All the details about inversions you thought you don't have to know

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Summary

- What is our model, what is our data, and how are they connected?
- Technical execution
- Main problems and limitations
- Cross talk between the parameters
- Is there hope?

What is 'inversion'?

Fitting the data (Stokes spectrum) with a model (model of the atmosphere + atomic physics).

Atomic physics is considered given (unless you want to fit abundances too).

We minimize a merit function, most often it is χ^2:

$$\chi^2 = \sum_{s}^{4} \sum_{i}^{N_\lambda} (I_{s,i}^{\mathrm{obs}} - I_{s,i}^{\mathrm{calc}})^2 / \sigma_{s,i}^2$$

Model → data

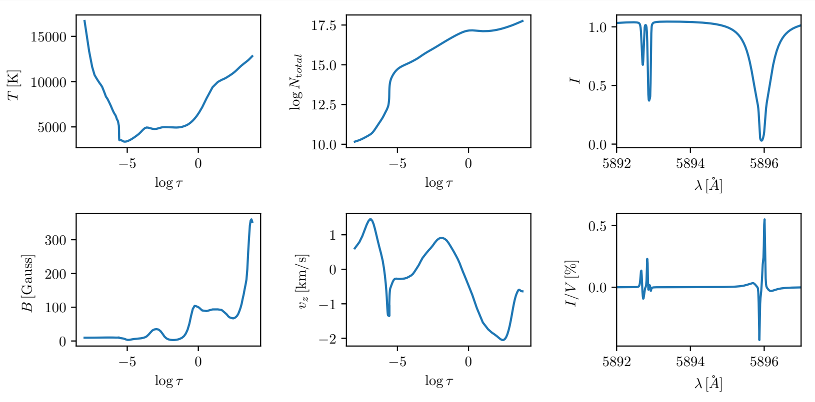

Figure 1: Temperature, particle density, magnetic field and velocity (left,middle) and the resulting spectra and the circular polarization (right)

Data → model

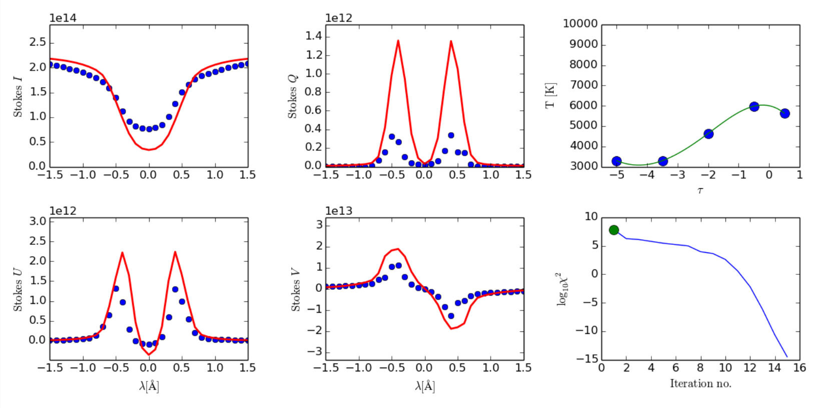

Figure 2: An example of a fitting procedure. The atmosphere is adjusted until the fit between observed (blue) and fitted (red) profiles is achieved.

Cool story, how does it look in practice?

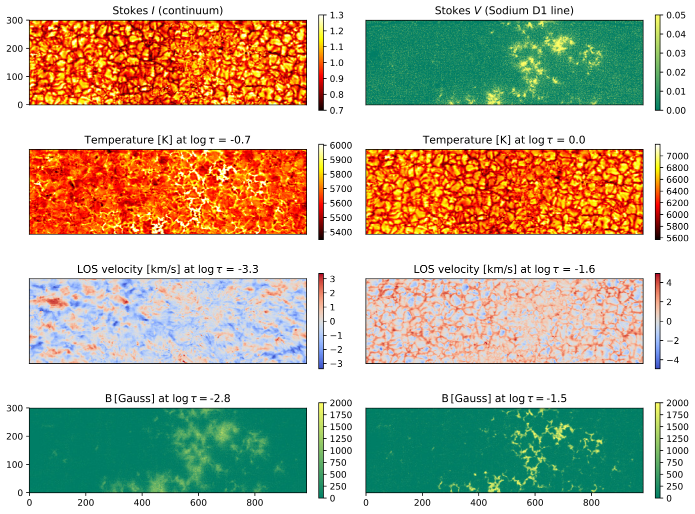

Figure 3: Spatial distributions of the observables (top two panels), and inferred model parameters (bottom six panels)

A simple LTE line formation

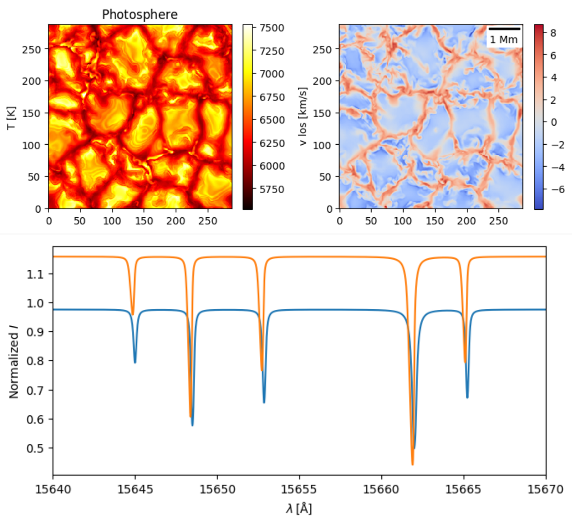

Figure 4: Fe lines at $1.56\mu\mathrm{m}$ formed in a MURAM model atmosphere

A simple LTE line formation

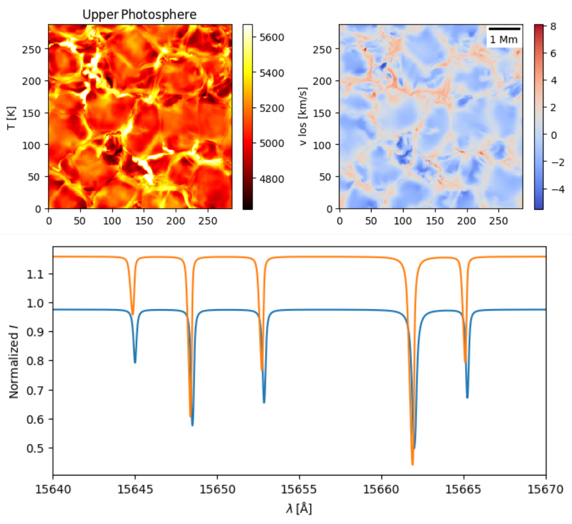

Figure 5: Fe lines at $1.56\mu\mathrm{m}$ formed in a MURAM model atmosphere

The spectra captures the depth dependence through RTE

Simple, scalar form:

$$\frac{dI(z,\lambda)}{dz} = -\chi(z,\lambda)I(z,\lambda) + \eta(z,\lambda).$$

Easier to treat in this form:

$$\frac{dI_\lambda(\tau_\lambda)}{d\tau_\lambda} = I_\lambda(\tau_\lambda) - S_\lambda(\tau_\lambda).$$

And the so called formal solution is:

$$I_\lambda^+ = \int_0^\infty S(\tau_\lambda)e^{-\tau_\lambda}d\tau_\lambda.$$

This is why we call it 'inversion'.

Let's make a simple inversion code together

Let's describe how our line opacity looks like:

$$\tau_\lambda = \tau \times (1 + r\phi_\lambda)$$

Assume that we know exact values of r and ϕ_λ, and that S is independent of λ. Now, numerically:

$$I^+_{\lambda_i} = \sum_j^{ND} w_{i,j} S_j$$

$w_{i,j}$ follow from the numerical solution of RTE. Let's say we know them. We now have a line formation model, linear in S.

Generative model:

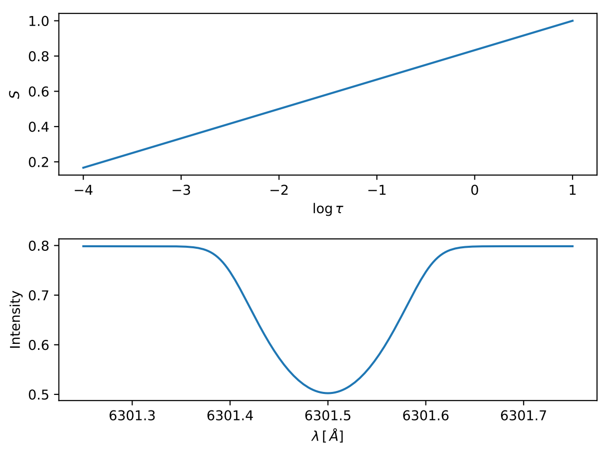

Figure 6: Toy model for spectral line formation. Decreasing source function toward the surface results in an absorption line.

Inversion

Now, let's try to retreive the original run of the Source function from this 'observed' spectra (this back-and-forth shenanigan is a common thing).

$$I_i = \sum_j w_{ij} S_j$$

$$\boldsymbol{S} = \hat{w}^{-1} \boldsymbol{I}$$

We have ≈ 200 wavelengths, to retreive ≈ 50 depths, how hard can this be?

It’s not hard, it’s impossible

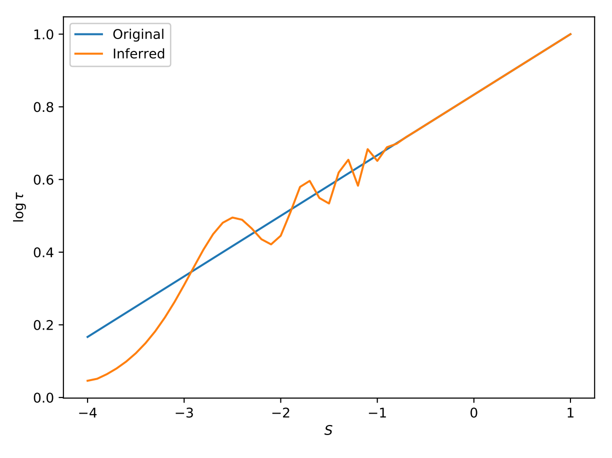

Figure 7: The original and the inferred run of the source function, under the assumption that each depth point is a free parameter

How do inversions work in the 'real life'

Generative model is more physically realistic and more complicated:

$$f\left(T(z), p_{\mathrm{gas}}(z), v_z(z), \vec{B}(z)\right) = \boldsymbol{I}(\lambda)$$

$$\boldsymbol{I} = (I, Q, U, V)$$

We are hopefully able to calculate response functions (recall Han's talk):

$$\mathcal{R}_{i,s,j} = \frac{\partial I_{i,s}}{\partial q_j}$$

Then a proposed correction follows from:

$$\hat{\mathcal{R}}\,\boldsymbol{\delta q} = \boldsymbol{I}^{\mathrm{obs}} - \boldsymbol{I}^{\mathrm{calc}}$$

Problems

- A lot of unknowns (j runs over all atmosphere)
- Different parameters do the same things
- Noise in the observations (and systematics!)
- Local minima

A lot of unknowns → nodes

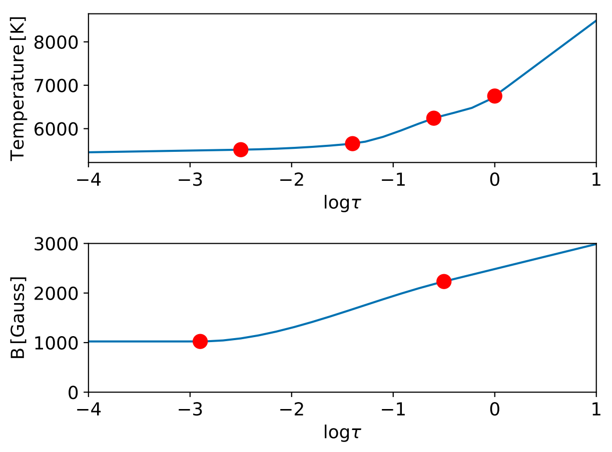

Figure 8: Nodes (red circles), are free parameters, points in between are interpolated

The nodes

We decreased the number of unknowns. More importantly we decreased their "degeneracy".

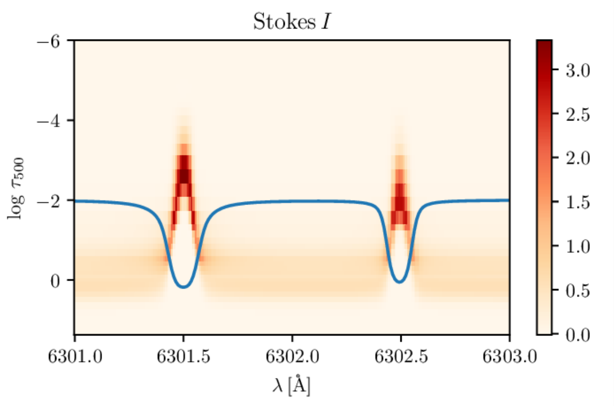

Figure 9: Response function of 6300 Å line pair to temperature.

Reminder on what inversions do:

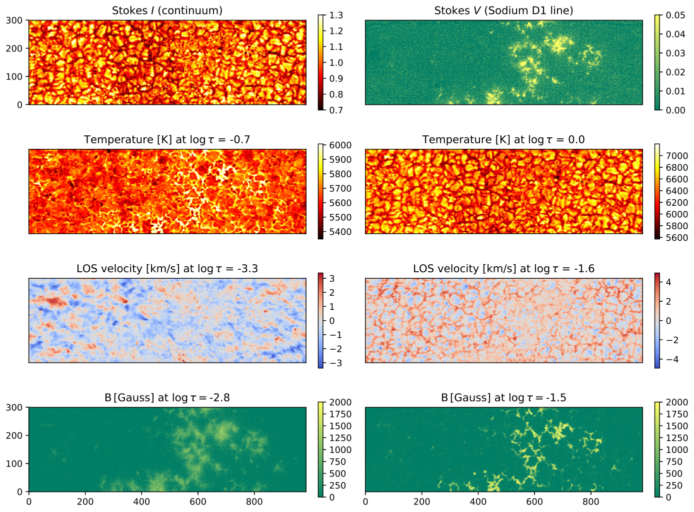

Figure 10: Spatial distributions of the observables (top two panels), and model parameters (bottom six panels)

These are great results!

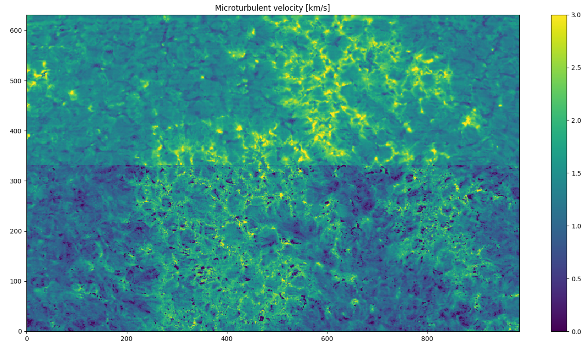

Figure 11: Map of microturbulent velocity for two runs with different initial conditions

Subtle but visible consequences

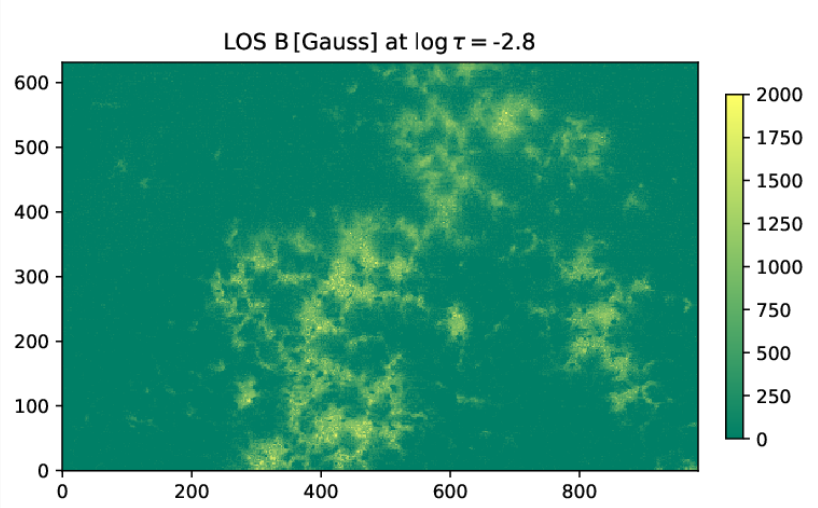

Figure 12: Vertical magnetic field map

This is not the first time people pointed this out

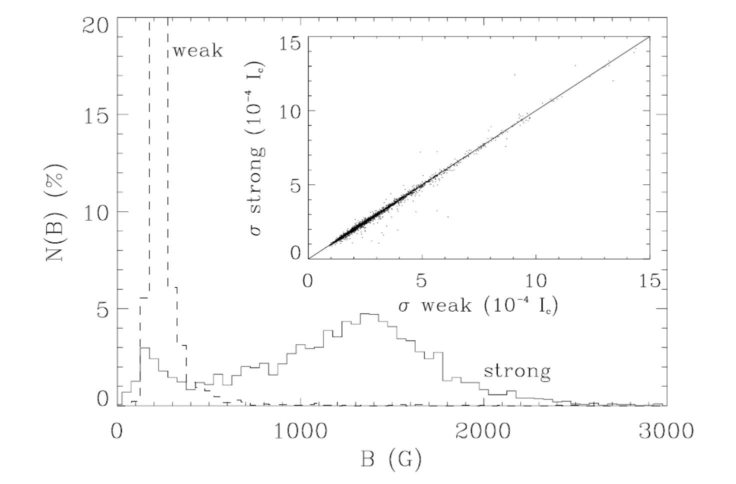

Figure 13: Magnetic field histograms from two different initializations (Martinez Gonzalez et al. 2006).

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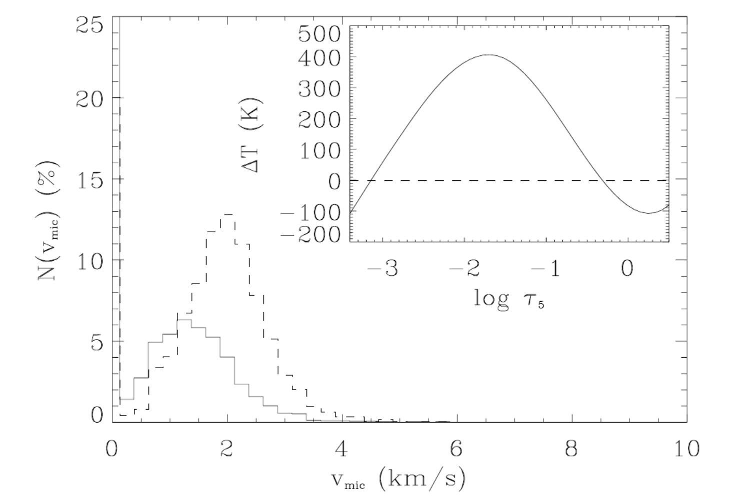

Figure 14: Microturbulent velocity histograms from two different initializations (Martinez Gonzalez et al. 2006).

What is going on here?

Parameters are degenerate. B, v_{turb} and T all broaden the line. How to decipher this?

(In principle, los velocity variations can also broaden the line!)

It is hard, we need a lot of lines with different sensitivities.

This is done surprisingly rarely, do this with DKIST please!

Conclusions

Inversion is an ill-posed problem.

Number of actual observables is much smaller than the number of points in λ (Roberto's talk).

There are multiple solutions and it is not clear how to discriminate between the models (read papers by Andrés!)

They are also time demanding, but that is a story for the beer afterward ;)