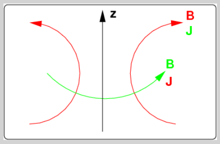
α^2-dynamo



Induction of field parallel to current (producing helical field!)

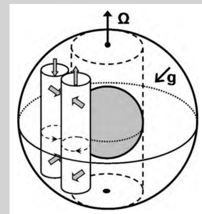
$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times (\alpha \overline{\boldsymbol{B}}) = \alpha \mu_0 \overline{\boldsymbol{j}}$$

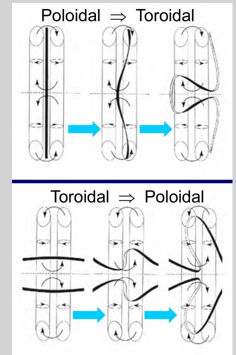
Dynamo cycle:

$$\boldsymbol{B}_t \stackrel{\alpha}{\longrightarrow} \boldsymbol{B}_p \stackrel{\alpha}{\longrightarrow} \boldsymbol{B}_t$$

- Poloidal and toroidal field of similar strength
- In general stationary solutions

α^2 model for Geodynamo

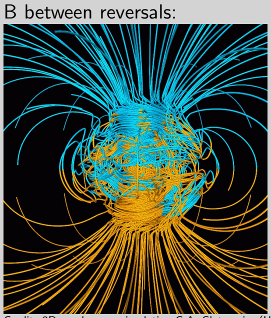




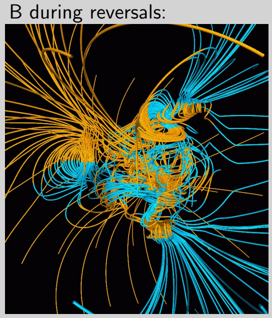
- Strong influence of rotation
  $\tau_{rot} = 1$ day, $\tau_c \sim 1000$ years
  (Sun: $\tau_{rot} = 27$ days, $\tau_c \sim$ weeks)
- Flow organization: Taylor columns
- Secondary flow along columns
  (boundary effect) $\rightarrow$ helicity

α^2 model for Geodynamo

B between reversals:

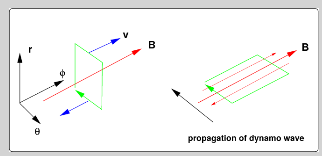


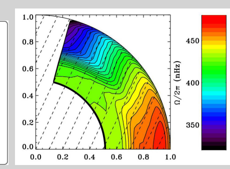
B during reversals:



Credit: 3D geodynamo simulation G.A. Glatzmaier (UCSC)

$\alpha\Omega$-, $\alpha^2\Omega$-dynamo





Dynamo cycle:

$$\boldsymbol{B}_t \stackrel{\alpha}{\longrightarrow} \boldsymbol{B}_p \stackrel{\Omega,\,\alpha}{\longrightarrow} \boldsymbol{B}_t$$

- Toroidal field much stronger that poloidal field
- In general traveling (along lines of constant Ω) and periodic solutions

Movie α-effect Movie Ω-effect

$\alpha\Omega$-dynamo

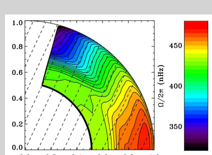
$$\frac{\partial B}{\partial t} = r \sin \boldsymbol{B}_p \cdot \boldsymbol{\nabla}\Omega + \eta_t \left(\Delta - \frac{1}{(r\sin\theta)^2}\right) B$$
$$\frac{\partial A}{\partial t} = \alpha B + \eta_t \left(\Delta - \frac{1}{(r\sin\theta)^2}\right) A$$

- Cyclic behavior:

$$P \propto (\alpha |\boldsymbol{\nabla}\Omega|)^{-1/2}$$

- Propagation of magnetic field along contourlines of Ω "dynamo-wave"
- Direction of propagation "Parker-Yoshimura-Rule":

$$\boldsymbol{s} = \alpha \boldsymbol{\nabla}\Omega \times \boldsymbol{e}_\phi$$



Movie: $\alpha\Omega$-dynamo

$\alpha\Omega$-dynamo with meridional flow

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right) = r \sin \boldsymbol{B}_p \cdot \boldsymbol{\nabla}\Omega + \eta_t \left(\Delta - \frac{1}{(r\sin\theta)^2}\right) B$$
$$\frac{\partial A}{\partial t} + \frac{1}{r\sin\theta} \boldsymbol{v}_p \cdot \boldsymbol{\nabla}(r\sin\theta A) = \alpha B + \eta_t \left(\Delta - \frac{1}{(r\sin\theta)^2}\right) A$$

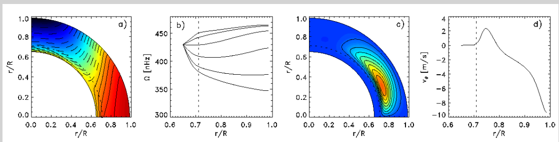
- If η_t is sufficiently small, such that:

$$\tau_d = D_{CZ}^2/\eta_t > D_{CZ}/v_m \longrightarrow \eta_t < v_m D_{CZ}$$

  the meridional flow v_m can control the cycle period and propagation of the magnetic activity
- Additional advection like effects can arise from the γ-effect, they can be accounted for by formally substituting:

$$\boldsymbol{v}_m \longrightarrow \boldsymbol{v}_m + \gamma$$

$\alpha\Omega$-dynamo with meridional flow



Meridional flow:

- Poleward at top of convection zone
- Equatorward at bottom of convection zone

Effect of advection:

- Equatorward propagation of activity
- Correct phase relation between poloidal and toroidal field
- Circulation time scale of flow sets dynamo period
- Requirement: Sufficiently low turbulent diffusivity

Movie: Flux-transport-dynamo (M. Dikpati, HAO)

$\Omega \times J$ dynamo

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \nabla \times [\delta \times (\nabla \times \overline{\boldsymbol{B}})] \sim \nabla \times (\Omega \times \overline{\boldsymbol{j}}) \sim \frac{\partial \overline{\boldsymbol{j}}}{\partial z}$$

- similar to α-effect, but additional z-derivative of current
- couples poloidal and toroidal field
- δ^2 dynamo is not possible:

$$\overline{\boldsymbol{j}} \cdot \overline{\mathcal{E}} = \overline{\boldsymbol{j}} \cdot (\boldsymbol{\delta} \times \overline{\boldsymbol{j}}) = 0$$

- δ-effect is controversial (not all approximations give a non-zero effect)
- in most situations α dominates

Dynamos and magnetic helicity

Magnetic helicity (integral measure of field topology):

$$H_m = \int \boldsymbol{A} \cdot \boldsymbol{B} \, dV$$

has following conservation law (no helicity fluxes across boundaries):

$$\frac{d}{dt} \int \boldsymbol{A} \cdot \boldsymbol{B} \, dV = -2\mu_0 \, \eta \int \boldsymbol{j} \cdot \boldsymbol{B} \, dV$$

Decomposition into contributions from small and large scale magnetic field:

$$\begin{aligned} \frac{d}{dt} \int \overline{\boldsymbol{A}} \cdot \overline{\boldsymbol{B}} \, dV &= +2 \int \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} \, dV - 2\mu_0 \, \eta \int \overline{\boldsymbol{j}} \cdot \overline{\boldsymbol{B}} \, dV \\ \frac{d}{dt} \int \overline{\boldsymbol{A}' \cdot \boldsymbol{B}'} \, dV &= -2 \int \overline{\boldsymbol{\mathcal{E}}} \cdot \overline{\boldsymbol{B}} \, dV - 2\mu_0 \, \eta \int \overline{\boldsymbol{j}' \cdot \boldsymbol{B}'} \, dV \end{aligned}$$

Dynamos and magnetic helicity

Dynamos have helical fields:

- α-effect induces magnetic helicity of same sign on large scale
- α-effect induces magnetic helicity of opposite sign on small scale

Asymptotic staturation

$$\begin{aligned} \overline{\boldsymbol{j}' \cdot \boldsymbol{B}'} &= -\overline{\boldsymbol{j}} \cdot \overline{\boldsymbol{B}} \longrightarrow \frac{|\overline{B}|}{|B'|} \sim \sqrt{\frac{L}{l_c}} \\ \overline{\boldsymbol{j}' \cdot \boldsymbol{B}'} &= -\frac{\alpha \overline{\boldsymbol{B}}^2}{\mu_0 \eta} + \frac{\eta_t}{\eta} \overline{\boldsymbol{j}} \cdot \overline{\boldsymbol{B}} \end{aligned}$$

Non-kinematic effects

Proper way to treat them: 3D simulations

- Still very challenging
- Has been successful for geodynamo, but not for solar dynamo

Semi-analytical treatment of Lorentz-force feedback in mean field models:

- Macroscopic feedback: Change of the mean flow (differential rotation, meridional flow) through the mean Lorentz-force

$$\overline{\boldsymbol{f}} = \overline{\boldsymbol{j}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{j}' \times \boldsymbol{B}'}$$

- Mean field model including mean field representation of full MHD equations:
  Movie: Non-kinematic flux-transport dynamo
- Microscopic feedback: Change of turbulent induction effects (e.g. α-quenching)

Microscopic feedback

Feedback of Lorentz force on small scale motions:

- Intensity of turbulent motions significantly reduced if $\frac{1}{2\mu_0}B^2 > \frac{1}{2}\varrho v_{rms}^2$. Typical expression used

$$\alpha = \frac{\alpha_k}{1 + \frac{\overline{\boldsymbol{B}}^2}{B_{eq}^2}}$$

  with the equipartition field strength $B_{eq} = \sqrt{\mu_0 \varrho} v_{rms}$
- Similar quenching also expected for turbulent diffusivity
- Additional quenching of α due to topological constraints possible (helicity conservation)
  Controversial !

Microscopic feedback

Symmetry of momentum and induction equation $\boldsymbol{v}' \leftrightarrow \boldsymbol{B}'/\sqrt{\mu_0\varrho}$:

$$\begin{aligned}
\frac{d\boldsymbol{v}'}{dt} &= \frac{1}{\mu_0\varrho}(\overline{\boldsymbol{B}}\cdot\nabla)\boldsymbol{B}' + \dots \\
\frac{d\boldsymbol{B}'}{dt} &= (\overline{\boldsymbol{B}}\cdot\nabla)\boldsymbol{v}' + \dots \\
\overline{\boldsymbol{\mathcal{E}}} &= \overline{\boldsymbol{v}'\times\boldsymbol{B}'}
\end{aligned}$$

Strongly motivates magnetic term for α-effect (Pouquet et al. 1976):

$$\alpha = \frac{1}{3}\tau_c\left(\frac{1}{\varrho}\overline{\boldsymbol{j}'\cdot\boldsymbol{B}'} - \overline{\boldsymbol{\omega}'\cdot\boldsymbol{v}'}\right)$$

- Kinetic α: $\overline{\boldsymbol{B}} + \boldsymbol{v}' \longrightarrow \boldsymbol{B}' \longrightarrow \overline{\boldsymbol{\mathcal{E}}}$
- Magnetic α: $\overline{\boldsymbol{B}} + \boldsymbol{B}' \longrightarrow \boldsymbol{v}' \longrightarrow \overline{\boldsymbol{\mathcal{E}}}$

Microscopic feedback

From helicity conservation one expects

$$\overline{\boldsymbol{j}'\cdot\boldsymbol{B}'} \sim -\alpha\overline{\boldsymbol{B}}^2$$

leading to algebraic quenching

$$\alpha = \frac{\alpha_k}{1 + g\,\frac{\overline{\boldsymbol{B}}^2}{B_{eq}^2}}$$

With the asymptotic expression (steady state)

$$\overline{\boldsymbol{j}'\cdot\boldsymbol{B}'} = -\frac{\alpha\overline{\boldsymbol{B}}^2}{\mu_0\eta} + \frac{\eta_t}{\eta}\overline{\boldsymbol{j}}\cdot\overline{\boldsymbol{B}}$$

we get

$$\alpha = \frac{\alpha_k + \frac{\eta_t^2}{\eta}\frac{\mu_0\overline{\boldsymbol{j}}\cdot\overline{\boldsymbol{B}}}{B_{eq}^2}}{1 + \frac{\eta_t}{\eta}\frac{\overline{\boldsymbol{B}}^2}{B_{eq}^2}}$$

Microscopic feedback

Catastrophic α-quenching ($R_m \gg 1$!) in case of steady state and homogeneous $\overline{\boldsymbol{B}}$:

$$\alpha = \frac{\alpha_k}{1 + R_m\frac{\overline{\boldsymbol{B}}^2}{B_{eq}^2}}$$

If $\overline{\boldsymbol{j}}\cdot\overline{\boldsymbol{B}} \neq 0$ (dynamo generated field) and η_t unquenched:

$$\alpha \approx \eta_t\,\mu_0\frac{\overline{\boldsymbol{j}}\cdot\overline{\boldsymbol{B}}}{\overline{\boldsymbol{B}}^2} \sim \frac{\eta_t}{L} \sim \frac{\eta_t}{l_c}\frac{l_c}{L} \sim \alpha_k\frac{l_c}{L}$$

- In general α-quenching dynamic process: linked to time evolution of helicity
- Boundary conditions matter: Loss of small scale current helicity can alleviate catastrophic quenching
- Catastrophic α-quenching turns large scale dynamo into slow dynamo

3D simulations

Why not just solving the full system to account for all non-linear effects?

- Most systems have $R_e \gg R_m \gg 1$, requiring high resolution
- Large scale dynamos evolve on time scales $\tau_c \ll t \ll \tau_\eta$, requiring long runs compared to convective turn over
- 3D simulations successful for geodynamo
  - $R_m \sim 300$: all relevant magnetic scales resolvable
  - Incompressible system
- Solar dynamo: Ingredients can be simulated
  - Compressible system: density changes by 10^6 through convection zone
  - Boundary layer effects: Tachocline, difficult to simulate (strongly subadiabatic stratification, large time scales)
  - How much resolution required? (CZ about $\sim 10^9$ Mm^3, 1 Mm resolution $\sim 1000^3$ numerical problem)
  - Small scale dynamos can be simulated (for $P_m \sim 1$)

Where did the "first" magnetic field come from?

Meanfield induction equation linear in $\overline{\boldsymbol{B}}$: possible solution.

$$\frac{\partial\overline{\boldsymbol{B}}}{\partial t} = \nabla\times\left[\alpha\overline{\boldsymbol{B}} + (\overline{\boldsymbol{v}}+\boldsymbol{\gamma})\times\overline{\boldsymbol{B}} - (\eta+\eta_t)\nabla\times\overline{\boldsymbol{B}}\right]$$

$\overline{\boldsymbol{B}} = 0$ is always a valid solution!
Generalized Ohm's law with electron pressure term:

$$\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B} + \frac{1}{\sigma}\boldsymbol{j} - \frac{1}{\varrho_e}\nabla p_e\,.$$

leads to induction equation with inhomogeneous source term "Biermann Battery":

$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B} - \eta\nabla\times\boldsymbol{B}) + \frac{1}{\varrho_e^2}\nabla\varrho_e\times\nabla p_e\,.$$

Where did the "first" magnetic field come from?

Early universe:

- Ionization fronts from point sources (quasars) driven through an inhomogeneous medium: $1/\varrho_e^2\nabla\varrho_e\times\nabla p_e$ can lead to about $10^{-23}G$
- Collapse of intergalactic medium to form galaxies leads to 10^{-20} G
- Galactic dynamo (growth rate $\sim 3Gy^{-1}$) leads to 10^{-6} G after 10 Gy (today)

Source term is working all the time

- $\nabla\varrho_e\times\nabla p_e/\varrho^2$ at edge of solar granules induces field of about 10^{-6} G (Khomenko et al. 2017)

Summarizing remarks

Destruction of magnetic field:

- Turbulent diffusivity: cascade of magnetic energy from large scale to dissipation scale (advection+reconnection)
- Enhances dissipation of large field by a factor R_m

Creation of magnetic field:

- Small scale dynamo (non-helical)
  - Amplification of field at and below energy carrying scale of turbulence
  - Stretch-twist-fold-(reconnect)
  - Produces non-helical field and does not require helical motions
  - Controversy: behavior for $P_m \ll 1$
- Large scale dynamo (helical)
  - Amplification of field on scales larger than scale of turbulence
  - Produces helical field and does require helical motions
  - Requires rotation + additional symmetry direction (controversial $\Omega \times J$ effect does not require helical motions)
  - Controversy: catastrophic vs. non-catastrophic quenching