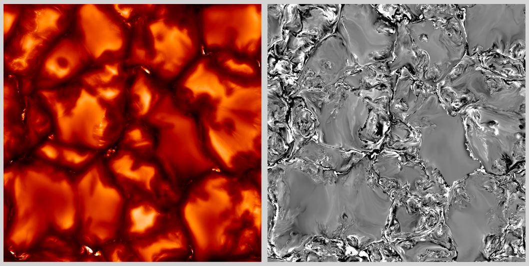
ASTR 7500: Solar & Stellar Magnetism

Hale CGEG Solar & Space Physics



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Lecture 9 Tues 19 Feb 2013

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Scope of this lecture

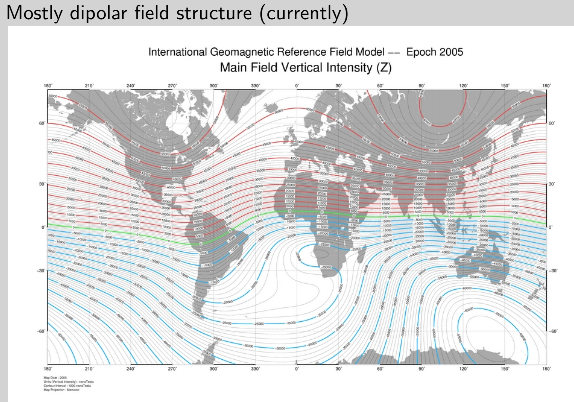
- Processes of magnetic field generation and destruction in turbulent plasma flows
- Introduction to general concepts of dynamo theory (this is not a lecture about the solar dynamo!)
- Outline
  - Intro: Magnetic fields in the Universe
  - MHD, induction equation
  - Some general remarks and definitions regarding dynamos
  - Small scale dynamos
  - Large scale dynamos (mean field theory)
    - Kinematic theory
    - Characterization of possible dynamos
    - Non-kinematic effects
  - Concluding remarks

Magnetic fields in the Universe

- Earth
  - Field strength ~ 0.5G
  - Magnetic field present for $\sim 3.5 \cdot 10^9$ years, much longer than Ohmic decay time ($\sim 10^4$ years)
  - Strong variability on shorter time scales (10^3 years)
- Mercury, Ganymede, (Io), Jupiter, Saturn, Uranus, Neptune have large scale fields
- Sun
  - Magnetic fields from smallest observable scales to size of sun
  - 22 year cycle of large scale field
  - Ohmic decay time $\sim 10^9$ years (in absence of turbulence)
- Other stars
  - Stars with outer convection zone: similar to sun
  - Stars with outer radiation zone: primordial fields, field generation in convective core
- Galaxies
  - Field strength $\sim \mu$G
  - Field structure coupled to observed matter distribution

Geomagnetism

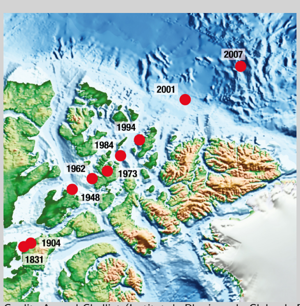
Mostly dipolar field structure (currently)



Credit: NOAA NGDC

Geomagnetism

Short-term variation on scales of hundreds of years

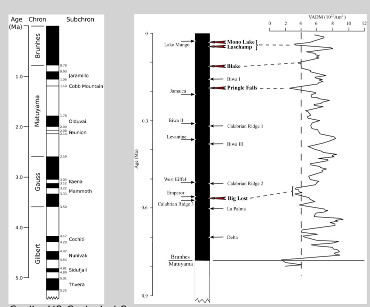


Credit: Arnaud Chulliat (Institut de Physique du Globe de Paris)

- Independent movement of the poles
- South and North pole are in general not opposite to each other (higher multipoles)
- Movements up to 40 km/year (~ 1 mm/sec)

Geomagnetism

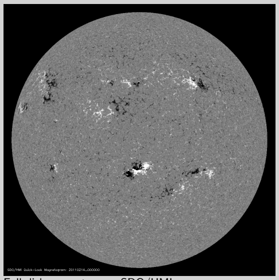
Long-term variation on scales of thousands to millions of years (deduced from volcanic rocks and sediments)



Credit: US Geological Survey

- Mostly random changes of polarity
- A given polarity for $\sim 100,000$ years
- Fast switches ~ 1000 years
- Strong variation of dipole moment and failed reversals

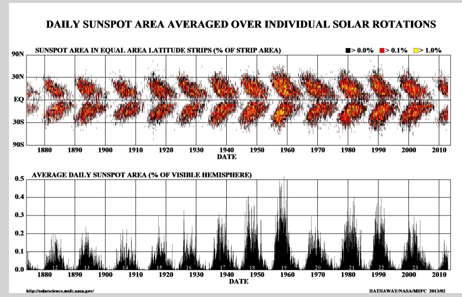
Solar Magnetism



Full disk magnetogram SDO/HMI

- Up to 4kG (sunspot umbra) field in solar photosphere
- Structured over the full range of observable scales from 100 km to size of Sun
- Large scale field shows symmetries with respect to equator

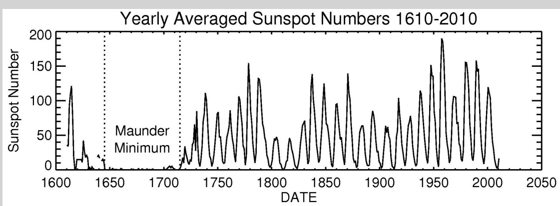
Solar Magnetism



Movie

- Large scale field exhibits $\sim$ 22 year magnetic cycle
- 11 year cycle present in large scale flow variations (meridional flow and differential rotation)

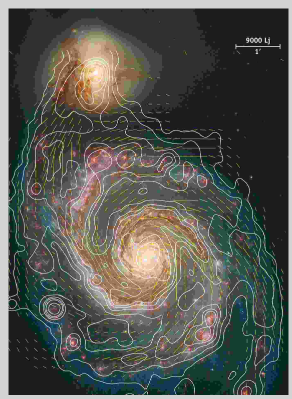
Solar Magnetism



Credit: NASA

- Cycle interrupted by grand minima with duration of up to 100 years
- Similar overall activity has been present for past $\sim 100,000$ years (tree ring and ice core records of cosmogenic isotopes: C-14 and Be-10).

Galactic magnetism



M51, Credit: MPI for Radioastronomy, Germany

- Magnetic field derived from polarization of radio emission
- μG field strength
- Magnetic field follows spiral structure to some extent
- Optically thin dynamo - Dynamo region can be observed!

Magnetic fields in the Universe

- Objects from size of a planet to galaxy clusters have large scale ($\sim$ size of object) magnetic fields
- Physical properties of object differ substantially
  - 1,000 km to 100,000 LJ
  - liquid iron to partially ionized plasma
  - spherical to disk-shaped
  - varying influence of rotation (but all of them are rotating)
  - $R_m \sim 10^3 \dots 10^{18}$
  - ....
- Is there a common origin of magnetic field in these objects?
- Can we understand this on basis of MHD?

MHD equations

The full set of MHD equations combines the induction equation with the Navier-Stokes equations including the Lorentz-force:

$$\frac{\partial \varrho}{\partial t} = -\nabla \cdot (\varrho \mathbf{v})$$

$$\varrho \frac{\partial \mathbf{v}}{\partial t} = -\varrho(\mathbf{v}\cdot\nabla)\mathbf{v} - \nabla p + \varrho \mathbf{g} + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \boldsymbol{\tau}$$

$$\varrho \frac{\partial e}{\partial t} = -\varrho(\mathbf{v}\cdot\nabla)e - p\nabla\cdot\mathbf{v} + \nabla\cdot(\kappa\nabla T) + Q_\nu + Q_\eta$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v}\times\mathbf{B} - \eta\nabla\times\mathbf{B})$$

Assumptions:

- Validity of continuum approximation (enough particles to define averages)
- Non-relativistic motions, low frequencies
- Strong collisional coupling: validity of single fluid approximations, isotropic (scalar) gas pressure

MHD equations

Viscous stress tensor τ

$$\begin{aligned}\Lambda_{ik} &= \frac{1}{2}\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)\\ \tau_{ik} &= 2\varrho\nu\left(\Lambda_{ik}-\frac{1}{3}\delta_{ik}\nabla\cdot\mathbf{v}\right)\\ Q_\nu &= \tau_{ik}\Lambda_{ik}\ ,\end{aligned}$$

Ohmic dissipation Q_η

$$Q_\eta = \frac{\eta}{\mu_0}(\nabla\times\mathbf{B})^2\ .$$

Equation of state

$$p = \frac{\varrho\, e}{\gamma-1}\ .$$

ν, η and κ: viscosity, magnetic diffusivity and thermal conductivity
μ_0 denotes the permeability of vacuum

Kinematic approach

- Solving the 3D MHD equations is not always feasible
- Semi-analytical approach preferred for understanding fundamental properties of dynamos
- Evaluate turbulent induction effects based on induction equation for a given velocity field
  - Velocity field assumed to be given as 'background' turbulence, Lorentz-force feedback neglected (sufficiently weak magnetic field)
  - What correlations of a turbulent velocity field are required for dynamo (large scale) action?
  - Theory of onset of dynamo action, but not for non-linear saturation
- More detailed discussion of induction equation

Ohm's law

Equation of motion for drift velocity $\mathbf{v}_d$ of electrons

$$n_e m_e\left(\frac{\partial v_d}{\partial t}+\frac{v_d}{\tau_{ei}}\right) = n_e q_e(\mathbf{E}+\mathbf{v}_d\times\mathbf{B})-\nabla p_e$$

τ_{ei}: collision time between electrons and ions
n_e: electron density
q_e: electron charge
m_e: electron mass
p_e: electron pressure
With the electric current: $\mathbf{j}=n_e\,q_e\,\mathbf{v}_d$ this gives the generalized Ohm's law:

$$\frac{\partial\mathbf{j}}{\partial t}+\frac{\mathbf{j}}{\tau_{ei}} = \frac{n_e q_e^2}{m_e}\mathbf{E}+\frac{q_e}{m_e}\mathbf{j}\times\mathbf{B}-\frac{q_e}{m_e}\nabla p_e$$

Simplifications:

- $\tau_{ei}\,\omega_L \ll 1$, $\omega_L = eB/m_e$: Larmor frequency
- neglect ∇p_e
- low frequencies (no plasma oscillations)

Ohm's law

Simplified Ohm's law

$$\mathbf{j} = \sigma\mathbf{E}$$

with the plasma conductivity

$$\sigma = \frac{\tau_{ei} n_e q_e^2}{m_e}$$

The Ohm's law we derived so far is only valid in the co-moving frame of the plasma. Under the assumption of non-relativistic motions this transforms in the laboratory frame to

$$\mathbf{j} = \sigma\,(\mathbf{E}+\mathbf{v}\times\mathbf{B})$$

Induction equation*

Using Ampere's law $\nabla\times\mathbf{B}=\mu_0\mathbf{j}$ yields for the electric field in the laboratory frame

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B}+\frac{1}{\mu_0\sigma}\nabla\times\mathbf{B}$$

leading to the induction equation

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} = \nabla\times(\mathbf{v}\times\mathbf{B}-\eta\nabla\times\mathbf{B})$$

with the magnetic diffusivity

$$\eta = \frac{1}{\mu_0\sigma}\ .$$

Advection, diffusion, magnetic Reynolds number

L: typical length scale U: typical velocity scale L/U: time unit

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{v}\times\mathbf{B}-\frac{1}{R_m}\nabla\times\mathbf{B}\right)$$

with the magnetic Reynolds number

$$R_m = \frac{U\,L}{\eta}\ .$$

Advection, diffusion, magnetic Reynolds number

$R_m \ll 1$: diffusion dominated regime

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \Delta \mathbf{B} \; .$$

Only decaying solutions with decay (diffusion) time scale

$$\tau_\mathrm{d} \sim \frac{L^2}{\eta}$$

Object	$\eta[\mathrm{m^2/s}]$	$L[\mathrm{m}]$	$U[\mathrm{m/s}]$	R_m	τ_d
earth (outer core)	2	10^6	10^{-3}	300	10^4 years
sun (plasma conductivity)	1	10^8	100	10^{10}	10^9 years
sun (turbulent conductivity)	10^8	10^8	100	100	3 years
liquid sodium lab experiment	0.1	1	10	100	10 s

Advection, diffusion, magnetic Reynolds number

$R_m \gg 1$ advection dominated regime (ideal MHD)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Equivalent expression

$$\frac{\partial \mathbf{B}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}\, \nabla \cdot \mathbf{v}$$

- advection of magnetic field
- amplification by shear (stretching of field lines)
- amplification through compression

Advection, diffusion, magnetic Reynolds number

Incompressible fluid ($\nabla \cdot \mathbf{v} = 0$):

$$\frac{d\mathbf{B}}{dt} = (\mathbf{B} \cdot \nabla)\mathbf{v}$$

Velocity shear in the direction of **B** plays key role. Mathematically similar equation for compressible fluid (Walen equation):

$$\frac{d}{dt}\frac{\mathbf{B}}{\varrho} = \left(\frac{\mathbf{B}}{\varrho} \cdot \nabla\right)\mathbf{v}$$

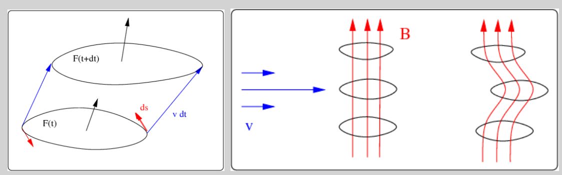
Vertical flux transport in statified medium:

- $B \sim \varrho$ — no expansion in direction of **B**
- $B \sim \varrho^{2/3}$ — isotropic expansion
- $B \sim \varrho^{1/2}$ — 2D expansion in plane containing **B**
- $B = const.$ — only expansion in direction of **B**

Alfven's theorem

Let Φ be the magnetic flux through a surface F with the property that its boundary ∂F is moving with the fluid:

$$\Phi = \int_F \mathbf{B} \cdot d\mathbf{f} \longrightarrow \frac{d\Phi}{dt} = 0$$



- Flux is 'frozen' into the fluid
- Field lines 'move' with plasma

Dynamos: Motivation

- For $\mathbf{v} = 0$ magnetic field decays on timescale $\tau_d \sim L^2/\eta$
- Earth and other planets:
  - Evidence for magnetic field on earth for $3.5 \cdot 10^9$ years while $\tau_d \sim 10^4$ years
  - Permanent rock magnetism not possible since $T > T_\mathrm{Curie}$ and field highly variable $\longrightarrow$ field must be maintained by active process
- Sun and other stars:
  - Evidence for solar magnetic field for $\sim 300\,000$ years (^{10}Be)
  - Most solar-like stars show magnetic activity independent of age
  - Indirect evidence for stellar magnetic fields over life time of stars
  - But $\tau_d \sim 10^9$ years!
  - Primordial field could have survived in radiative interior of sun, but convection zone has much shorter diffusion time scale ~ 10 years (turbulent diffusivity)

Mathematical definition of dynamo

S bounded volume with the surface ∂S, **B** maintained by currents contained within S, $B \sim r^{-3}$ asymptotically,

$$\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) && \text{in } S \\
\nabla \times \mathbf{B} &= 0 && \text{outside } S \\
[\mathbf{B}] &= 0 && \text{across } \partial S \\
\nabla \cdot \mathbf{B} &= 0
\end{aligned}$$

$\mathbf{v} = 0$ outside S, $\mathbf{n} \cdot \mathbf{v} = 0$ on ∂S and

$$E_\mathrm{kin} = \int_S \frac{1}{2}\varrho \mathbf{v}^2 \, dV \le E_\mathrm{max} \quad \forall\, t$$

v is a dynamo if an initial condition $\mathbf{B} = \mathbf{B}_0$ exists so that

$$E_\mathrm{mag} = \int_{-\infty}^{\infty} \frac{1}{2\mu_0}\mathbf{B}^2 \, dV \ge E_\mathrm{min} \quad \forall\, t$$

Mathematical definition of dynamo

- Is this dynamo different from those found in powerplants?
  - Both have conducting material and relative motions (rotor/stator in powerplant vs. shear flows)
- Difference mostly in one detail:
  - Dynamos in powerplants have wires (very inhomogeneous conductivity), i.e. the electric currents are strictly controlled
  - Mathematically the system is formulated in terms of currents
  - A short circuit is a major desaster!
  - For astrophysical dynamos we consider homogeneous conductivity, i.e. current can flow anywhere
  - Mathematically the system is formulated in terms of **B** (**j** is eliminated from equations whenever possible).
  - A short circuit is the normal mode of operation!
- Homogeneous vs. inhomogeneous dynamos

Large scale/small scale dynamos

Decompose the magnetic field into large scale part and small scale part (energy carrying scale of turbulence) $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'$:

$$E_{\rm mag} = \int \frac{1}{2\mu_0} \overline{\mathbf{B}}^2 \, dV + \int \frac{1}{2\mu_0} \overline{\mathbf{B}'^2} \, dV \, .$$

- Small scale dynamo: $\overline{\mathbf{B}}^2 \ll \overline{\mathbf{B}'^2}$
- Large scale dynamo: $\overline{\mathbf{B}}^2 \geq \overline{\mathbf{B}'^2}$

Almost all turbulent (chaotic) velocity fields are small scale dynamos for sufficiently large R_m, large scale dynamos require additional large scale symmetries (see second half of this lecture)

What means large/small in practice (Sun)?

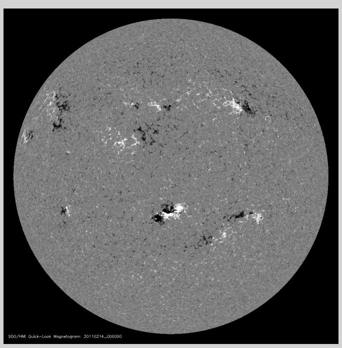


Figure: Full disk magnetogram SDO/HMI

What means large/small in practice (Sun)?

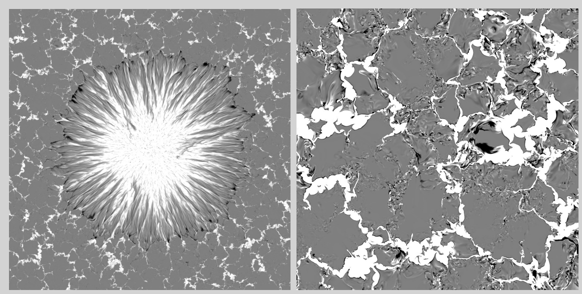


Figure: Numerical sunspot simulation. Dimensions: Left 50x50 Mm, Right: 12.5x12.5 Mm

Small scale dynamo action

Lagrangian particle paths:

$$\frac{d\mathbf{x}_1}{dt} = \mathbf{v}(\mathbf{x}_1, t) \qquad \frac{d\mathbf{x}_2}{dt} = \mathbf{v}(\mathbf{x}_2, t)$$

Consider small separations:

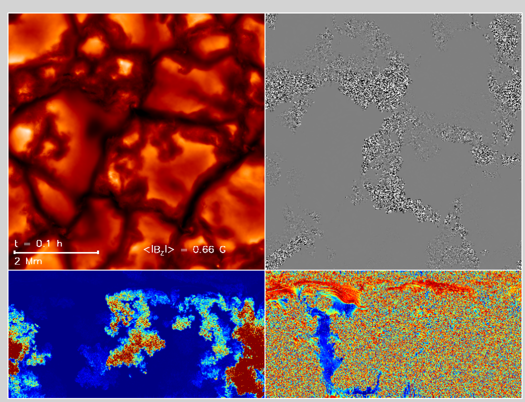
$$\boldsymbol{\delta} = \mathbf{x}_1 - \mathbf{x}_2 \qquad \frac{d\boldsymbol{\delta}}{dt} = (\boldsymbol{\delta} \cdot \boldsymbol{\nabla})\mathbf{v}$$

Chaotic flows have exponentially growing solutions. Due to mathematical simularity the equation:

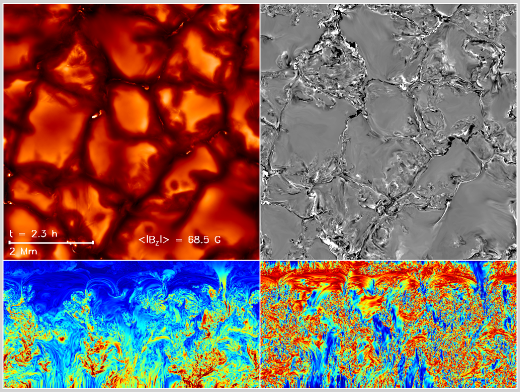
$$\frac{d}{dt}\frac{\mathbf{B}}{\varrho} = \left(\frac{\mathbf{B}}{\varrho} \cdot \boldsymbol{\nabla}\right)\mathbf{v}$$

has exponentially growing solutions, too. We neglected here η, exponentially growing solutions require $R_m > \mathcal{O}(100)$.

SSD in solar photosphere: kinematic phase

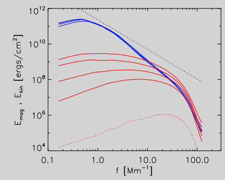


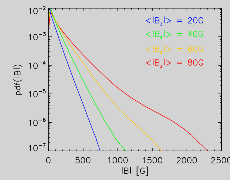
SSD in solar photosphere: saturated phase



Movie

SSD in solar photosphere: power spectra





Movie

- Kinematic phase: Magnetic energy peaks at smallest resolved scales (here 30 km (4 km numerical resolution, would be 100 – 1000 m for the Sun
- Saturated phase: Magnetic energy peaks at granular scales (mostly flat spectrum at large scales). Dynamo action moved toward larger scales, where most of the kinetic energy sits (downflow lanes $\sim$ 300 km)