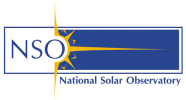
Introduction to Solar Radiative Transfer
II Non-LTE Radiative Transfer

Han Uitenbroek
National Solar Observatory/Sacramento Peak
Sunspot NM

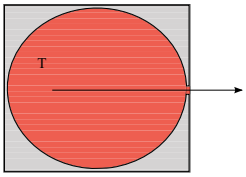


George Ellery Hale CGEP, CU Boulder
Lecture 14, Mar 7, 2013

Overview

- I Basic Radiative Transfer
  Intensity, emission, absorption, source function, optical depth, transfer equation
- II Detailed Radiative Processes
  Spectral lines, radiative transitions, collisions, polarization, Non-LTE radiative transfer, molecular concentrations
- III Observations of Solar Radiation
  Solar telescopes, spectroscopy, polarimetry

Local Thermodynamic Equilibrium (LTE)



- Radiation field is given by Planck function
- Velocities are given by Maxwellian distribution
- Ionization and excitation are given by Saha–Boltzmann statistics

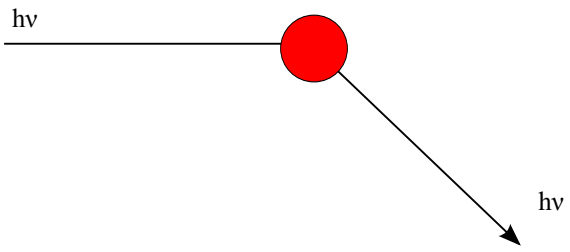
Basic Radiative Transfer: Radiation Field

Remember:

Angle-averaged mean intensity:

$$J_\nu(\vec{r},t) \equiv \frac{1}{4\pi}\int I_\nu \,\mathrm{d}\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\nu \sin\theta \,\mathrm{d}\theta \,\mathrm{d}\varphi$$

Scattering



Basic Radiative Transfer: Scattering

Absorption:

$$\mathrm{d}I_\nu \equiv -\sigma_\nu I_\nu \mathrm{d}s$$

Emission:

$$\mathrm{d}I_\nu = \sigma J_\nu \mathrm{d}s; \qquad \text{(isotropic scattering)}$$

Scatering source function:

$$S_\nu = \sigma_\nu J_\nu/\sigma_\nu = J_\nu$$

In the case of pure scattering the source function is solely determined by the radiation field and, therefore, completely decoupled from local conditions in the atmosphere, resulting in possible departures from Local Thermodynamic Equilibrium (LTE).

Radiative Transfer with Absorption and Scattering

$$\eta_\nu = \alpha_\nu B_\nu + \sigma_\nu J_\nu$$
$$\chi_\nu = \alpha_\nu + \sigma_\nu$$

Total source function:

$$\begin{aligned} S_\nu &\equiv \frac{\eta_\nu}{\chi_\nu} \\ &= \frac{\sigma_\nu}{\alpha_\nu + \sigma_\nu} J_\nu + \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu} B_\nu; \qquad \epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu} \\ &= (1 - \epsilon_\nu) J_\nu + \epsilon_\nu B_\nu \\ &= (1 - \epsilon_\nu) \Lambda_\nu \left[S_\nu \right] + \epsilon_\nu B_\nu \end{aligned}$$

Simple Solution: Lambda Iteration

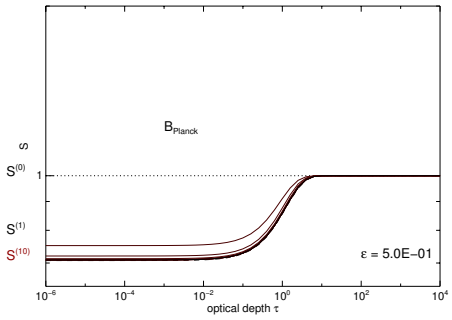
Operator equation for source function:

$$S_\nu = (1 - \epsilon_\nu) \Lambda_\nu \left[S_\nu \right] + \epsilon_\nu B_\nu$$

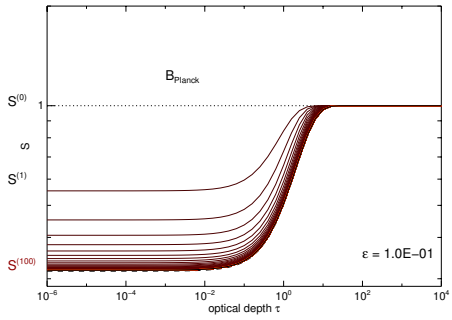
Simple iterative solution:

$$S_\nu^{(0)} = B_\nu$$
$$S_\nu^{(n)} = (1 - \epsilon_\nu) \Lambda_\nu \left[S_\nu^{(n-1)} \right] + \epsilon_\nu B_\nu$$

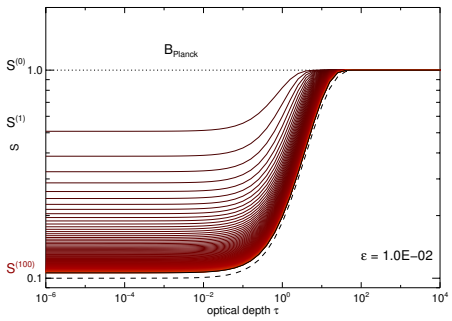
Lamda Iteration: $\epsilon = 0.5$



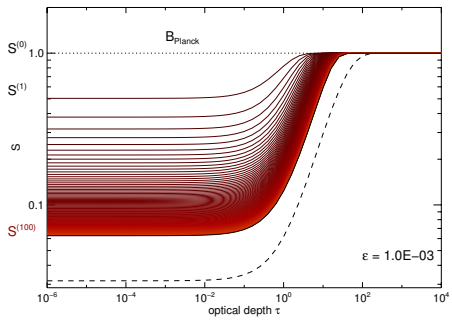
Lamda Iteration: $\epsilon = 0.1$



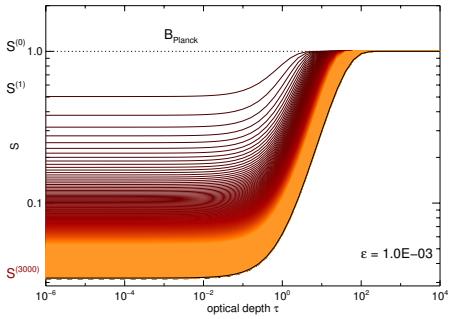
Lamda Iteration: $\epsilon = 0.01$



Lamda Iteration: $\epsilon = 0.001$



Lamda Iteration: $\epsilon = 0.001$, 3000 Iterations



Accelerated Lambda Iteration

$$S_\nu = (1 - \epsilon_\nu)\Lambda_\nu \left[S_\nu\right] + \epsilon_\nu B_\nu$$

Auer, in "Numerical Radiative Transfer", 1987, ed. W. Kalkofen, p. 101

For one wavelength, this is a matrix equation in depth points:

$$S_k = (1 - \epsilon_k)\Lambda_\nu \left[S_\nu\right]_k + \epsilon_k B_k$$

We could solve this equation easily if the Λ operator where just a multiplication, i.e., if it where a local operator. Use the local part of the operator, i.e, its diagonal Λ^* (see Olson, Auer & Buchler, 1986, JQSRT 35, 431).

Accelerated Lambda Iteration

Split off the diagonal part:

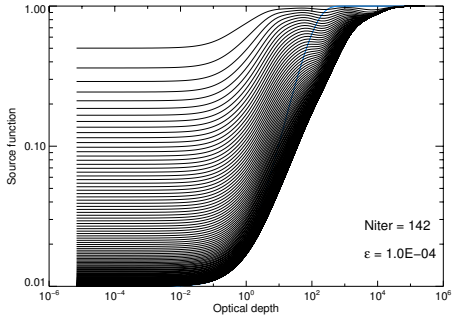
$$J_k = \Lambda_\nu \left[S_\nu\right]_k \equiv \Lambda^*_k S_k + (\Delta J)_k \qquad \rightarrow \qquad \Delta J_k = \Lambda \left[S\right]_k - \Lambda^*_k S_k$$
$$S_k = (1 - \epsilon_k)\left\{\Lambda^*_k S_k + \Delta J_k\right\} + \epsilon_k B_k$$

New iterative scheme:

$$\Delta J^{(n)} = \Lambda^{(n)}\left[S^{(n)}\right] - \Lambda^* S^{(n)}$$
$$S_k^{(n+1)} = \frac{(1 - \epsilon_k)\Delta J_k^{(n)} + \epsilon_k B_k}{1 - (1 - \epsilon_k)\Lambda^*_k}$$

We can invert the diagonal part now directly and only have to lambda iterate the weaker off-diagonal contributions.

Accelerated Lamda Iteration: $\epsilon = 0.0001$



Accelerated Lamda Iteration: $\epsilon = 0.0001$ with convergence extrapolation

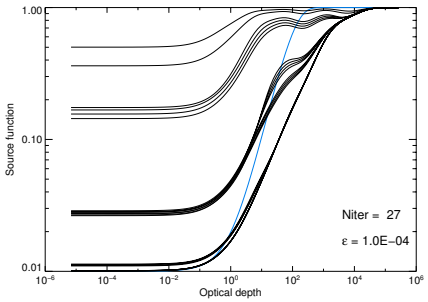
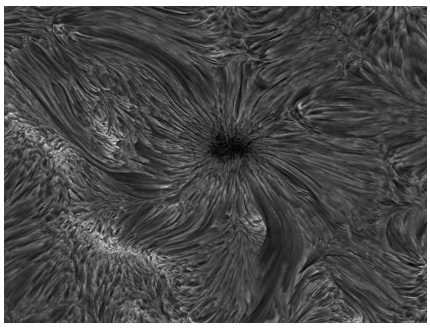
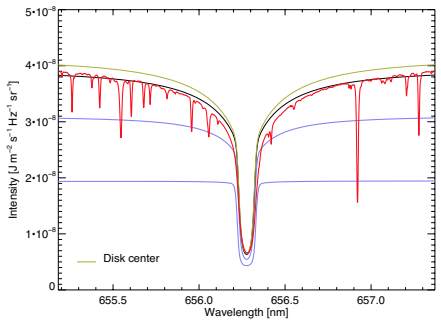


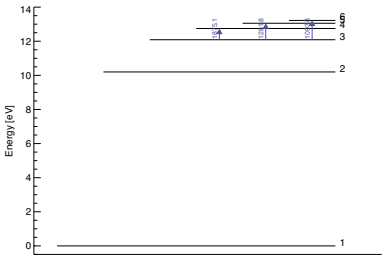
Image of the Sun in the light of Hα



Hα Spectral Line

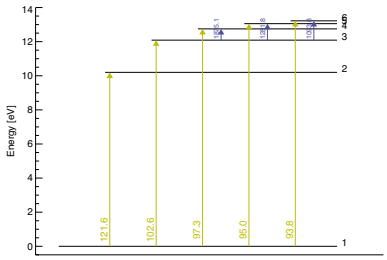


Termdiagram and Transitions in Hydrogen



$$\Delta E = h\nu = \frac{hc}{\lambda}$$

Termdiagram and Transitions in Hydrogen



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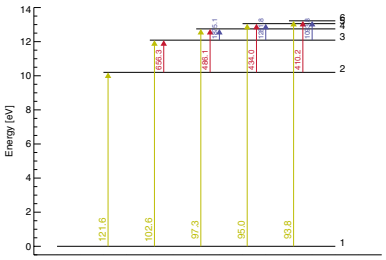
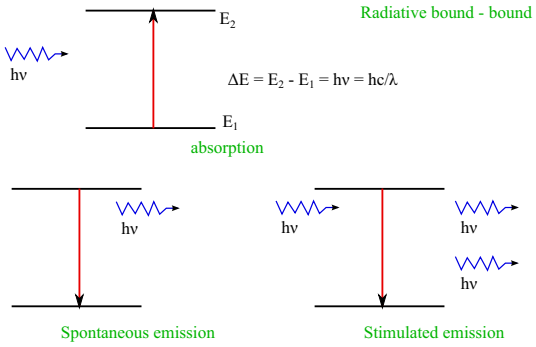
Termdiagram and Transitions in Hydrogen

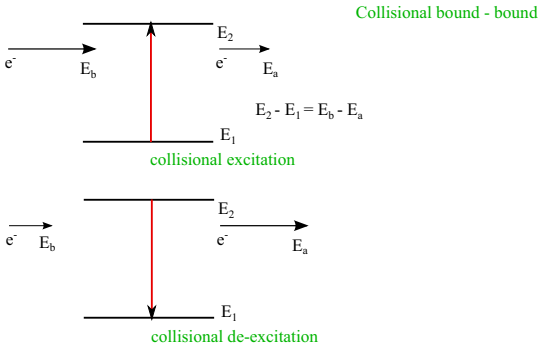


$$\Delta E = h\nu = \frac{hc}{\lambda}$$

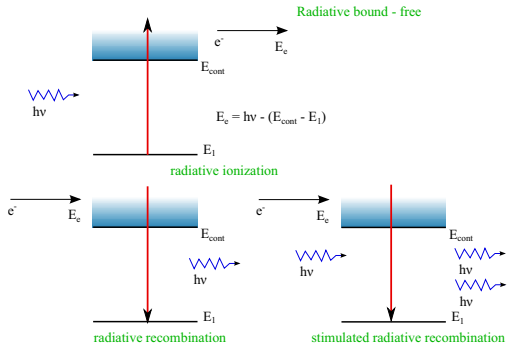
Radiative bound–bound transitions



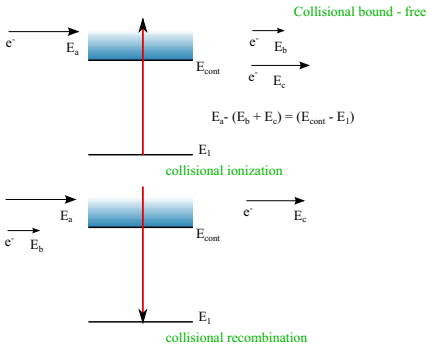
Collisional bound–bound transitions



Radiative bound–free transitions



Collisional bound–free transitions



Absorption and emission coefficients for bound-bound transitions

Spontaneous emission $j \to i$:

$$j_\nu^{\rm spont} = n_j (A_{ji} h\nu_{ij}/4\pi)\phi_\nu$$

Stimulated emission $j \to i$:

$$j_\nu^{\rm stim} = n_j (B_{ji} h\nu_{ij}/4\pi)\phi_\nu I_\nu, \qquad A_{ji} = (2h\nu^3/c^2) B_{ji}$$

Absorption $i \to j$:

$$\alpha_\nu = n_i (B_{ij} h\nu_{ij}/4\pi)\varphi_\nu, \qquad g_i B_{ij} = g_j B_{ji}$$

Source function of bound–bound transition

Transfer equation:

$$\begin{aligned}\frac{{\rm d}I_\nu}{{\rm d}s} &= j_\nu^{\rm spont} + j_\nu^{\rm stim} - \alpha_\nu I_\nu \\ &= n_j (A_{ji} h\nu_{ij}/4\pi)\phi_\nu - h\nu_{ij}/4\pi\phi_\nu (n_i B_{ij} - n_j B_{ji}) I_\nu\end{aligned}$$

Source function:

$$\begin{aligned}S_\nu &= \frac{j_\nu}{\alpha_\nu} = \frac{n_j A_{ji}}{n_i B_{ij} - n_j B_{ji}} \\ &= \frac{2h\nu_{ij}^3}{c^2}\frac{n_j}{g_j/g_i n_i - n_j} \qquad = (1-\epsilon)\overline{J} + \epsilon B_\nu; \qquad \epsilon \equiv \frac{C_{ji}}{C_{ji} + A_{ji} + B_{ji}\overline{J}}\end{aligned}$$

Radiative rates

Radiative excitation

$$\begin{aligned}R_{ij} &= B_{ij}\overline{J}\frac{h\nu}{4\pi}\int {\rm d}\Omega \int \frac{{\rm d}\nu}{h\nu} I_\nu \phi_\nu \\ &= B_{ij}\overline{J}; \qquad\qquad \overline{J} \equiv \frac{1}{4\pi}\int {\rm d}\Omega \int {\rm d}\nu I_\nu \phi_\nu\end{aligned}$$

Radiative de-excitation

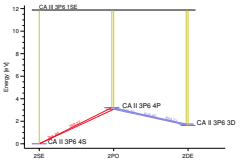
$$\begin{aligned}R_{ij} &= Aji + B_{ji}\frac{h\nu}{4\pi}\int {\rm d}\Omega \int \frac{{\rm d}\nu}{h\nu} I_\nu \phi_\nu \\ &= A_{ji} + B_{ji}\overline{J}\end{aligned}$$

Basic Equation: Statistical Equilibrium

Consider and atom (or molecule) with levels $i = 0, \ldots, N-1$.

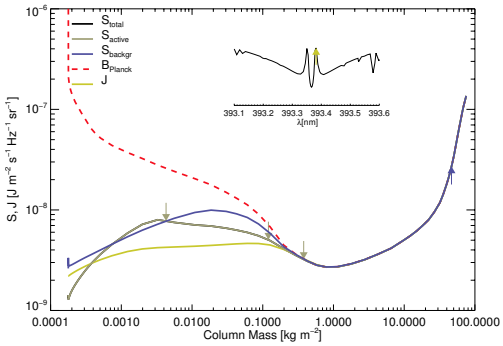
Statistical equilibrium for level i:

$$\sum_{j=0}^{N-1} n_j (Cji + Rji) = \sum_{j=0}^{N-1} n_i (Cij + Rij)$$



The set of equations for all levels forms a, generally non-linear, and non-local, set of equations for the population numbers n_i

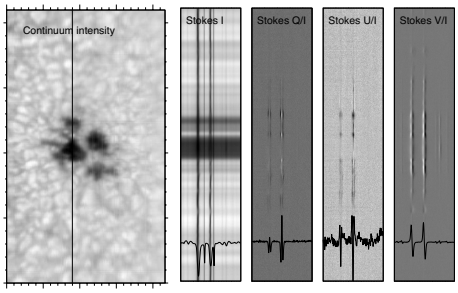
Ca II source function



When is Non-LTE Transfer Important?

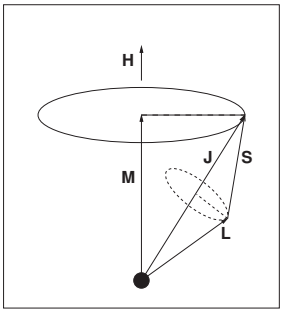
- When density is high and collisions are frequent enough, population numbers are determined by local conditions, and given by the Saha-Boltzmann relations at the kinetic temperature of the gas.
- The radiation field is then given by the Planck function
- As densities drop with height, collisions become less frequent, and radiative transitions become relatively more important.
- Populations are now determined by non-local conditions, namely the radiation field that comes from different places in the atmosphere.
- Need to find a global solution, not only in space, but also in wavelength.

How Do We Extract Physics from Observations?

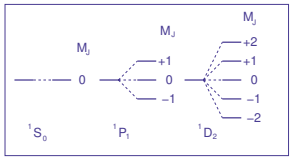


DLSP at the DST, courtesy Alexandra Tritschler (NSO/SP)

The Zeeman effect in atoms



$$M_J = -J, -J+1, \ldots, J-1, J$$
$$J_z = M_J \hbar$$
$$E = E_0 + g_L M_J \mu_B H$$



Equation of Polarized Radiative Transfer

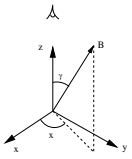
Transfer Equation:

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}s} = -\mathbf{K}\mathbf{I} + \mathbf{j}$$

$$\mathbf{I} = (I, Q, U, V)^\dagger, \quad \text{(Stokes vector)}$$
$$\mathbf{j} = (j_c + j_l \mathbf{\Phi})\mathbf{e}_0, \quad \mathbf{e}_0 = (1, 0, 0, 0)^\dagger$$
$$\mathbf{K} = \alpha_c \mathbf{1} + \alpha_l \mathbf{\Phi}, \quad \text{(Absorption matrix)}$$

Line Absorption Matrix

$$\mathbf{\Phi} = \begin{pmatrix} \phi_I & \phi_Q & \phi_U & \phi_V \\ \phi_Q & \phi_I & \psi_V & -\psi_U \\ \phi_U & -\psi_V & \phi_I & \psi_Q \\ \phi_V & \psi_U & -\psi_Q & \phi_I \end{pmatrix}$$



$$\phi_I = \phi_\Delta \sin^2\gamma + \tfrac{1}{2}(\phi_+ + \phi_-), \quad \phi_\Delta = \tfrac{1}{2}\left[\phi_0 - \tfrac{1}{2}(\phi_+ + \phi_-)\right]$$
$$\phi_Q = \phi_\Delta \sin^2\gamma \cos 2\chi$$
$$\phi_U = \phi_\Delta \sin^2\gamma \sin 2\chi$$
$$\phi_V = \tfrac{1}{2}(\phi_+ - \phi_-)\cos\gamma$$

Molecular Spectral Lines in the Solar Spectrum

- Molecules are abundant in the solar atmosphere, in particular in cooler areas like Sunspot umbrae
- The G band is one of the most used pass bands in solar high resolution imaging. It its majot source of opacity are lines of the CH molecule.
- CO lines are a major contributor to radiative cooling of the atmosphere in the infrared.
- Molecules are sensitive to the Zeeman effect, and have much more diverse sensitivity than atomic lines. This can be used to advantage.

Concentration of Molecules

Chemical equilibrium:

$$\frac{n_A n_B}{n_{AB}} = \left(\frac{2\pi m_{AB} kT}{h^2}\right)^{3/2} e^{-D/kT} \left(\frac{U_A U_B}{Q_{AB}}\right)$$

$$m_{AB} = \frac{m_A m_B}{m_A + m_B}$$

Non-linear set of coupled equations:

$$n_{AB} - n_A n_B \Phi_{AB}(T) = 0$$

$$n_A + n_{AB} = A_A n_H$$

$$n_B + n_{AB} = A_B n_H$$

Use Newton-Raphson Method to Solve

Set of non-linear chemical equilibrium equations:

$$f(\vec{n}) = \vec{a}$$

Iterative solution:

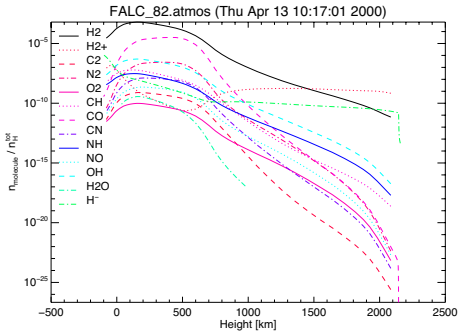
$$f(\vec{n}^{(n)} + \delta\vec{n}) = \vec{a}$$

$$f(\vec{n}^{(n)}) + \delta\vec{n}\frac{\partial f}{\partial n} \approx \vec{a}$$

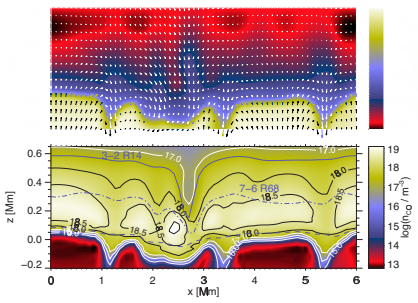
$$\delta\vec{n} = \left(\vec{a} - f(\vec{n}^{(n)})\right) / \frac{\partial f}{\partial n}$$

$$\vec{n}^{(n+1)} = \vec{n}^{(n)} + \delta\vec{n}$$

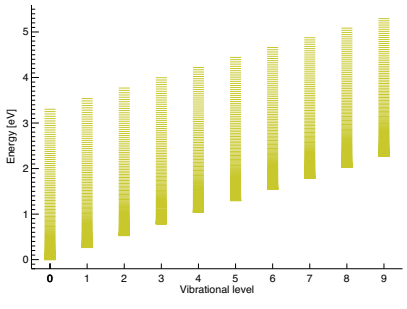
Molecular Concentrations in the Solar Atmosphere



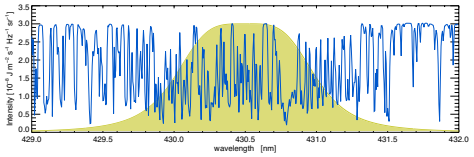
CO Concentration in Vertical Cross Section



Energy levels of the CO Molecule



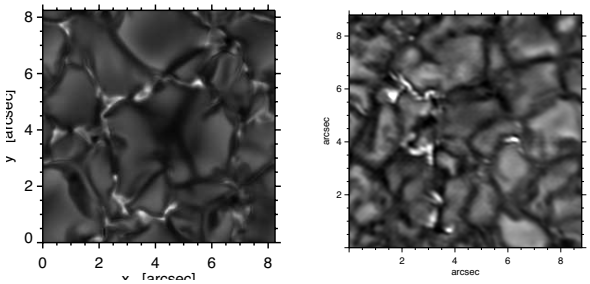
Filter Integrated Intensity



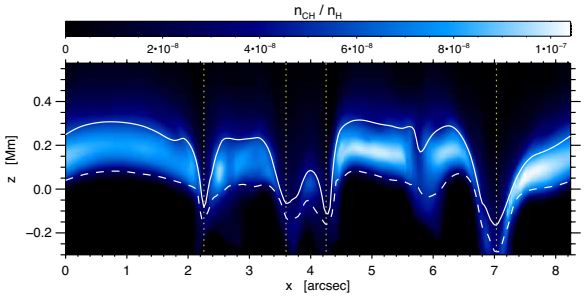
Filter signal:

$$f = \int_0^\infty I_\lambda F_\lambda d\lambda$$

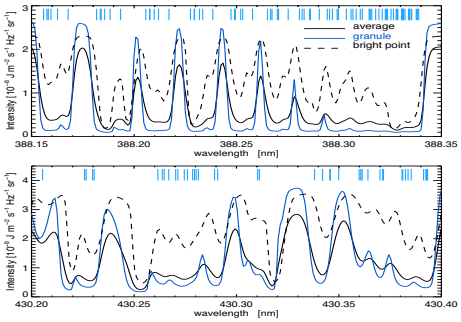
High resolution imaging in the G-band



Concentration CH Molecules in Magneto-Convection Slice

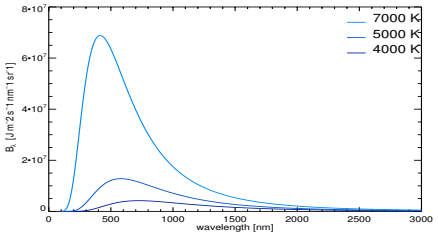


Detailed Spectra of Granule and Bright Point



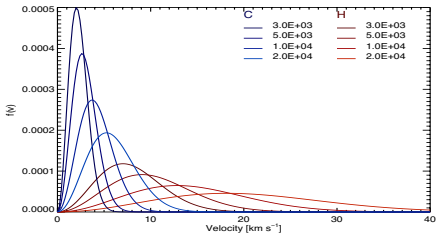
End Part II

Planck Function



$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Maxwellian Velocity Distribution



$$f(v)\mathrm{d}v = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp(-mv^2/2kT)\, 4\pi v^2 \mathrm{d}v$$

Saha–Boltzmann statistics

Boltzmann distribution for excitation:

$$\left[\frac{n_j}{n_i}\right]_{\mathrm{LTE}} = \frac{g_j}{g_i} e^{-\Delta E_{ji}/kT}$$

Saha distribution for ionization:

$$\left[\frac{n_{r+1,1}}{n_{r,1}}\right]_{\mathrm{LTE}} = \frac{1}{N_{\mathrm{e}}} \frac{2g_{r+1,1}}{g_{r,1}} e^{-\Delta\chi_r/kT}$$

Back

Formal Solution as Operator Equation

$$\begin{aligned} J_\nu &= \frac{1}{4\pi} \int I_\nu(\tau, \vec{l})\, \mathrm{d}\Omega \\ &= \frac{1}{4\pi} \int \mathrm{d}\Omega \int_{\tau_\nu}^{\infty} S_\nu(t, \vec{l})\, e^{-(t-\tau_\nu)}\, \mathrm{d}t \\ &= \Lambda_\nu\, [S_\nu] \end{aligned}$$

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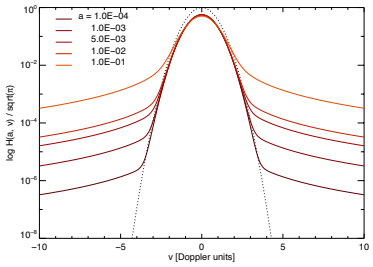
Voigt Functions



$$\phi(\nu - \nu_0) = \frac{H(a, v)}{\sqrt{\pi}\Delta\nu_D}$$

$$\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

$$a = \frac{\Gamma}{4\pi\Delta\nu_D}$$

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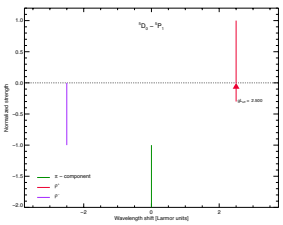
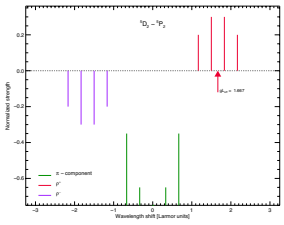
Splitting pattern for Fe I 630.25 nmlevel=2 and 630.15 nm

level=1





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Splitting Patterns for Satellite Branch ($\Delta N \neq \Delta J$)