Introduction to Solar Radiative Transfer
I Basic Radiative Transfer

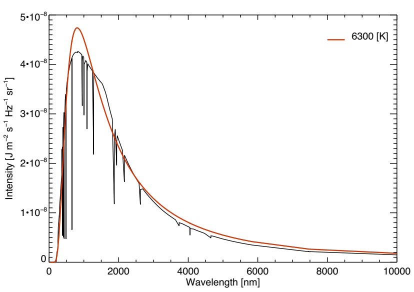
Han Uitenbroek
National Solar Observatory/Sacramento Peak
Sunspot NM

George Ellery Hale CGEP, CU Boulder
Lecture 13, Mar 5 2013

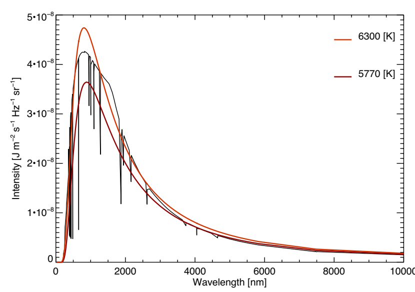
Why Radiative Transfer?

- In general we cannot visit the astronomical objects we are interested in, and thus cannot take in-situ measurements
- Instead, to determine the object's properties, we have to rely on the information carried to us by the electromagnetic radiation emitted and/or reflected by the object.
- Multi-wavelength (spectroscopic) observations and analysis are the only available means to determine the physical conditions of astronomical objects.
- To analyze spectroscopic data meaningfully we need to understand how physical information is encoded in the radiation (Radiative Transfer).
- We need to understand how the radiative signal is modified as it travels to our instruments and is detected with them.

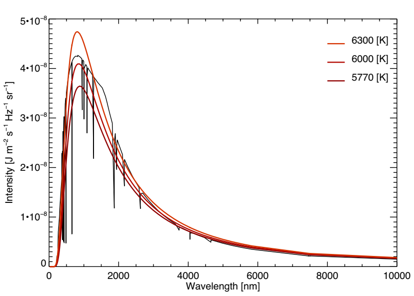
The Solar Spectrum and Surface Temperature



The Solar Spectrum and Surface Temperature



The Solar Spectrum and Surface Temperature



Overview

- I Basic Radiative Transfer
  Intensity, emission, absorption, source function, optical depth, transfer equation, line formation
- II Detailed Radiative Processes
  Spectral lines, radiative transitions, collisions, polarization, Non-LTE radiative transfer
- III Observations of Solar Radiation
  Solar telescopes, spectroscopy, polarimetry

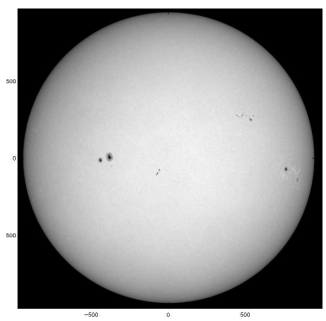
Bibliography

- **Rutten:** Radiative Transfer in Stellar Atmospheres (http://esoads.eso.org/abs/2003rtsa.book.....R)
- **Rybicki and Lightman:** Radiative Processes in Astrophysics
- **Mihalas:** Stellar Atmospheres
- **Shu:** The Physics of Astrophysics. I. Radiation
- **Gray:** Observation and Analysis of Stellar Photospheres
- **del Toro Iniesta:** Introduction to Spectropolarimetry
- **Allen:** Astrophysical Quantities

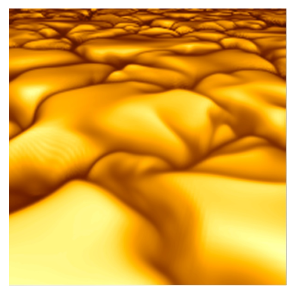
Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines
- **1814** Fraunhofer rediscovers lines. Assigns names e.g., C (Hα), D (Na I), G (CH molecules), F (Hβ), and H (Ca II)
- **1823** Herschel realized spectra contain information on composition of source from flame spectra
- **1842** Becquerel photographs spectra, discovers lines in the UV, beyond the visible
- **1858** Bunsen and Kirchhoff discover wavelength correspondence between bright flame emission and dark solar absorption lines. Start of quantitative spectroscopy.

The Sun

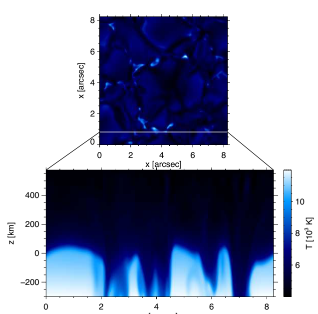


Solar atmosphere is also very strongly time dependent

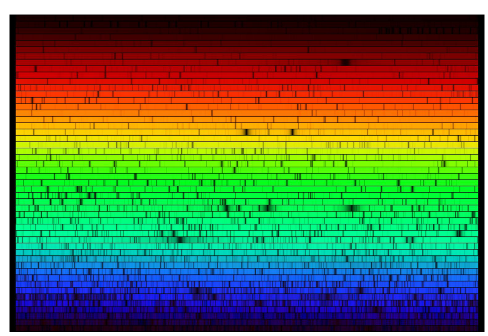


Courtesy: Mats Carlsson

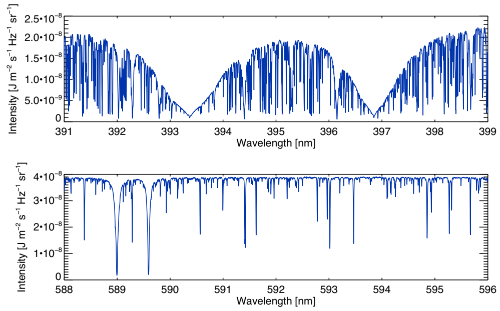
A vertical cross section through a 3-D Convection Simulation



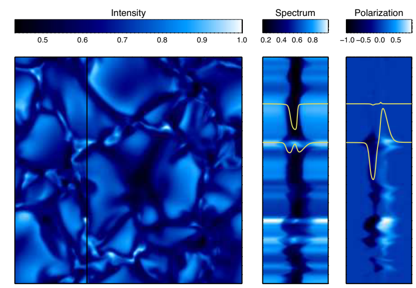
The Visible Solar Spectrum



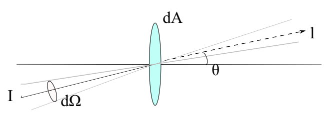
Solar Spectrum in the Blue and Red



Spatially Resolved Spectral Lines



Basic Radiative Transfer: Radiation Field



Specific intensity I_ν is the radiative energy that flows, at the location $\vec{r}$, per second, per wavelength interval, and per solid angle, in the direction $\vec{l}$ through the surface area dA' perpendicular to $\vec{l}$. Intensity is conserved with distance in the absence of emission and absorption or scattering processes.

Specific Intensity:

$$\mathrm{d}E_\lambda^{\mathrm{rad}} \equiv I_\lambda(\vec{r}, \vec{l}, t)\,\mathrm{d}t\,\mathrm{d}A'\,\mathrm{d}\lambda\,\mathrm{d}\Omega = I_\lambda(\vec{r}, \vec{l}, t)\,\mathrm{d}t\,\cos\theta\,\mathrm{d}A\,\mathrm{d}\lambda\,\mathrm{d}\Omega$$

Units: J s^{-1} m^{-2} nm^{-1} ster^{-1}

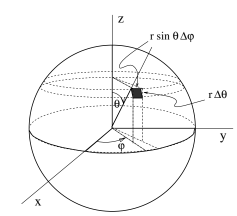
Basic Radiative Transfer: Mean Intensity

Angle-averaged Mean intensity:

$$J_\nu(\vec{r}, t) \equiv \frac{1}{4\pi}\int I_\lambda\,\mathrm{d}\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} I_\lambda \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$$

Units: J s^{-1} m^{-2} nm^{-1} ster^{-1}

Unlike the Specific Intensity the Angle-averaged Mean Intensity is not conserved with distance



Basic Radiative Transfer: Flux

Flow of radiative energy through a surface.
Flux:

$$\mathcal{F}_\lambda(\vec{r}, \vec{n}, t) \equiv \int I_\lambda \cos\theta\,\mathrm{d}\Omega = \int_0^{2\pi}\int_0^{\pi} I_\lambda \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \qquad (1)$$

Units: J s^{-1} m^{-2} nm^{-1}

Flux in radial direction:

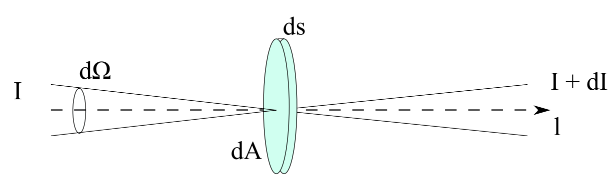
$$\begin{aligned}\mathcal{F}_\lambda(z) &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_\lambda \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi + \int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} I_\lambda \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi \qquad (2)\\ &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_\lambda \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi - \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_\lambda(\pi-\theta) \cos\theta\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi\\ &\equiv \mathcal{F}_\lambda^+(z) - \mathcal{F}_\lambda^-(z)\end{aligned}$$

Basic Radiative Transfer: Absorption

Absorption α_λ:

$$I_\lambda(s + \mathrm{d}s) = I_\lambda(s) + \mathrm{d}I_\lambda = I_\lambda - \alpha_\lambda I_\lambda \mathrm{d}s$$

Units: m^{-1}

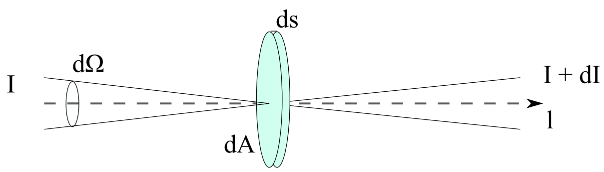


Basic Radiative Transfer: Emission

Emission j_λ:

$$I_\lambda(s+\mathrm{d}s) = I_\lambda(s) + \mathrm{d}I_\lambda = I_\lambda + j_\lambda(s)\mathrm{d}s$$

Units: J m^{-3} s^{-1} nm^{-1} ster^{-1}



Basic Radiative Transfer: Source Function

Source function:

$$S_\lambda \equiv j_\lambda / \alpha_\lambda$$

Units: J s^{-1} m^{-2} nm^{-1} ster^{-1}

For multiple proceses active at the same wavelength:

$$S_\lambda^{\mathrm{tot}} = \sum j_\lambda / \sum \alpha_\lambda$$

$$S_\lambda^{\mathrm{tot}} = \frac{j_\lambda^c + j_\lambda^l}{\alpha_\lambda^c + \alpha_\lambda^l} = \frac{S_\lambda^c + \eta_\lambda S_\lambda^l}{1+\eta_\lambda}, \quad \eta_\lambda \equiv \alpha_\lambda^l / \alpha_\lambda^c$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$\mathrm{d}I_\lambda(s) = I_\lambda(s+\mathrm{d}s) - I_\lambda(s) = j_\lambda(s)\mathrm{d}s - \alpha_\lambda(s) I_\lambda(s)\mathrm{d}s \quad (3)$$

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}s} = j_\lambda - \alpha_\lambda I_\lambda$$

$$\frac{\mathrm{d}I_\lambda}{\alpha_\lambda \mathrm{d}s} = \frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = S_\lambda - I_\lambda$$

Optical length and thickness:

$$\mathrm{d}\tau_\lambda \equiv \alpha_\lambda(s)\mathrm{d}s \quad (4)$$

$$\tau_\lambda(D) = \int_0^D \alpha_\lambda(s)\mathrm{d}s$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = S_\lambda - I_\lambda \quad (5)$$

$$I_\lambda(\tau_\lambda) = I_\lambda(0)e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S_\lambda(t) e^{-(\tau_\lambda - t)}\mathrm{d}t$$

Homogeneous medium:

$$I_\lambda(D) = I_\lambda(0)e^{-\tau_\lambda(D)} + S_\lambda\left(1 - e^{-\tau_\lambda(D)}\right) \quad (6)$$

Optically thick: $I_\lambda(D) \approx S_\lambda$
Optically thin: $I_\lambda(D) \approx I_\lambda(0) + [S_\lambda - I_\lambda(0)]\,\tau_\lambda(D)$

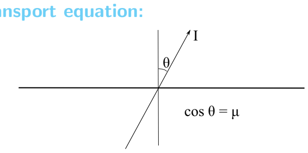
Basic Radiative Transfer: Through an Atmosphere

Optical path:

$$\mathrm{d}\tau_{\mu\lambda} = \alpha_\lambda \mathrm{d}s \equiv -\alpha_\lambda \frac{\mathrm{d}z}{\mu}$$

Standard plane parallel transport equation:

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_{\mu\lambda}} = \mu\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = I_\lambda - S_\lambda$$



Basic Radiative Transfer: Eddington–Barbier

Emergent intensity at the surface:

$$I_\lambda^+(\tau_\lambda = 0, \mu) = \int_0^\infty S_\lambda(t) e^{-t/\mu}\mathrm{d}t/\mu$$

Substitute power series:

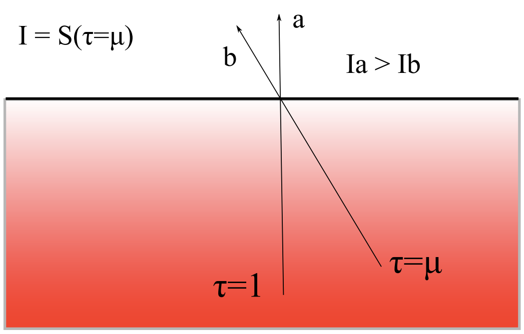
$$S_\lambda(\tau_\lambda) = \sum_{n=0}^N a_n \tau_\lambda^n \qquad (\text{using}: \int_0^\infty e^{-t} t^n \mathrm{d}t = !n)$$

$$I_\lambda^+(\tau_\lambda = 0, \mu) = a_0 + a_1\mu + 2a_2\mu^2 + \ldots + n! a_N \mu^N$$

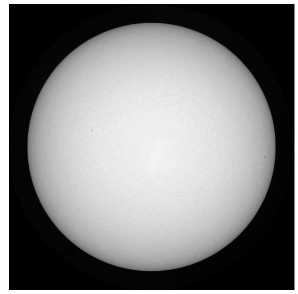
Eddington–Barbier relation:

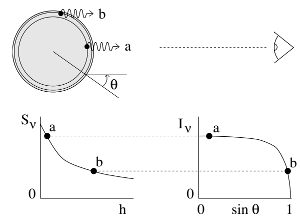
$$I_\lambda^+(\tau_\lambda = 0, \mu) \approx S_\lambda(\tau_\lambda = \mu)$$

Eddingtomn-Barbier approximation



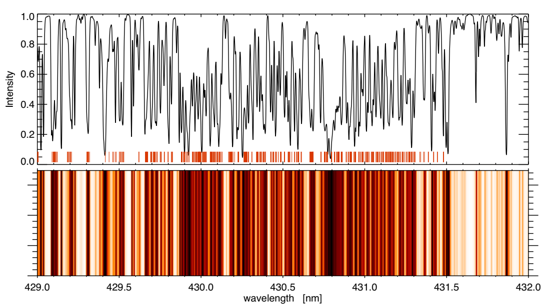
Basic Radiative Transfer: Limb Darkening



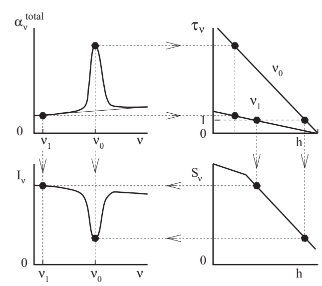


$r/R = \sin\theta$

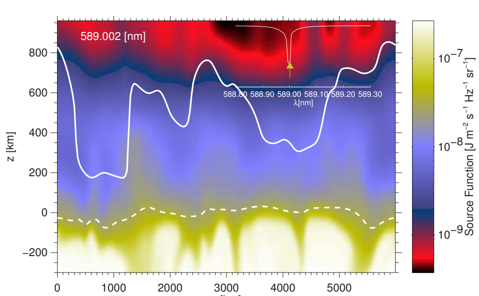
Absorption lines in the solar spectrum



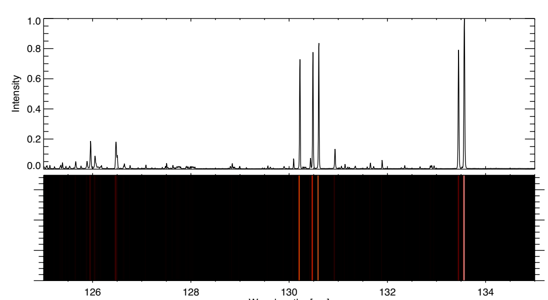
Why do we get spectral lines in absoption?



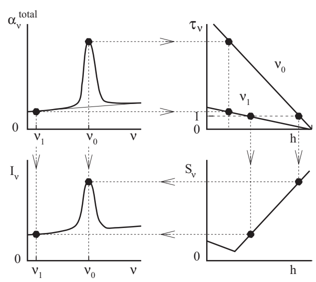
Optical depth unity in the Na I D_2 line



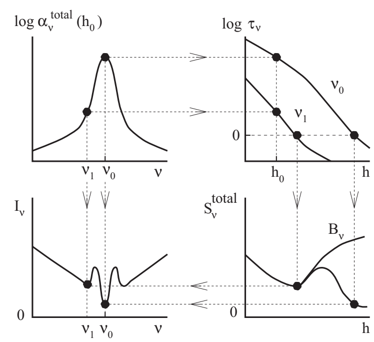
In the UltraViolet the Spectral Lines are in Emission



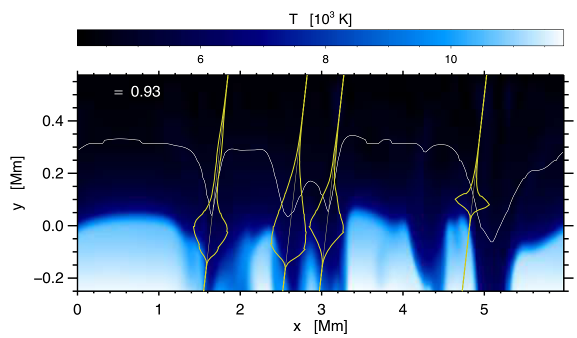
Why do we get spectral lines in emission?



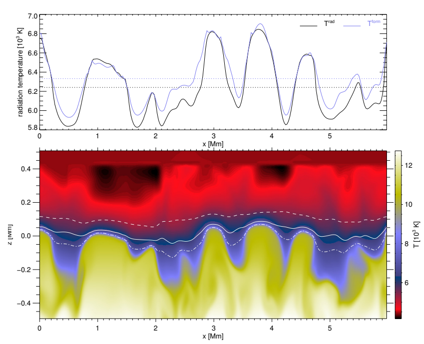
Why do we get spectral lines in emission and absorption?



Eddington–Barbier is an approximation!!



Eddington–Barbier is an approximation!!



Continuum processes

Outside spectral lines the solar plasma has significant opacity in so called continuum processes. They are called this way because their opacity varies very slowly with wavelength.

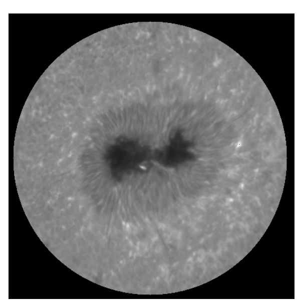
- Atomic Bound–free and free–free transitions
- H^- bound–free and free-free
- Thomson scattering

$$\alpha_e^T = N_e \sigma_e = N_e \frac{8\pi}{3 m_e^4 c^2} \frac{q_e^4}{(4\pi\epsilon_0)^2}$$

- Rayleigh scattering

$$\alpha^R(\omega) = \sigma_e f_{ij} \omega^4 / (\omega_{ij}^2 - \omega^2)^2$$

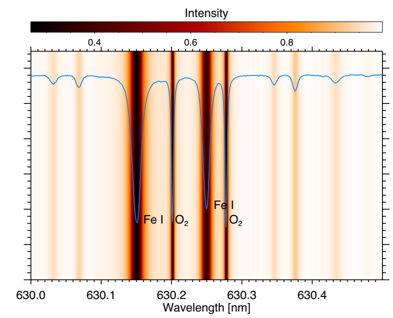
There is a lot of information in spectral lines



Uitenbroek & Tritschler, IBIS DST

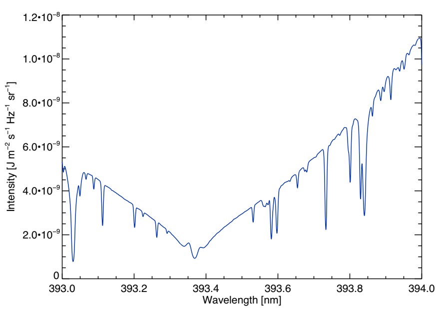
End Part I

Molecular Oxygen in the Earth Atmosphere



Back

Differences in spectral lines



Back

Invariance of Specific Intensity along Rays

Specific Intensity has been defined in such a way as to be independent of the source and the observer.

$$dE_\lambda = I_\lambda \cos\theta \, dt \, dA \, d\lambda \, d\Omega = I'_\lambda \cos\theta' \, dt \, dA' \, d\lambda \, d\Omega'$$

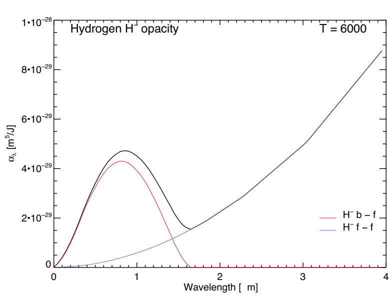
$$d\Omega = dA' \cos\theta' / R^2$$

$$d\Omega' = dA \cos\theta / R^2$$

$$I_\lambda = I'_\lambda$$

Back

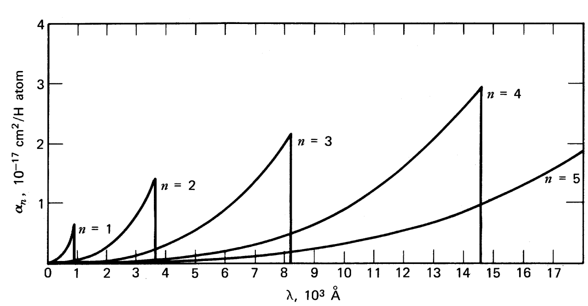
H^- Opacity



$\Delta E^{\mathrm{bf}} = 0.754$ eV

Back

Bound–Free Cross Sections of Hydrogen



Back