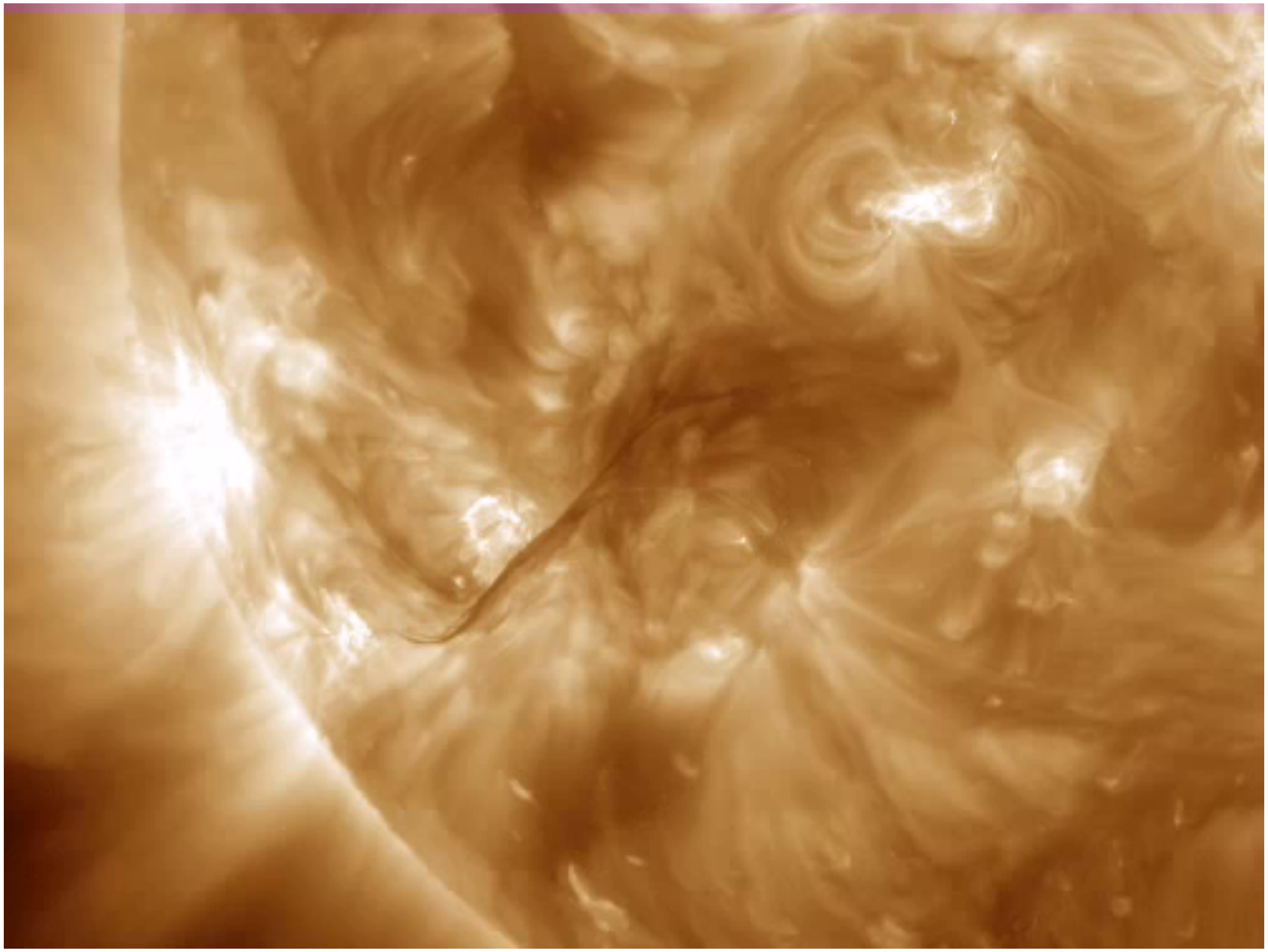
Energy build-up & release – eruptions

MHD instabilities

Lecture 2

Jan. 23, 2017



Our mission: explain what has happened here
Our tool: **magnetohydrodynamics** **(**MHD)

MHD describes…

- Plasma as a single fluid (combining e^-s & ions)
- Fluid fully described by densities*
  - Mass density $\rho(\mathbf{x},t)$
  - Momentum density $\rho(\mathbf{x},t)\,\mathbf{u}(\mathbf{x},t)$

    $\mathbf{u}(\mathbf{x},t)$ is C.M. velocity – **NOT** particle velocity
  - Pressure $p(\mathbf{x},t) = p_e + p_i$

    thermal energy density = $(3/2)p$
- Slow dynamics ($\tau \gg 10^{-3}$ sec for Sun)
- Good conductor: $\mathbf{E}' = \mathbf{E} + \mathbf{u} \times \mathbf{B} \simeq 0$

  (no charge density! "quasi-neutrality")

* instead of particle information – often particle velocities have Maxwellian dist'n w/ moments specified by ρ, $\mathbf{u}$, and p

MHD equations

cgs

Dynamical evolution of fluid densities & **B**

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

mass continuity

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = -\nabla p + \tfrac{1}{c}\mathbf{J}\times\mathbf{B} + \rho\mathbf{g}$$

momentum

$$\frac{\partial p}{\partial t} + \mathbf{u}\cdot\nabla p = -\tfrac{5}{3}p\nabla\cdot\mathbf{u} + \tfrac{2}{3}\eta_e\left|\mathbf{J}\right|^2 + \cdots$$

energy

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\left(c\eta_e\mathbf{J} - \mathbf{u}\times\mathbf{B}\right)$$

induction
(Faraday+Ohm)

$$\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

Ampere

J is **NOT** related to **u**

MHD equations

Dynamical evolution of fluid densities & **B**

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot\left(\rho\,\mathbf{u}\right)$$

fluid pressure

magnetic (Lorentz)

mass × acceleration (of CM)

$$\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u} = -\nabla p + \tfrac{1}{c}\mathbf{J}\times\mathbf{B} + \rho\mathbf{g}$$

gravity

$$\frac{\partial p}{\partial t}+\mathbf{u}\cdot\nabla p = -\tfrac{5}{3}\,p\nabla\cdot\mathbf{u} + \tfrac{2}{3}\eta_e\left|\mathbf{J}\right|^2 + \cdots$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\left(c\eta_e\mathbf{J}-\mathbf{u}\times\mathbf{B}\right)$$

$$\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

MHD equations

Dynamical evolution of fluid densities & **B**

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \mathbf{u} \right)$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} \;=\; -\nabla p + \tfrac{1}{c} \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\tfrac{5}{3} p \nabla \cdot \mathbf{u} \;+\; \tfrac{2}{3} \eta_e \left| \mathbf{J} \right|^2 + \cdots$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \underbrace{\left(c\eta_e \mathbf{J} - \mathbf{u} \times \mathbf{B} \right)}$$

Faraday

$$\mathbf{J} = \frac{c}{4\pi} \nabla \times \mathbf{B}$$

$$c\mathbf{E} = c\mathbf{E}' - \mathbf{u} \times \mathbf{B} = c\eta_e \mathbf{J} - \mathbf{u} \times \mathbf{B}$$

Ohm

MHD equations

Dynamical evolution of fluid densities & **B**

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \left(\rho \mathbf{u} \right)$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} \; = \; -\nabla p + \tfrac{1}{4\pi} \left(\nabla \times \mathbf{B} \right) \times \mathbf{B} + \rho \mathbf{g}$$

$$\frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p = -\tfrac{5}{3} p \nabla \cdot \mathbf{u} \; + \; \tfrac{\eta}{6\pi} \left| \nabla \times \mathbf{B} \right|^2 + \; \cdots$$

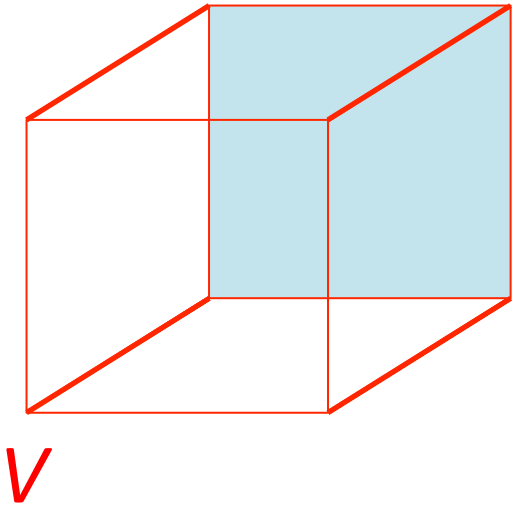
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right) + \eta \nabla^2 \mathbf{B}$$

magnetic diffusivity

Use Ampere to eliminate **J** from dynamical eqs. – closed eqs. for densities & **B** alone

$$\eta = \frac{c^2}{4\pi} \eta_e \quad [\ \mathrm{cm}^2\ \mathrm{s}^{-1}\ ]$$

Integrated densities



mass

$$M = \int_V \rho(\mathbf{x})\, d^3x$$

momentum

$$\mathbf{P} = \int_V \rho(\mathbf{x})\,\mathbf{u}(\mathbf{x})\, d^3x$$

Integrated densities

$$\mathbf{g} = -\nabla\Psi$$

Total energy

thermal

magnetic

gravitational

$$E = \int_V \left[\tfrac{1}{2}\rho|\mathbf{u}|^2 + \tfrac{3}{2}p + \frac{1}{8\pi}|\mathbf{B}|^2 + \rho\Psi \right] d^3x$$

bulk kinetic (i.e. KE of fluid **NOT** of particles)

Conservation of MHD energy

$$E = \int_V \left[\tfrac{1}{2}\rho|\mathbf{u}|^2 + \tfrac{3}{2}p \, + \, \tfrac{1}{8\pi}|\mathbf{B}|^2 + \rho\Psi \right] d^3x$$

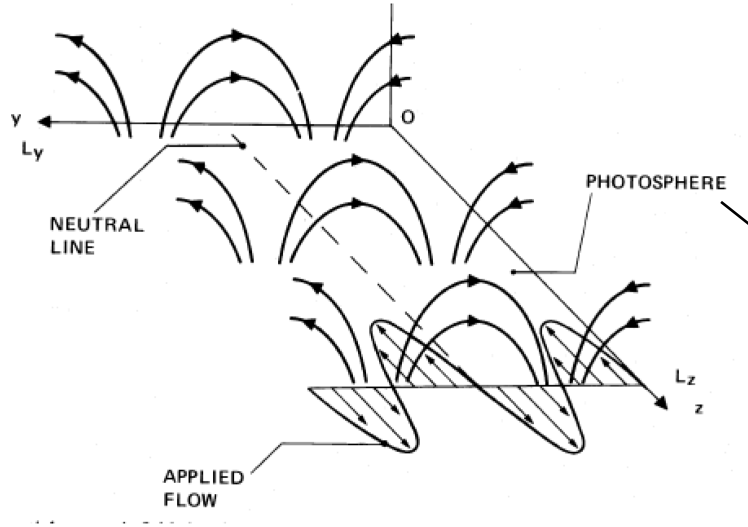
Take d/dt, bring into integral, use MHD equations, integrate by parts ....

$$\frac{dE}{dt} = -\oint_{\partial V} \left[\tfrac{1}{2}\rho|\mathbf{u}|^2\,\mathbf{u} + \tfrac{5}{2}p\mathbf{u} \, + \, \tfrac{1}{4\pi}\mathbf{B}\times(\mathbf{u}\times\mathbf{B}) + \rho\Psi\mathbf{u} \right] \cdot d\mathbf{a}$$

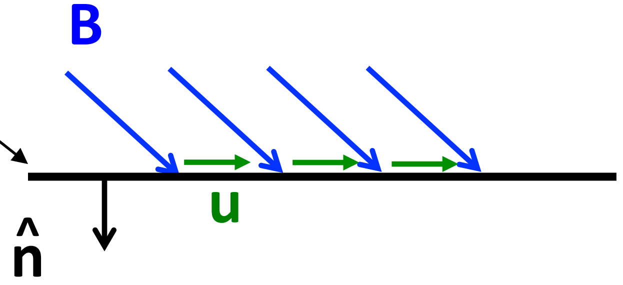
enthalpy flux

Poynting flux

- All fluxes vanish if **u** = 0 on bndry ∂V



Pos. work stressing field from bndry



$$\frac{dE}{dt} = -\oint_{\partial V}\left[\tfrac{1}{2}\rho\left|\mathbf{u}\right|^2\mathbf{u} + \tfrac{5}{2}p\mathbf{u} \;+\; \tfrac{1}{4\pi}\mathbf{B}\times\left(\mathbf{u}\times\mathbf{B}\right) + \rho\Psi\mathbf{u}\right]\cdot d\mathbf{a}$$

enthalpy flux

Poynting flux

- All fluxes vanish if **u** = 0 on bndry ∂V
- Tangential bndry motion only: **u.da** = $u_n$da = 0 ➔

$$\frac{dE}{dt} = \tfrac{1}{4\pi}\oint_{\partial V}\left(\mathbf{u}_\perp\cdot\mathbf{B}_\perp\right)B_n\,da$$

Work done by stressing field from bndry

Integrated densities

$$\mathbf{g} = -\nabla\Psi$$

Total energy

thermal magnetic gravitational

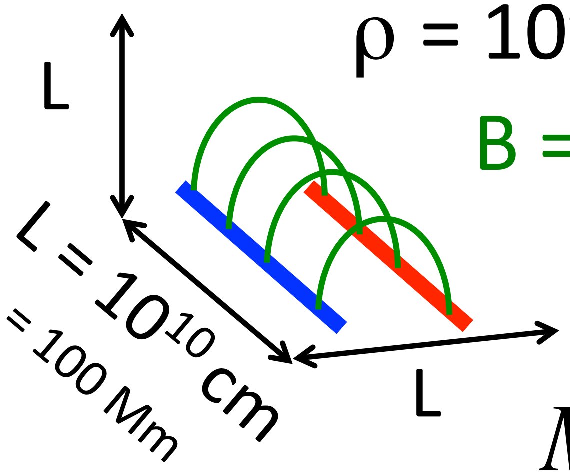
$$E = \int_V \left[\tfrac{1}{2}\rho|\mathbf{u}|^2 + \tfrac{3}{2}p + \tfrac{1}{8\pi}|\mathbf{B}|^2 + \rho\,\Psi \right] d^3x$$

bulk kinetic (i.e. KE of fluid **NOT** of particles)

$$\frac{\text{thermal}}{\text{magnetic}} \sim \frac{p}{B^2/8\pi} \equiv \beta \quad \sim 10^{-3} \text{ in corona}$$

$\Longrightarrow$ neglect thermal (& grav.)

focus on magnetic & kinetic



ρ = 10^{-15} g/cm^3 (n_e= 10^9 cm^{-3})

B = 100 G

p = 0.1 erg/cm^3 (T = 10^6 K)

$$M = \int_V \rho(\mathbf{x})\, d^3x = \rho\, L^3 = 10^{15}\ \mathrm{g}$$

$$E = \int_V \left[\tfrac{1}{2}\rho|\mathbf{u}|^2 + \tfrac{3}{2}p + \tfrac{1}{8\pi}|\mathbf{B}|^2 + \rho\Psi \right] d^3x$$

E_T ~ $pL^3 = 10^{29}$ erg

E_{grav} ~ $MgL/2 = 10^{29}$ erg

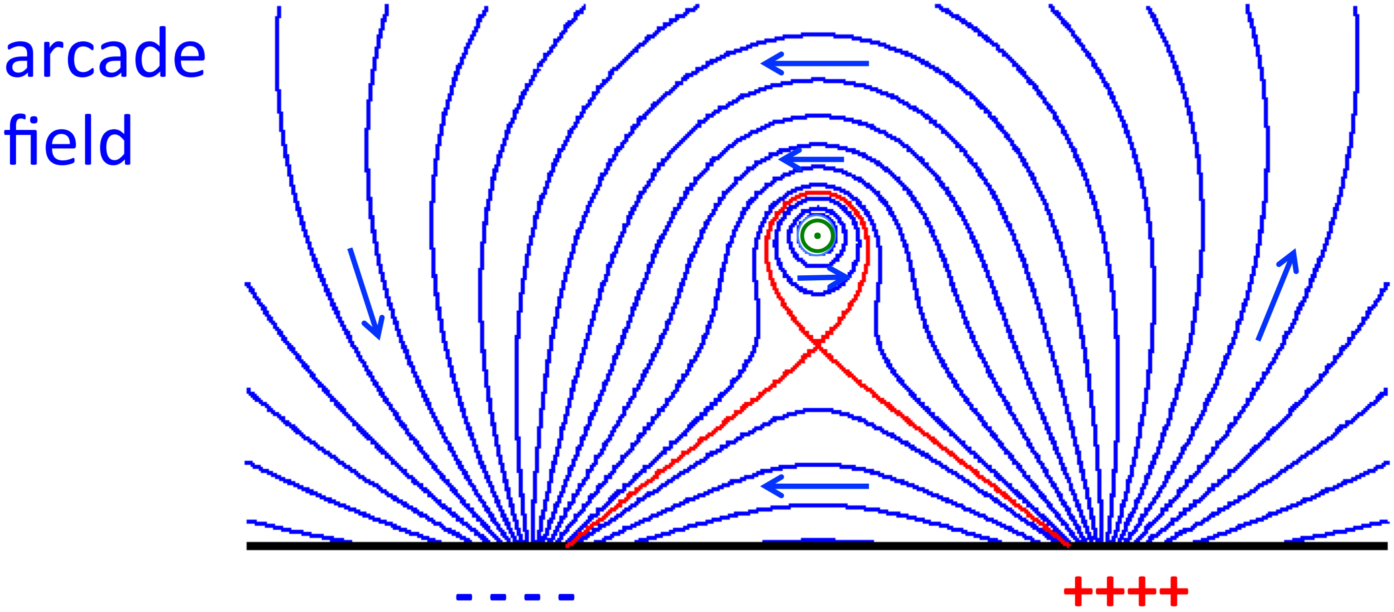
E_{mag} ~ $B^2L^3/8\pi = 4\times10^{32}$ erg

ejected @ u = 10^8 cm/s : E_{kin} ~ $Mu^2/2 = 5\times10^{30}$ erg

CME model ingredients

- Current = twist in flux rope
- Increasing twist (current) ➔ instability/LOE
- Eruption converts energy: magnetic → kinetic

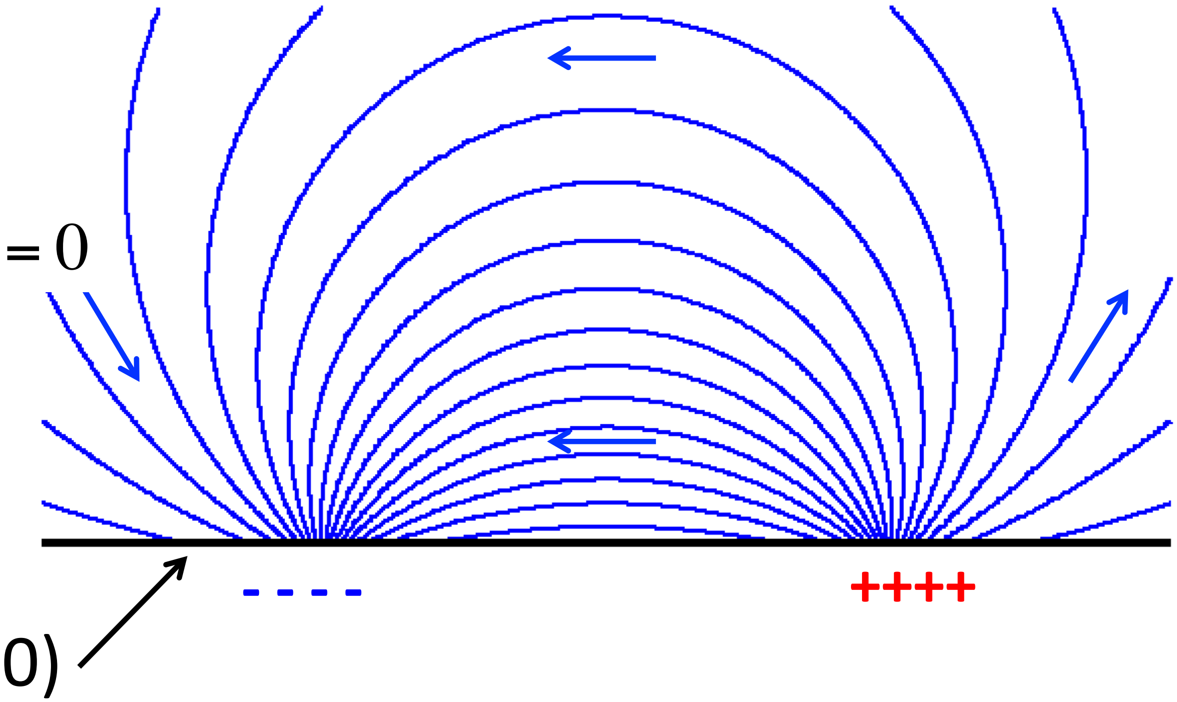
Simple example*



w/ flux rope – line current

*based on van Tend & Kuperus 1978
and Forbes & Isenberg 1991

Simple example



arcade
field

$\mathbf{J} = \nabla \times \mathbf{B} = 0$

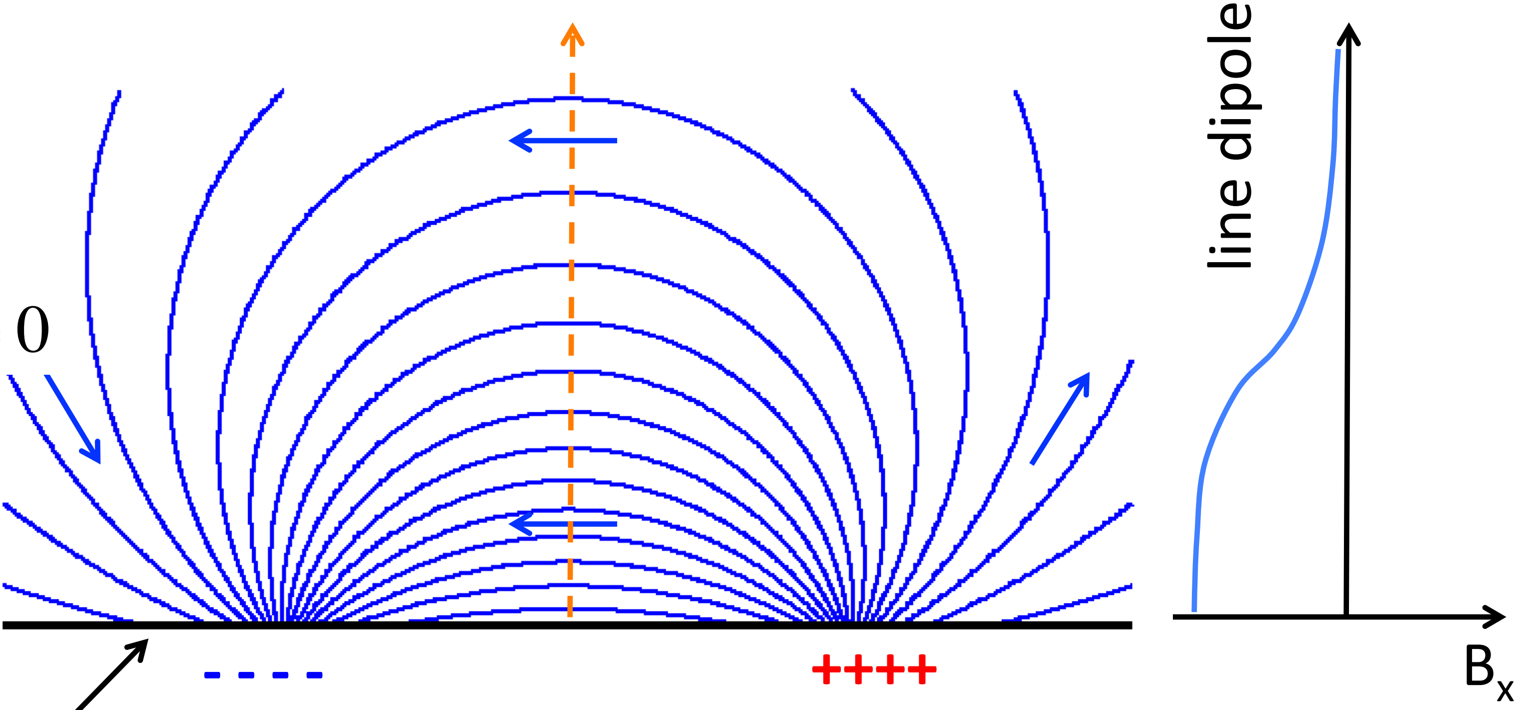
fix $B_z(x,0)$

Simple example

arcade field

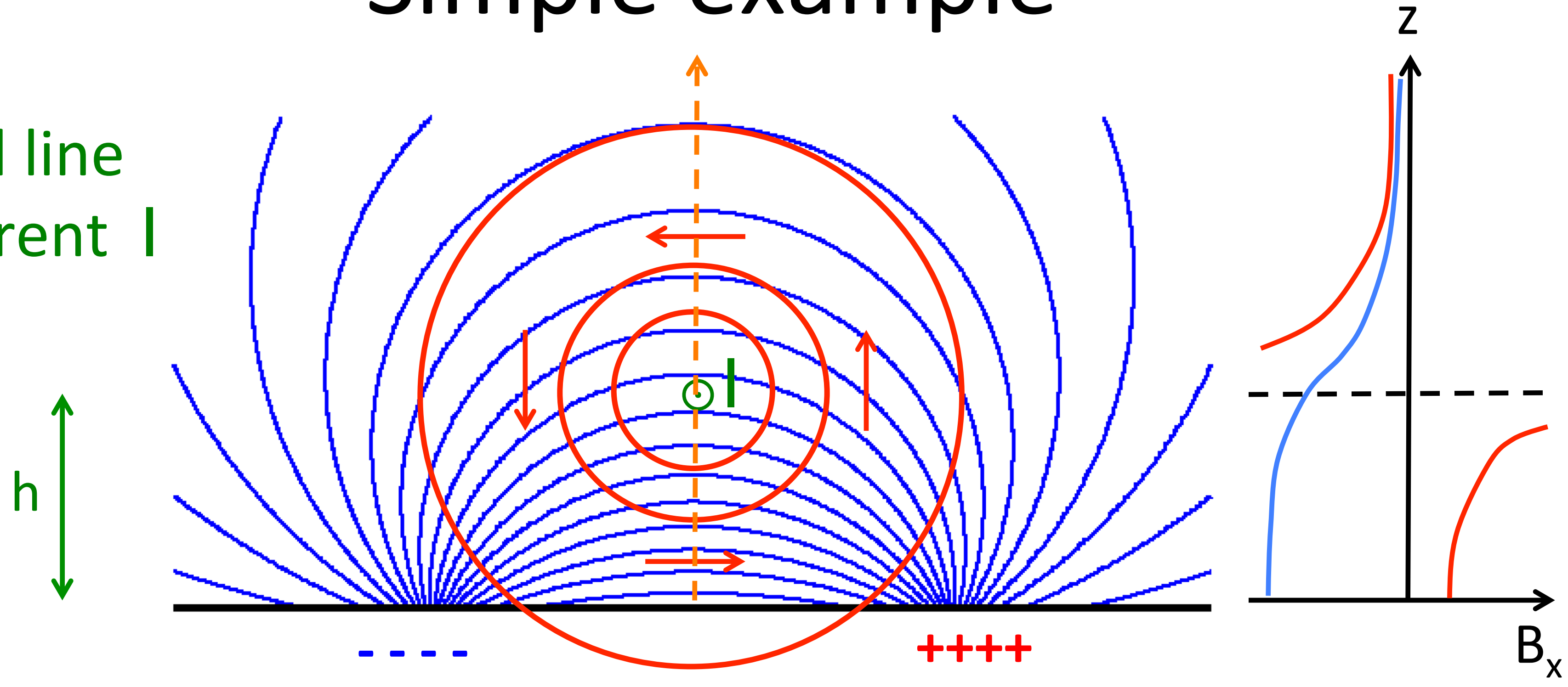
$\mathbf{J} = \nabla \times \mathbf{B} = 0$

fix $B_z(x,0)$



Simple example

add line current I



line current: $B_x = \dfrac{I}{h - z}$

Simple example

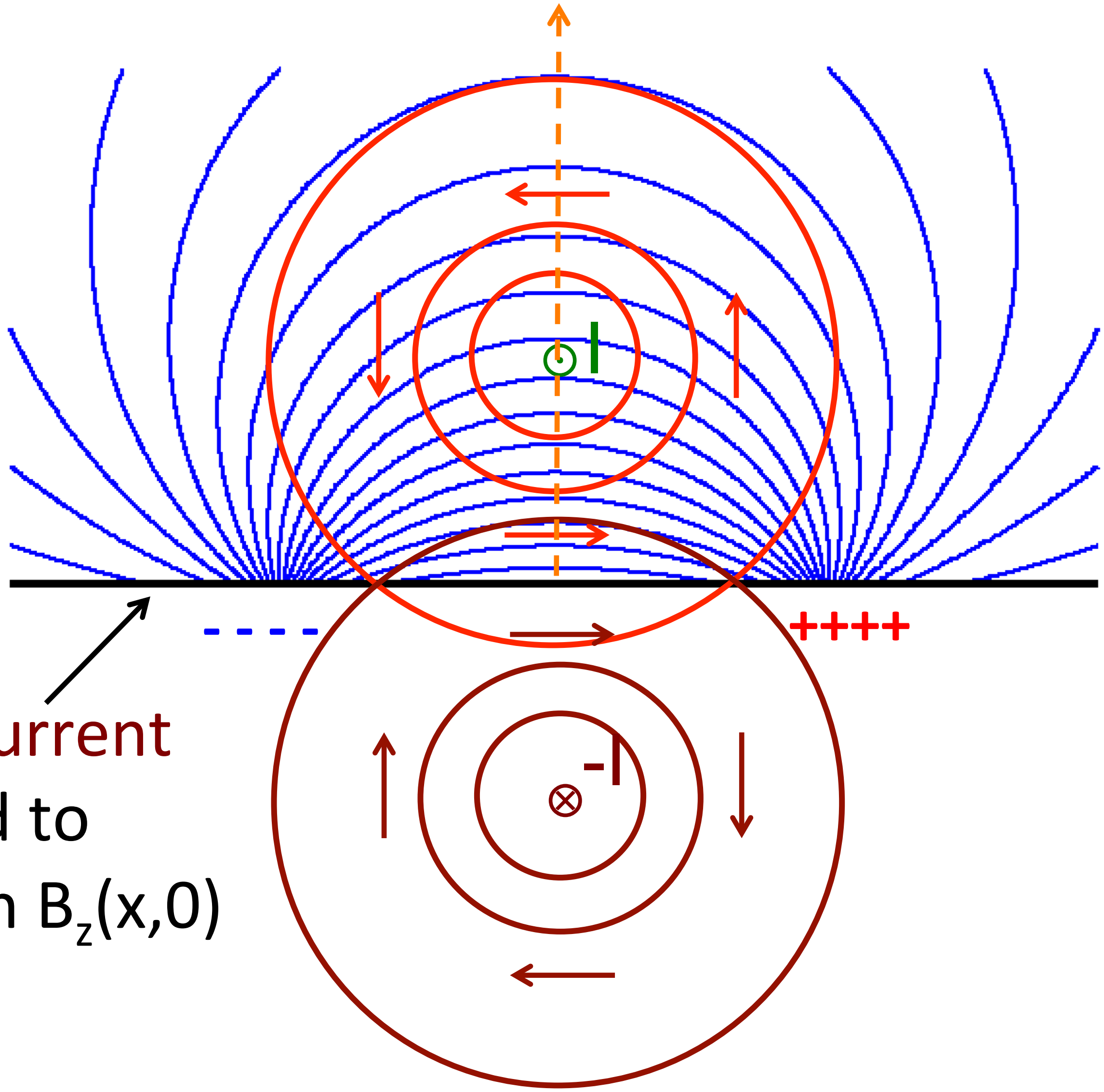
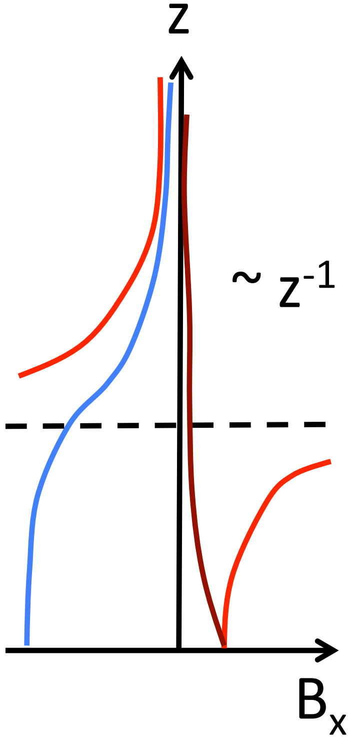
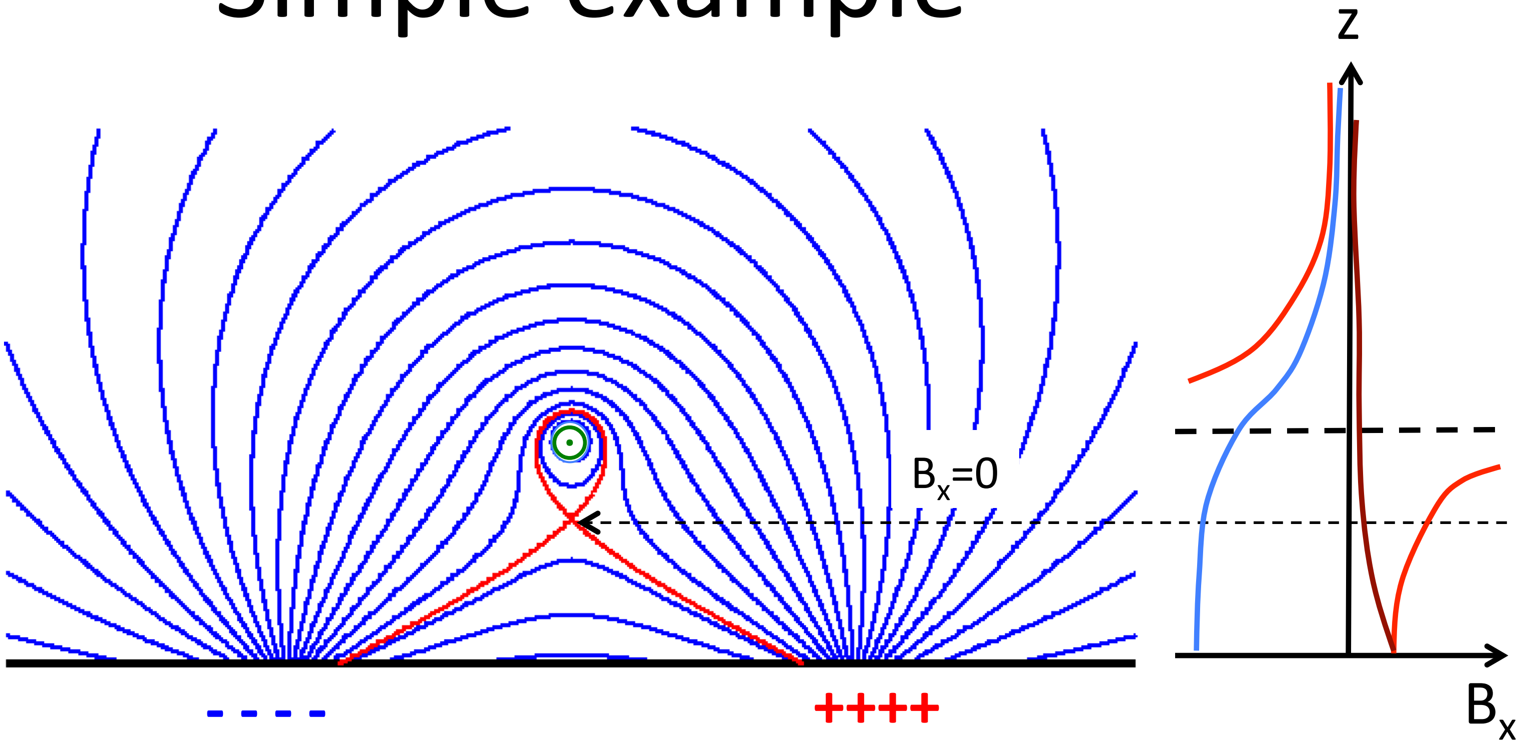


Image current required to maintain $B_z(x,0)$



$$B_x = \frac{I}{h+z}$$

Simple example

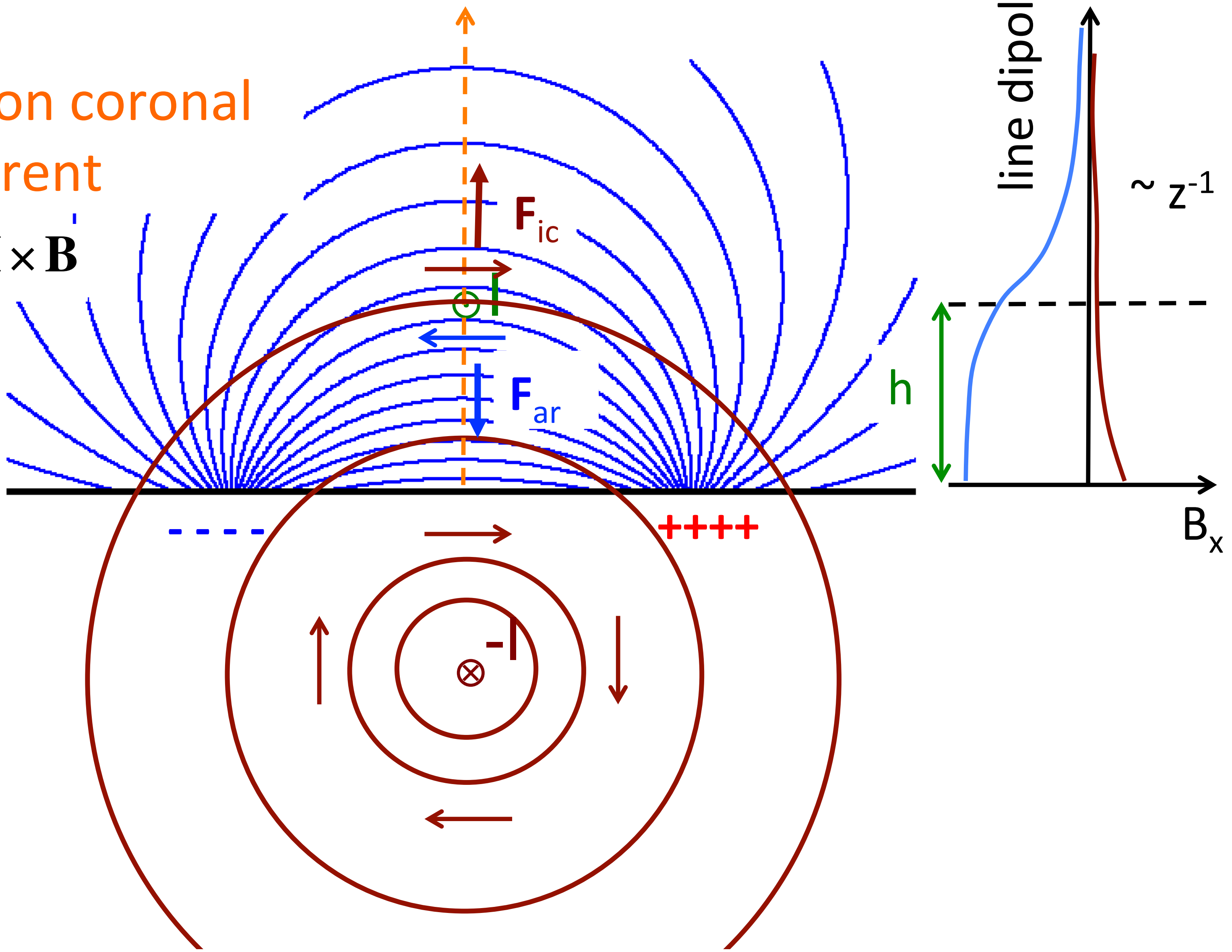


Sum all contributions

Simple example

Forces on coronal line current

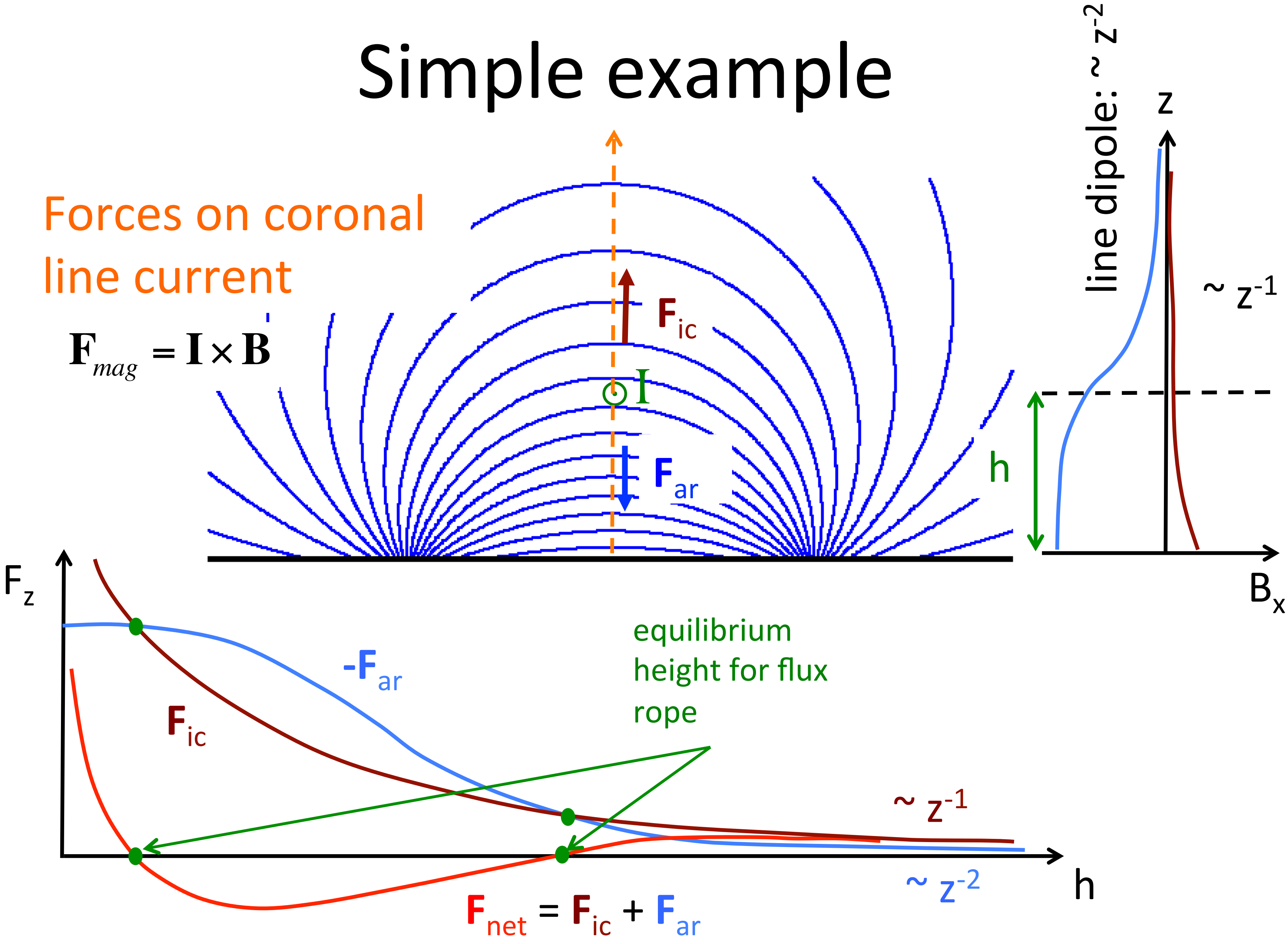
$$\mathbf{F}_{mag} = \mathbf{I} \times \mathbf{B}$$



Simple example

Forces on coronal line current

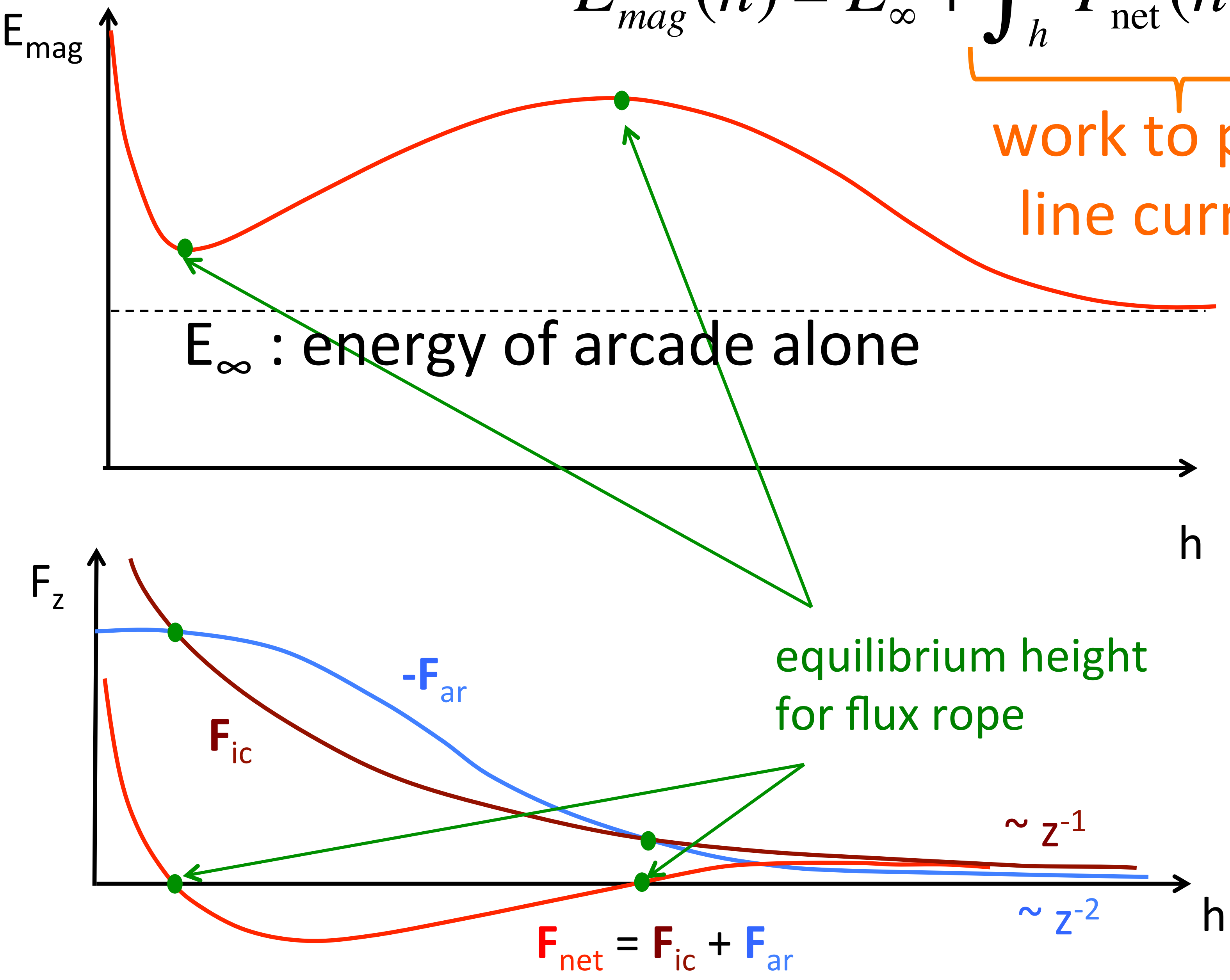
$$\mathbf{F}_{mag} = \mathbf{I} \times \mathbf{B}$$



equilibrium height for flux rope

$\mathbf{F}_{net} = \mathbf{F}_{ic} + \mathbf{F}_{ar}$

$$E_{mag}(h) = E_\infty + \underbrace{\int_h^\infty F_{\rm net}(h')\,dh'}_{\text{work to place line current}}$$

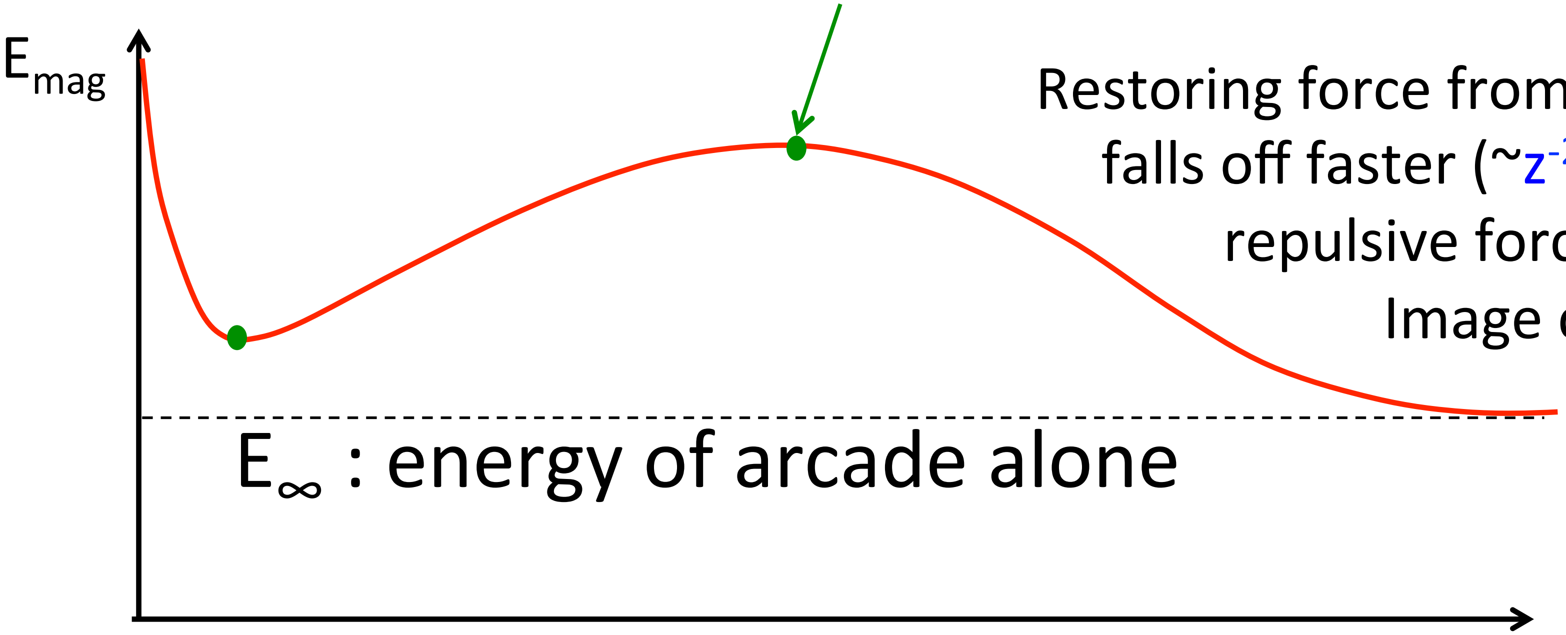


E_∞ : energy of arcade alone

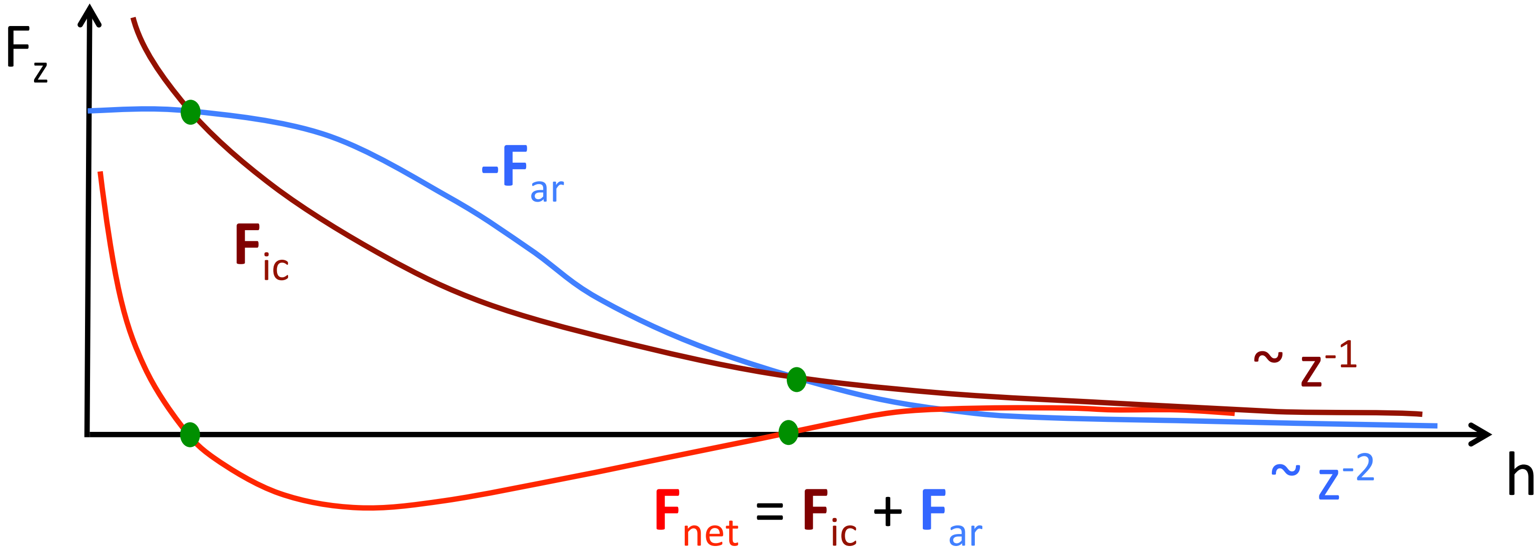
equilibrium height for flux rope

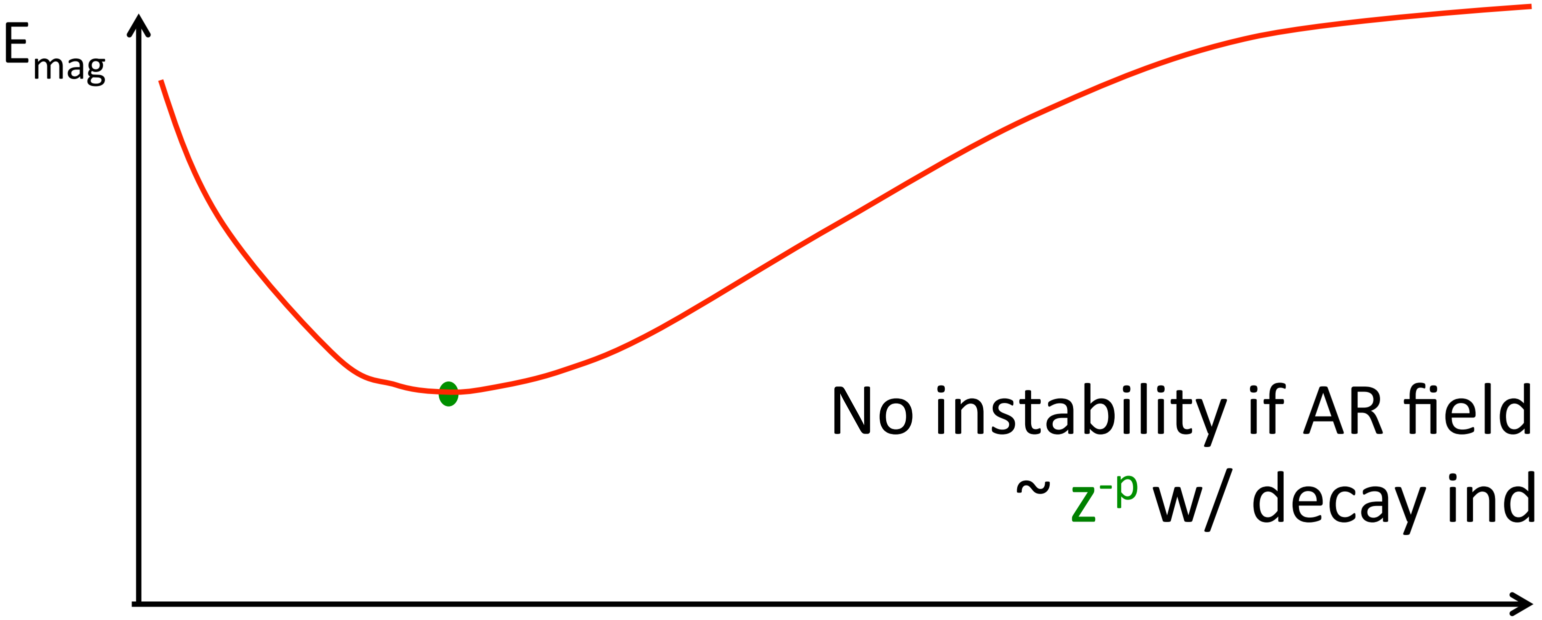
$\mathbf{F}_{net} = \mathbf{F}_{ic} + \mathbf{F}_{ar}$

equilibrium unstable to "torus instability":



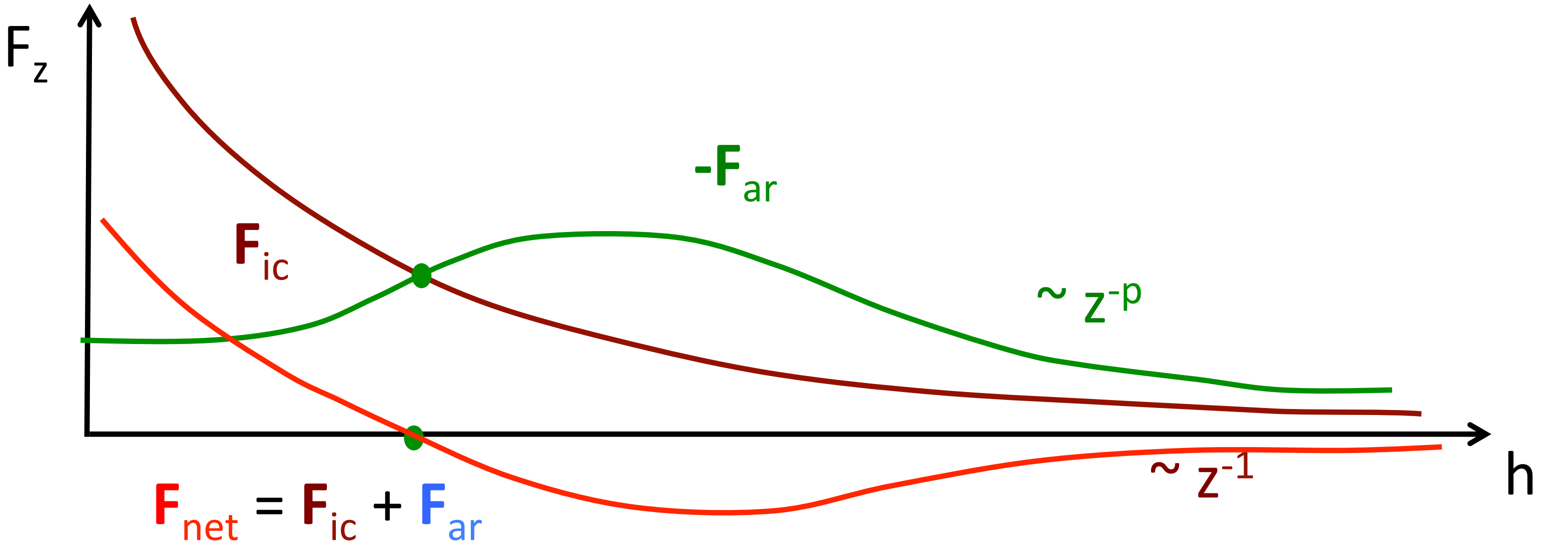
Restoring force from bipole falls off faster ($\sim z^{-2}$) than repulsive force from Image current ($\sim z^{-1}$)



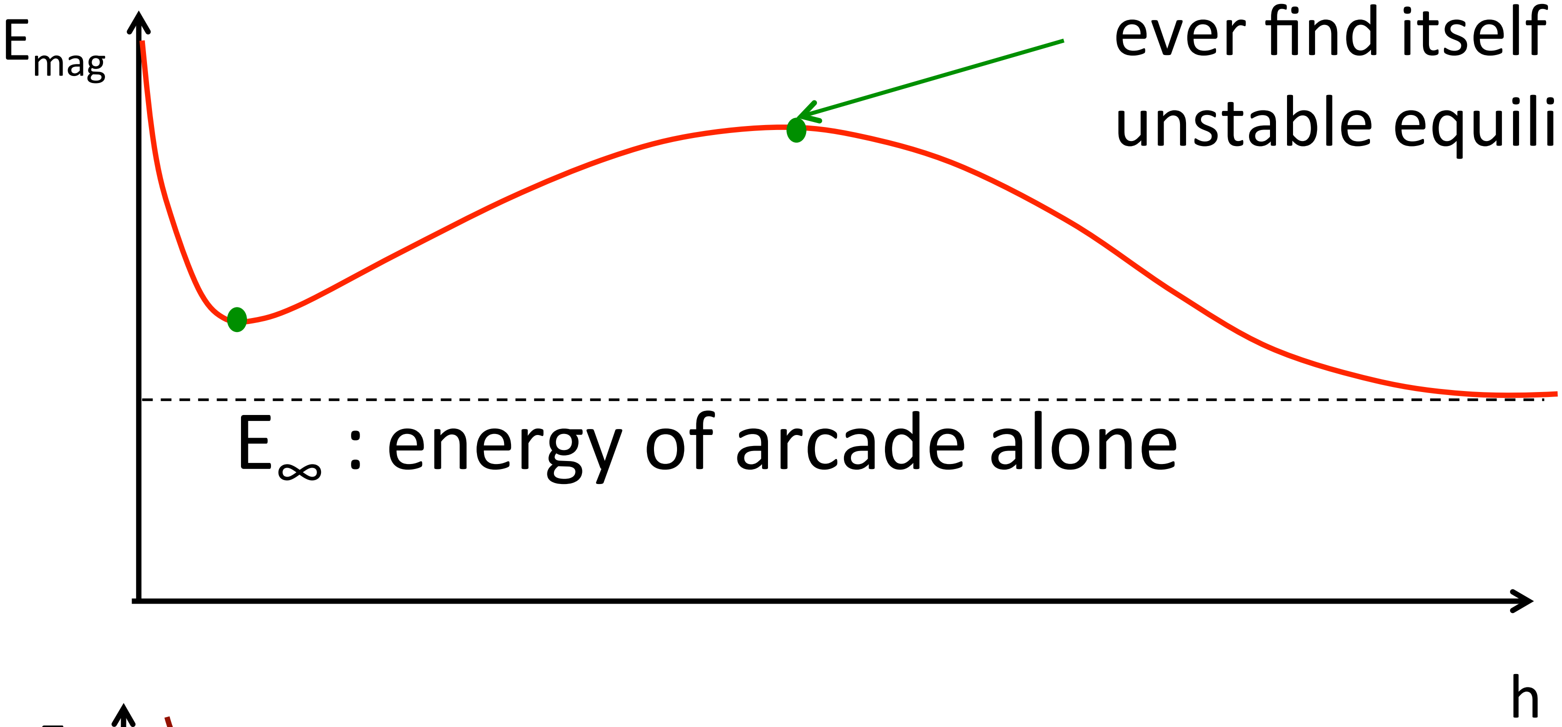


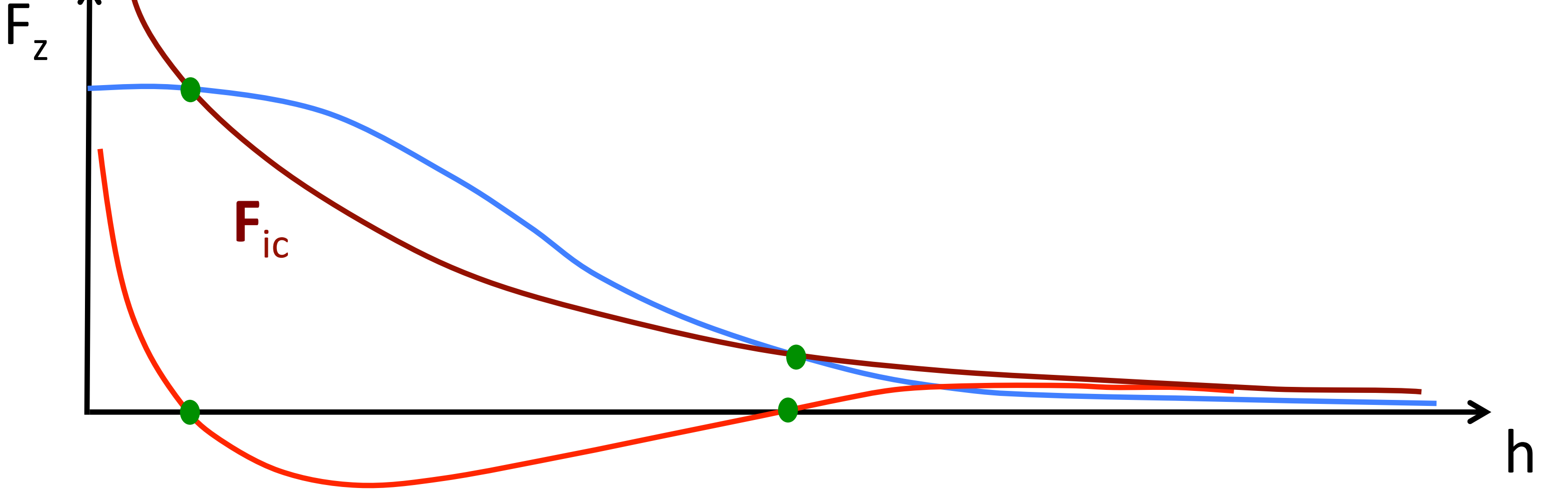
No instability if AR field falls off $\sim z^{-p}$ w/ decay index $p < 1$

(in cylindrical goem. critical index is $p < 1.5$)

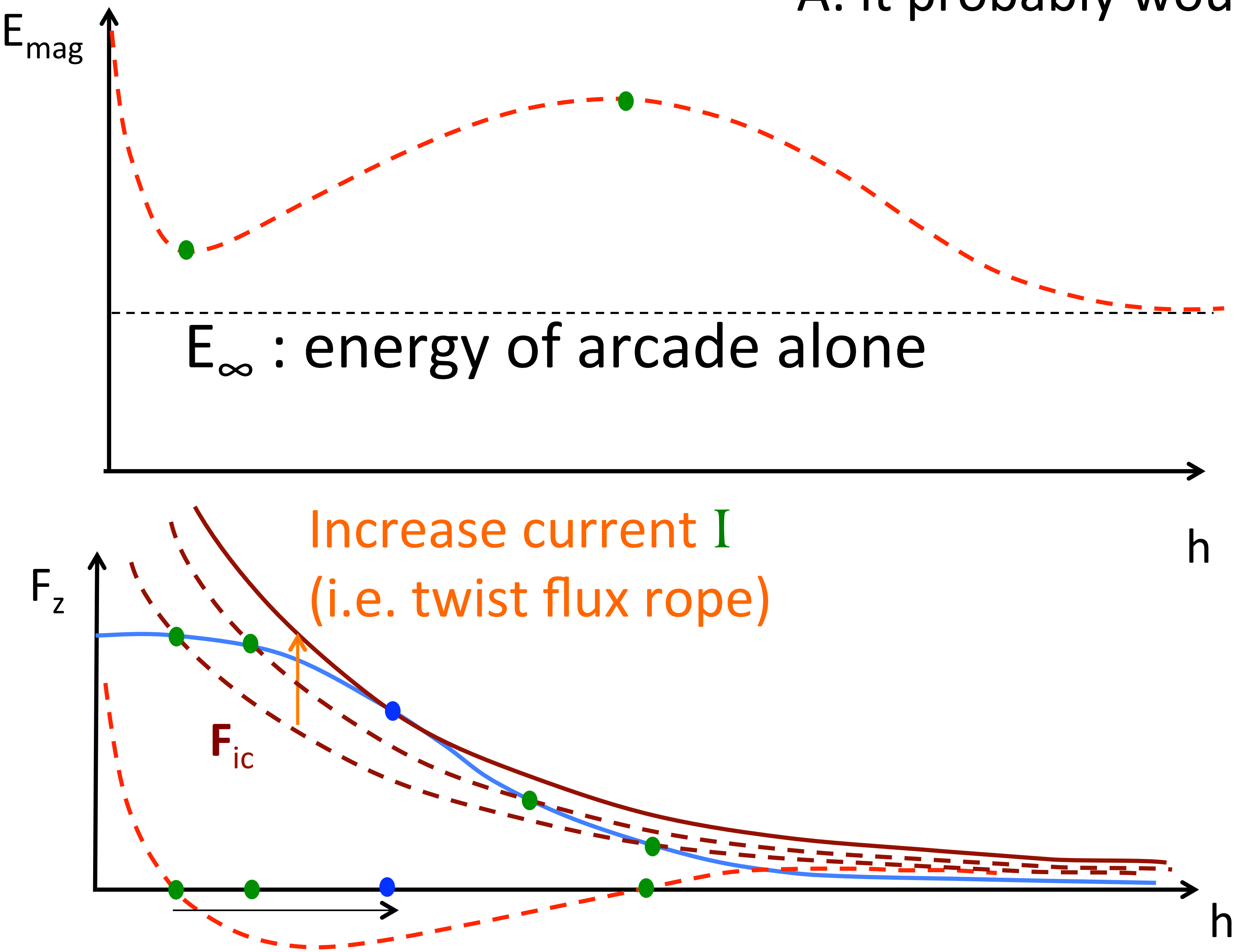


Q: How would system ever find itself in unstable equilibrium?





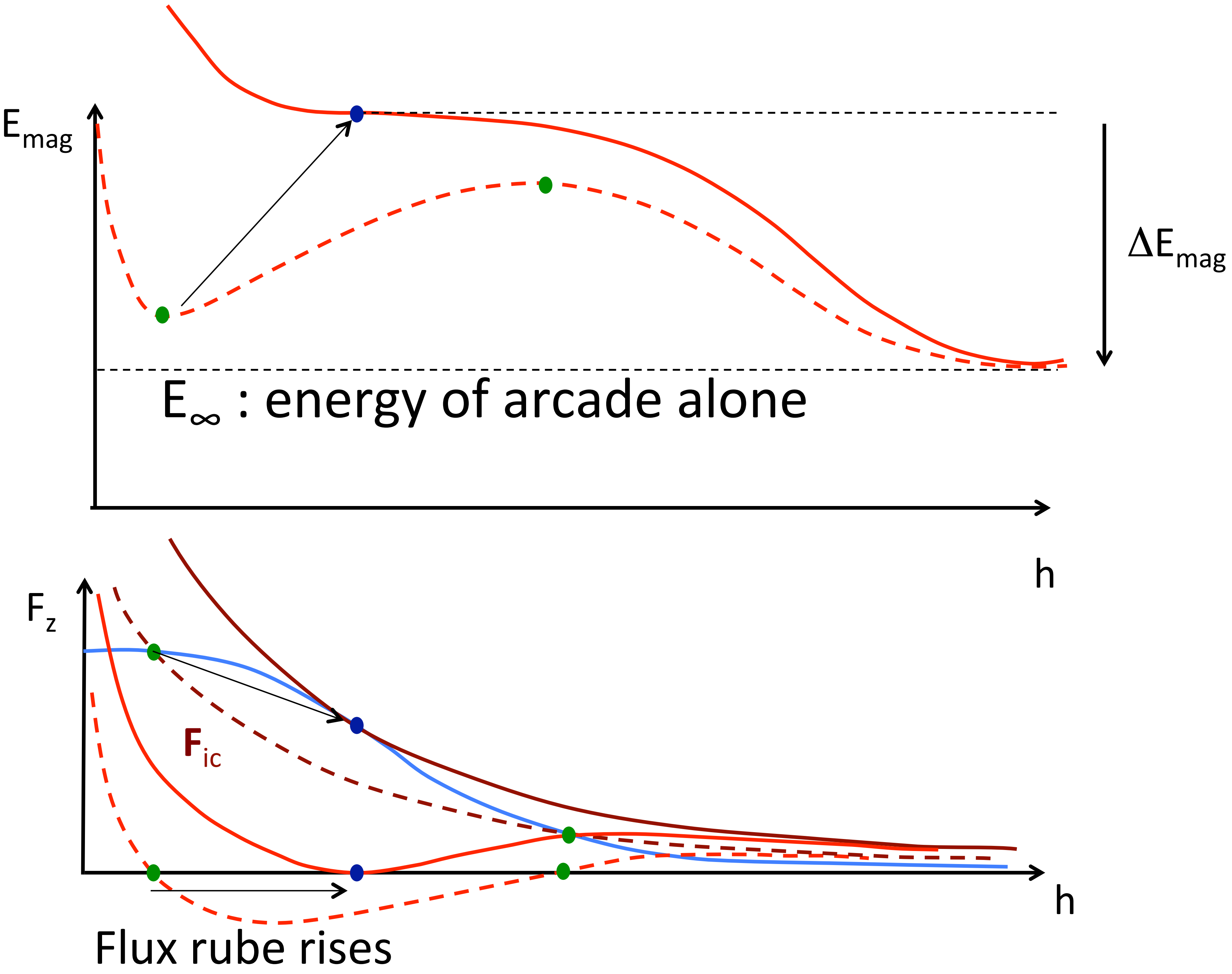
A: it probably wouldn't



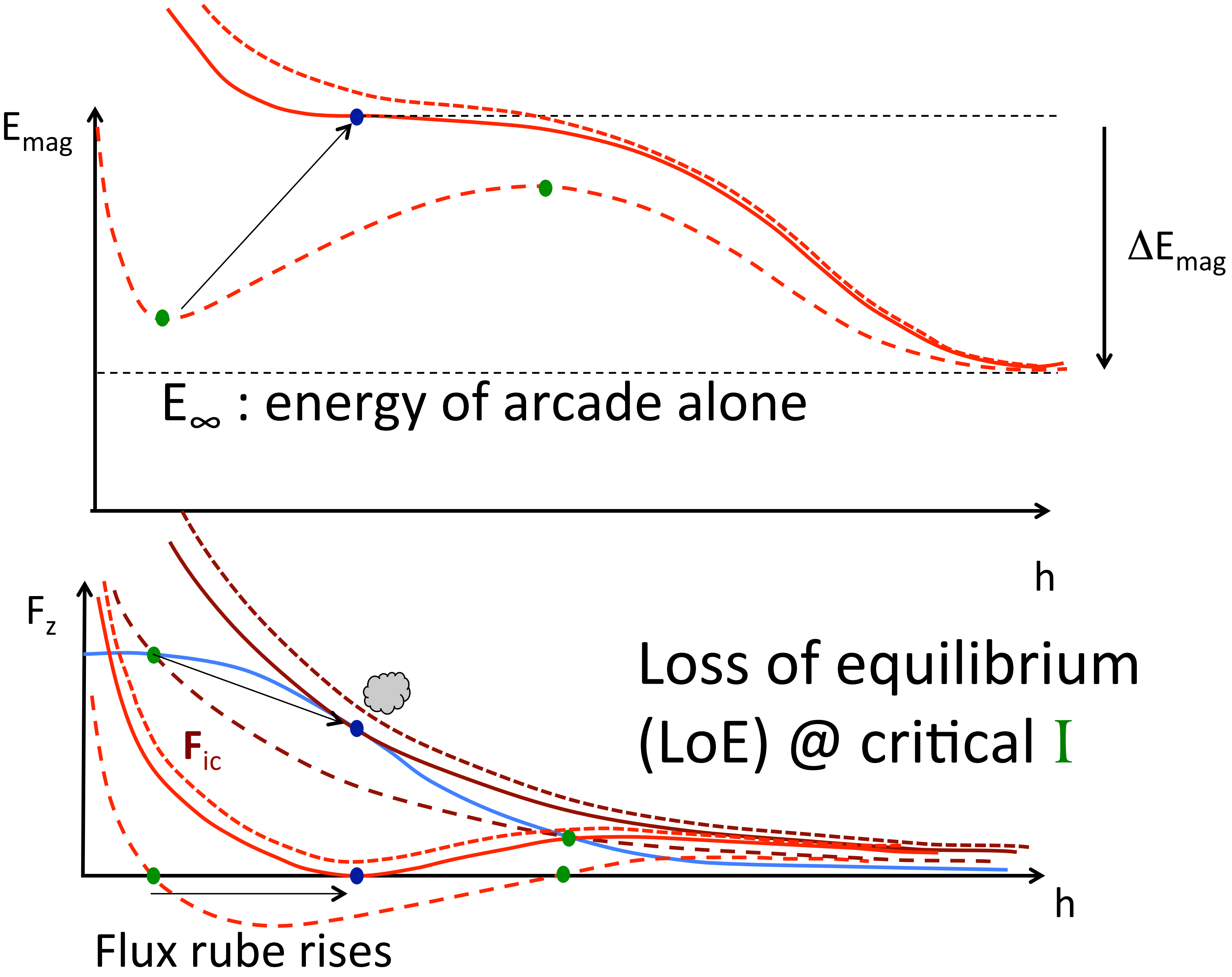
E_∞ : energy of arcade alone

Increase current I
(i.e. twist flux rope)

Flux rube rises


E_{mag}
ΔE_{mag}
E_∞ : energy of arcade alone
h
F_z
$\mathbf{F}_{ic}$
h
Flux rube rises

Loss of equilibrium (LoE) @ critical I



From Toy to Reality

I. Equilibrium

$$\overset{\text{equilibrium}}{\cancel{\rho\left(\frac{\partial}{\partial t}+\mathbf{u}\cdot\nabla\right)\mathbf{u}}} = -\overset{\beta \ll 1}{\cancel{\nabla p}}+\tfrac{1}{4\pi}\left(\nabla\times\mathbf{B}\right)\times\mathbf{B}+\overset{\beta \ll 1}{\cancel{\rho\mathbf{g}}}$$

➔ $\left(\nabla\times\mathbf{B}_0\right)\times\mathbf{B}_0=\mathbf{J}_0\times\mathbf{B}_0=0$ Force free field

➔ $\boxed{\nabla\times\mathbf{B}_0=\alpha(\mathbf{x})\,\mathbf{B}_0}$ $\nabla\times\mathbf{B}_0$ is parallel to $\mathbf{B}_0$

Special case: $\alpha = 0$

➔ $\nabla\times\mathbf{B}_0=\mathbf{J}_0=0$

$\mathbf{B}_0=-\nabla\chi$, $\nabla\cdot\mathbf{B}_0=-\nabla^2\chi=0$ **“potential field”**

Equilibria

Example 1: constant-α equilbria

vector
Helmholtz eq.

$$\nabla\times\mathbf{B}_0=\alpha\,\mathbf{B}_0 \quad \Rightarrow \quad \nabla\times\left(\nabla\times\mathbf{B}_0\right)=\boxed{-\nabla^2\mathbf{B}_0=\alpha^2\mathbf{B}_0}$$

Linear eq. ➔ constant-α

a.k.a.

“Linear force-free field” (LFFF)

Equilibria

Example 1: constant-α equilbria

vector
Helmholtz eq.

$$\nabla\times\mathbf{B}_0 = \alpha\,\mathbf{B}_0 \quad \Rightarrow \quad \nabla\times\left(\nabla\times\mathbf{B}_0\right) = \boxed{-\nabla^2\mathbf{B}_0 = \alpha^2\mathbf{B}_0}$$

Example 1a: cylindrical sym. = Lundquist flux rope

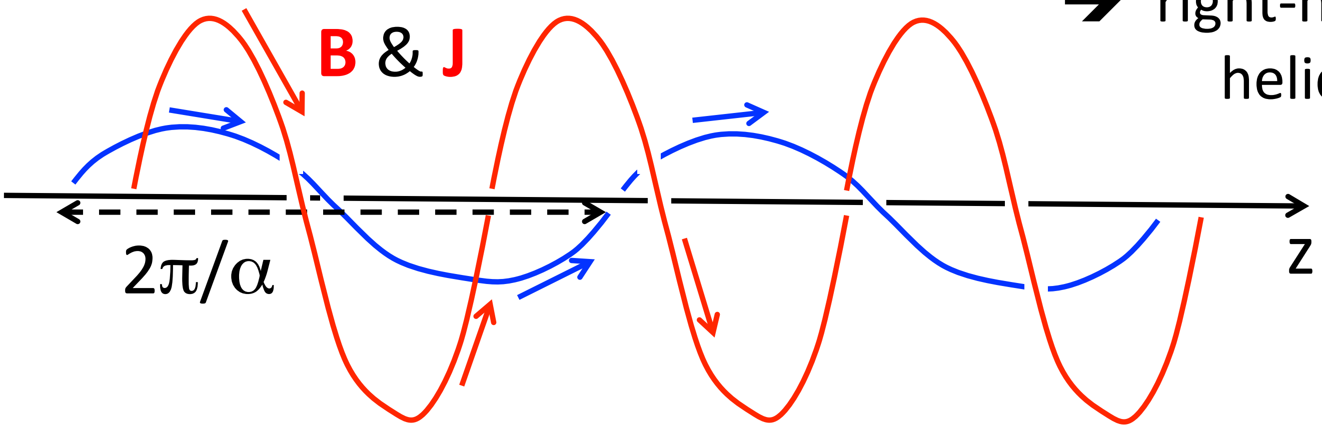
$$\nabla^2 B_z = -\alpha^2 B_z(r) \quad \Rightarrow \quad B_z(r) = B_0\,J_0(\alpha r)$$

Bessel Func.

$$\mathbf{B}_0(r) = B_0\left[J_0(\alpha r)\hat{\mathbf{z}} + J_1(\alpha r)\hat{\phi}\right]$$

α > 0

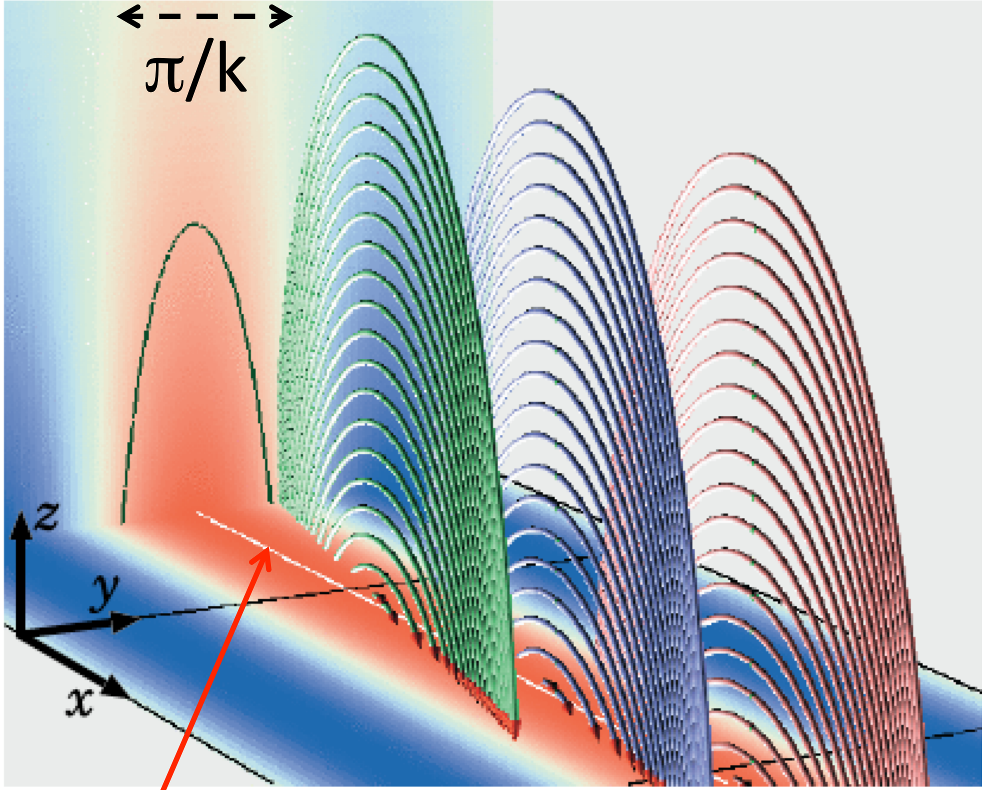
➔ right-handed
helices



Equilibria

Example 1b: Cartesian sym. = periodic arcade

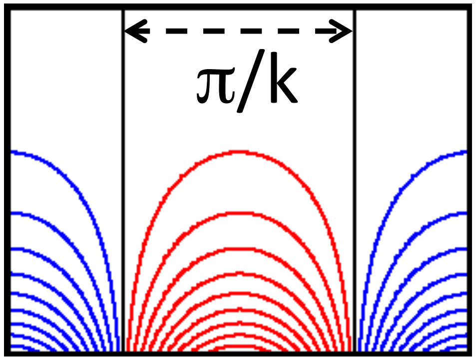
$$\nabla^2 B_x = -\alpha^2 B_x(y,z) \quad \rightarrow \quad B_x(y,z) \sim \cos(ky)\, e^{-z\sqrt{k^2-\alpha^2}}$$



Kusano et al. 2004

PIL: $B_z = 0$

$$\mathbf{B}_0 = B_0 \left\{ \frac{\alpha}{k}\cos(ky)\hat{\mathbf{x}} - \frac{\sqrt{k^2-\alpha^2}}{k}\cos(ky)\hat{\mathbf{y}} + \sin(ky)\hat{\mathbf{z}} \right\} e^{-z\sqrt{k^2-\alpha^2}}$$



From Toy to Reality

II. Perturb magnetic field

η << is very small (Rm >>1)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right) + \cancel{\eta} \nabla^2 \mathbf{B}$$

"ideal"

Make small displacement $\vec{\xi}(\mathbf{x}) = \mathbf{u}(\mathbf{x})\, \delta t$

$$\mathbf{B}(\mathbf{x}) = \mathbf{B}_0(\mathbf{x}) + \nabla \times \left(\vec{\xi} \times \mathbf{B}_0 \right) + \tfrac{1}{2} \nabla \times \left\{ \vec{\xi} \times \left[\nabla \times \left(\vec{\xi} \times \mathbf{B}_0 \right) \right] \right\} + \cdots$$

Perturbed field – **no field lines broken**

From Toy to Reality

III. Perturbation to Energy

$$E_m = \frac{1}{8\pi}\int \left|\mathbf{B}(\mathbf{x})\right|^2 d^3x$$

introduce $\mathbf{B}(\mathbf{x}) = \mathbf{B}_0(\mathbf{x}) + \nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right) + \tfrac{1}{2}\nabla\times\left\{\vec{\xi}\times\left[\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right]\right\} + \cdots$

Expand, integrate by parts, …

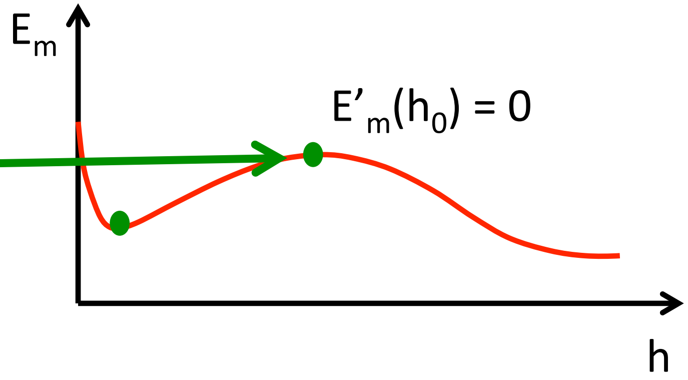
$$E_m = \frac{1}{8\pi}\int \left|\mathbf{B}_0\right|^2 d^3x - \frac{1}{4\pi}\int \left[\left(\nabla\times\mathbf{B}_0\right)\times\mathbf{B}_0\right]\cdot\vec{\xi}\, d^3x$$

$$+\frac{1}{8\pi}\int\left\{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2 - \left(\vec{\xi}\times\nabla\times\mathbf{B}_0\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\}d^3x + \cdots$$

From Toy to Reality

III. Perturbation to Energy

First variation: vanishes if $\mathbf{B}_0(\mathbf{x})$ is equilibrium



$$\boxed{\left(\nabla\times\mathbf{B}_0\right)\times\mathbf{B}_0 = 0}$$

$$E_m = \frac{1}{8\pi}\int\left|\mathbf{B}_0\right|^2 d^3x \boxed{-\frac{1}{4\pi}\int\left[\left(\nabla\times\mathbf{B}_0\right)\times\mathbf{B}_0\right]\cdot\vec{\xi}\, d^3x}$$

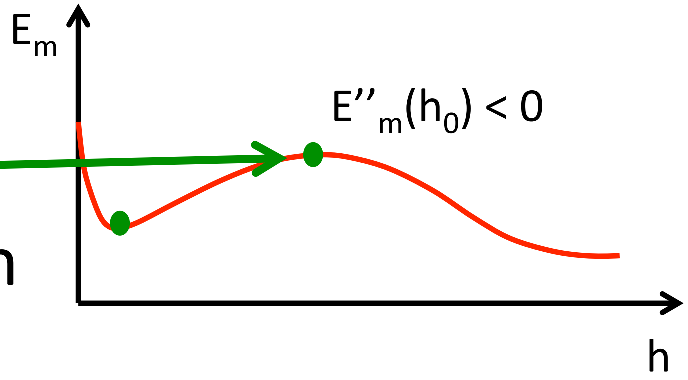
$$+\frac{1}{8\pi}\int\left\{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2-\left(\vec{\xi}\times\nabla\times\mathbf{B}_0\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\}d^3x+\cdots$$

From Toy to Reality

III. Perturbation to energy Energy

second variation:

equilibrium is unstable if **any** perturbation $\xi(\mathbf{x})$ can make $\delta^2 W < 0$



$$E_m = \frac{1}{8\pi}\int |\mathbf{B}_0|^2 d^3x - \frac{1}{4\pi}\int \left[(\nabla\times\mathbf{B}_0)\times\mathbf{B}_0\right]\cdot\vec{\xi}\, d^3x$$

$$\boxed{+\frac{1}{8\pi}\int\left\{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2 - \left(\vec{\xi}\times\nabla\times\mathbf{B}_0\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\}d^3x} + \cdots$$

$\delta^2 W\{ \xi(\mathbf{x}) \}$

Q: can we find perturbation $\boldsymbol{\xi}(\mathbf{x})$ to make this negative?

$$\delta^2 W\left\{\vec{\xi}(\mathbf{x})\right\} = \frac{1}{8\pi}\int\left\{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2 - \left(\vec{\xi}\times\nabla\times\mathbf{B}_0\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\}d^3x$$

Q: can we find perturbation $\boldsymbol{\xi}(\mathbf{x})$ to make this negative?

$$\delta^2 W\left\{\vec{\xi}(\mathbf{x})\right\} = \frac{1}{8\pi}\int\left\{ \underset{\text{always} > 0}{\boxed{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2}} - \left(\vec{\xi}\times\underset{\mathbf{J}_0}{\boxed{\nabla\times\mathbf{B}_0}}\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\} d^3x$$

- Not if $\nabla\times\mathbf{B}_0 = 0$ --- potential fields are stable
- Only if $\nabla\times\mathbf{B}_0 = \mathbf{J}_0$ is “large enough”

Take-home message: To be unstable a force-free equilibrium must have **current** exceeding some threshold value

Application to constant-α equilibria: $\nabla \times \mathbf{B}_0 = \alpha\, \mathbf{B}_0$

$$\delta^2 W\left\{\vec{\xi}(\mathbf{x})\right\} = \frac{1}{8\pi}\int\left\{\left|\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right|^2 - \alpha\left(\vec{\xi}\times\mathbf{B}_0\right)\cdot\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right)\right\}d^3x$$

Find* ξ & μ s.t. $\boxed{\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right) = \mu\left(\vec{\xi}\times\mathbf{B}_0\right)}$

$$\delta^2 W = \frac{\mu(\mu-\alpha)}{8\pi}\int\left|\vec{\xi}\times\mathbf{B}_0\right|^2 d^3x$$

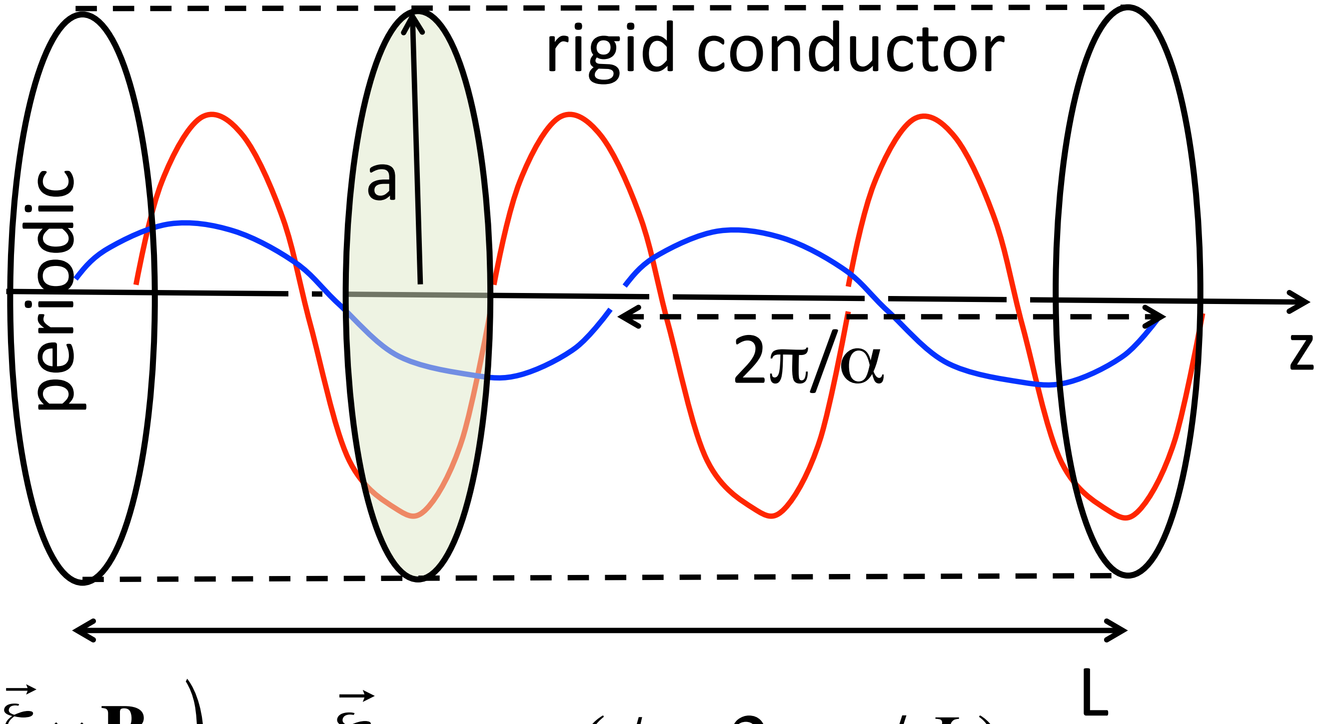
δ^2W < 0 if we can find* $0 < \mu < \alpha$

* Will depend on equilibrium, boundary conditions, and α

Example 1a:
Lundquist
flux rope

$$\mathbf{B}_0(r) = B_0\left[J_0(\alpha r)\hat{\mathbf{z}} + J_1(\alpha r)\hat{\phi}\right]$$

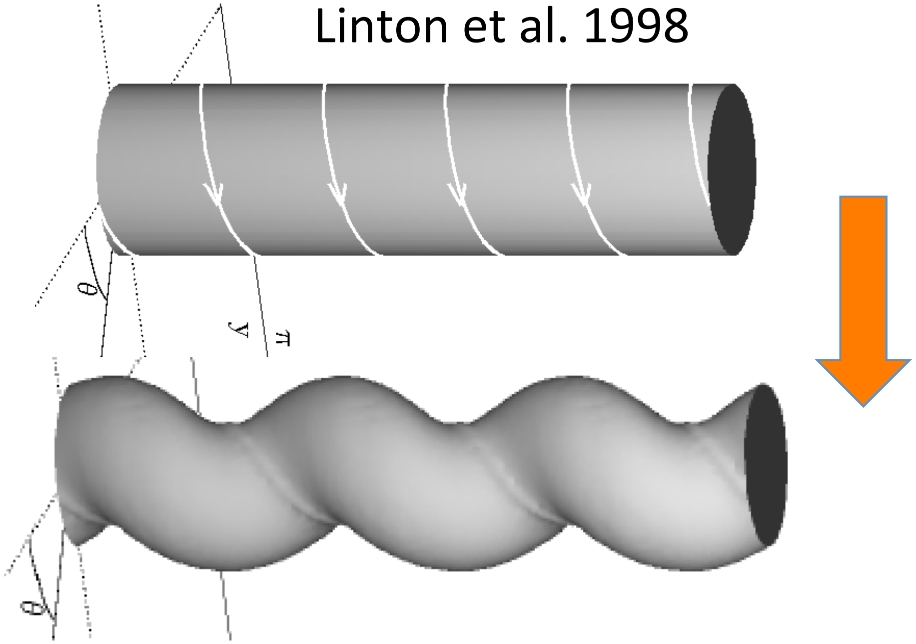
Take eg.
L = 4a



$$\nabla \times \left(\vec{\xi} \times \mathbf{B}_0\right) = \mu\left(\vec{\xi} \times \mathbf{B}_0\right) \qquad \vec{\xi} \sim \cos(\phi - 2\pi z / L)$$

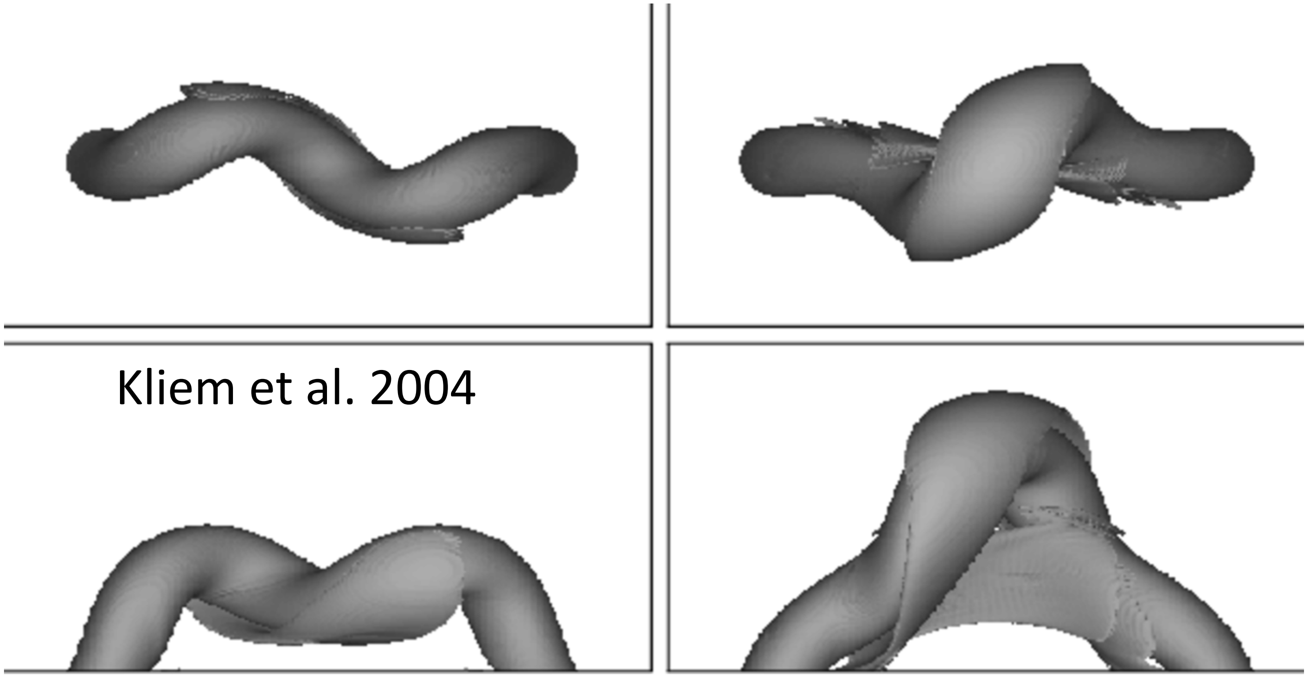
"kink" mode (m=1)

$$\mu = \frac{12.62}{L}$$ Unstable when $$\alpha > \frac{12.62}{L}$$ ➔ $$\frac{\alpha L}{2\pi} = 2.01$$ turns



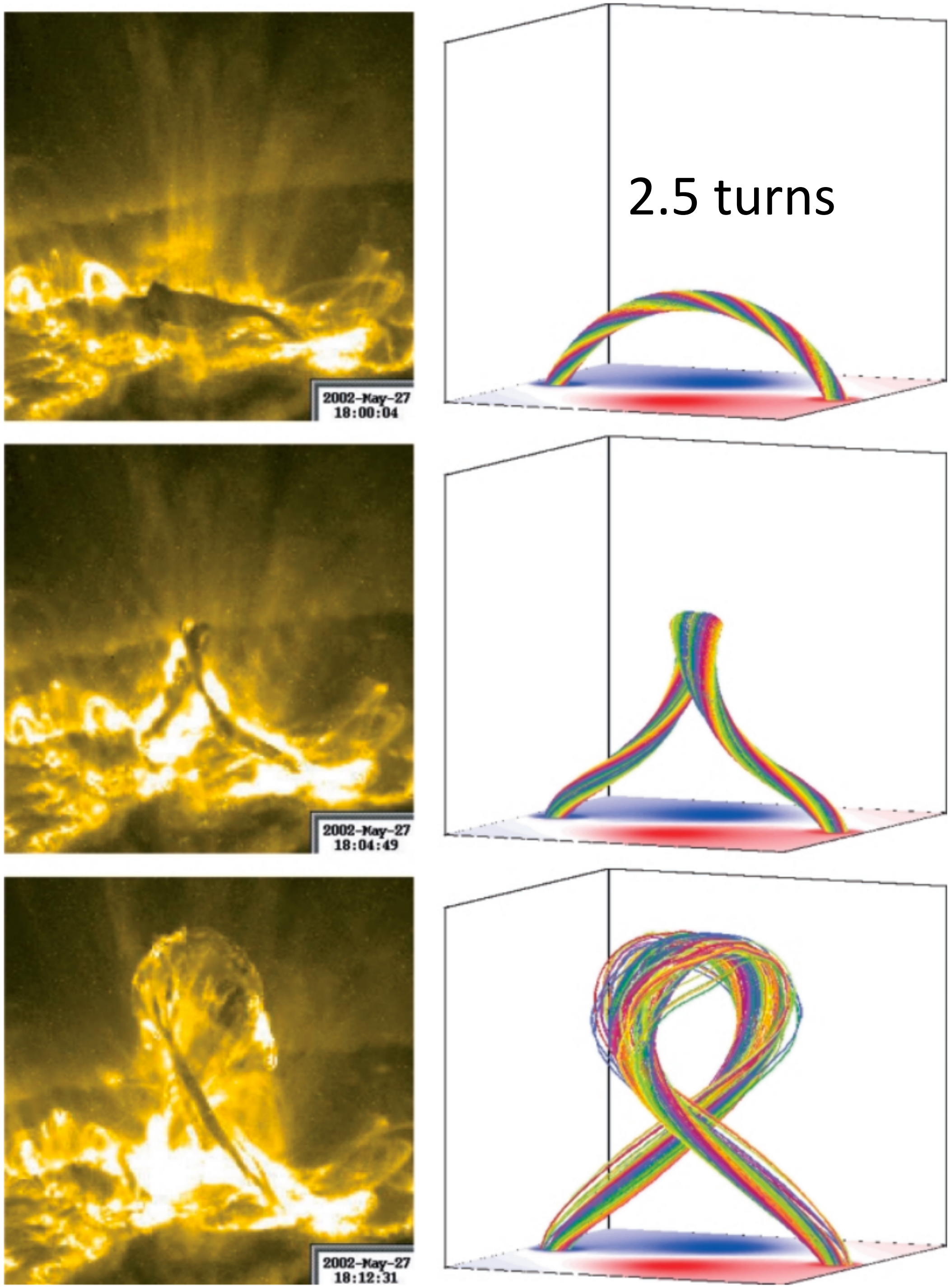
Linton et al. 1998

Kink mode instability



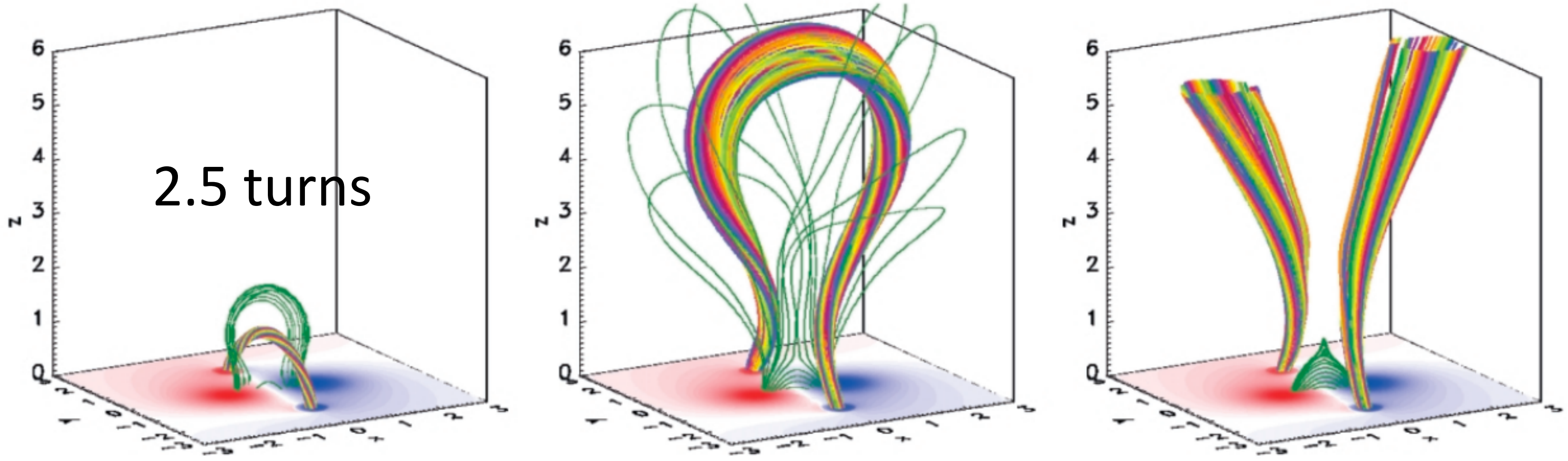
Kliem et al. 2004

Non-eruption observed by TRACE on 2002-May-27



Torok & Kliem 2005

Weaker field above ➔ full eruption



2.5 turns

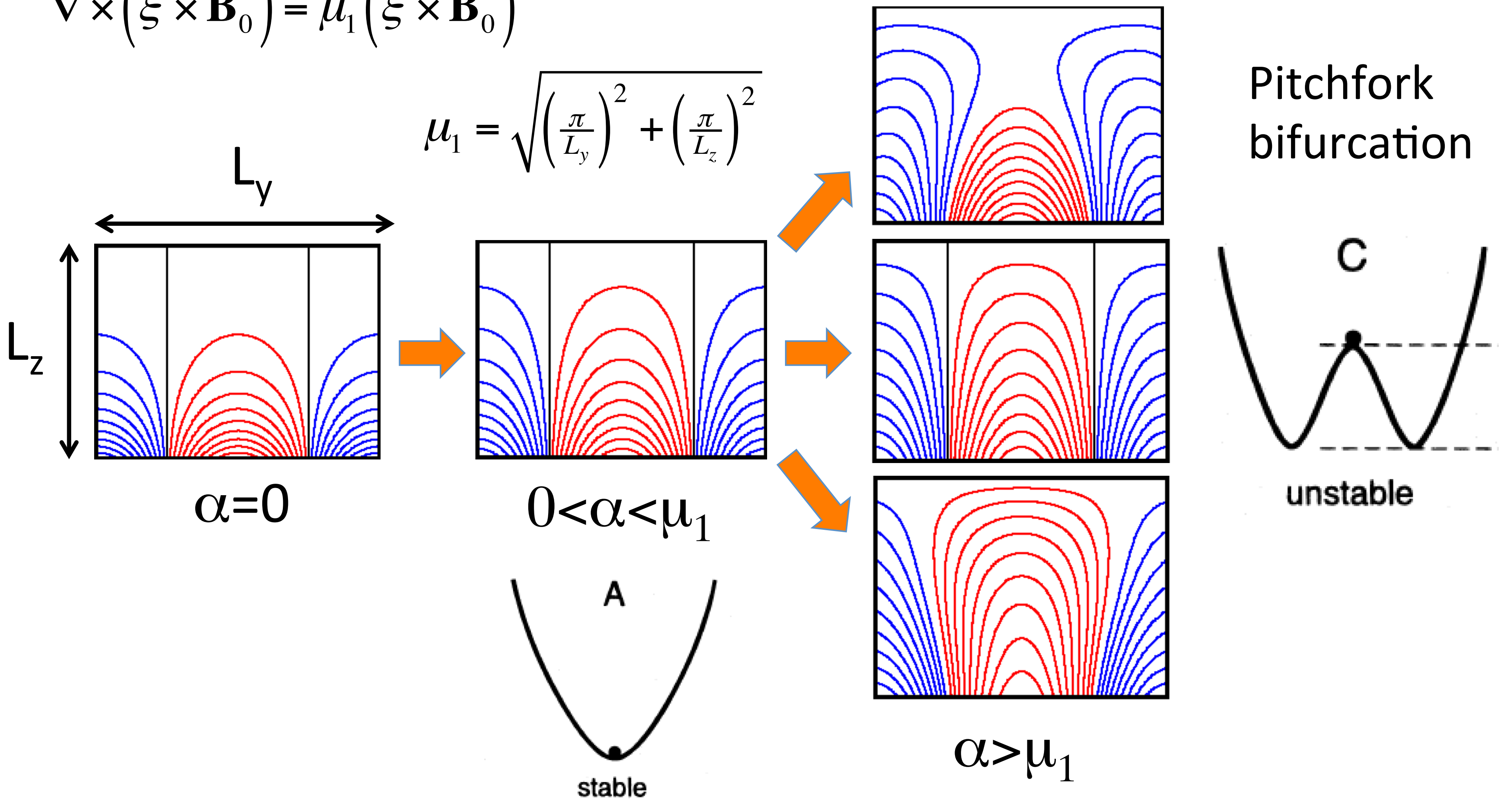
Torok & Kliem 2005

Example 1b: Cartesian sym. = periodic arcade

$$\vec{\xi}\times\mathbf{B}_0 = \frac{\pi}{L_z}\cos\left(\frac{\pi y}{L_y}\right)\cos\left(\frac{\pi z}{L_z}\right)\hat{\mathbf{y}} + \frac{\pi}{L_y}\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right)\hat{\mathbf{z}} + \mu_1\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right)\hat{\mathbf{x}}$$

$$\nabla\times\left(\vec{\xi}\times\mathbf{B}_0\right) = \mu_1\left(\vec{\xi}\times\mathbf{B}_0\right)$$

$$\mu_1 = \sqrt{\left(\frac{\pi}{L_y}\right)^2 + \left(\frac{\pi}{L_z}\right)^2}$$



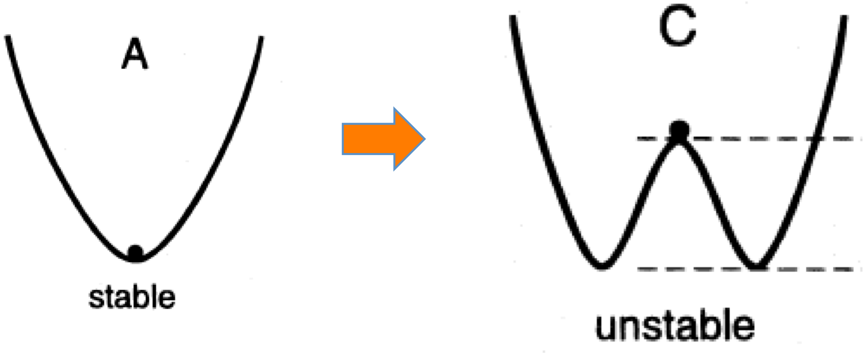
Pitchfork bifurcation

Instability

- Unstable equilibrium

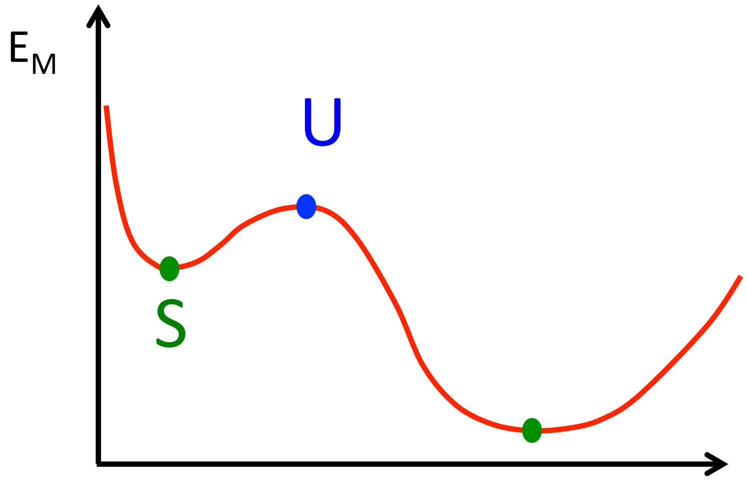
  $\delta^2 W < 0$ (ideal instability: $\eta=0$)

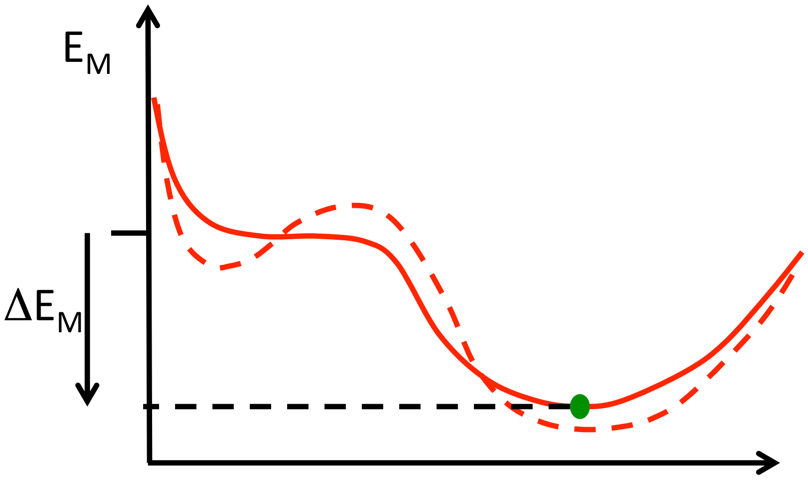
- How does system end up in unstable equilib?
- Evolution can change stable → unstable
  - Termed a bifurcation
  - Pitchfork bif'n: 2+ new stable equilibria are "born"
  - system **usually** drops into new stable equilibrium
  - Slow evolution → **small energy release**



Loss of equilibrium

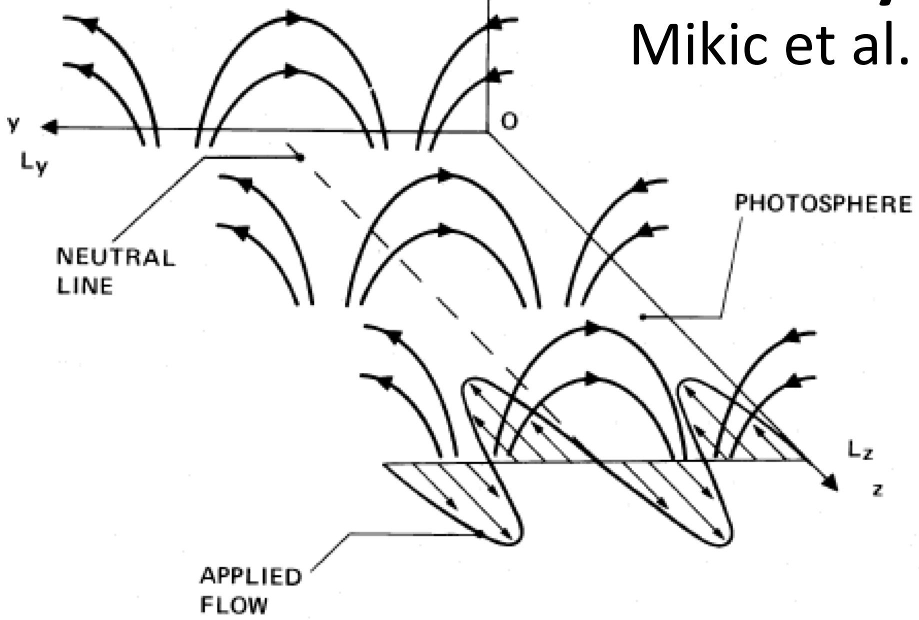
- Saddle-node bifurcation
  - Stable & unstable equilibria annihilate
- Energy release: indep’t of evolution speed

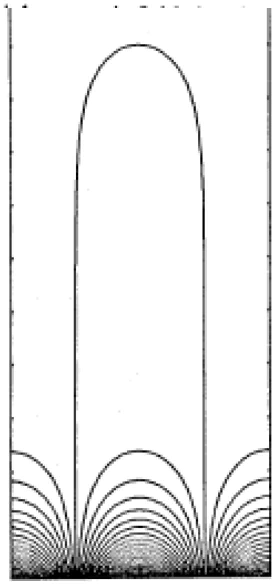




Instability in action

Mikic et al. 1988

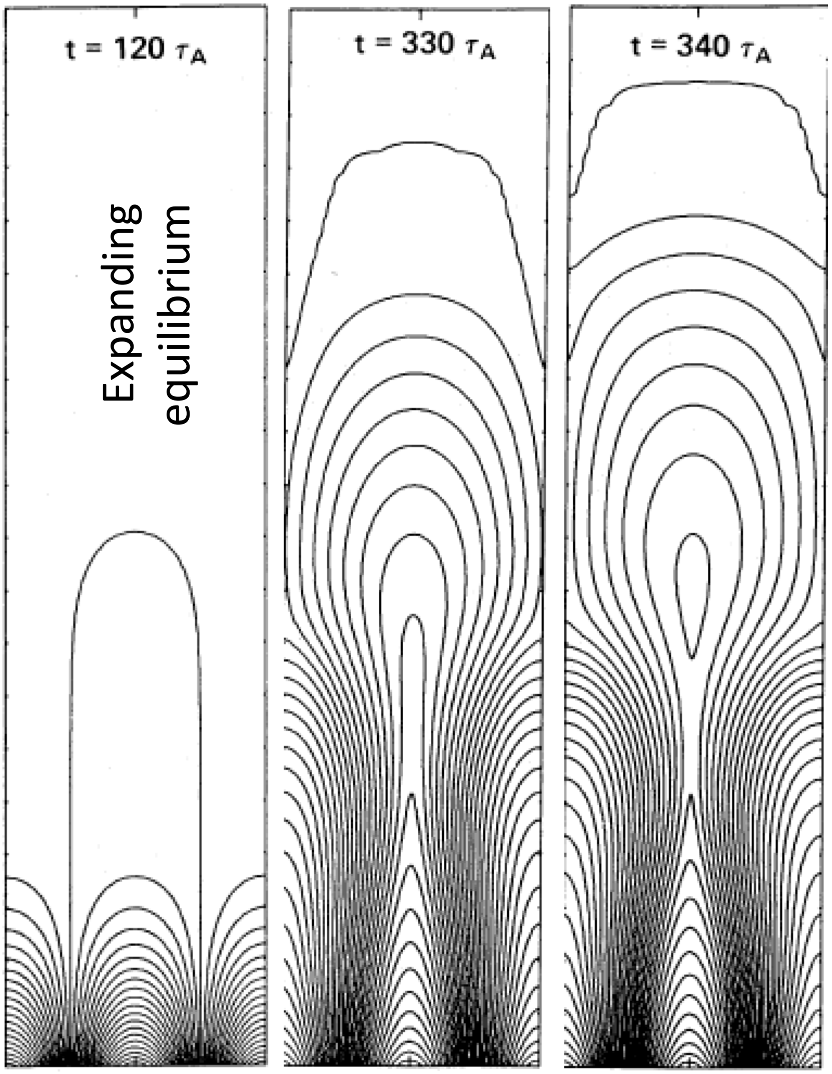




α=0

- Start w/ potential field (α=0)
- Increase current by imposing **slow** shear @ bottom bndry
- Cross stability threshold (probably not LOE)

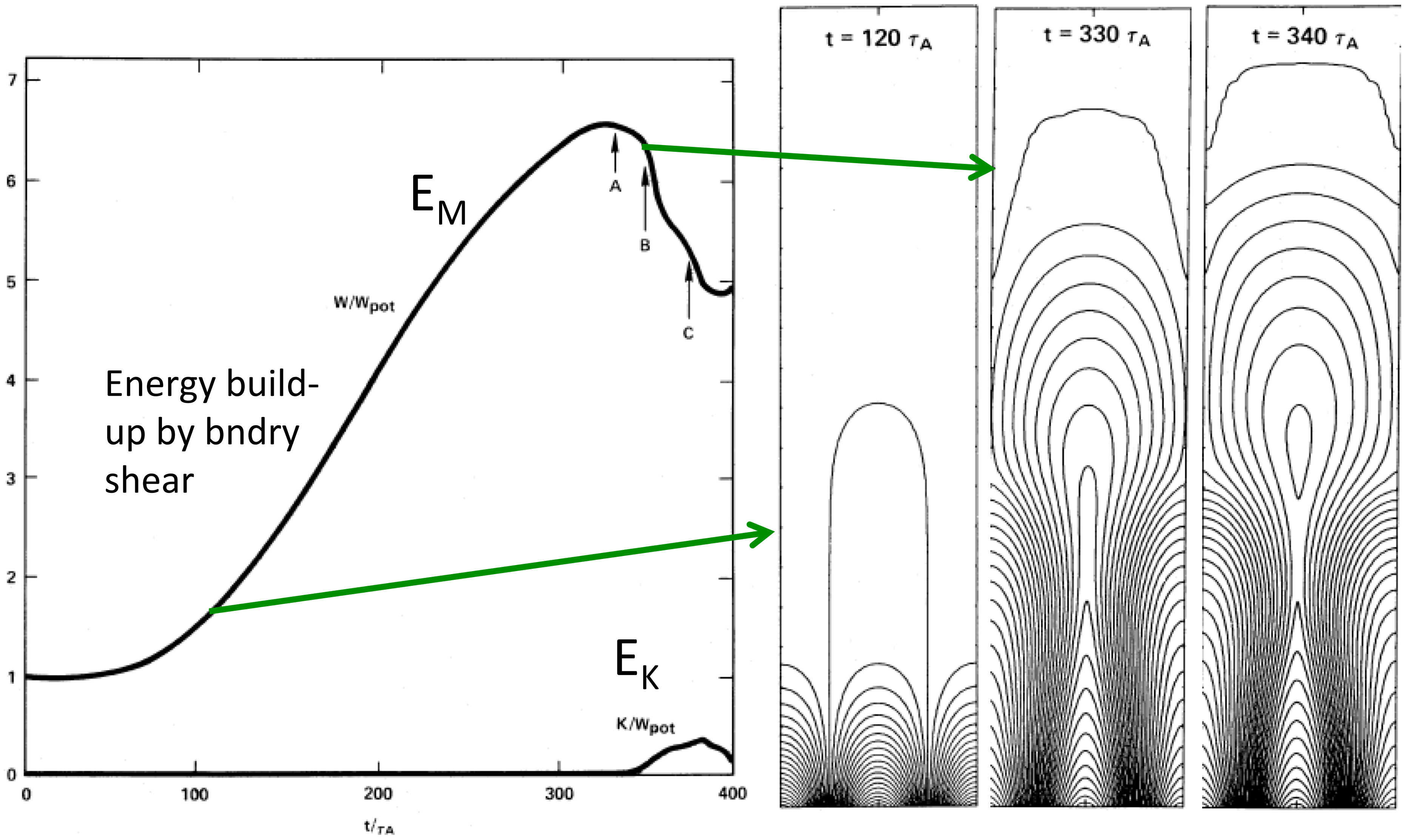
Reconnection produces island



Expanding equilibrium

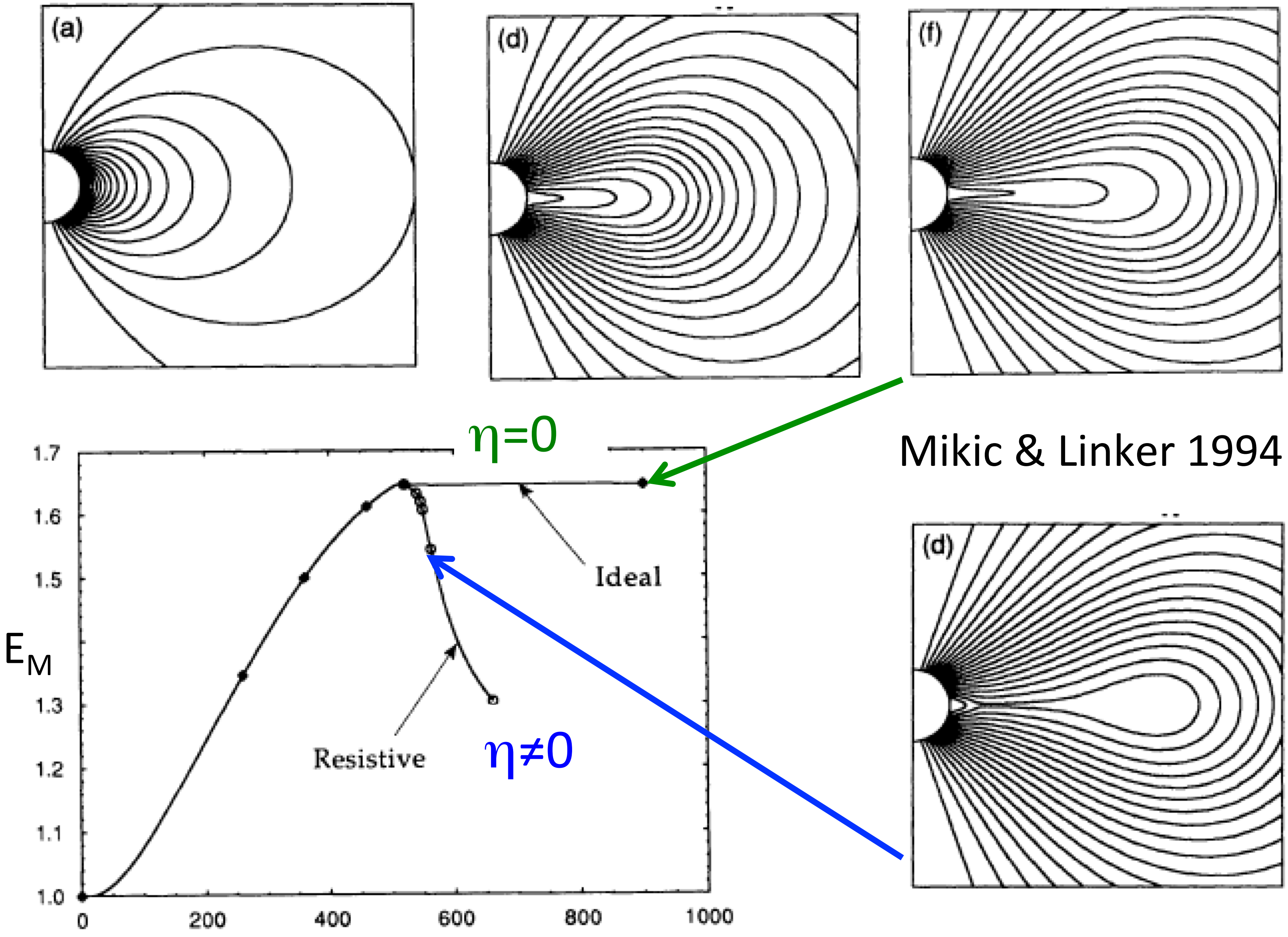
Instability in action

Mikic et al. 1988



Energy build-up by bndry shear

Instability?

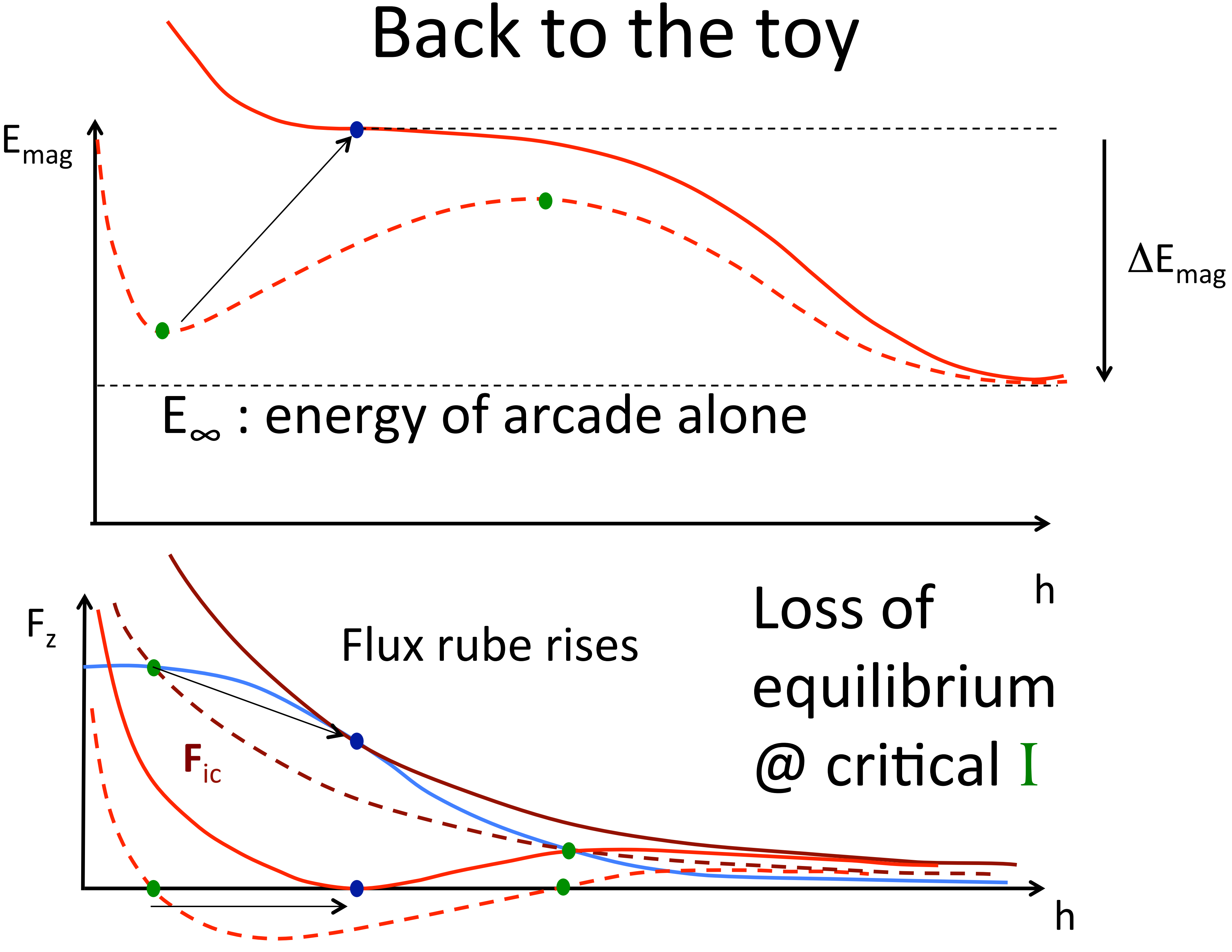


$\eta=0$

Mikic & Linker 1994

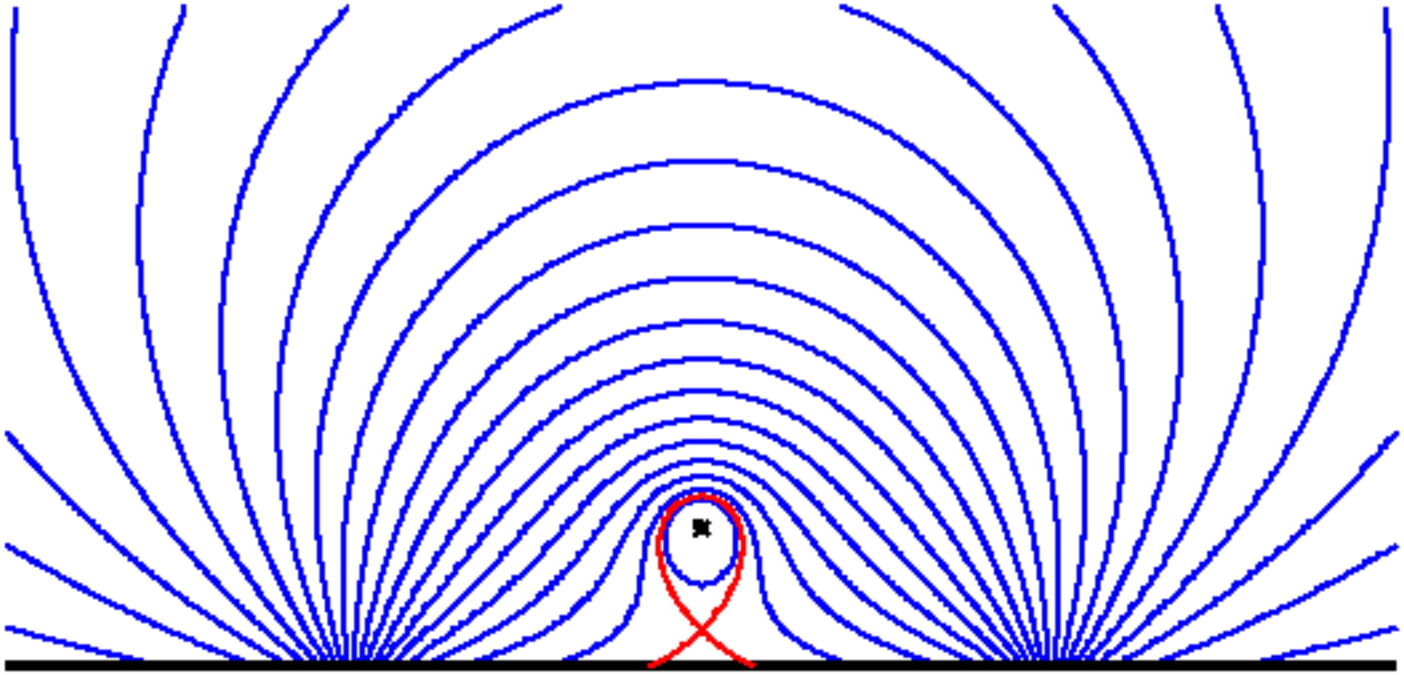
$\eta\neq0$

Back to the toy

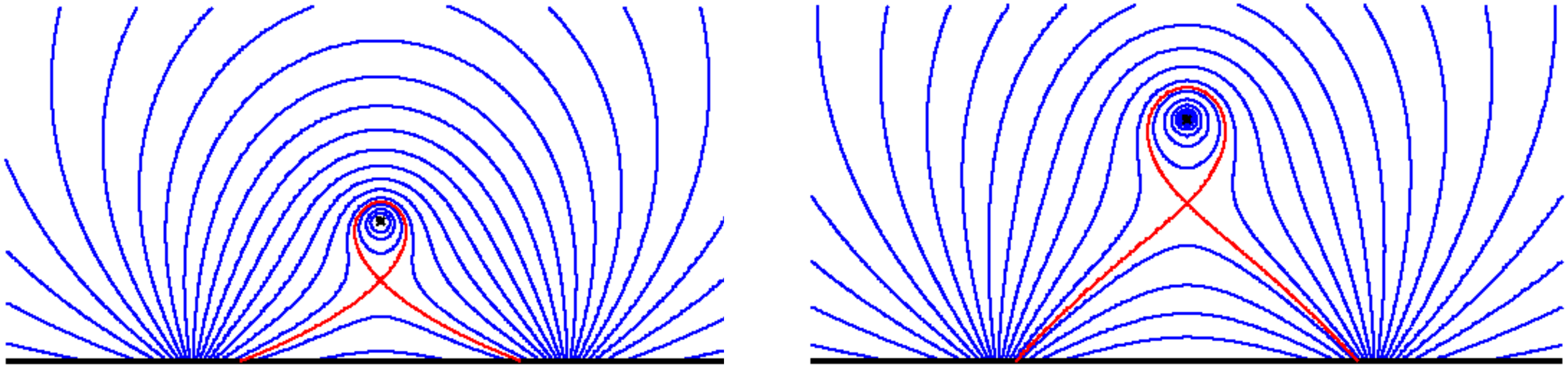


Loss of equilibrium @ critical I

Eruption



Scenario requires **E**



$$\frac{d\Phi}{dt} = -\oint \mathbf{E}\cdot d\mathbf{l} = -E_y(h)\,L_y \neq 0$$

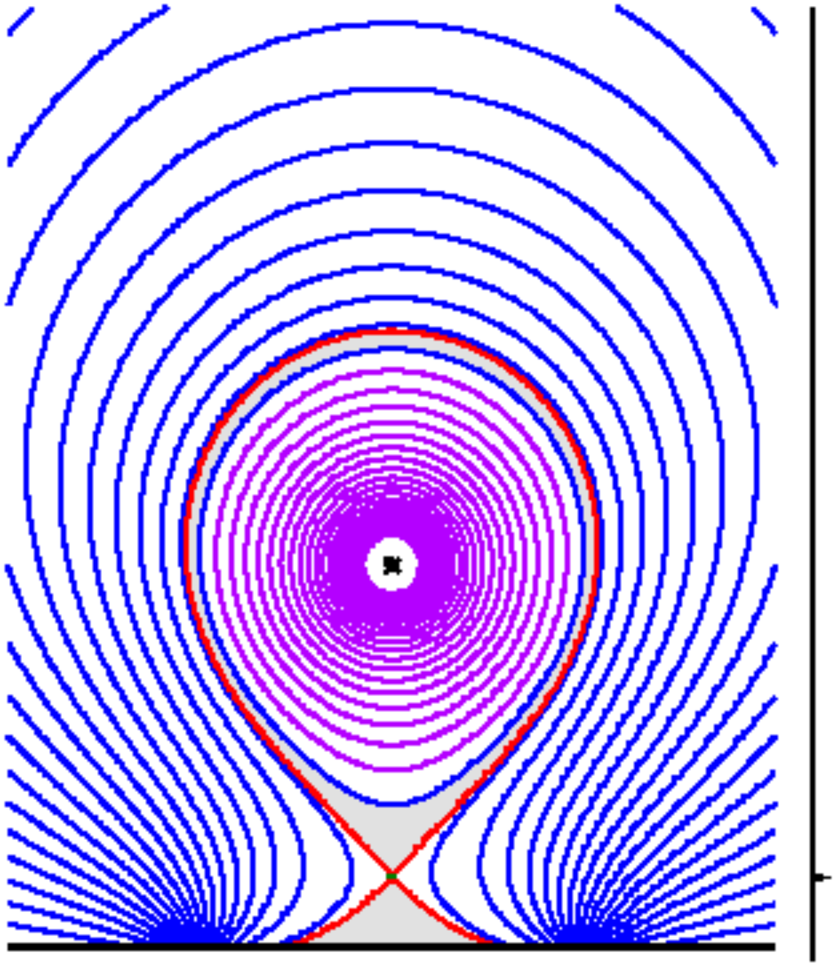
$$\mathbf{E} = \mathbf{E}' - \cancel{\mathbf{u}\times\mathbf{B}} = \mathbf{E}' \neq 0$$ @ X-point

BUT $$\mathbf{E}' = \eta\mathbf{J} \approx 0$$

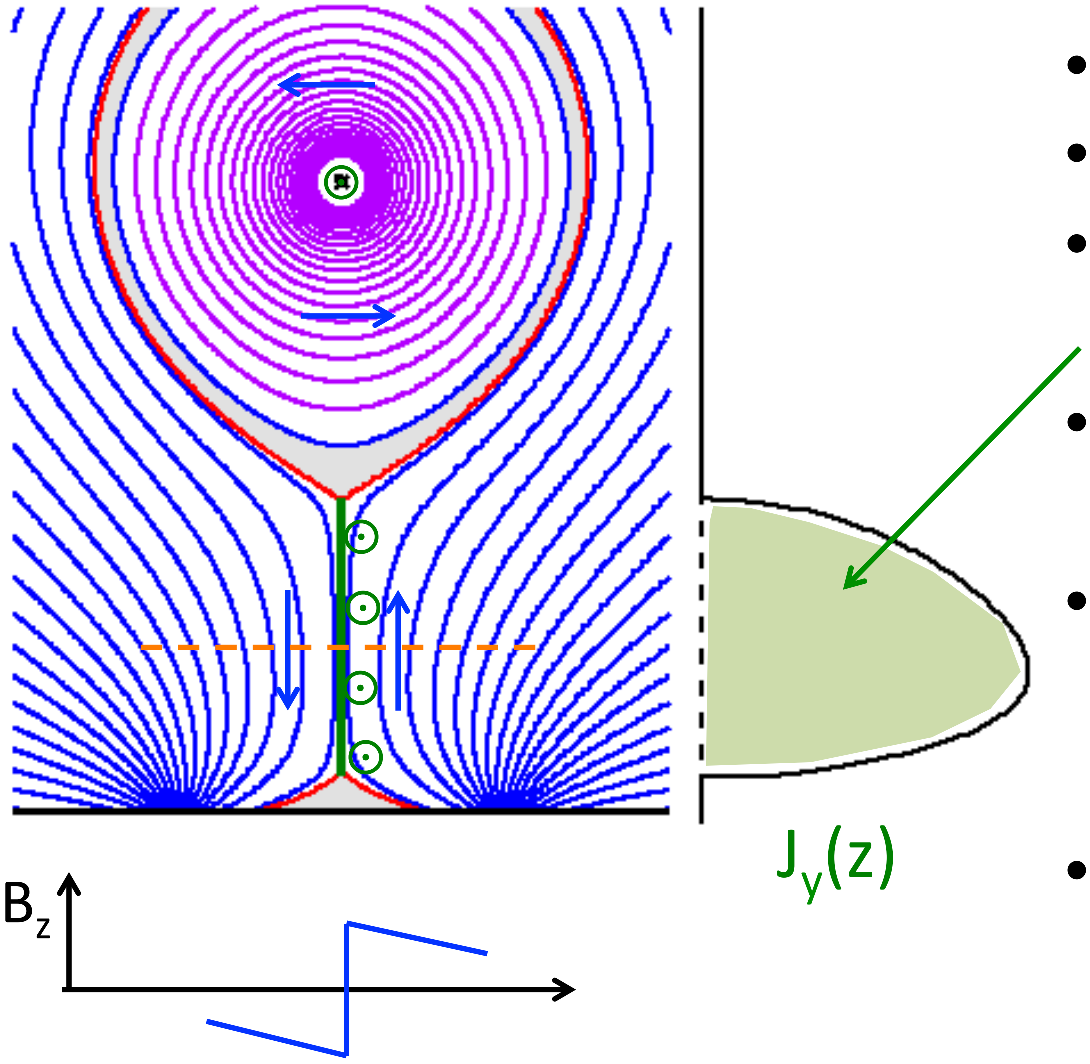
Since η is small

$$Rm = \frac{uL}{\eta} \sim 10^{10} >> 1$$

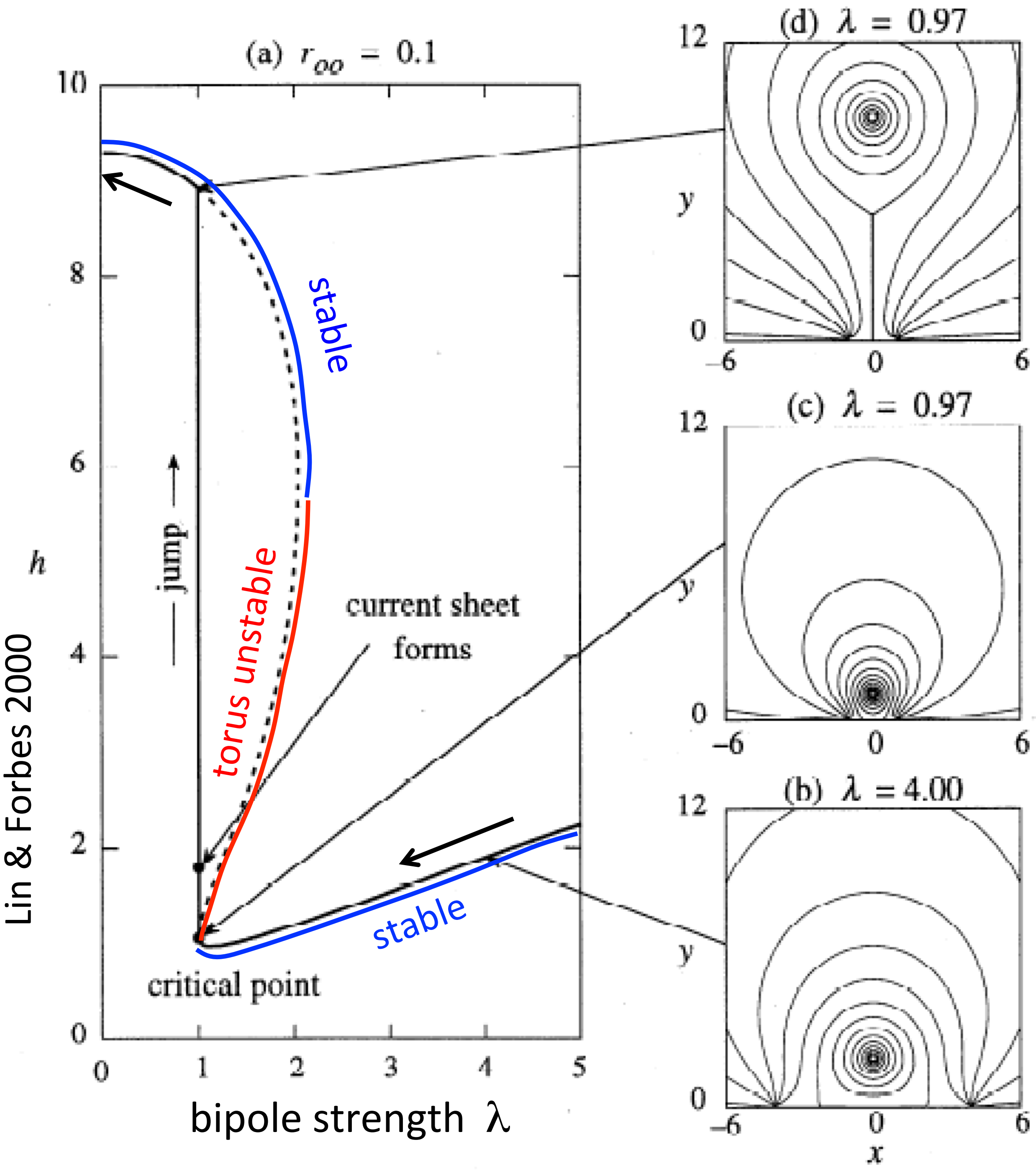
Eruption w/ E’=0



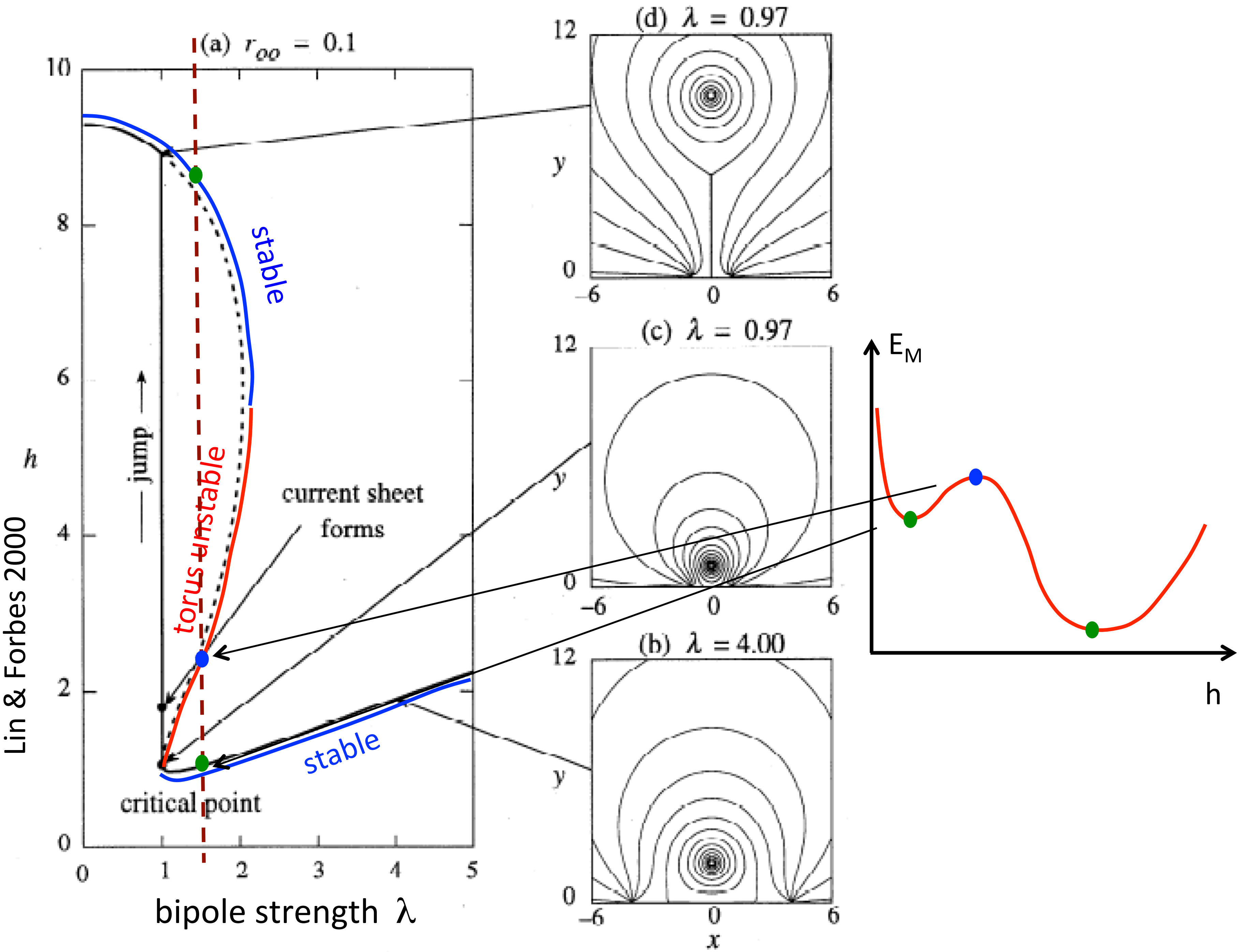
Current sheet



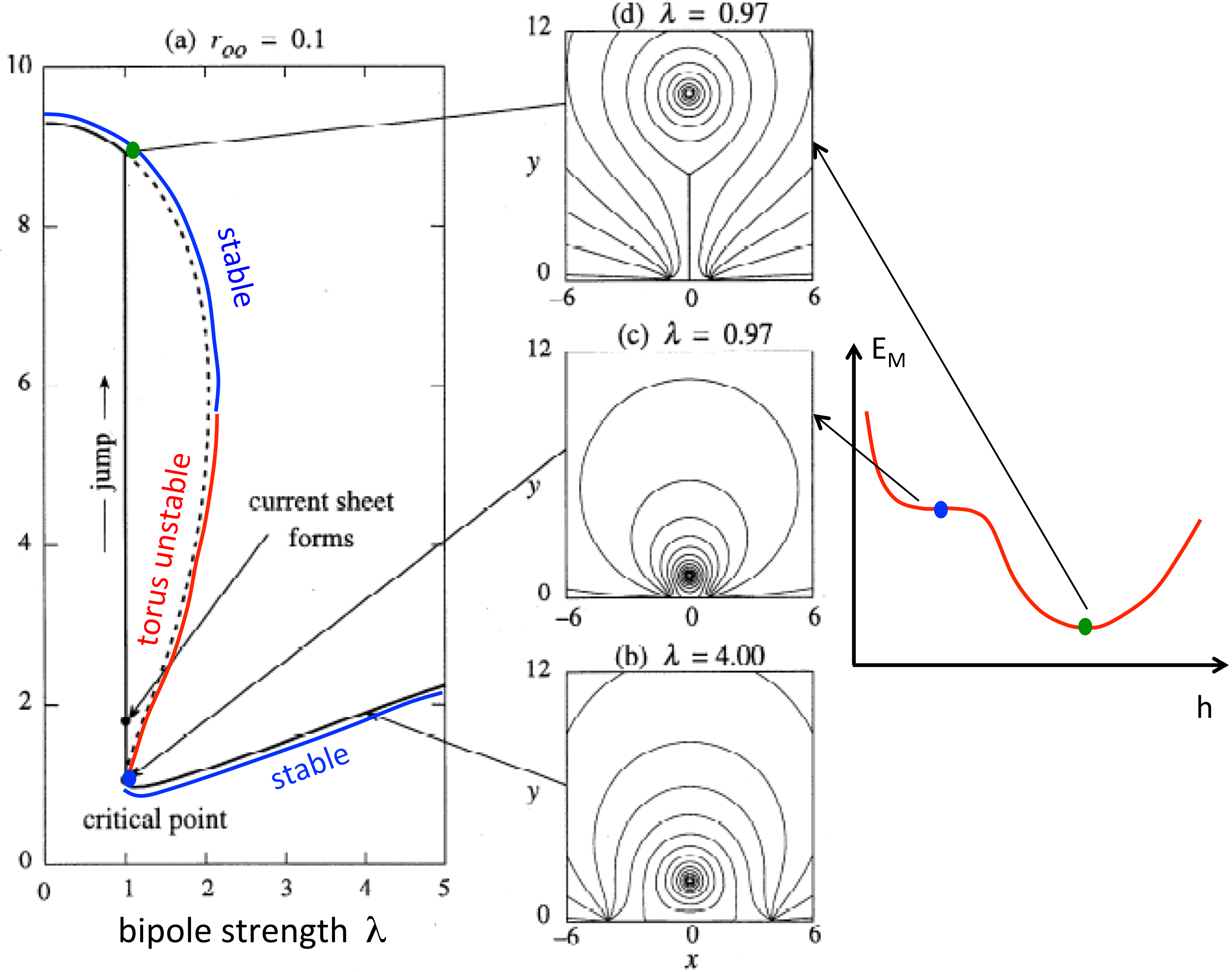
- B_z discontinuous
- X-point → CS
- Integral: total current in sheet
- Exerts downward force on line current
- Balances upward force from image: CS is new equilibrium
- Equivalent: tension from overlying field holds flux rope down



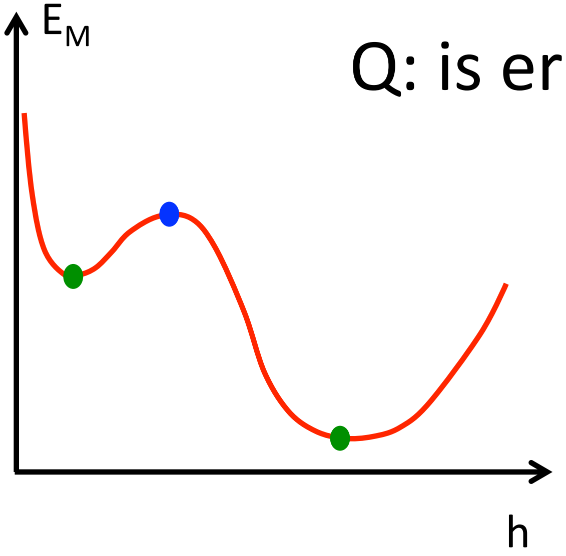
Lin & Forbes 2000



Lin & Forbes 2000



Lin & Forbes 2000

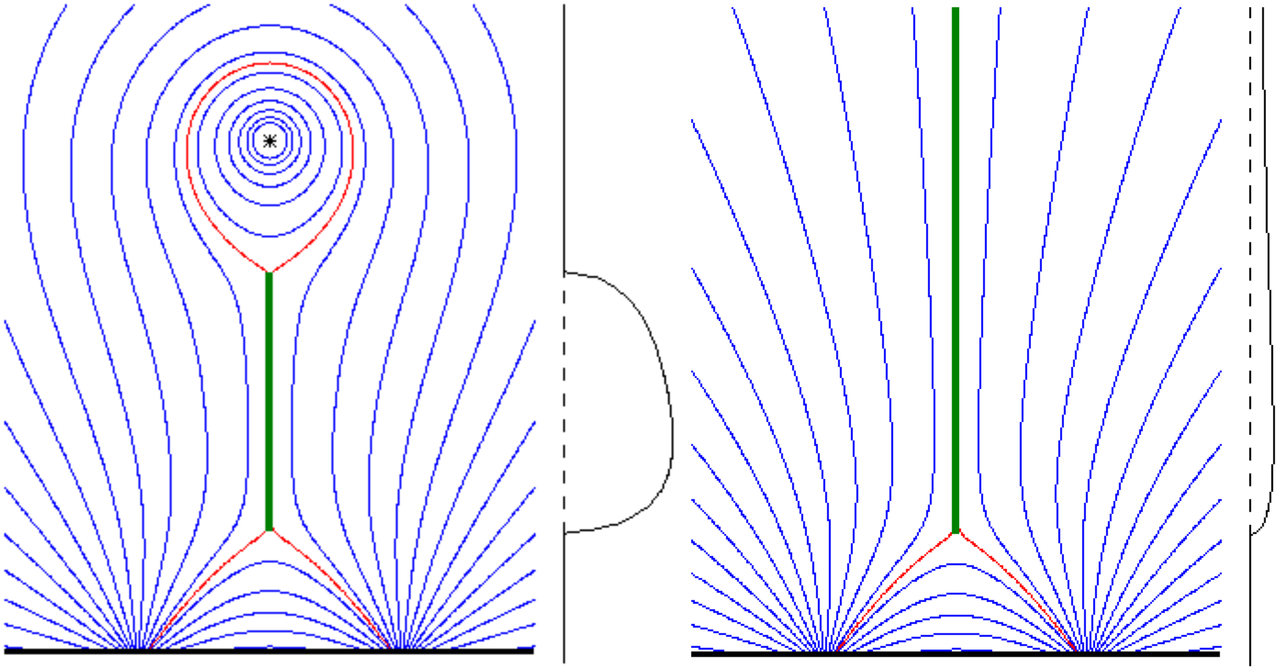


Q: is eruption (h→∞) possible w/ **E'**=0?

A: **No.** h→∞ produces open field

- r →∞ appears like monopole
- $B_{mp} \sim r^{-1}$

$$E_{\text{open}} = \tfrac{1}{8\pi}\int B^2 r\,dr\,d\phi \sim \int^{\infty}\frac{dr}{r} \to \infty$$



Aly-Sturrock conjecture:

$E_M < E_{\text{open}}$

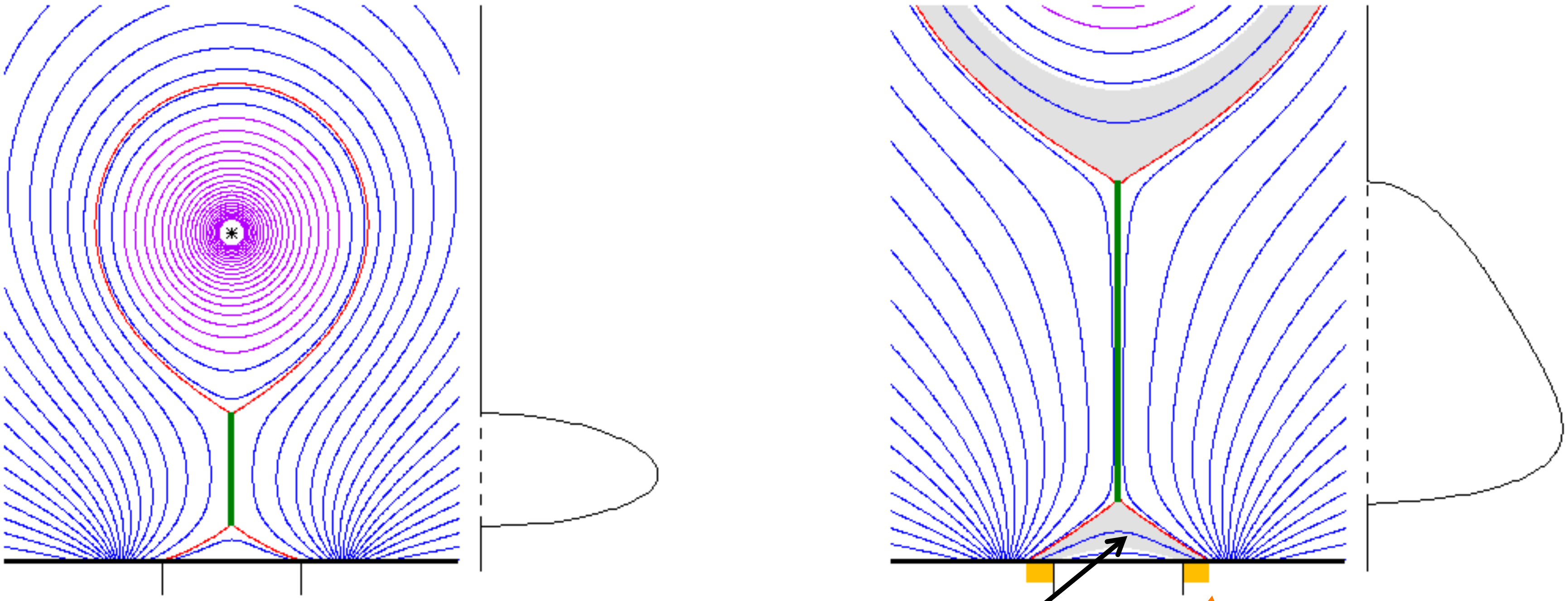
NB: spherical

$B_{mp} \sim r^{-2}$

$E_{\text{open}} < \infty$

Eruption via reconnection

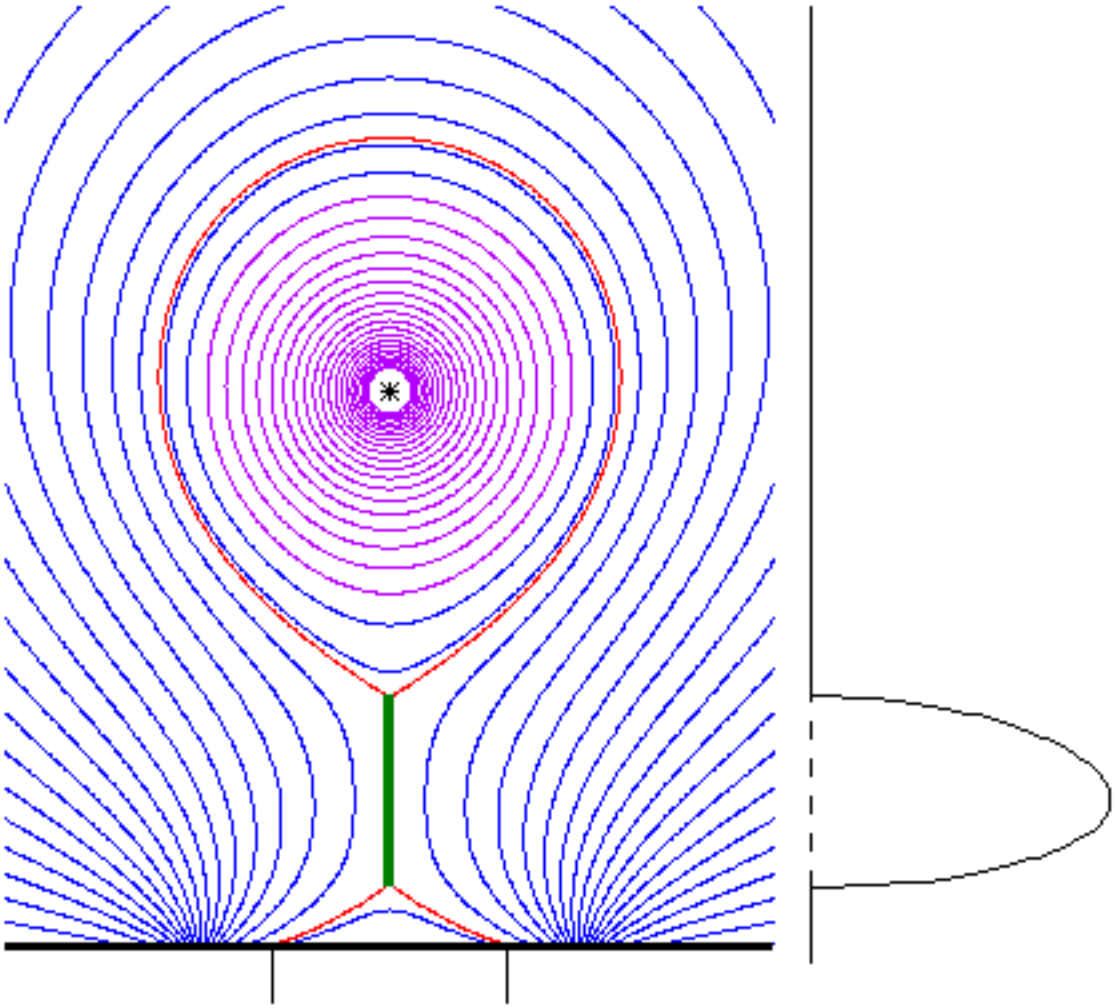
Assume $E' \neq 0$ @ CS (more later) ➔



- Φ beneath CS increases
- Downward force decreases

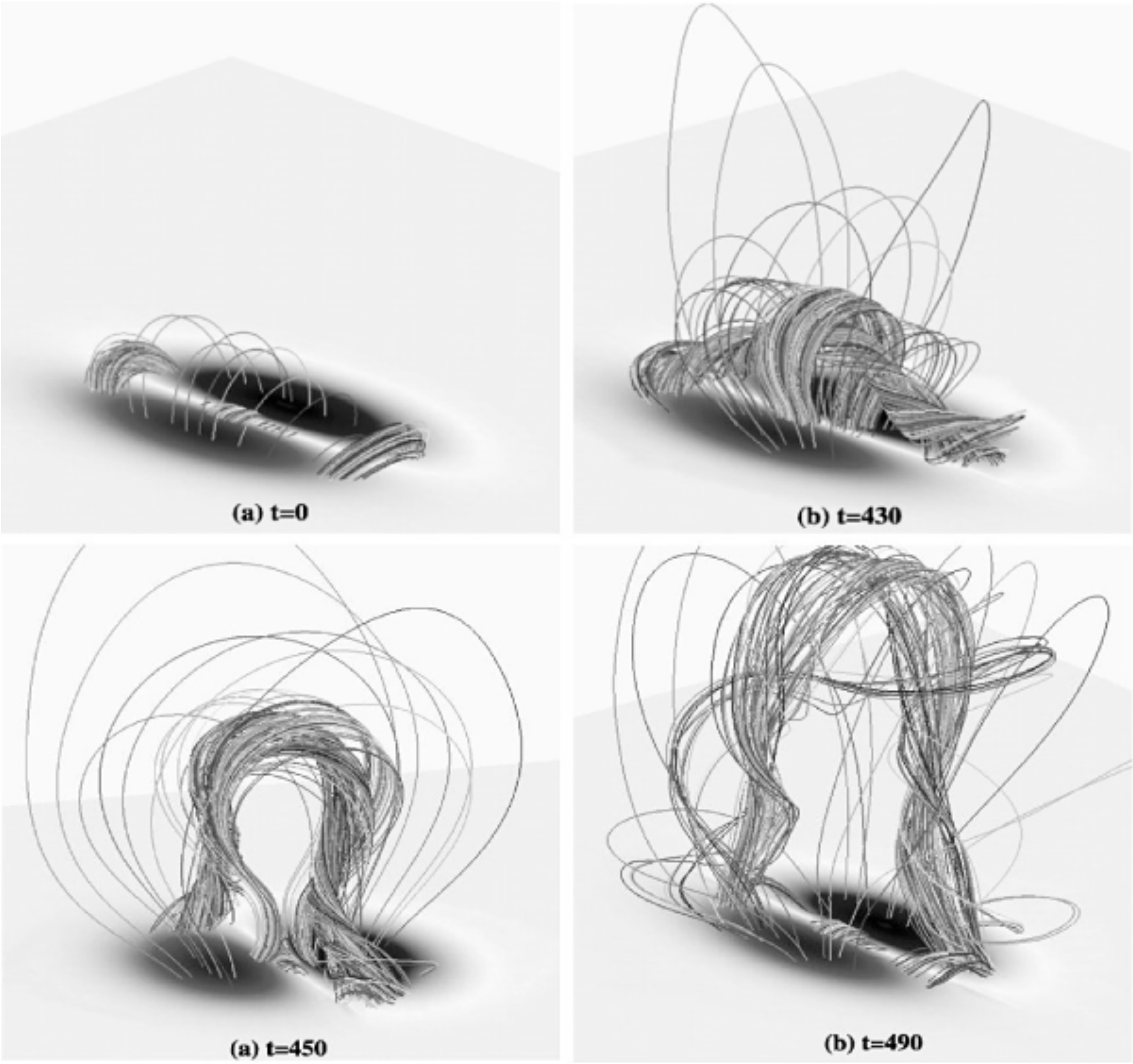
  (reconnection reduces overlying flux)
- Flux rope rises
- Flare signatures produced by E'

Eruption via reconnection



CME model ingredients

- Current = twist in flux rope
- Increasing twist (current) ➔ instability/LOE
- Eruption converts energy: magnetic → kinetic
- Frozen-flux leads to current sheet (CS)
- Reconnection @ CS eliminates overlying flux
  - Can lead to instability
  - Can permit further eruption
  - Will produce flare signatures (later)



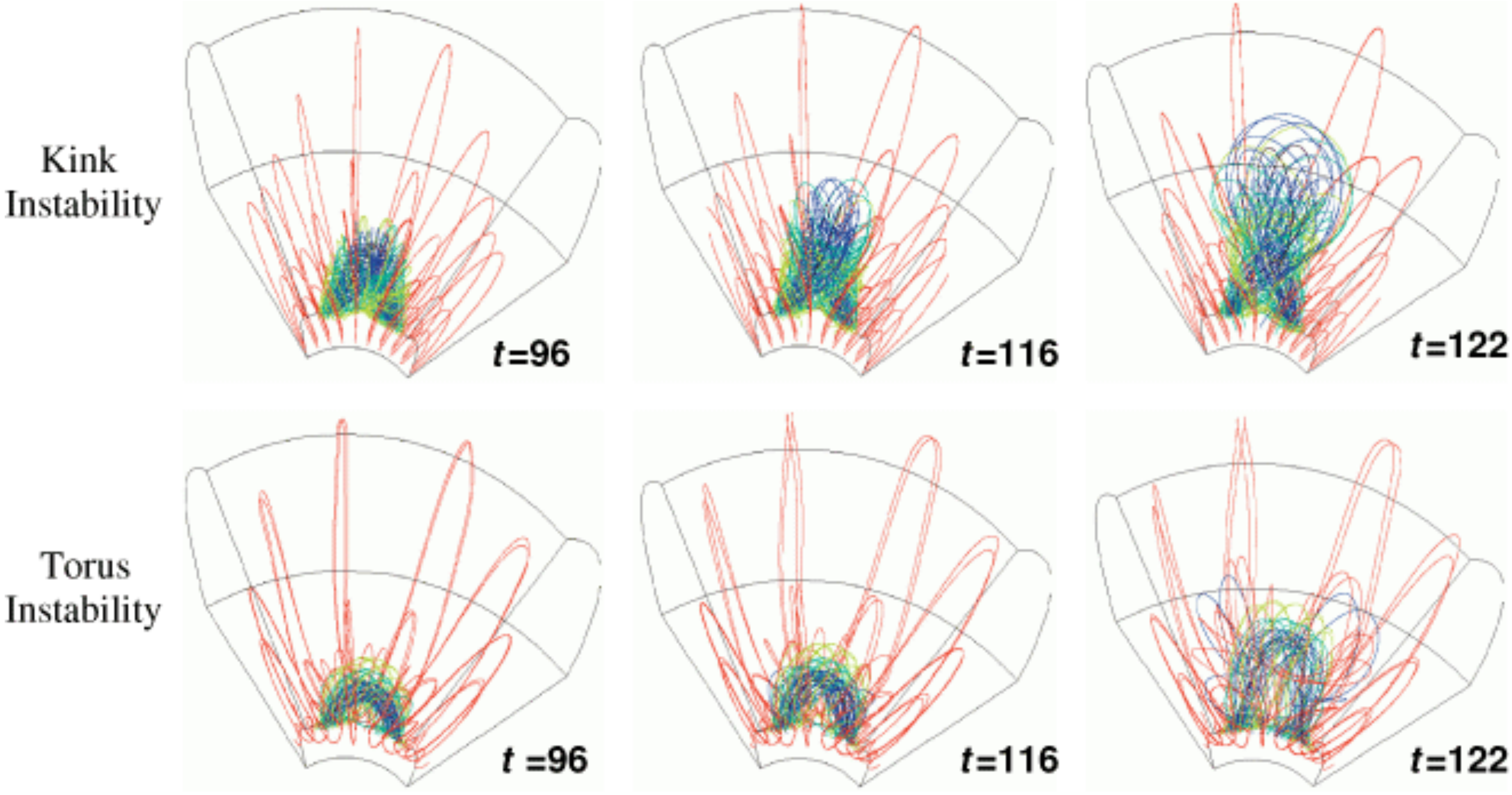
(a) $t=0$

(b) $t=430$

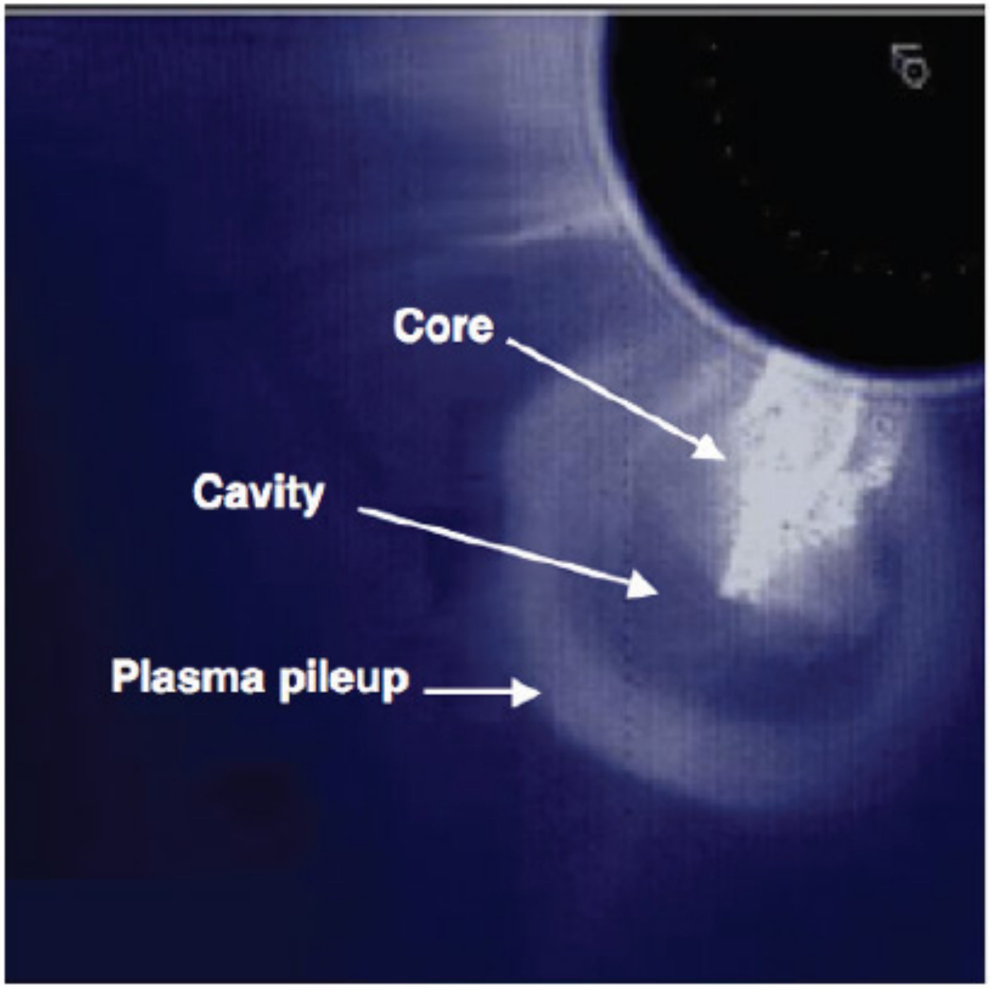
(a) $t=450$

(b) $t=490$

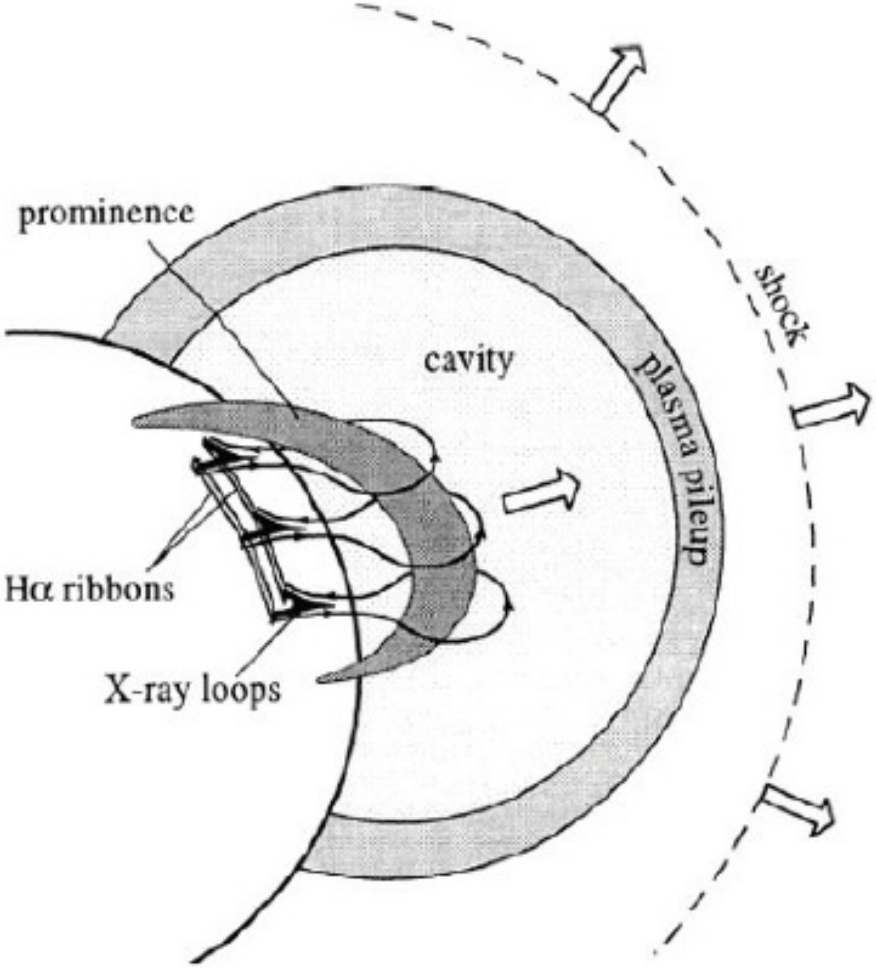
Amari et al. 2001



From Chen 2011 adapted from Fan & Gibson 2007



Van Driel-Gesztelyi & Culhane 2009



Forbes 2000

Summary

- Eruption governed by MHD
- MHD instability (or LoE) releases magnetic energy → kinetic energy
- LoE can lead to sig. energy release
- Reconnection prob. plays important role

Next

What is reconnection & what role does it play?