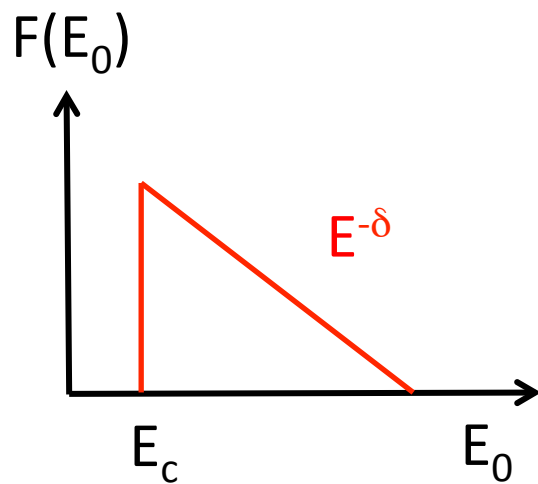
Comparison to 0d model





Chr-spheric deposition

$$h \sim F_{\text{fl}}(t) \frac{n_e(s)}{\mu_0 N_c} \left(\frac{N}{N_c \mu_0} \right)^{-\delta/2}$$



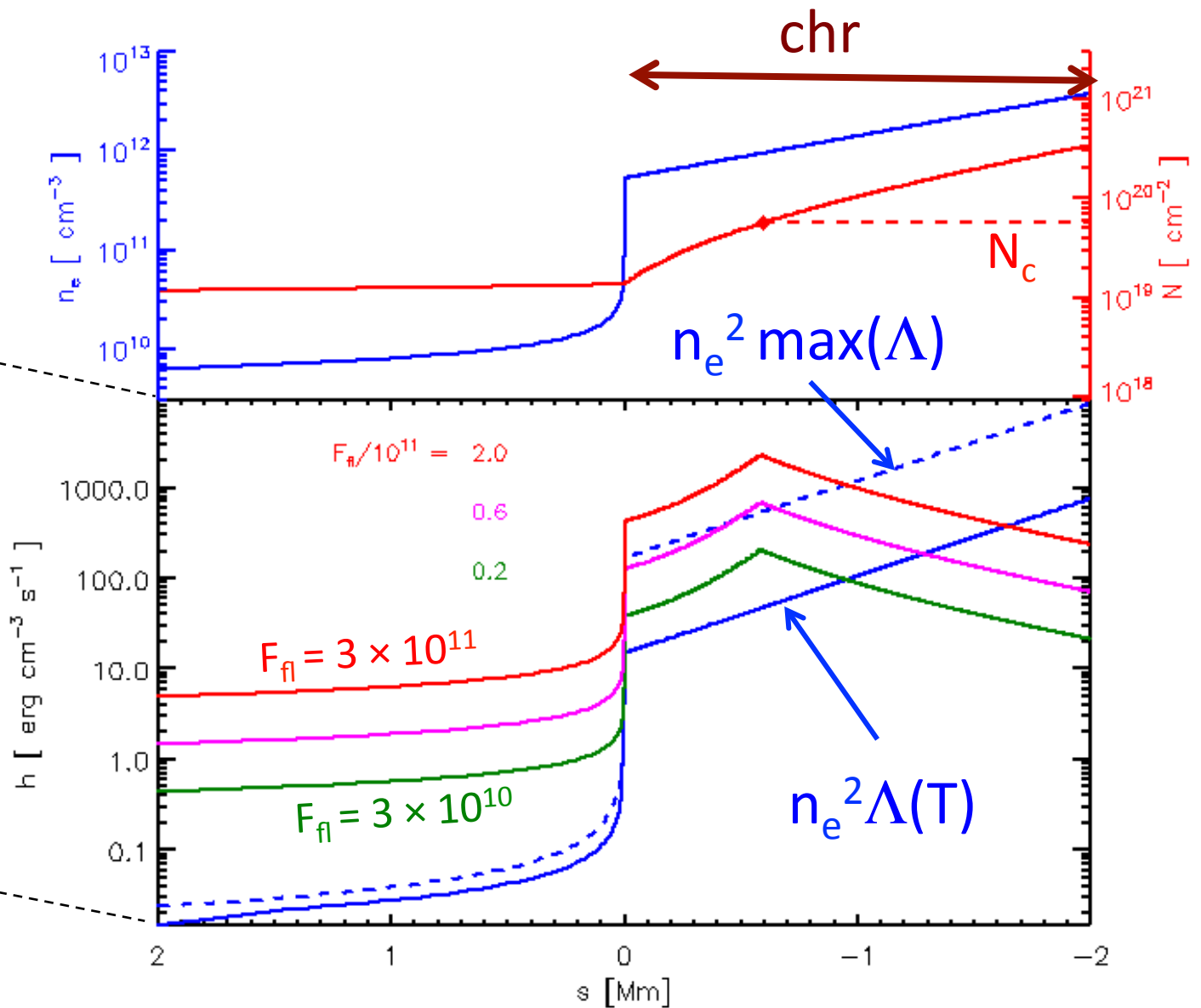
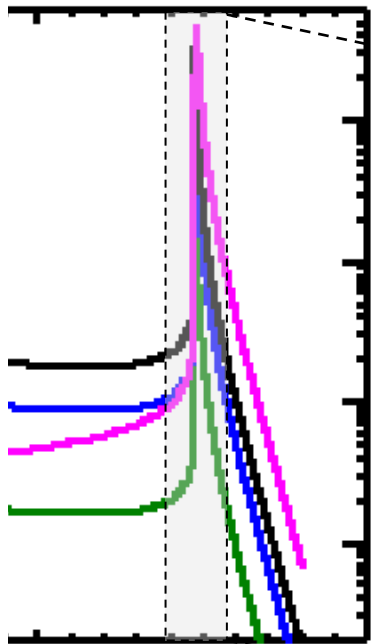
Chr-spheric deposition

$E_c = 20 \text{ keV}$

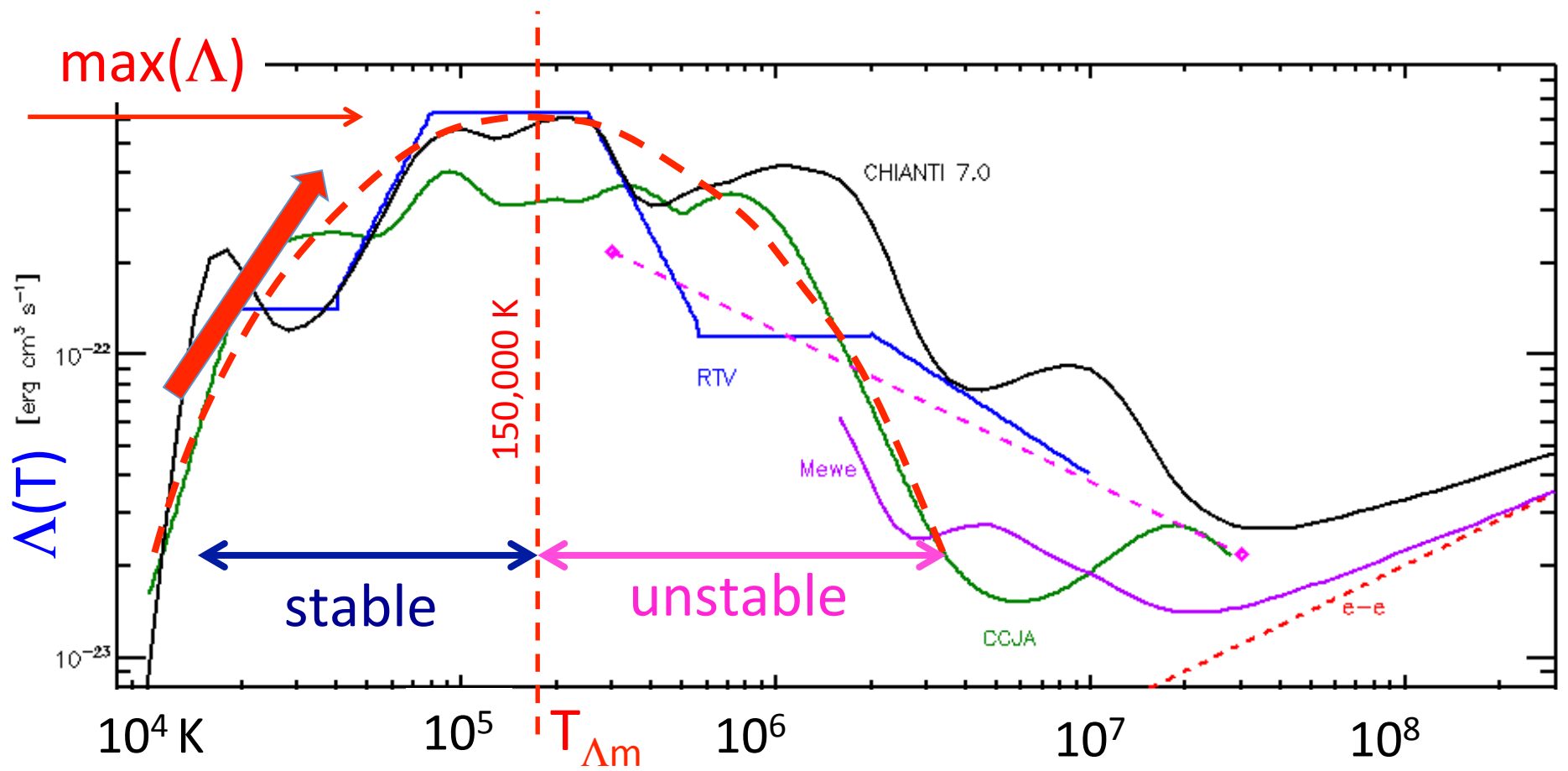
$\delta = 4$

beamed:

$\langle \mu_0 \rangle = 1$



Radiative instability (@ constant n_e)



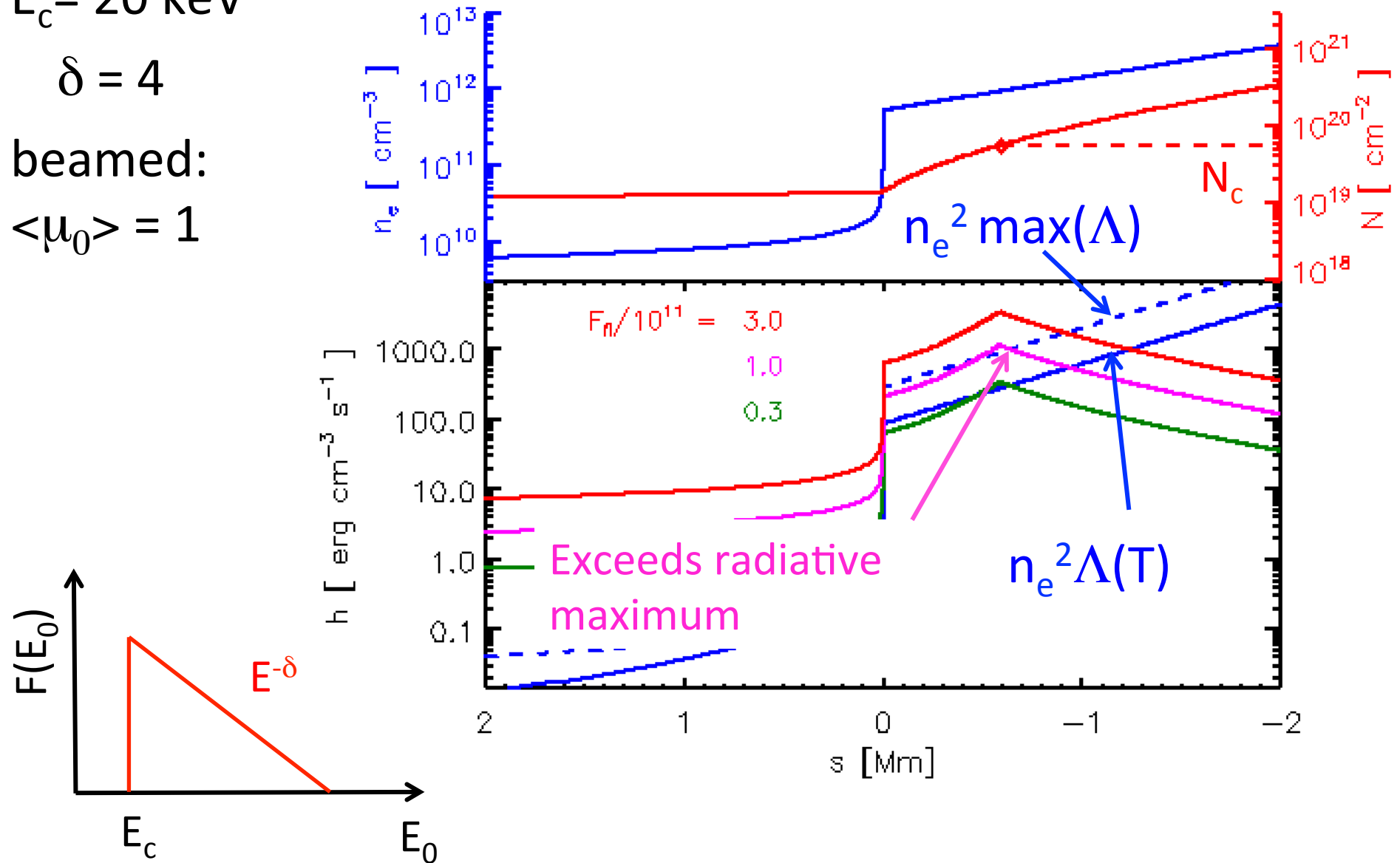
Chr-spheric deposition

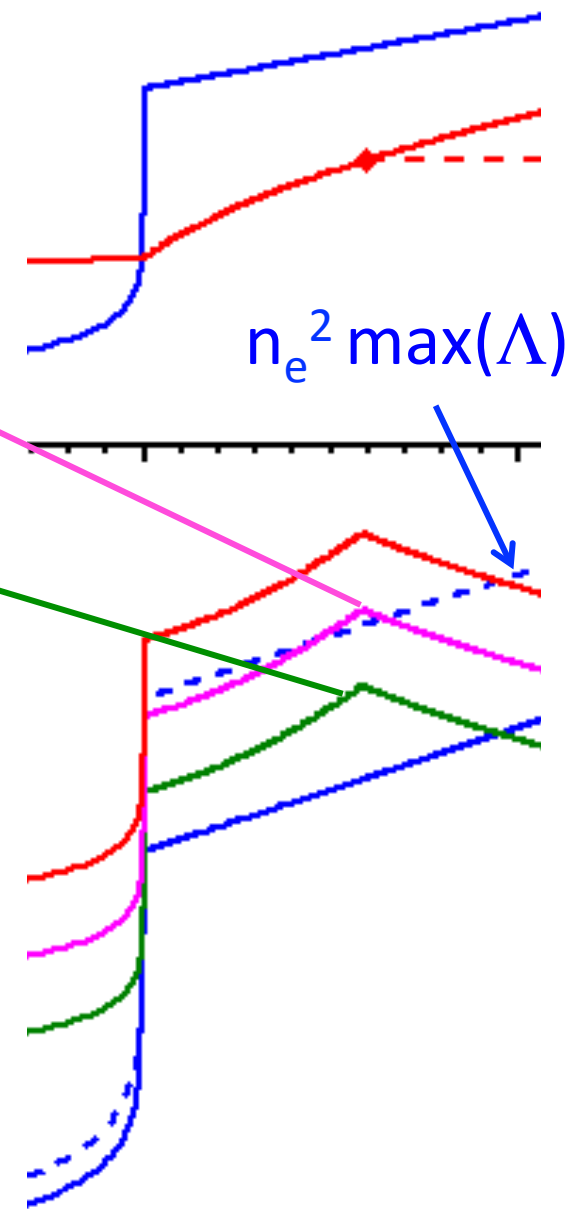
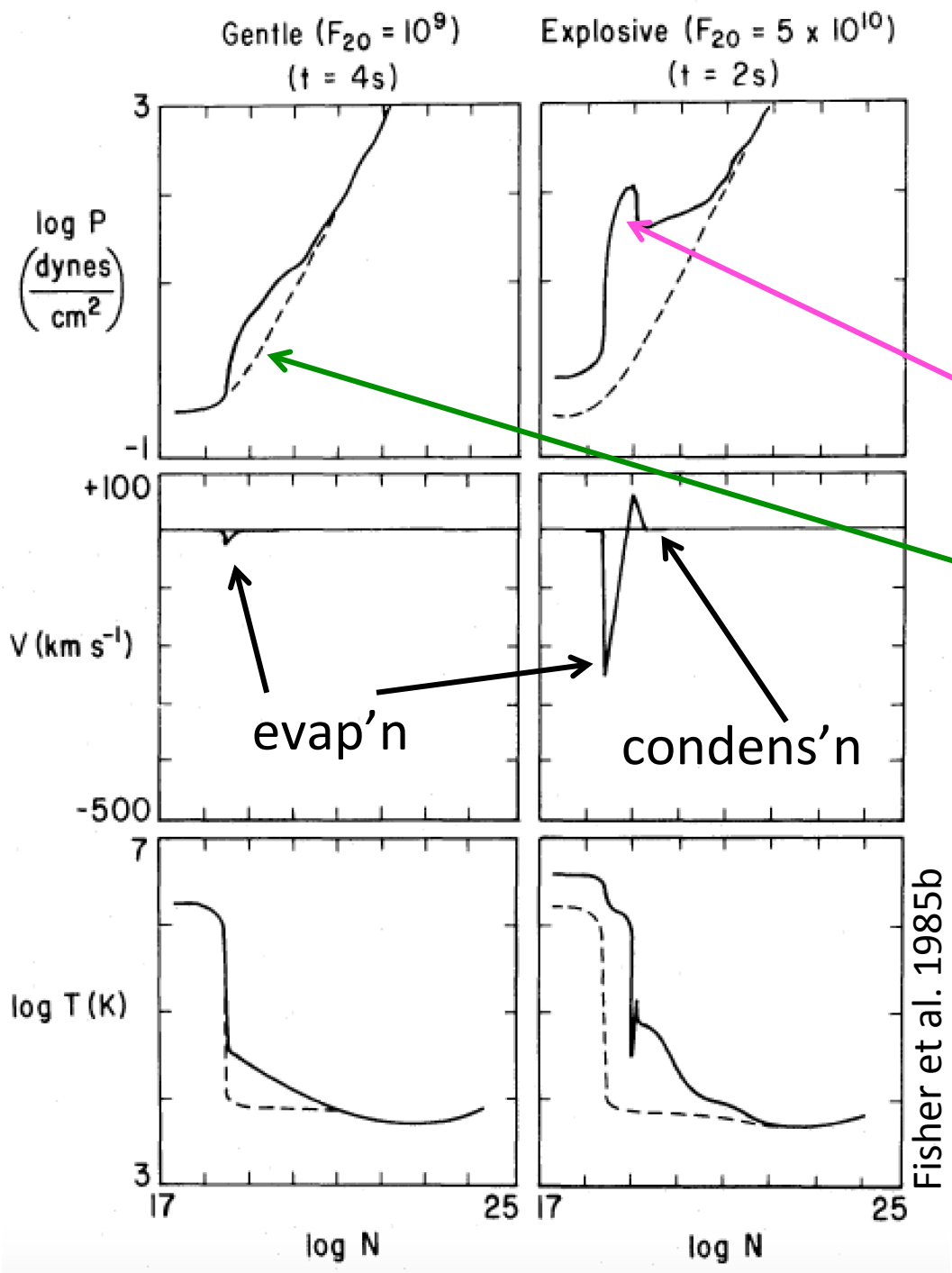
$E_c = 20$ keV

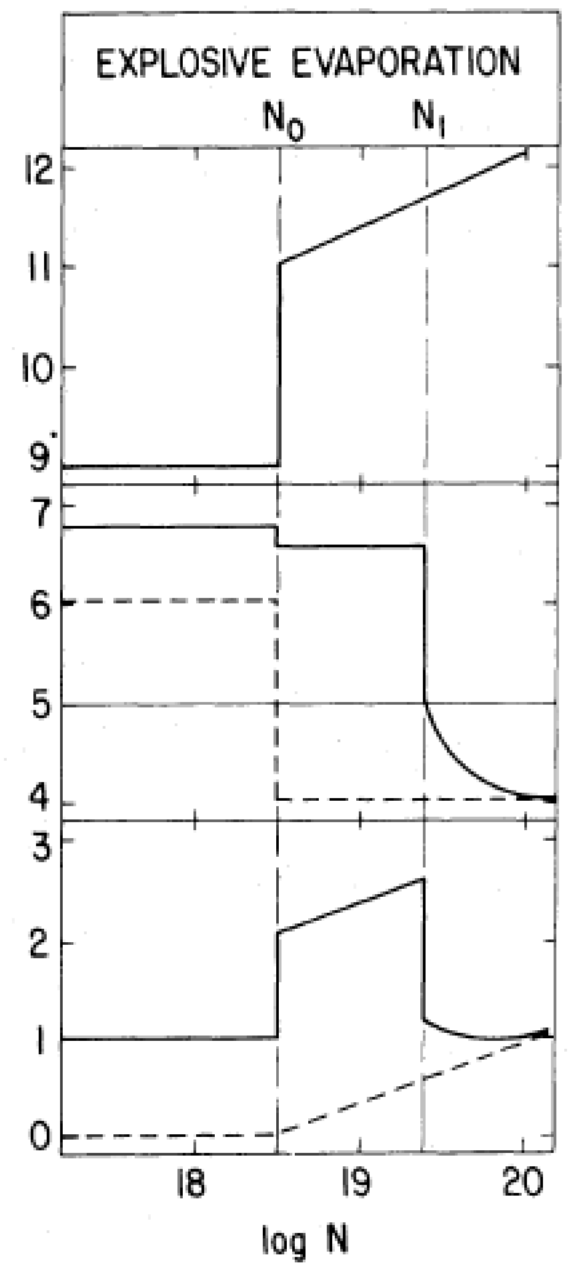
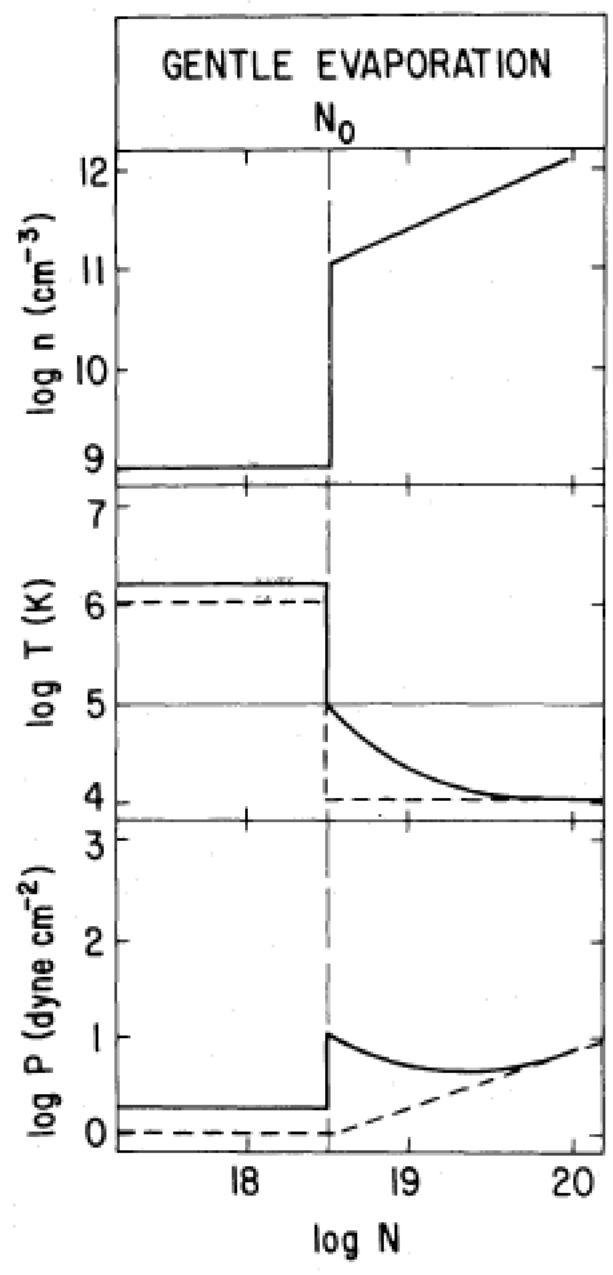
$\delta = 4$

beamed:

$\langle \mu_0 \rangle = 1$







Fisher et al. 1985b

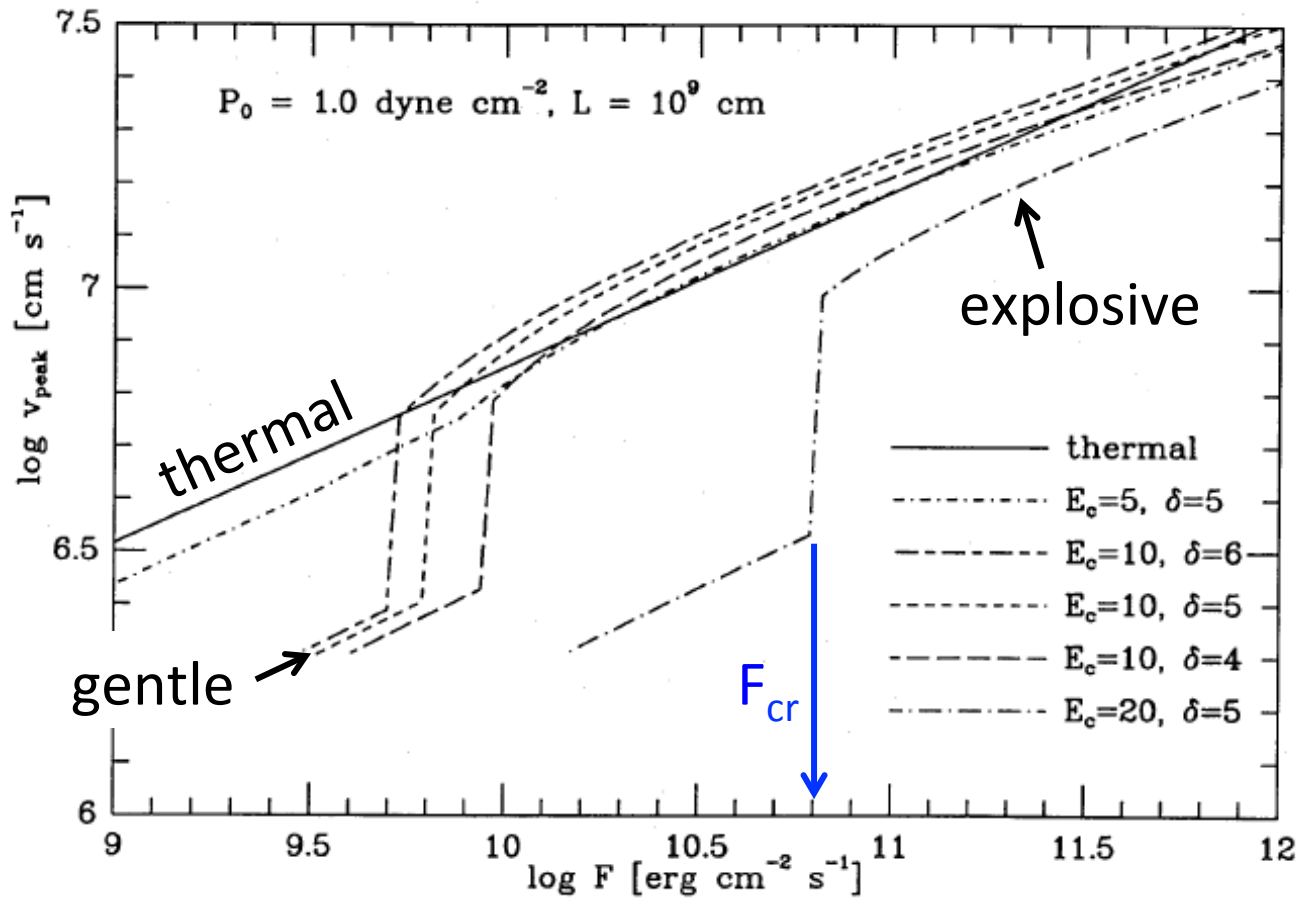
F_{cr} : local heating
exceeds radiation

$F_{fl} < F_{cr}$: gentle evaporation

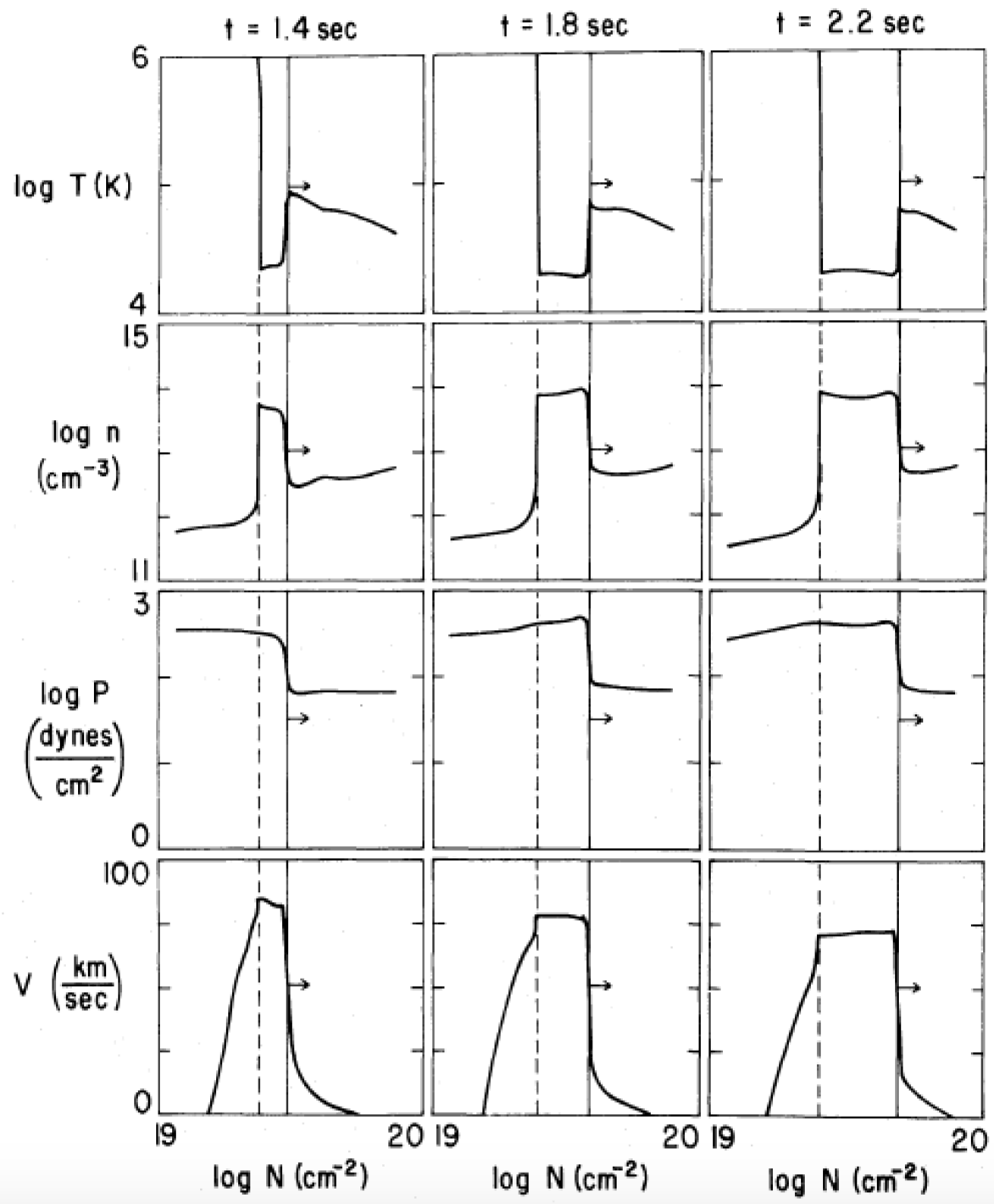
$F_{fl} > F_{cr}$: explosive evaporation

Fisher et al. 1985b

$$F_{cr} \approx \frac{N_* \max(\Lambda)}{2k_b T_{ch}} (p_{cor} + m_p g N_c) = 2 \times 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1} \left[p_{cor, \text{cgs}} + 0.64 \left(\frac{E_c}{10 \text{ keV}} \right)^2 \right]$$



Fisher 1989



Fisher et al. 1985b

Summary

- 1d dynamical evolution better models dynamics
 - Evaporation w/ significant KE
 - Chromospheric Energy Deposition by NT e^-
- 0d picture becomes accurate when KE has subsided – cooling phase