Solar Focus Meeting: Spectro-Polarimetric Inversions

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Program for Today

Solar Focus on Spectral Inversions:

1:30 – 2:00 Introduction – Han Uitenbroek (NSO)

2:00 – 2:30 All the details about inversions you thought you don't have to know – Ivan Milic (CU/LASP, NSO)

2:30 – 3:00 Break

3:00 – 3:15 HAZEL Inversion of the Filament Observed by the DST/FIRS on May 29/30, 2017 – Shuo Wang (NSMU, via Zoom)

3:15 – 3:45 Stokes Inversion via Principal Component Analysis - Roberto Casini (HAO)

3:45 – 4:00 Wrap-Up – Han Uitenbroek (NSO)

4:00 – 4:30 Socializing

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- The properties of the atmospehere are mapped into the (polarized) spectrum and Spectral Inversions seek to uncover the reverse of this mapping in a mathematically rigorous way.

The Equations of Radiative Transfer

Absorption and emission coefficient:

$$\mathrm{d}I_\lambda = j_\lambda \mathrm{d}s$$
$$\mathrm{d}I_\lambda = -\alpha_\lambda I_\lambda \mathrm{d}s$$

Transport along a ray:

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}s} = j_\lambda - \alpha_\lambda I_\lambda$$
$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = I_\lambda - S_\lambda$$

Optical depth and source function:

$$\mathrm{d}\tau_\lambda \equiv -\alpha_\lambda \mathrm{d}s$$
$$S_\lambda \equiv j_\lambda / \alpha_\lambda$$

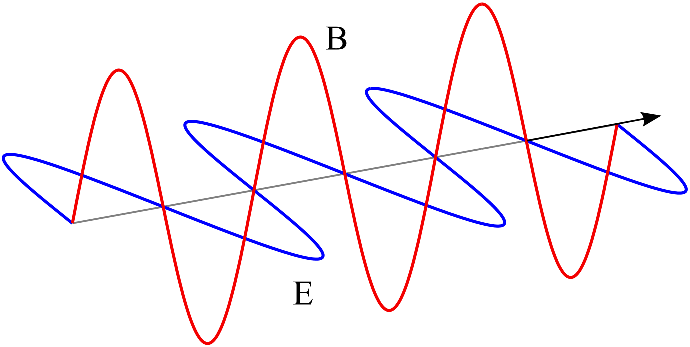
Basic Radiative Transfer: Formal Solution

Emergent intensity from a semi-infinite medium:

$$\frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_\lambda} = I_\lambda - S_\lambda$$

$$I_\lambda(\tau_\lambda = 0) = \int_0^\infty S_\lambda(\tau) e^{-\tau} \mathrm{d}\tau$$

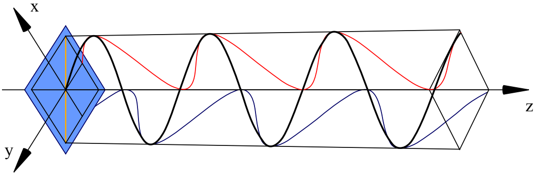
A plane electromagnetic wave



$$E(\vec{r}, t) = (A \sin(kz - \omega t), 0, 0)$$

General description of polarized light

$$E(\vec{r}, t) = (A_x \cos(kz - \omega t), A_y \cos(kz - \omega t + \phi), 0)$$



Linear Polarization:

$$A_x = A_y$$
$$\phi = 0$$

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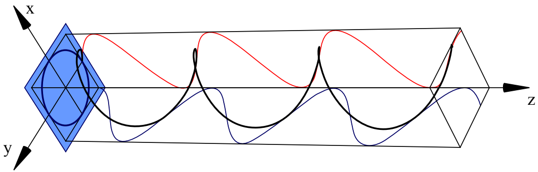
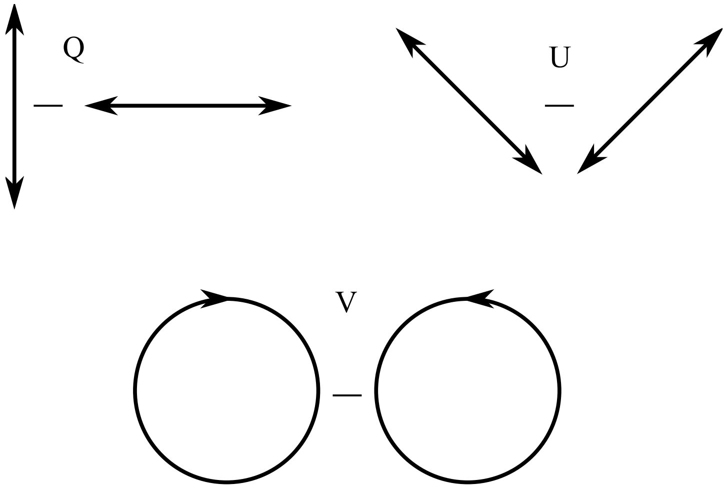


Circular Polarization:

$$A_x = A_y$$
$$\phi = 90$$

Stokes parameters



Equation of Polarized Radiative Transfer

Transfer Equation:

$$\frac{\mathrm{d}\mathbf{I}}{\mathrm{d}s} = -\mathbf{K}\mathbf{I} + \mathbf{j}$$

Stokes vector and absorption matrix

$$\mathbf{I} = [I, Q, U, V]^T$$
$$\mathbf{j} = j_\lambda \mathbf{\Phi}_\lambda \mathbf{e}_0, \quad \mathbf{e}_0 = (1,0,0,0)^\dagger$$
$$\mathbf{K} = \alpha_\lambda \mathbf{\Phi}_\lambda$$

Absorption matrix:

$$\mathbf{\Phi} = \begin{pmatrix} \phi_I & \phi_Q & \phi_U & \phi_V \\ \phi_Q & \phi_I & \psi_V & -\psi_U \\ \phi_U & -\psi_V & \phi_I & \psi_Q \\ \phi_V & \psi_U & -\psi_Q & \phi_I \end{pmatrix}$$

Dependence of Transfer Equattion on Physical Properties

Physical properties, absorption and emission coefficient

$$\bar{a} = [a_1(\tau), a_2(\tau), \ldots, a_N(\tau)]^T$$
$$\alpha_\lambda = \alpha(\lambda; \bar{a})$$
$$j_\lambda = j(\lambda; \bar{a})$$

Equation of transfer and formal solution

$$\frac{\mathrm{d}I}{\mathrm{d}\tau} = I(\lambda; \bar{a}) - S(\lambda; \bar{a})$$
$$I(\lambda; \bar{a}) = \int_0^\infty S(\lambda; \bar{a}) e^{-\tau(\lambda; \bar{a})} \mathrm{d}\tau$$

Response to Perturbations of Physical Properties

Let's investigate what happens to the intensity $I(\lambda; \overline{a})$ when we perturb the physical quantities $\overline{a}$ by writing down the partial derivative of I with respect to single quantity a_j.

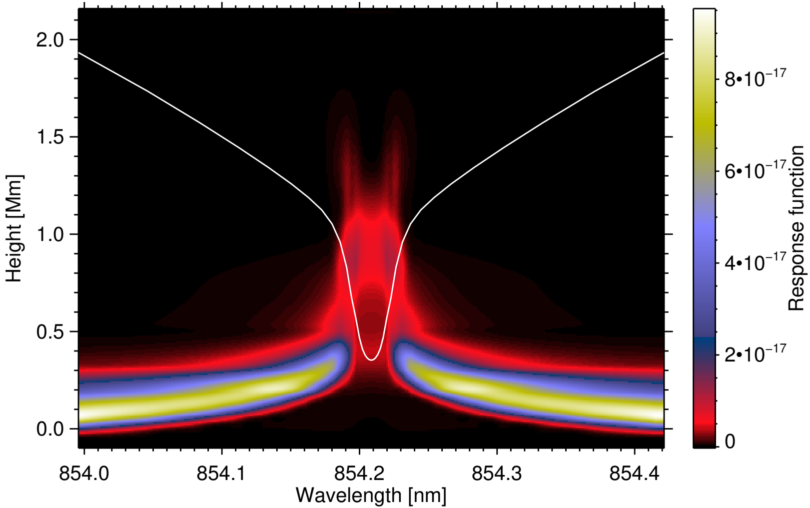
$$\mathrm{d}\frac{\partial I}{\partial a_j}/\mathrm{d}s = \frac{\partial j}{\partial a_j} - \left(\frac{\partial \alpha}{\partial a_j} I + \alpha \frac{\partial I}{\partial a_j} \right)$$

$$\mathrm{d}\frac{\partial I}{\partial a_j}/\mathrm{d}\tau = \frac{\partial I}{\partial a_j} - \frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial a_j} I - \frac{\partial j}{\partial a_j} \right)$$

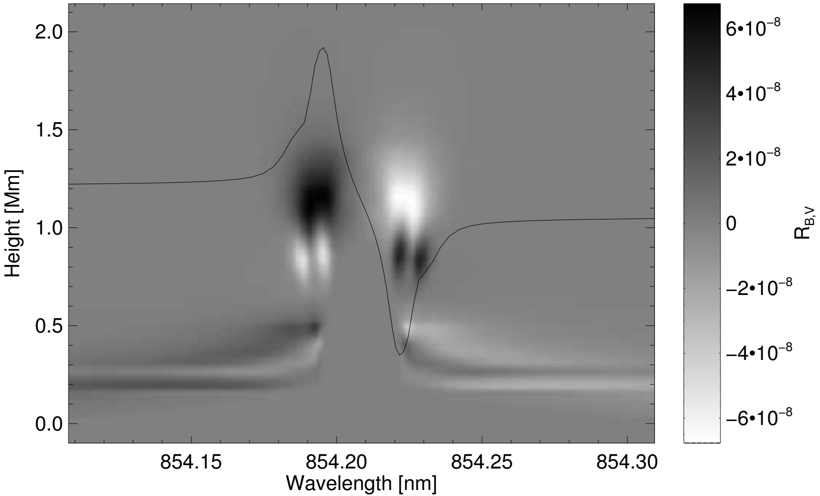
$$\delta I = \sum_1^N \frac{\partial I}{\partial a_j} \delta a_j = \sum_1^N \int_0^\infty \left\{ \frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial a_j} I - \frac{\partial j}{\partial a_j} \right) \right\} e^{-\tau} \delta a_j \mathrm{d}\tau$$

$$= \sum_1^N \int_0^\infty R_j \delta a_j \mathrm{d}\tau$$

Response Function of Ca II 854.21 nm to Perturbation in T



Response function Ca II 854.2 Stokes V to B



The Merit Function

Comparing the synthetic spectrum from a given estimate of the atmosphere with the observed:

$$\chi^2 = \frac{1}{N_f - N_u} \sum_{i=1}^{M} \left[I^{\mathrm{obs}}(\lambda_i) - I^{\mathrm{synth}}(\lambda_i; \overline{a}) \right]^2$$

$$\delta\chi^2 = \frac{2}{N_f - N_u} \sum_{i=1}^{M} \left[I^{\mathrm{obs}}(\lambda_i) - I^{\mathrm{synth}}(\lambda_i; \overline{a}) \right] \delta I_i$$

$$= \frac{2}{N_f - N_u} \sum_{i=1}^{M} \left\{ \left[I^{\mathrm{obs}}(\lambda_i) - I^{\mathrm{synth}}(\lambda_i; \overline{a}) \right] \sum_{1}^{N} \int_0^\infty R_j(\lambda_i; \overline{a}) \delta a_j \mathrm{d}\tau \right\}$$

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6. Do to step 2 and iterate.