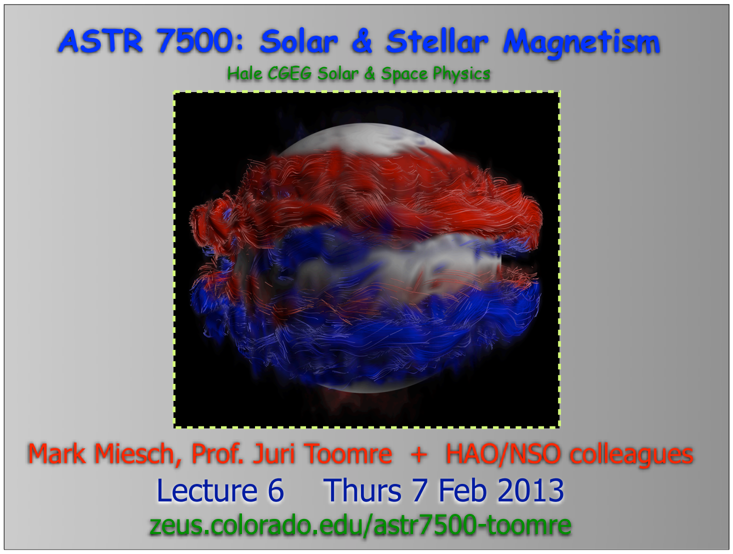
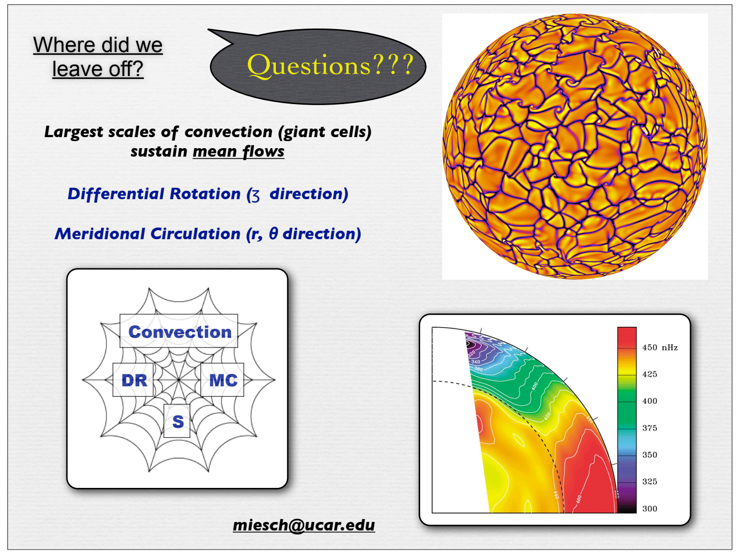
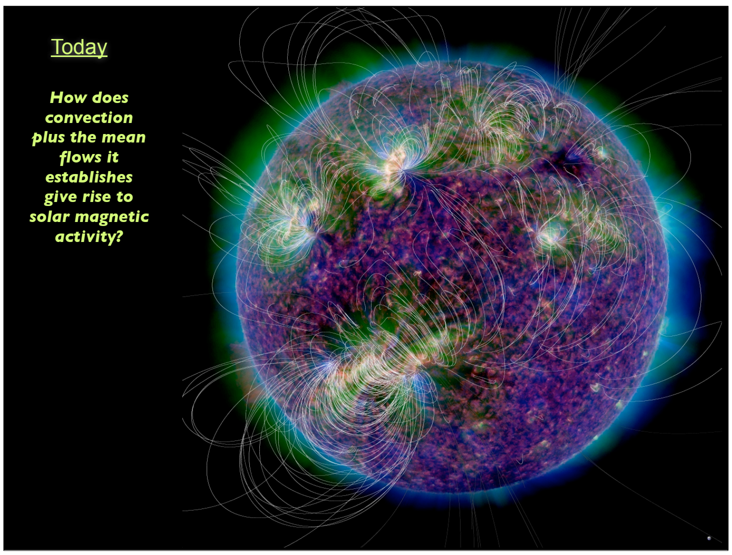
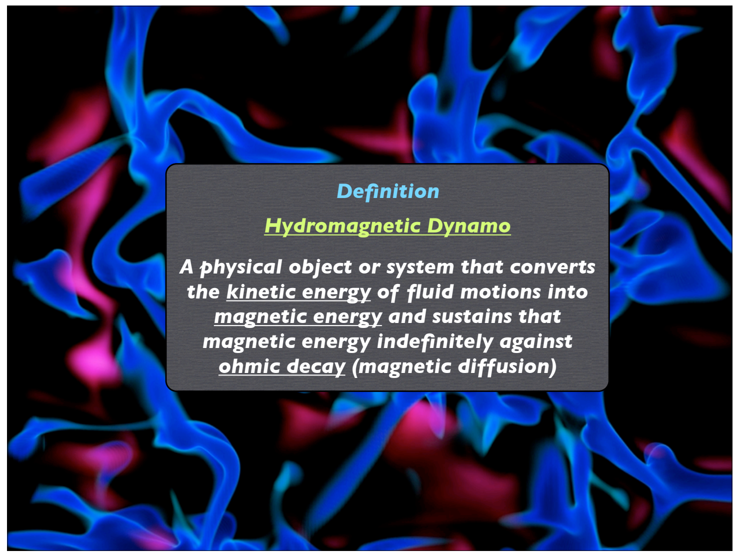
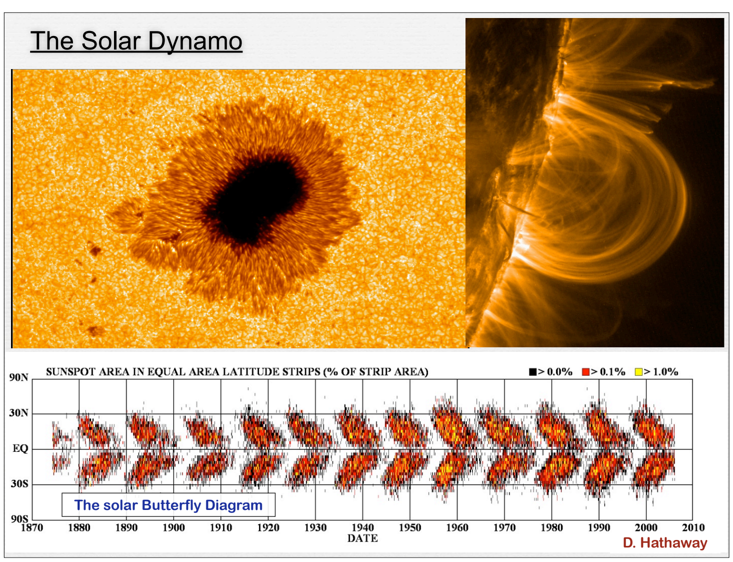
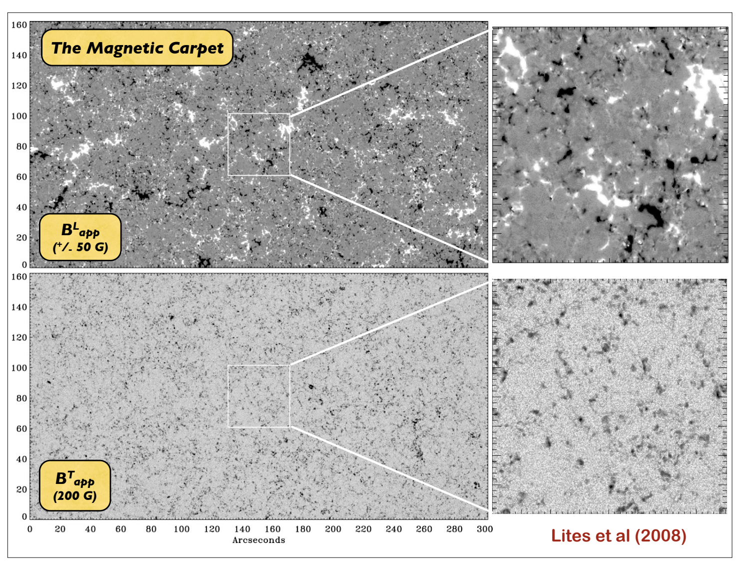

ASTR 7500: Solar & Stellar Magnetism
Hale CGEG Solar & Space Physics
Mark Miesch, Prof. Juri Toomre + HAO/NSO colleagues
Lecture 6 Thurs 7 Feb 2013
zeus.colorado.edu/astr7500-toomre


Where did we leave off?
Questions???
Largest scales of convection (giant cells) sustain mean flows
Differential Rotation (ɀ direction)
Meridional Circulation (r, θ direction)
Convection
DR
MC
S
450 nHz
425
400
375
350
325
300
miesch@ucar.edu

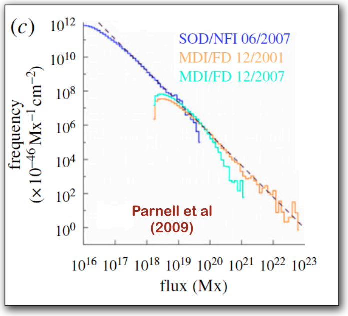

Today
How does convection plus the mean flows it establishes give rise to solar magnetic activity?

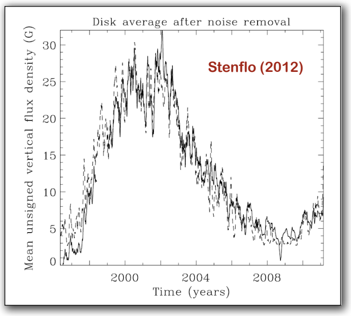

Definition
Hydromagnetic Dynamo
A physical object or system that converts the kinetic energy of fluid motions into magnetic energy and sustains that magnetic energy indefinitely against ohmic decay (magnetic diffusion)


The Solar Dynamo
SUNSPOT AREA IN EQUAL AREA LATITUDE STRIPS (% OF STRIP AREA)
■> 0.0% ■> 0.1% ■> 1.0%
90N
30N
EQ
30S
90S
1870 1880 1890 1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000 2010
DATE
The solar Butterfly Diagram
D. Hathaway


The Magnetic Carpet
B^L_{app} (+/- 50 G)
B^T_{app} (200 G)
160 140 120 100 80 60 40 20 0
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
Arcseconds
Lites et al (2008)

Large and Small Scales are Linked!

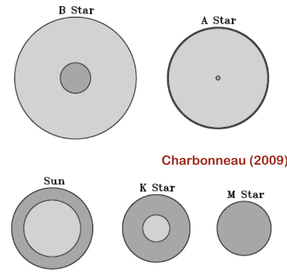




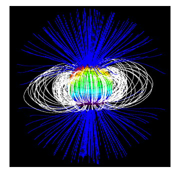
Continuous, apparently self-similar distribution of flux from the largest sunspots to the smallest resolved scales

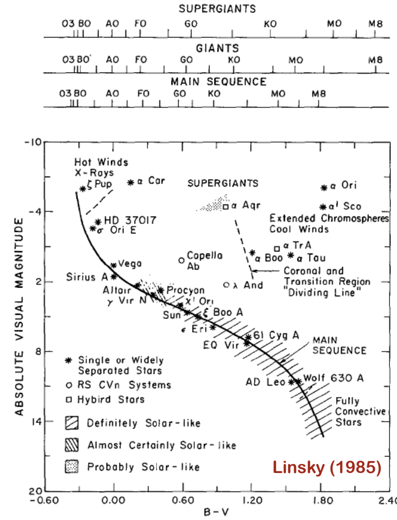
Magnetic flux on scales 2000 → 0.2 Mm Tracks the solar Activity cycle

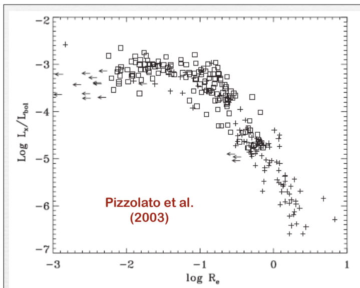
Convection Breeds Magnetism

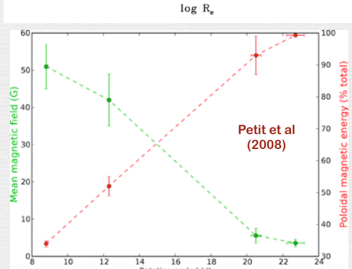


Donati et al (2006)







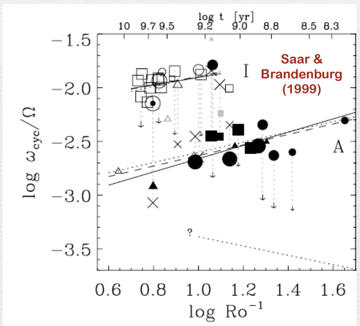


Rotation Promotes Magnetism

$$\frac{L_x}{L_{bol}} \propto (\Omega\tau_c)^2 \propto \mathrm{Ro}^{-2}$$

Saturation at large Ω

P_{cyc} vs P_{rot} less clear



Generation of Magnetic Fields: The MHD Magnetic Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\nabla\times\mathbf{B})$$

$$R_m = \frac{UL}{\eta}$$

Follows from Faraday's Law of Induction

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

And Ohm's Law (with a Galilean transformation)

$$\mathbf{J} = \sigma\,(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

MHD assumptions

Highly ionized, Quasi-neutral

High collision frequency/short mean-free paths

(high density, temperature)

sub-relativistic bulk velocity

Lesson #1 in Solar Dynamo Theory: If the velocity is specified (ki... the induction equation is ...

Note: this is the definition of

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{I}\ldots$$

Note one important implication in particular:

Kinematic dynamo models can be used to predict cycle period but <u>not</u> cycle amplitude

Amplitude requires an additional ingredient that you dial in: nonlinear quenching

Exception: Data assimilation (Dikpati & Gilman 2006)

Profound implications (imme...

Asking whether or not a given (steady) v... dynamo then reduces to a line...

Solutions are a linear superposition of different modes, each with its own (complex) eigenvalue and eigenfunction

Real part of eigenvalue indicates whether the solution exponentially grows or exponentially decays

Imaginary part determines whether or not the solution is oscillatory (cyclic)

Lesson #2 in Solar Dynamo Theory: No real dynamo in nature is kinematic

Profound pain in the neck (..or opportunity, depending on your perspective)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}$$

This suggests two classes of dynamos:

<u>Essentially kinematic:</u> *Make it stop!*

Small seed field that is initially kinematic (too weak to induce a significant Lorentz force) grows exponentially until it becomes big enough to modify the velocity field

This brings up the crucial issue of: Dynamo Saturation

<u>Essentially Nonlinear:</u> *Make it go!*

The velocity field that gives rise to the dynamo mechanism depends on the existence of the field

The focus then shifts toward: Dynamo Excitation

Lagrangian Chaos

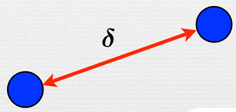
Chaotic fluid trajectories amplify magnetic fields

(provided that chaotic stretching wins the battle against ohmic diffusion)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

$$\frac{D\mathbf{B}}{Dt} = \frac{\partial \mathbf{B}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{v} - \mathbf{B}(\nabla \cdot \mathbf{v}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

If $\nabla \cdot \mathbf{v} = \eta = 0$ **then**

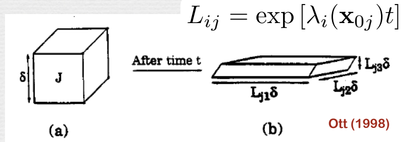


l = Local Lyapunov exponents

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \nabla)\mathbf{v}$$

$$\frac{d\boldsymbol{\delta}}{dt} = (\boldsymbol{\delta} \cdot \nabla)\mathbf{v}$$

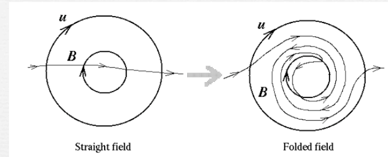
$$L_{ij} = \exp[\lambda_i(\mathbf{x}_{0j})t]$$



$$\frac{d\delta_i(\mathbf{x}_0, t)}{dt} = \mathcal{J}_{ij}(\mathbf{x}_0, t)\, \delta_j(\mathbf{x}_0, t) \qquad \lambda_1 + \lambda_2 + \lambda_3 = 0$$

Spatially smooth, temporally chaotic flows work best

$$R_m = \frac{UL}{\eta} \qquad P_m = \frac{\nu}{\eta}$$



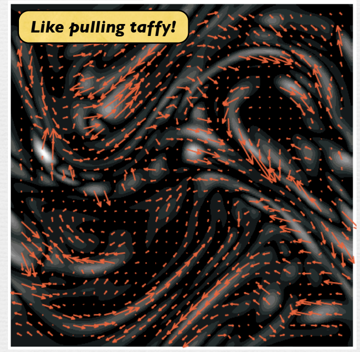
Schekochihin et al (2004)

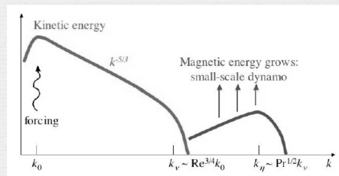
If $P_m > 1$ then turbulent dynamos build fields on sub-viscous scales

Magnetic energy peaks near resistive scale

Turbulent flows beget turbulent fields!

Like pulling taffy!





Folded Field Structure

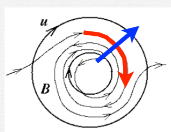
$$k_{\parallel} = \left(\frac{\langle |(\boldsymbol{B} \cdot \nabla)\boldsymbol{B}|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2}$$

$$k_{\boldsymbol{B} \times \boldsymbol{J}} = \left(\frac{\langle |\boldsymbol{B} \times \boldsymbol{J}|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2}$$

$$k_{\boldsymbol{B} \cdot \boldsymbol{J}} = \left(\frac{\langle |\boldsymbol{B} \cdot \boldsymbol{J}|^2 \rangle}{\langle B^4 \rangle} \right)^{1/2}$$

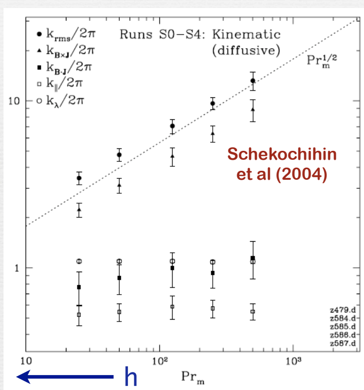
$$k_{\mathrm{rms}} = \left(\frac{\langle |(\nabla \boldsymbol{B})|^2 \rangle}{\langle B^2 \rangle} \right)^{1/2}$$

$$k_{\lambda} = \left(\frac{\langle |(\nabla \boldsymbol{v})|^2 \rangle}{\langle v^2 \rangle} \right)^{1/2}$$



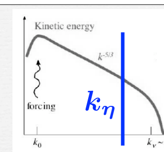
$$k_{\parallel} \sim k_{\lambda}$$

$$k_{B \times J} \sim R_m^{1/2}$$



But Stars have $P_m < 1$!

Now chaotic streching must overcome ohmic diffusion and turbulent diffusion



Still, the dynamo prevails if R_m is large enough

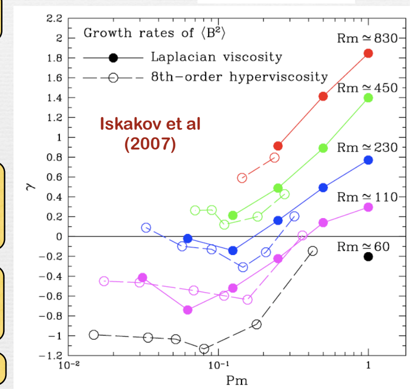
$$E_k \sim k^{-p}$$

$$\gamma \sim k v_k \sim k^{(3-p)/2}$$

Rough velocity fields ($p < 3$) Smallest eddies are best at amplifying field because they have the fastest turnover time

Magnetic energy still peaks near the resistive scale, at least in the kinematic regime

Small-scale Fields!



Local Dynamo Action in the Sun and Stars

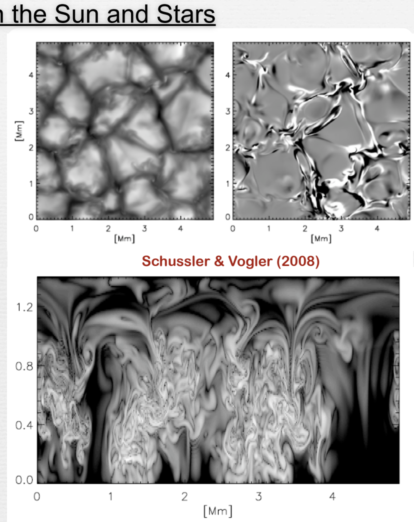
Granulation: τ ~ 10-15 min
Giant Cells: τ ~ days - months

Granulation may generate field locally by chaotic stretching with little regard for the deeper convection zone

Flux expulsion and reconnection produce strong horizontal fields near photosphere

Magnetic pumping of flux through lower boundary can inhibit the surface dynamo in simulations

In the Sun the local dynamo is likely intimately coupled to the global dynamo



Types of Dynamos

define Small-scale dynamo

Generates magnetic fields on scales smaller than the velocity field

$$\ell_B \leq \ell_v$$

define Large-scale dynamo

Generates magnetic fields on scales larger than the velocity field

$$\ell_B >> \ell_v$$

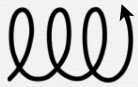
Are local solar/stellar dynamos small-scale dynamos?

Probably - but intimately coupled to deep CZ

Are global solar/stellar dynamos large-scale dynamos?

Probably - but v-B correlations induced by large-scale convective modes or instabilities may contribute to global field generation

Recipe for Building Large-Scale Fields



- **Lagrangian Chaos**
  - Builds magnetic energy
- **Rotational Shear**
  - Builds large-scale toroidal flux (W-effect)
  - Enhances dissipation of small-scale fields
  - Promotes magnetic helicity flux
- **Helicity**
  - Rotation and stratification generate kinetic helicity
  - Kinetic helicity generates magnetic helicity
  - Upscale spectral transfer of magnetic helicity generates large-scale fields
    - Local transfer: inverse cascade of magnetic helicity
    - Nonlocal transfer: a-effect

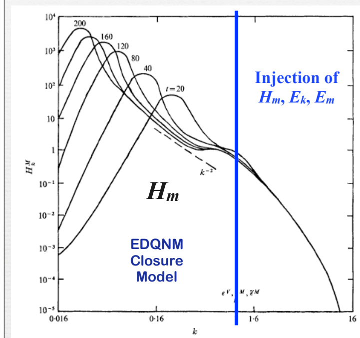
$$H_k = \langle \boldsymbol{\omega} \cdot \boldsymbol{v} \rangle$$
$$H_m = \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle$$
$$H_c = \langle \boldsymbol{J} \cdot \boldsymbol{B} \rangle$$

$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$
$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$
$$\boldsymbol{J} = \frac{c}{4\pi} \boldsymbol{\nabla} \times \boldsymbol{B}$$

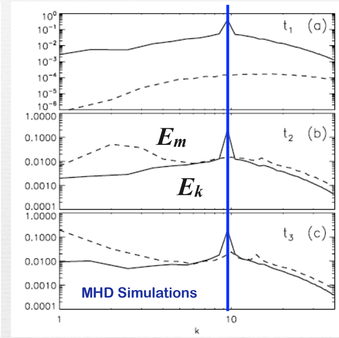
Specific manifestations of a more general (and more profound) phenomenon

Inverse Cascade of Magnetic Helicity

Injection of E_k, H_k



Pouquet, Frisch & Leorat (1976)



Alexakis, Mininni & Pouquet (2006)

Magnetic Helicity is conserved in the limit $\eta \longrightarrow 0$

Provides an essential link between large and small scales

If you twist the field on small scales, large scales will respond

Large Scale Dynamos: The Mean Induction Equation

Go back to our basic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \eta \boldsymbol{\nabla}^2 \mathbf{B}$$

Now just average over longitude and rearrange a bit
(other averages are possible but we'll stick to this for simplicity)

Note: ***The B field in the Sun is clearly not axisymmetric. Still, the solar cycle clearly has an axisymmetric component so that's a good place to start***

The equation for the mean field comes out to be

$$\lambda = r \sin\theta$$

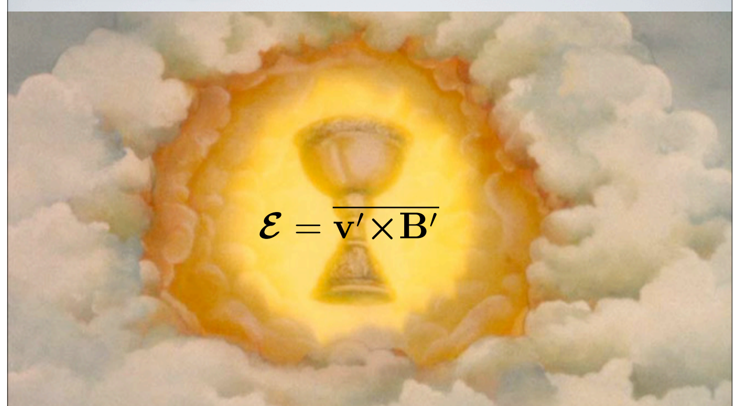
$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \lambda \overline{\mathbf{B}}_p \cdot \boldsymbol{\nabla}\Omega \; \hat{\phi} + \boldsymbol{\nabla} \times (\overline{\mathbf{v}}_m \times \overline{\mathbf{B}}) + \eta \nabla^2 \overline{\mathbf{B}} + \boldsymbol{\nabla} \times \boldsymbol{\mathcal{E}}$$

⋆ -effect — ***Meridional circulation*** — ***Diffusion (molecular)*** — ***Fluctuating emf***

No assumptions made up to this point beyond the basic MHD induction equation

Straightforward to show that if E=0, the dynamo dies (Cowling's theorem)

The Fluctuating emf



How can a non-axisymmetric flow across magnetic field lines produce an axisymmetric field?

$$\mathbf{v}' = \mathbf{v} - \overline{\mathbf{v}}$$
$$\mathbf{B}' = \mathbf{B} - \overline{\mathbf{B}}$$

The α-effect

Helical motions (lift, twist) can induce an emf that is parallel to the mean field

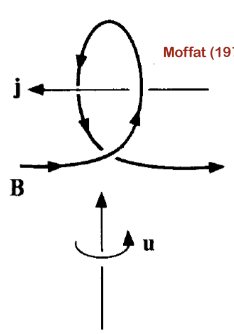
$$\mathcal{E} = \overline{\mathbf{v}' \times \mathbf{B}'} = \alpha \overline{\mathbf{B}}$$

This creates mean poloidal (r, θ) field from toroidal (ვ) field

which closes the Dynamo Loop

Mean Toroidal Field (east-west)

Mean Poloidal Field (r, θ plane) (north/south, up/down)



Moffat (1978)

Linked to kinetic, magnetic helicity

Linked to large-scale dynamo action

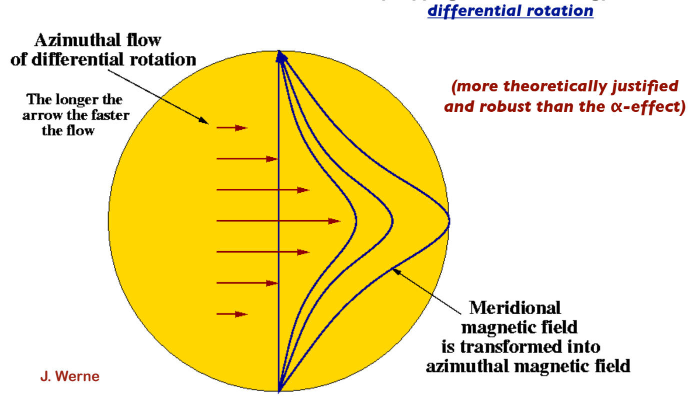
Illustrates the 3D nature of dynamos

The ⋆ -effect

Converts poloidal to toroidal field and amplifies it

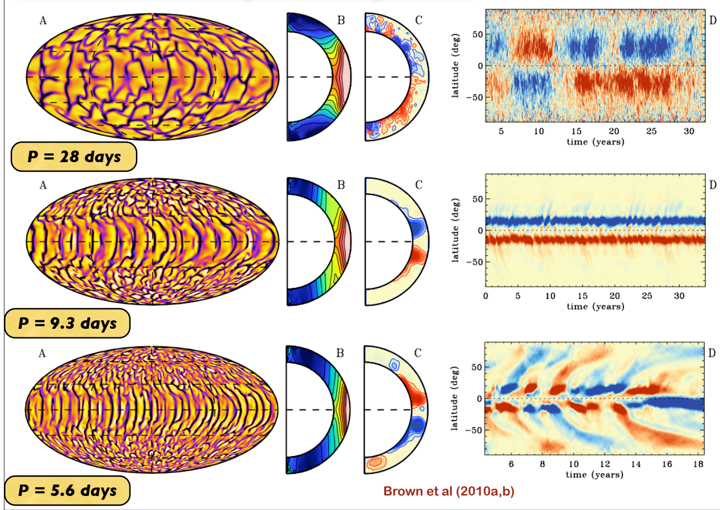
...by tapping the kinetic energy of the differential rotation

(more theoretically justified and robust than the α-effect)



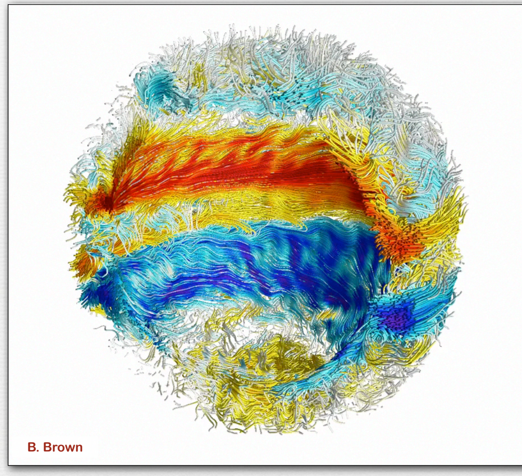
J. Werne

Rotation promotes magnetic self-organization



Brown et al (2010a,b)

Magnetic Cycles in a Convective Dynamo



"Magnetic Wreathes"