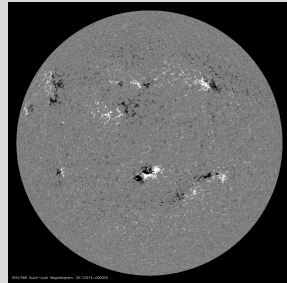
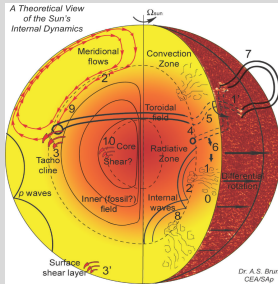


# PHYS 7810 Special Topics in Physics: Physics of the Solar Atmosphere

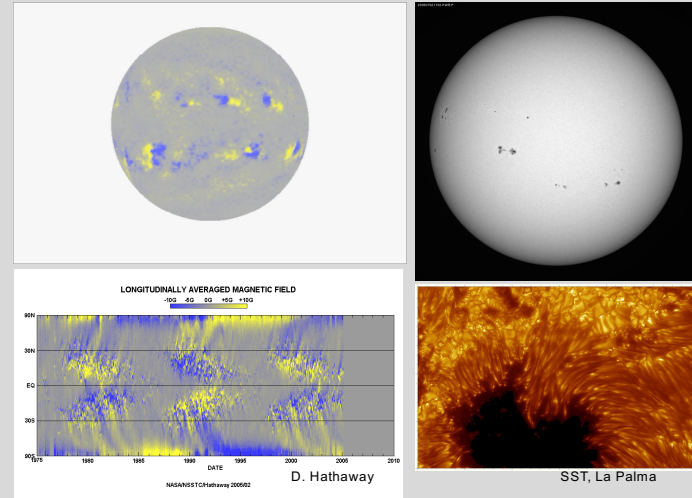


Matthias Rempel

Lecture 5 Tues Jan 29 2019

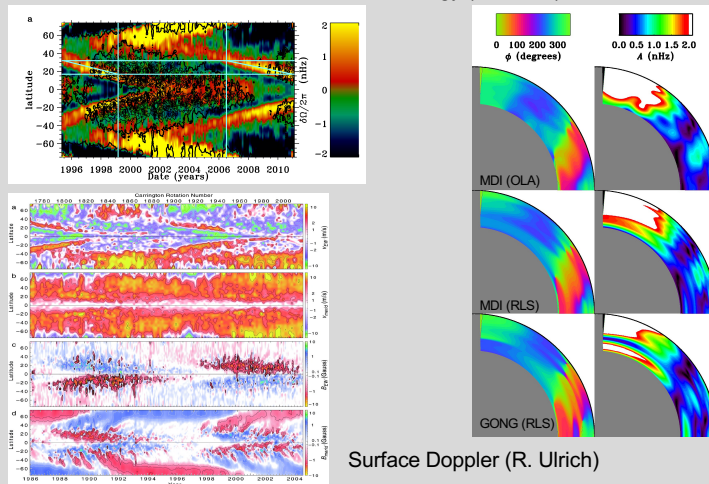
<https://www.nso.edu/students/collage/collage-2019/>

## Solar magnetic field



## Large scale flow variations

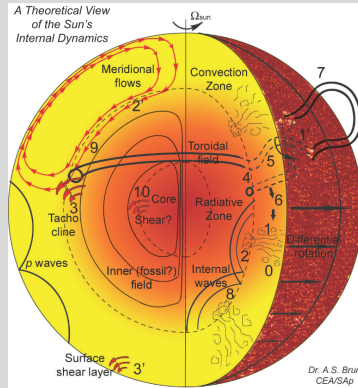
Global Helioseismology (R. Howe)



## Solar dynamo models – what is the goal?

- What is a solar dynamo model supposed to do?
  - 1) Show a “solar-like” activity pattern in terms of:
    - Cyclic behavior with equator-ward propagation of activity
    - Surface flux evolution consistent with observations
    - Large scale flow variations consistent with observations
  - 2) Show a “solar-like” amplitude variation from cycle to cycle
  - 3) Allow prediction of future activity
- Most models struggle already with point 1)
  - Focus this lecture on 1)
  - 2) and 3) can provide additional constraints on dynamo models

## The basic dynamo ingredients



- Large-scale flows
  - Differential rotation
  - Meridional flow
  - Mean and (cyclic) variation
- Turbulent induction
  - Transport
    - Advective
    - Diffusive
  - $\alpha$ -Effects
    - Key terms that enable dynamo action
- Flux emergence
  - Links dynamo to photospheric field observations
  - Might play role in dynamo process itself
    - Babcock-Leighton mechanism

## Numerical modeling approaches

- Meanfield models
  - Solve equations for mean flows, mean magnetic field only
  - Inexpensive, but need good model for correlations of small scale quantities (e.g. turbulent angular momentum transport), see extensive work by Rüdiger & Kitchatinov
  - Can address the full problem, but not from first principles (models have many degrees of freedom and tunable parameters)
- 3D numerical simulations
  - Solve the full set of equations (including small and large scale flows, magnetic field) from first principles
  - Very expensive:
    - Low resolution runs for long periods >10 years
    - High resolution for short periods
  - Good understanding of differential rotation, ingredients of solar dynamo, no complete model yet
- Advances in computing infrastructure shift balance toward 3D simulations, but we need both!

## Mean field models

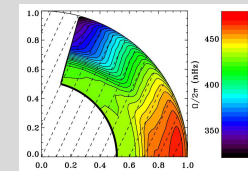
- Mean field models consider only average quantities
  - Sunspots are a key feature of the solar cycle, but they are averaged away
- Mean field models make strong assumptions that are not well justified from first principles
- Too many degrees of freedom require “educated guesses”

$$(\mathbf{v}' \times \mathbf{B}')_i = a_{ik} \bar{B}_k + b_{ijk} \frac{\partial \bar{B}_j}{\partial x_k}$$

- Contains 36!!! (mostly unknown) functions of  $r$  and  $\vartheta$ , in most models only 2 are considered and even that allows for a lot of freedom
- Computing mean field coefficients from 3D simulations (Schrinner et al. 2007, Ghizaru et al. 2011) shows that in general almost all of them are important!
- Mean field models allow us to study certain scenarios or they allow to analyze a complicated 3D simulation, but one has to be very lucky to find the “correct” model for the solar cycle without additional knowledge
- Non-linear feedback difficult to implement

## Solar dynamo models

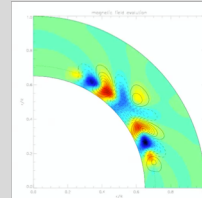
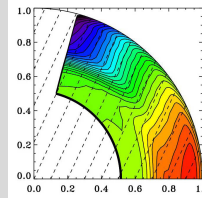
- Mean field models
  - Convection zone dynamos
  - Tachocline/interface dynamos
  - Near surface shear layer dynamos
  - Flux transport dynamos
- Main uncertainties
  - Location of dynamo
  - Poloidal field regeneration ( $B_r$ ,  $B_\vartheta$  from  $B_\phi$ :  $\alpha$ -effect)
  - Turbulent transport (magnetic pumping, turbulent diffusion vs. magnetic buoyancy)
  - Role of meridional flow (propagation of activity belt)



## Mean field dynamo

### Thin layer dynamo

- Overshoot/tachocline dynamo
  - Radial shear,  $\alpha\Omega$ -type dynamo, latitudinal propagating dynamo wave
  - Negative  $\alpha$  in northern hemisphere for equatorward propagation
- Surface shear layer?
- Main problem:
  - Typically very short latitudinal wave length (several overlapping cycles)



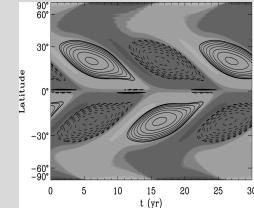
### Distributed dynamo

- Interface dynamo
  - $\Omega$ -effect in tachocline,  $\alpha$ -effect in CZ, introduced to avoid problems with strong  $\alpha$ -quenching
  - Solutions very sensitive to details

## Mean field dynamo

### Distributed dynamo

- Flux transport dynamo
  - Advective transport of field by meridional flow
  - Propagation of AR belt advection effect
  - Cycle length linked to overturning time scale of meridional flow
- Central assumption:
  - Proper meridional flow profile (mostly single flow cell poleward at top, equatorward near bottom of CZ)
  - Weak turbulent transport processes
  - Babcock-Leighton  $\alpha$ -effect
- Overall:
  - Most successful in reproducing solar like behavior



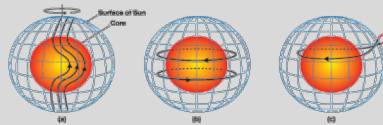
Dikpati et al. 2004

## Schematic of a Babcock-Leighton flux transport model

(Durney, Choudhuri, Schüssler, Dikpati, Nandi, Charbonneau, Gilman, Rempel, Hotta)

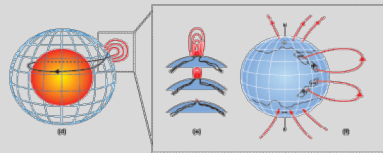
### Differential rotation

- Toroidal field production
- Stored at base of CZ
- Rising flux tubes

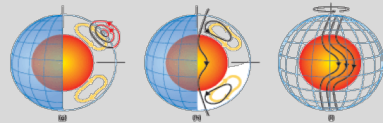


### Babcock-Leighton $\alpha$ effect

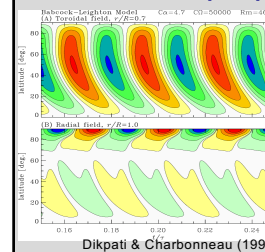
- Tilt angle of AR
- Leading spots have higher probability to reconnect across equator



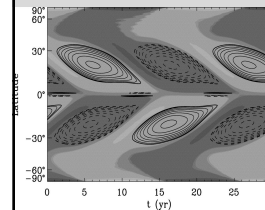
### Transport of magnetic field by meridional flow



## Solution properties flux transport dynamo



Dikpati & Charbonneau (1999)



Dikpati et al. (2004)

### Good agreement with basic cycle properties

- Equatorward propagation
- Weak cycle overlap
- Correct phase relation between poloidal and toroidal field

### Less good agreement

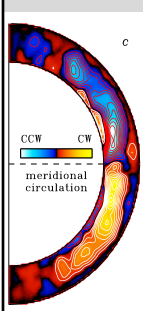
- Poleward extension of butterfly diagram?
- Polar surface field typically too strong
- Symmetry of solution (quadrupole preferred)

### More complicated ingredients can improve agreement

- Strong variation of magnetic diffusivity in CZ
- Additional  $\alpha$ -effect at base of CZ

### Expense: Strong sensitivity to many not well known ingredients

## Meridional flow structure, assumptions flux transport dynamo



3D simulation  
Miesch et al. (2008)

### Observations

- Poleward near surface (surface Doppler and local helioseismology agree well)
- Recent results indicate shallow return flow (Hathaway 2011)?

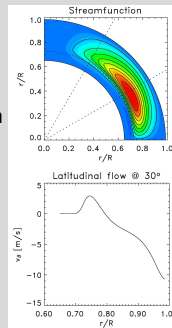
### Theory

- Mean field models: single flow cell, related to inward transport of angular momentum
- 3D: low res runs multi cellular, recent high res single cell, results not yet converged

### Advection dominated regime difficult to realize:

$$\eta_{turb} \propto H_p V_{rms}$$

$$V_{merid} \propto V_{rms}^2 / V_{rot}$$



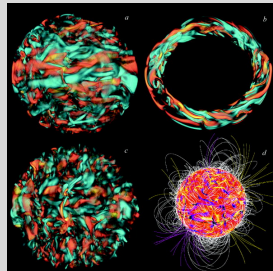
Mean field model  
Rempel (2005)

## 3D simulations

- Solve the full set of equations (including small and large scale flows, magnetic field) from first principles
  - No shortcuts, have to solve for the full problem including differential rotation and meridional flow
  - Non-linear effects automatically included
- Intrinsic limitations
  - Boundary conditions (radial direction)
    - Tachocline at base of CZ
    - Top boundary typically 20 Mm beneath photosphere
  - Cannot capture solar Re and Rm, how to treat small scales
    - DNS: resolve dissipation range with artificially increased diffusivities
    - (I)LES: do only the minimum required to maintain numerical stability
- Very expensive
  - Low resolution runs for long periods >10 years
  - High resolution for short periods
- Good understanding of differential rotation, ingredients of solar dynamo, no complete model yet

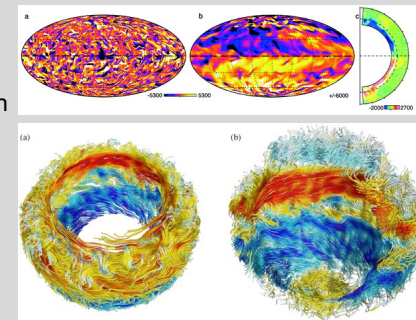
## 3D dynamo simulations

- 1981 Gilman & Miller
  - First 3D convective dynamos in a spherical shell (Boussinesq)
- 1983 Gilman
  - Dynamo simulations with reduced diffusivities
    - large scale field and periodic field reversal
    - poleward propagation
- 1985+ Glatzmaier ...
  - Mostly 3D geodynamo models
- 2004 Brun, Miesch, Toomre
  - Turbulent dynamo (anelastic)
    - 800 G peak toroidal field
    - Mean field 2% of energy
    - No cyclic behavior



## 3D dynamo simulations

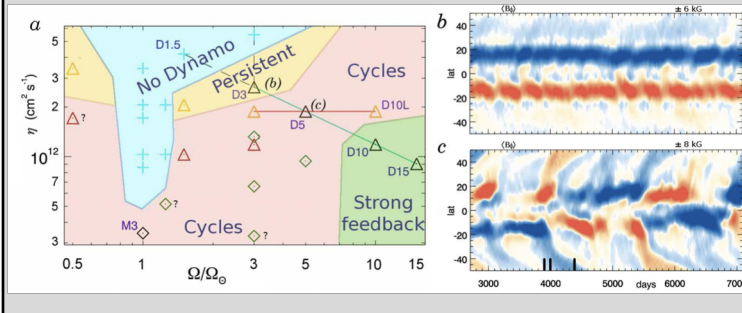
- 2006 Browning et al.
  - Addition of tachocline
  - Organized ~5 kG field in stably stratified region
- 2008+ Brown et al.
  - Faster rotating stars
  - Strong field (~10 kG) maintained within CZ
  - Cyclic behavior for certain parameter choices (faster rotation)



## Cyclic dynamo regimes

### ➤ 2011 Brown et al.

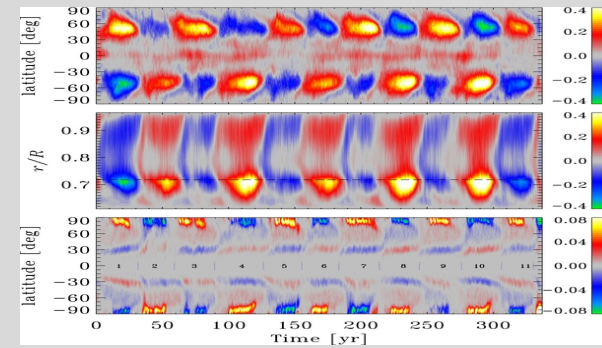
- Cyclic behavior typically found for sufficiently high Rm
  - Small diffusivity
  - Fast rotation



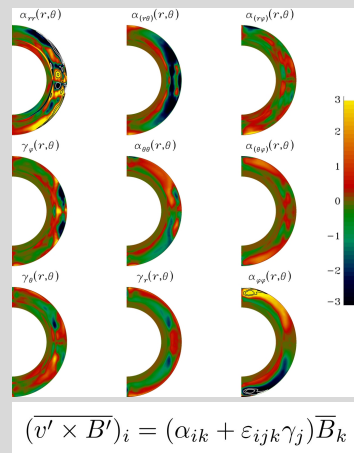
## 3D dynamo simulations

### ➤ 2010 Ghizaru et al., 2011 Racine et al.

- Cyclic  $\alpha\Omega$ -type dynamo 60 yr period
- Magnetic field generated near base of CZ



## Characterization of large-scale dynamo (Ghizaru et al. 2011)



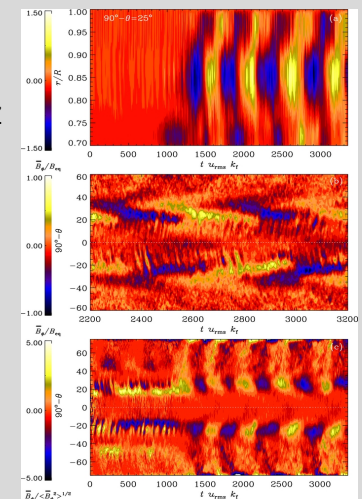
- All components have comparable amplitude
- Strongest effect present in diagonal elements, but substantial deviation from isotropy
  - $\alpha_{\phi\phi}$  and  $\alpha_{\phi\theta}$  show pattern that reflects expectation from helicity profiles, but not  $\alpha_{rr}$
- Turbulent pumping
  - Down- and equatorward in bulk on convection zone
  - Poleward near surface in high latitudes
  - Mimics /competes with meridional flow

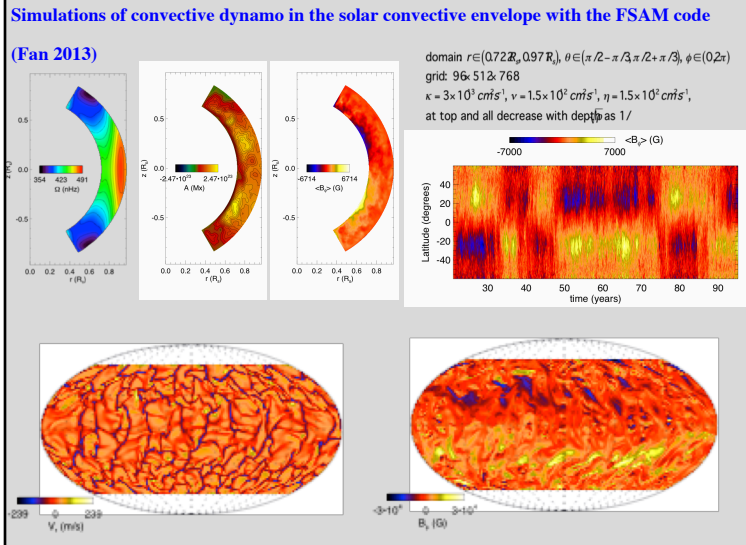
$$(\overline{v' \times B'})_i = (\alpha_{ik} + \varepsilon_{ijk} \gamma_j) \overline{B}_k$$

## 3D dynamo simulations

### ➤ Kapyla et al. (2012)

- 33 year period
- Field generated in bulk of CZ
- Equatorward propagation below 40 deg latitude
- Cycle length non-linear effect
  - Much shorter cycles during kinematic growth phase
  - “Phase transition” due to non-linear feedback





## 3D dynamo simulations

### Recent developments:

- Several independent groups find cyclic dynamos with periods in the 10-60 year range
- Some models with equatorward propagation of activity
- No simple explanation for cycle length and magnetic field patterns
  - Cycle length non-linear effect (longer cycles in saturated phase)
  - Not obvious if different models get similar solutions for the same reason

### Contrast to meanfield models:

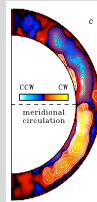
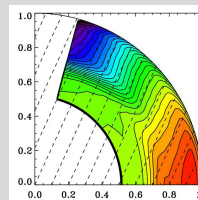
- In general no single dominant turbulent induction term (like a scalar  $\alpha$ -effect) that could capture the behavior
- Non-linear feedback more than just saturation effect (i.e. long cycle length only found in non-linear regime)

## What are the main uncertainties?

### Large scale flows:

- Differential rotation well known
  - Role of latitudinal vs. radial shear not clear
$$\Omega\text{-effect } B_p \cdot \nabla \Omega$$

$$\frac{\partial \Omega}{\partial r} > \frac{1}{r} \frac{\partial \Omega}{\partial \theta}, \text{ but typically } B_r < B_\theta$$
  - Role of tachocline (essential or does it just shape activity)
    - Fully convective stars show strong activity!
  - Variation of  $\Omega$  (torsional oscillations) very small
    - Weak magnetic feedback or DR strongly driven?
    - What does this tell us about saturation?
- Meridional flow
  - Poleward at surface
  - Flow structure in CZ?
  - Shallow return flow (Hathaway 2011)?



Miesch et al. 2008

## What are the main uncertainties?

### Turbulent induction/transport

- In most 3D simulations turbulence is more complicated than a combination of diffusion, advection and  $\alpha$ -effects
- Flux transport dynamos assume weak ( $< 10\%$  of MLT estimates) turbulent transport processes - is that reasonable?
  - $\eta$  has to be small, but not  $\nu$  and  $\kappa$  (need to transport energy and maintain DR)?
  - no clear indication from numerical experiments for asymmetric magnetic quenching of  $\eta$ ,  $\nu$  and  $\kappa$
- More general problem
  - Diffusivities of the order  $\eta_{turb} \propto H_p V_{rms}$  give too short cycles
  - Are longer cycles an intrinsically non-linear effect?
- How is the poloidal magnetic field maintained?
  - kinematic (turbulent)  $\alpha$ -effect?
  - magnetic saturation, role of magnetic helicity?
  - driven by magnetic instabilities?

## What are the main uncertainties?

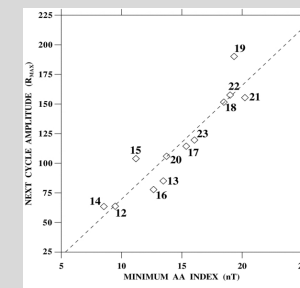
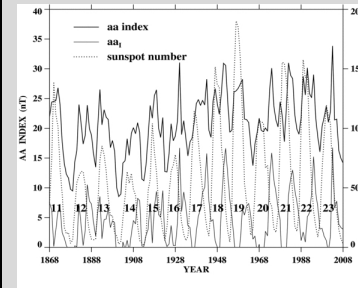
### ➤ Flux emergence process

- By-product of dynamo or essential part of dynamo process?
  - $10^{24}$  Mx is a lot of flux:  $10 \text{ kG} \times 100 \times 100 \text{ Mm}^2$
- Poloidal field in photosphere consequence of AR tilt angle
  - Babcock-Leighton  $\alpha$ -effect
  - Is that enough to drive the dynamo?
    - Polar flux  $\sim 10^{22}$  Mx about 1% of flux emerging in AR
    - How to get back to 100%
    - DR can do  $\sim 100!$

### ➤ What determines field amplitude

- Feedback on DR, meridional flow?
- Quenching of turbulent induction (magnetic helicity) ?

## What do the observations tell?



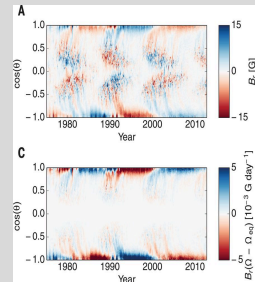
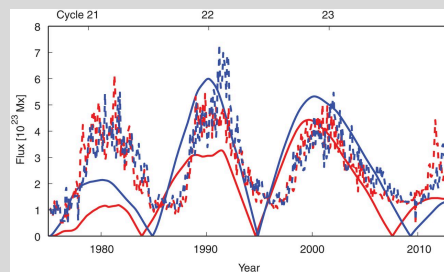
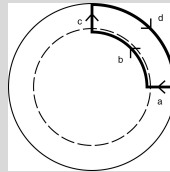
Wang & Sheeley 2009

- Geomagnetic activity related to solar high speed streams (solar minimum) and CMEs (solar max)
- High speed streams during minimum related to flux of polar caps  $\rightarrow$  poloidal field of sun during minimum
- Shows strong correlation with upcoming cycle amplitude

## Surface flux evolution and net toroidal flux

$$\frac{d\Phi_{\text{tor}}^N}{dt} = \frac{d}{dt} \left( \int_{\Sigma} B_{\theta} dS \right) = \int_{\Sigma} (\mathbf{U} \times \mathbf{B} + \langle \mathbf{u} \times \mathbf{b} \rangle - \eta \nabla \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\frac{d\Phi_{\text{tor}}^N}{dt} = \int_0^1 (\Omega - \Omega_{\text{eq}}) B_r R_{\odot}^2 d(\cos\theta) - \frac{\Phi_{\text{tor}}^N}{\tau}$$



Robert Cameron, and Manfred Schüssler Science  
2015;347:1333-1335

## Flux emergence and sunspot formation

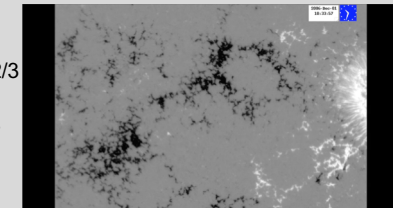
### ➤ General accepted view

- Magnetic flux rising toward surface from deep convection zone
- Observations show at first strong horizontal expansion of emerging flux

### ➤ Key question

- Transport of flux through convection zone and re-amplification in photosphere:

- Density contrast of  $10^6$ 
  - $B \sim \rho^{\epsilon}$   $\epsilon = 1/2 \dots 2/3$
  - $100 \text{ kG} \rightarrow 100 \text{ G}$
  - $100 \text{ G} \rightarrow 3 \text{ kG} ???$
- Vigorous convection



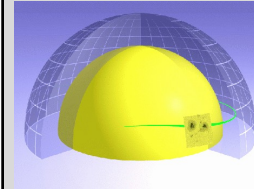
Flux emergence event observed with Hinode SOT

## Modeling of flux emergence

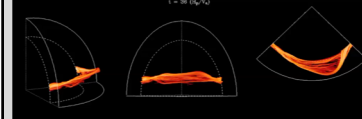
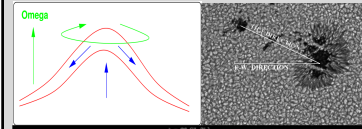
- Lower convection zone (up to ~ 20 Mm depth beneath photosphere)
  - Strongly subsonic velocities
  - Ideal gas equation of state sufficient
  - Size of flux tubes smaller than  $H_p$  and typical scale of convection
    - Flux tubes travel several times their diameter
    - Interaction with ambient flows (including flows created by rising flux) key to dynamics
    - Density contrast of 100 (out of  $10^6$ )
  - Modeling approaches
    - Thin flux tube approximation
    - 3D anelastic MHD models
    - Both with and without background convection

## Flux emergence in lower convection zone

(Caligari, Fan, Fisher, Moreno-Insertis, Schüssler ...)



Thin flux tube simulation: Caligari et al. (1995)

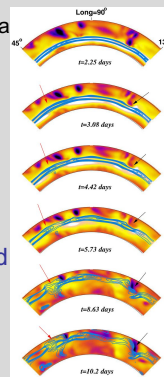


3D simulation: Fan (2008)

- Consistent results from thin tube and 3D simulations
- Coriolis force causes tilt of the top part of tube
- Explains asymmetry between leading/following spot
- Works best with ~ 100 kG flux tubes
  - Consistent with stability considerations in overshoot region
  - Too strong for dynamo models
- Twist required for 2D/3D simulations
  - Prevents fragmentation
  - Induces additional tilt (opposing that from Coriolis force)
  - Trade off between stability and tilt

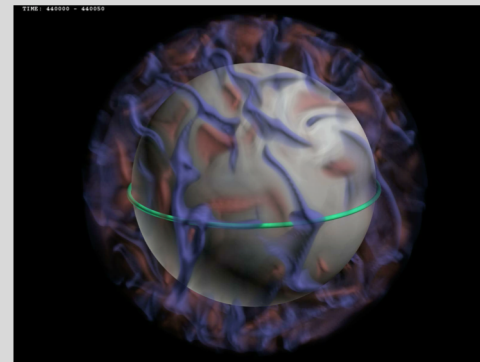
## Interaction with convection

- 3D anelastic MHD (Jouve & Brun 2009)
  - Self consistent interaction with convection, differential rotation and meridional flow (Global convection zone simulation)
  - Convective motions additional source of tilt, substantially shape tube during rise
  - Challenge: Focus on global picture limits resolution on the scale of flux tube, requires tubes with  $\gg 10^{22}$  Mx flux
- Thin flux-tubes rising in convective background
  - Take velocity from global CZ simulation
  - Treat flux tube as thin tube
  - Weber, Fan & Miesch 2011



Jouve & Brun (2009)

## Interaction with convection



Weber, Fan & Miesch (2011)

- Thin flux tube rising in convective envelope (taken from global 3D simulation)
- Flux tube evolution mostly dominated by convective time scales
- Less dependence on initial field strength
  - Best results for 40-50 kG (100-150 kG without convection)



## Flux emergence in upper most 20 Mm

### ➤ Upper convection zone

- Subsonic/supersonic transition velocities
- Partial ionization, 3D radiative transfer important
- Size of flux tubes larger than  $H_p$  and typical scale of convection
  - Flux 'tubes' travel about their diameter
  - Density contrast of  $10^4$  (out of  $10^6$ )
  - Dynamics dominated by strong expansion
  - Most weakening of field strength near surface
- Modeling approaches
  - Fully compressible MHD (with RT and realistic EOS)

### ➤ Currently treated independent from deep convection zone (computational constraints)

## Flux emergence, sunspot formation

Magnetogram  
( $\tau=1$ )

Domain size:  
150x75x16 Mm

$t = 0.0$  h  
30 Mm

$|B|$   
(vertical slice)

Rempel & Cheung 2014

## Flux emergence and active region formation a changing paradigm

### ➤ Early work based on rising thin flux tube

- Caligari, Schuessler, Moreno-Inertis, Ferriz-Mas, Fan, Fisher ....  
- 1993-1996
- Strong  $\sim 100$  kG flux tubes rise from base of CZ
  - Buoyancy instabilities in Overshoot region
  - Flux emergence in low latitudes
  - Tilt angles

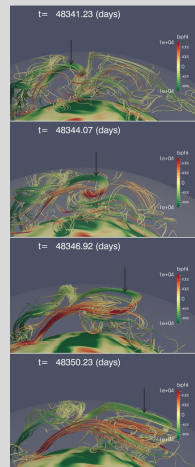
- Retrograde flows due to angular momentum conservation
- Asymmetric stretching of rising loop leads to stronger leading legs

### ➤ Thin flux tube models including ambient convection

- Weber et al. (2011 – 2015)
- Advective transport by ambient convection significant
- Less sensitive to initial field strength
- Tilt a combination of Coriolis forces and ambient helical flows

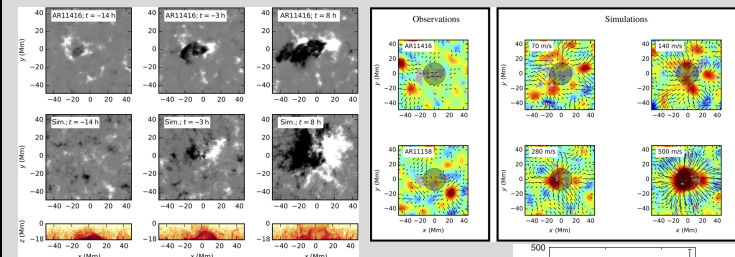
### ➤ 3D global dynamo models

- Nelson et al. 2014, Fan & Fang 2014
- Flux bundles originate within CZ  $\sim 10$  kG, non-axisymmetric zonal shear significant
- Convective/buoyant transport towards surface in giant cells
- Prograde flows in emerging flux regions



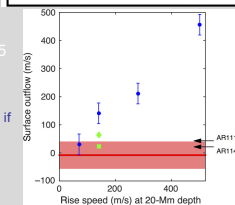
Fan & Fang (2014)

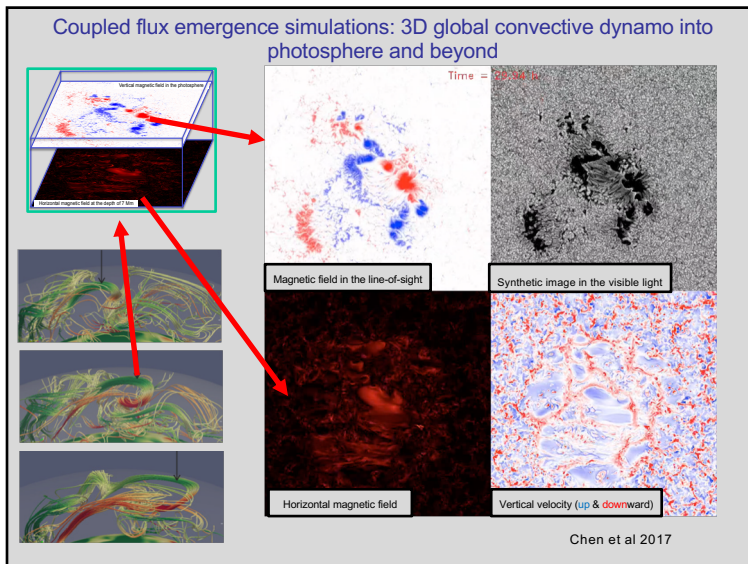
## Helioseismic constraints on emergence speeds in upper CZ



From Birch et al. 2015

- Observed AR (sample of  $\sim 100$ ) show little to no evidence of diverging flows in excess of those found in ambient convection
- Flux emergence models are only consistent with these constraints if the emergence speed is comparable or even smaller than typical convective upflows
- Flux emergence in the upper most 20 Mm of the CZ is mostly a passive process.





## PHYS 7810 Special Topics in Physics: Physics of the Solar Atmosphere

A Theoretical View of the Sun's Internal Dynamics

Meridional flows, Convection Zone, Toroidal field, Radiative Zone, Core Shear?, Inner (fossil?) field, Internal waves, Surface shear layer, Differential rotation, Tachocline, p waves

Dr. A.S. Brun  
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Matthias Rempel  
Lecture 6 Thurs Jan 31 2019

<https://www.nso.edu/students/collage/collage-2019/>

- ### Outline
- Energy transport in solar convection zone, photosphere and above
  - Origin and structure of magnetic field in the solar photosphere
  - Magnetic modulation of solar energy output

### Structure of the solar interior

A Theoretical View of the Sun's Internal Dynamics

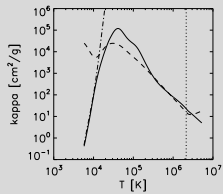
Meridional flows, Convection Zone, Toroidal field, Radiative Zone, Core Shear?, Inner (fossil?) field, Internal waves, Surface shear layer, Differential rotation, Tachocline, p waves

Dr. A.S. Brun  
CEAS/Sp

- Radiative interior  
– 0 ... 0.72 R
- Convection zone  
– 0.72 ... 1 R
- Solar magnetic activity  
– Dynamo process in convection zone

S. Brun

## Basic structure of convection zone



- Radiative energy flux (diffusion approximation)

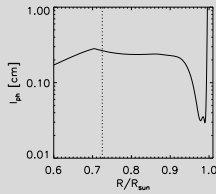
$$F_{rad} = \frac{1}{3} c l_{ph} \frac{d}{dr} a T^4$$

- Photon mean free path (kappa is here the Rosseland mean opacity)

$$l_{ph} = \frac{1}{\kappa \rho}$$

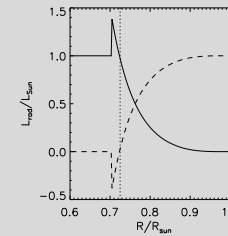
- Opacity sources

- $T > 3 \times 10^4$  K: free-free, bound-free transitions:  
 $\kappa \propto \rho T^{-3.5}$
- $T < 10^4$  K: negative H ion (0.75 eV)  
 $\kappa \propto \rho^{0.5} T^9$



Solar model: Kiefer & Stix

## Basic structure of convection zone



- Radiative energy flux (diffusion approximation)

$$F_{rad} = \frac{1}{3} c l_{ph} \frac{d}{dr} a T^4$$

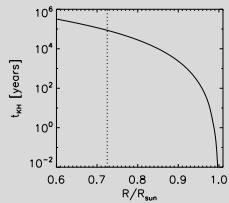
- Drop of radiation energy density makes RT inefficient ( $l_{ph} \sim \text{const}$ )

- Temperature gradient increases until convective instability sets in

- Convection zone in between 2 radiative boundary layers

- Bottom:  $L \sim 100$  Mm
- Top:  $L \sim 100$  km

## Time scales of convection zone



- Kelvin-Helmholtz time scale:

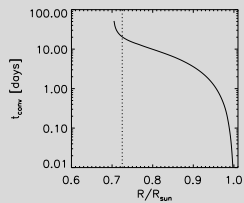
$$t_{KH}(r) = \frac{r}{L_{Sun}} \int_{r}^{R_{Sun}} 4\pi r'^2 E_{int} dr'$$

- Convective overturning time scale

$$t_{conv}(r) = \frac{H_p(r)}{v_{conv}(r)}$$

- Convection zone is a well mixed reservoir with a large heat capacity

- Thermal properties of CZ respond very slowly to a disturbance of the energy flux



## Key properties of convection zone

- Convection driven by strong cooling at the top and gentle heating at the bottom

- Resolving the top boundary is the key for understanding convective dynamics

- Convection zone has a large heat capacity and is well mixed

- $\Delta L \sim 0.1\%$  over solar cycle years does not lead to a significant temperature response in the convection zone

- Changes of  $F_{conv}$  in the bulk of the CZ likely do not lead to significant observable irradiance changes (this question is not fully settled)

## Modeling the solar photosphere

### ➤ Key ingredients:

- MHD
- Radiative transfer
  - 3D, i.e. angular dependence resolved
  - Frequency dependence of opacity (capture by a few opacity bins)
- Equation of state with partial ionization

### ➤ Open bottom boundary condition

- Cannot afford simulation the entire convection zone
- Use open bottom boundary conditions:
  - Convective energy flux across boundary
  - Downflows exit the domain with their thermal properties
  - Upflows have a prescribed fixed entropy

## Photospheric MHD

Fully compressible MHD

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\ \frac{\partial \rho \mathbf{v}}{\partial t} &= -\nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \mathbf{j} \times \mathbf{B} - \nabla P + \rho \mathbf{g} \\ \frac{\partial E_{\text{tot}}}{\partial t} &= -\nabla \cdot \left[ \mathbf{v} \left( E_{\text{tot}} + P + \frac{B^2}{8\pi} \right) - \frac{1}{4\pi} \mathbf{B}(\mathbf{v} \cdot \mathbf{B}) \right] + \rho \mathbf{v} \cdot \mathbf{g} + Q_{\text{rad}} \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B})\end{aligned}$$

With

$$E_{\text{tot}} = E_{\text{int}} + \frac{1}{2} \rho v^2 + \frac{B^2}{8\pi}$$

Energy flux

$$\mathbf{F} = \mathbf{v}(E_{\text{int}} + P) + \mathbf{v} \frac{1}{2} \rho v^2 + \frac{1}{4\pi} \mathbf{E} \times \mathbf{B}$$

Enthalpy    Kinetic energy    Poynting flux

## Photospheric radiative transfer

Radiative transfer equation ( $I$  specific intensity,  $\hat{\mathbf{n}}$  unit vector in ray direction)

$$\frac{dI_\nu}{ds}(\hat{\mathbf{n}}) = \kappa_\nu \rho (S_\nu - I_\nu(\hat{\mathbf{n}}))$$

Source function  $S_\nu = B_\nu(T)$  in local thermodynamic equilibrium (LTE)

Radiative energy flux

$$\mathbf{F}_\nu = \int_{4\pi} I_\nu(\hat{\mathbf{n}}) \hat{\mathbf{n}} d\Omega$$

Average intensity

$$J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu(\hat{\mathbf{n}}) d\Omega$$

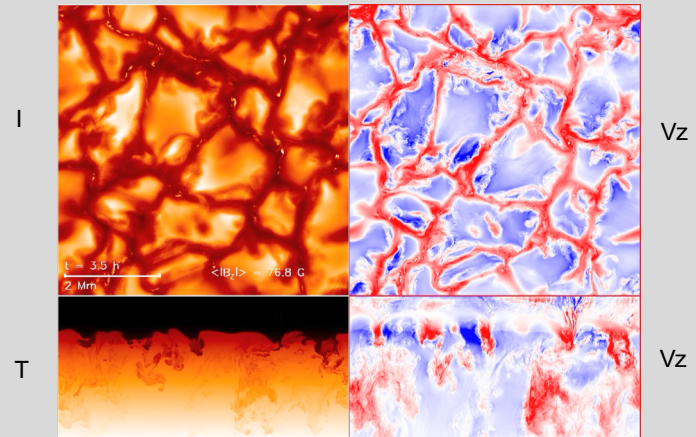
Radiative heating/cooling

$$Q_{\text{rad}} = - \int_\nu (\nabla \cdot \mathbf{F}_\nu) d\nu = 4\pi \rho \int_\nu \kappa_\nu (J_\nu - S_\nu) d\nu$$

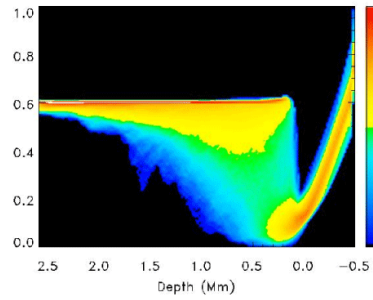
Numerical treatment

- Compute a discrete number of rays, typically 24 - 48
- Compute a discrete number of frequency bins, typically 1 - 12

## Example granulation simulation



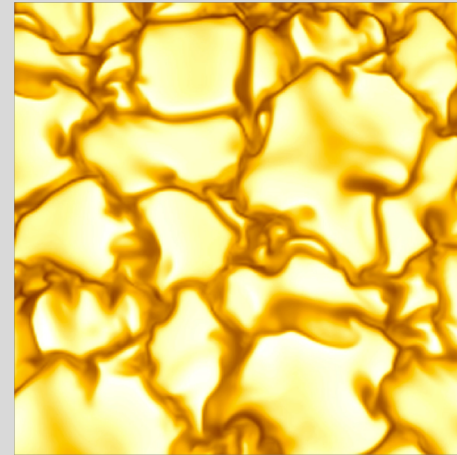
## Thermal properties of up and downflows



Stein et al. 1997

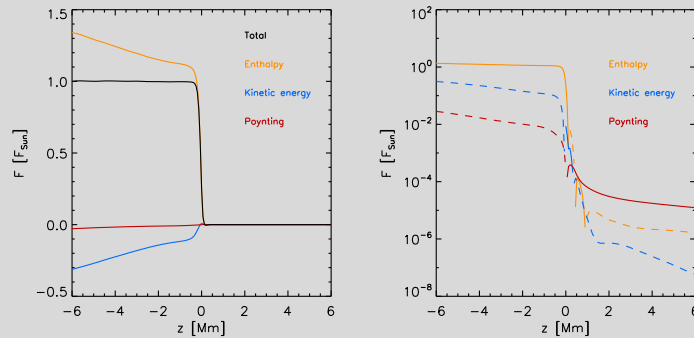
- Upflow entropy has little variation through the convection zone
  - Adiabatic stratification
- Downflows start with a low entropy at the top due to radiative energy loss
- Downflow entropy increases with depth due to mass entraining from upflow regions
  - Superadiabatic stratification
- Average stratification superadiabatic

## Corrugated photosphere



Mats Carlsson, Oslo

## Energy fluxes



$$\mathbf{F} = \mathbf{v}(E_{\text{int}} + P) + \mathbf{v} \frac{1}{2} \rho v^2 + \frac{1}{4\pi} \mathbf{E} \times \mathbf{B}$$

## Relation between intensity and photospheric properties

Radiative transfer equation (I drop here the  $\nu$  indices and angular dependence for clarity)

$$\frac{dI}{ds} = \kappa \rho (S - I)$$

Optical depth along ray (unit of  $\kappa \rho$  is  $\text{cm}^{-1}$ )

$$d\tau_s = -\kappa \rho ds$$

Alternate form of RT equation

$$\frac{dI}{d\tau_s} = I - S$$

Formal solution

$$I(\tau_1) = I(\tau_2)e^{-(\tau_2-\tau_1)} + \int_{\tau_1}^{\tau_2} S(x)e^{-(x-\tau_1)} dx$$

Observable Intensity ( $\tau_1 = 0$ ,  $\tau_2 = \infty$ )

$$I(0) = \int_0^{\infty} S(x)e^{-x} dx$$

Assume linear source function  $S = S_0 + S_1\tau$

$$I(0) = S(\tau_s = 1) = \frac{\sigma}{\pi} T^4(\tau_s = 1)$$

## Magnetic modulation of photospheric emission

- Long lived, large-scale magnetic field concentrations
  - Suppression of convective energy transport
  - Energy radiated away in photosphere cannot be replenished
  - $\alpha(\tau=1) = \frac{\sigma}{\pi} T^4(\tau=1)$  is reduced
  - Dark features, i.e. sunspot umbra

- Short lived, small-scale flux concentrations

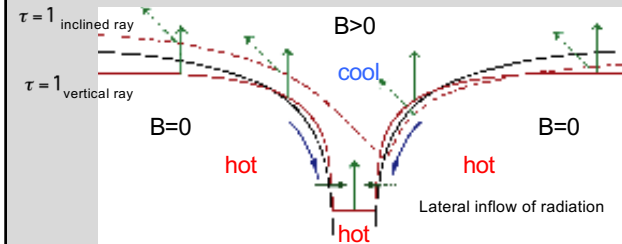
- Approximate pressure balance

$$\rho_i + \frac{B^2}{8\pi} = \rho_e \rightarrow \rho_i < \rho_e \rightarrow \rho_i < \rho_e$$

- Flux concentration more transparent, i.e. radiation escapes from a deeper layer where T is larger
- Lateral inflow of radiation keeps structure hot
- $\alpha(\tau=1) = \frac{\sigma}{\pi} T^4(\tau=1)$  is enhanced
- Brightpoints, faculae
- Typical required field strength ( $p_{\text{phot}} \sim 10^5 \text{ dyn/cm}^2$ )

$$B \approx \sqrt{8\pi p_{\text{phot}}} \approx 1 - 1.5 \text{ kG}$$

## Angular dependence of brightening

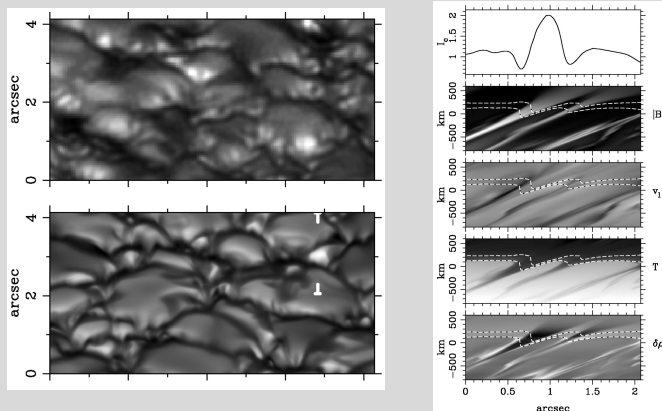


From Thaler & Spruit 2014

- Inclined rays more bright

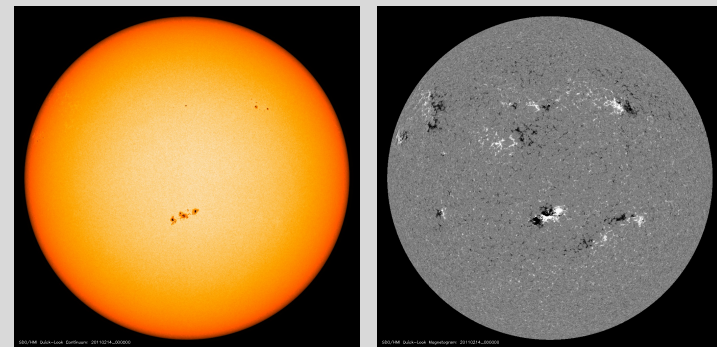
- Geometric projection effect
- $\tau = 1$  level deeper (i.e. hotter) near hot wall

## Simulated Faculae

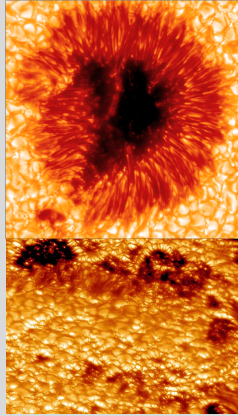


From Keller et al. 2004

## Photospheric magnetic field (solar surface dermatology)



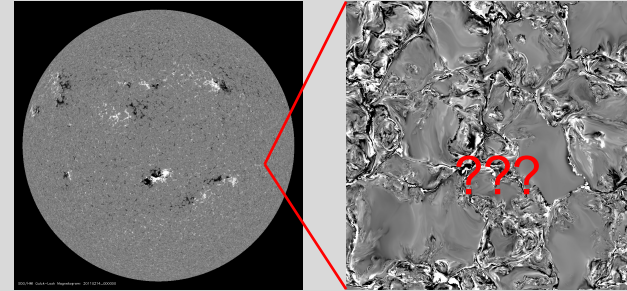
## Photospheric magnetic field



Swedish solar telescope

- Active regions
  - Solar cycle variation
  - Origin: Large scale dynamo
  - Sunspots:
    - dark
  - Plage:
    - dark pores
    - bright faculae
- Quiet Sun
  - No convincing evidence for a cycle variation
  - Magnetic field independent from large scale dynamo
  - Grand minimum = quiet sun??

## Quiet Sun magnetism



- Most of the solar surface is covered by "quiet Sun" at any time of the sunspot cycle
- Unsigned flux at  $\tau=1$  is a few times  $10^{24}$  Mx, i.e. comparable to the flux emerging in form of active regions throughout the cycle
- Where does this field come from and what does it tell us about the solar dynamo(s)?

## Quiet Sun – What are the open questions?

- How is the field distributed?
  - Spectral energy distribution
    - Preferred scale (i.e. "flux tubes" at 100 km) ?
  - Strength distribution
    - Fraction of kG field?
- Where does the field come from?
  - Remnant flux from active region decay
    - Only weak indication from observations
  - Small scale dynamo
    - Origin independent from solar cycle
    - Theoretical challenges

## What is a small scale dynamo?

$R_m \gg 1$  advection dominated regime (ideal MHD)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

Equivalent expression

$$\frac{\partial \mathbf{B}}{\partial t} = -(\mathbf{v} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{v} - \mathbf{B} \nabla \cdot \mathbf{v}$$

Equivalent expression

$$\frac{d \mathbf{B}}{dt} = \left( \frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}$$

Lagrangian particle paths:

$$\frac{dx_1}{dt} = \mathbf{v}(x_1, t) \quad \frac{dx_2}{dt} = \mathbf{v}(x_2, t)$$

Consider small separations:

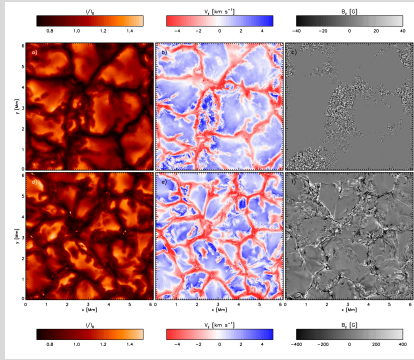
$$\delta = x_1 - x_2 \quad \frac{d\delta}{dt} = (\delta \cdot \nabla) \mathbf{v}$$

In a chaotic flow the separation grows exponentially (for small  $\delta$ ). Due to mathematical similarity the equation:

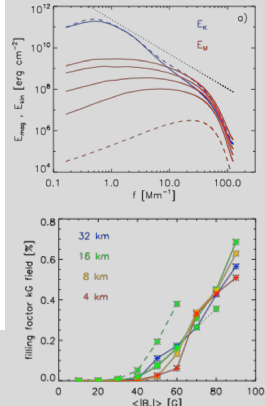
$$\frac{d \mathbf{B}}{dt} = \left( \frac{\mathbf{B}}{\rho} \cdot \nabla \right) \mathbf{v}$$

has exponentially growing solutions, too. We neglected here  $\eta$ , exponentially growing solutions require  $R_m > \mathcal{O}(100)$ .

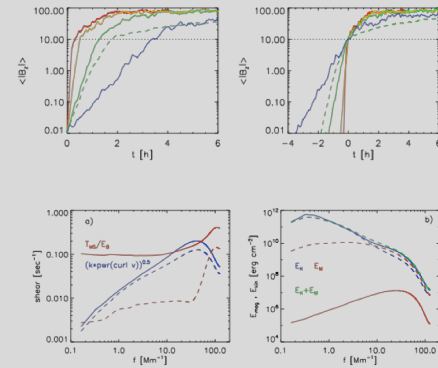
## Kinematic regime to saturation



- Magnetic field organization changes dramatically during saturation
  - Non-linear saturation begins for  $\langle |B_z| \rangle \sim 10$  G in photosphere
  - Sheet like appearance instead of "salt and pepper"
  - Peak of magnetic energy near granular scales
  - kG flux concentrations, bright points appear starting from  $\langle |B_z| \rangle \sim 30$  G

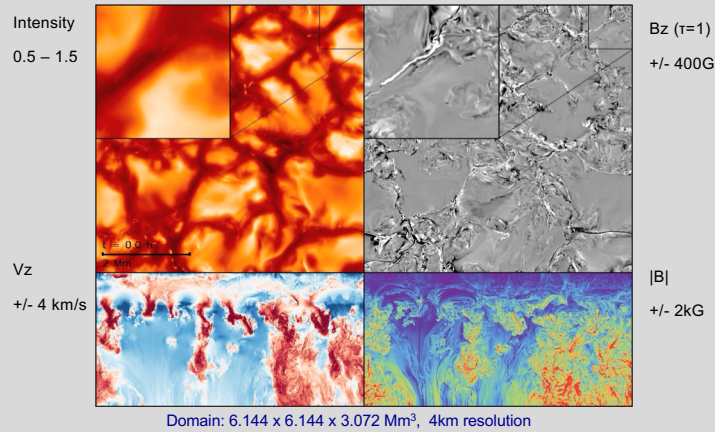


## Kinematic regime to saturation

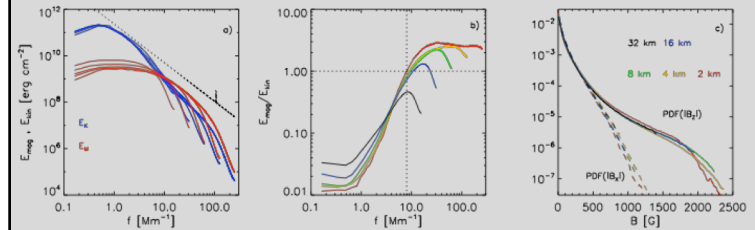


- Kinematic regime
  - $B < 0.01 B_{0s}$  (current simulations)
  - Equipartition with  $E_{kin}$  near magnetic dissipation scale
- $B > 0.1 B_{0s}$ 
  - Slow growth on a typical convective time scale
  - Organization of QS field on meso to supergranular scales expected
- Saturation process
  - Misalignment between  $B$  and shear
  - Only moderate reduction of  $E_{kin}$  and vorticity
- Observable quiet sun
  - Saturated regime of a small scale dynamo

## "Saturated" solution $\langle |B_z| \rangle \sim 80$ G



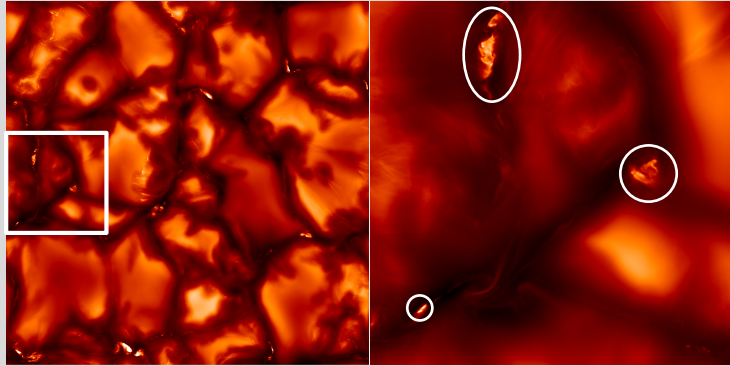
## Resolution dependence 32 ... 2 km



- Converged results using LES approach
  - No explicit viscosity or magnetic resistivity
  - Changing resolution by a factor of 16!
  - Domain sizes from 192x192x96 to 3072x3072x1536
- Does it converge toward the correct solution (computed with realistic viscosity, resistivity)?
  - Implicit magnetic Prandtl number  $\sim 1$
  - Sun (photosphere):  $P_m \sim 10^{-5}$
- Need either high resolution DNS or high resolution observations to confirm

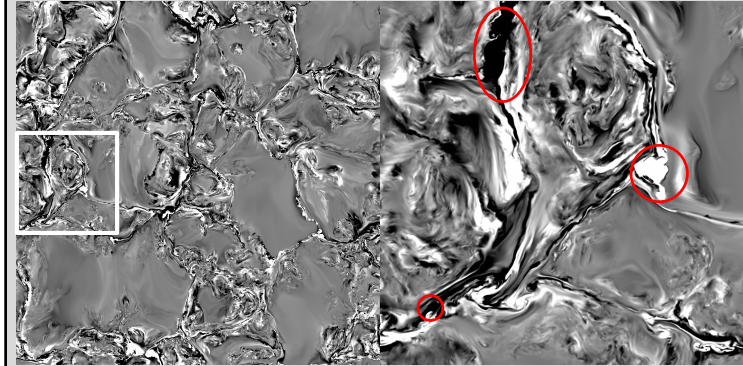


## “Sun” at 2 km resolution



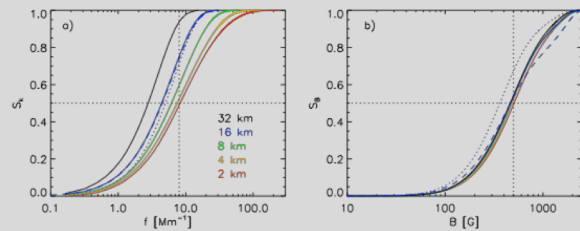
- Simulation domain:  $6.144 \times 6.144 \times 3.072 \text{ Mm}^3$
- Grid size:  $3072 \times 3072 \times 1536$

## “Sun” at 2 km resolution



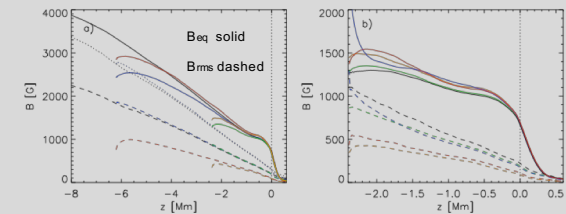
- Simulation domain:  $6.144 \times 6.144 \times 3.072 \text{ Mm}^3$
- Grid size:  $3072 \times 3072 \times 1536$

## Energy distribution in photosphere

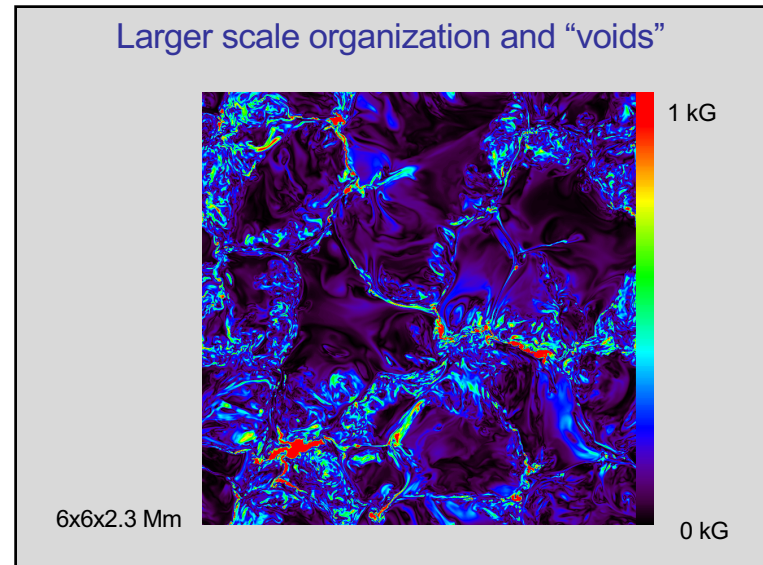
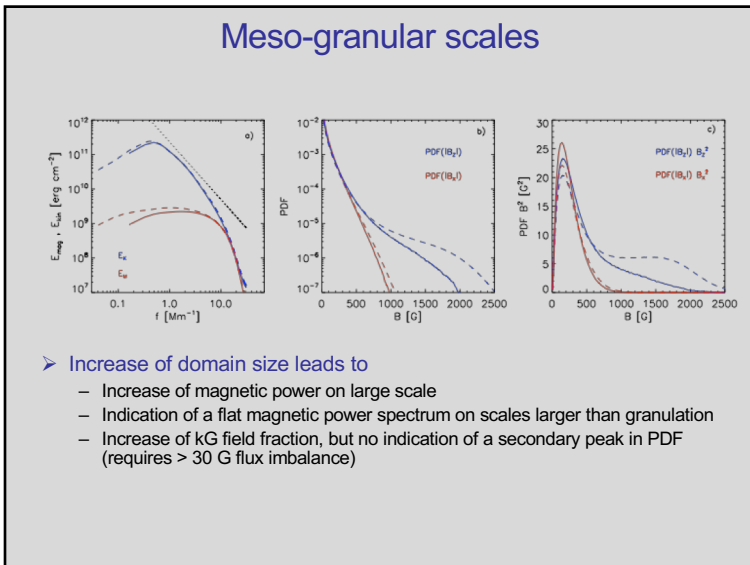
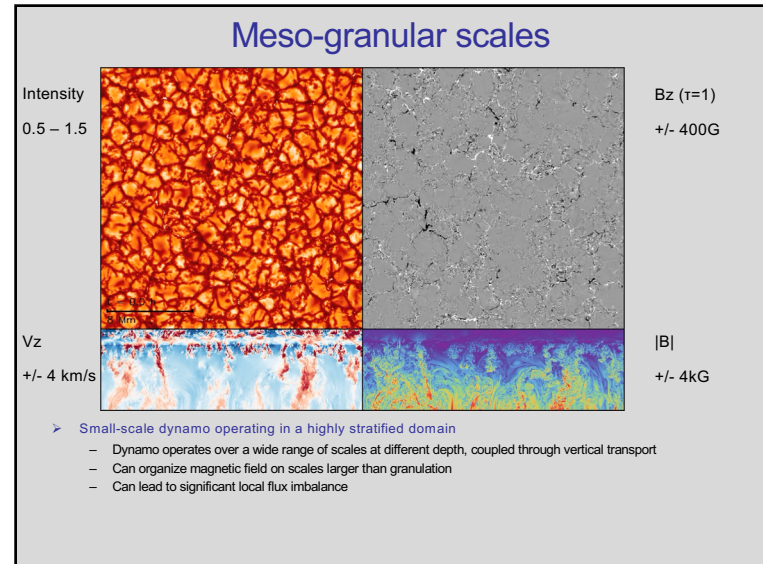
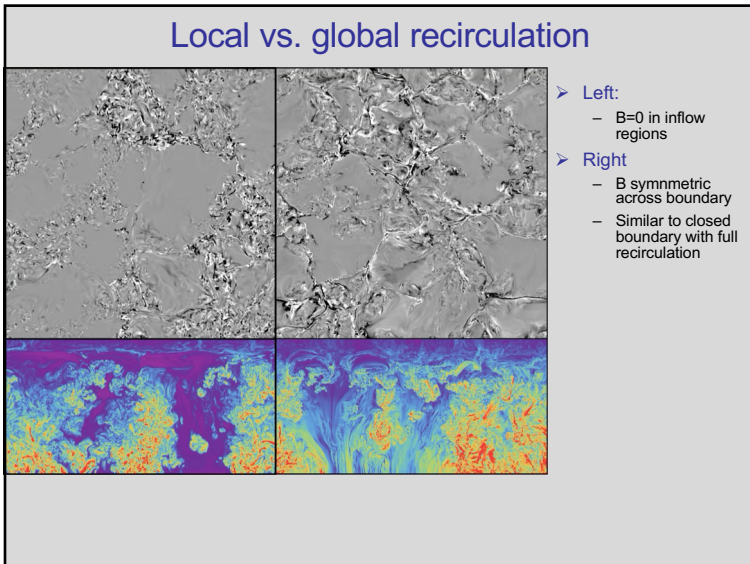


- ~50% of energy on scales smaller than 100 km
  - Need small (~8 km or smaller) grid spacing for properly resolving the spectral energy distribution
  - Hinode “sees” about 20% of the magnetic energy, DKIST could see more than 90%
- ~50% of energy from field weaker than 500 G
  - No resolution dependence, but domain size and overall field strength matters

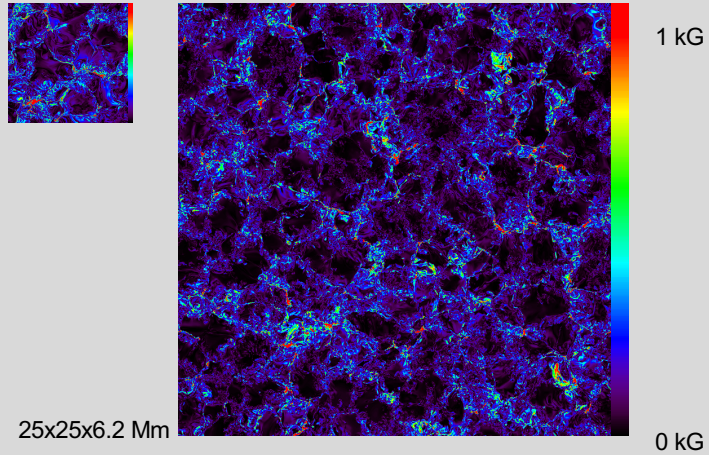
## Role of bottom boundary condition



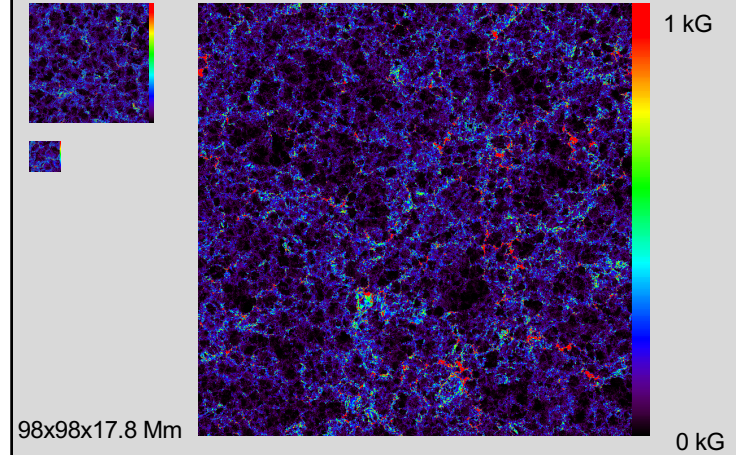
- Bottom boundary sets overall field strength reached in the photosphere in the range
  - $\langle |B_z| \rangle \sim 30 - 85 \text{ G}$
- “Lower” bound (30 G):
  - $B=0$  in inflow regions, or vertical field boundary condition
  - Dynamo lives from **local** recirculation due to turbulent upflow/downflow mixing
  - Stronger field requires **global** recirculation (i.e. closed bottom boundary condition, open boundary with horizontal magnetic field in upflow regions, conditions of deep CZ influences photosphere)
- “Upper” bound (85 G):
  - $B_{rms}$  increases at same rate as  $B_{eq}$



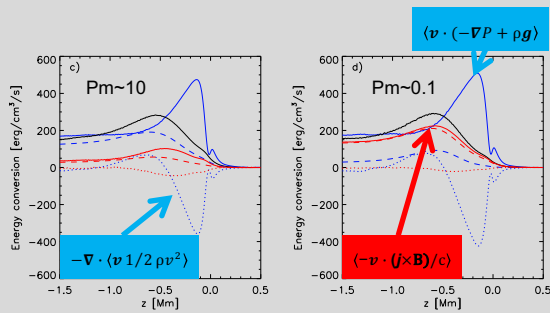
### Larger scale organization and “voids”



### Larger scale organization and “voids”



### SSD energetics



- About 150 erg/cm<sup>3</sup>/s “convective driving” available in upper CZ/photosphere to drive dynamo
- Energy transfer to magnetic energy strongly Pm dependent (Brandenburg 2011, 2014)
- Most efficient dynamos (in terms of energy conversion) found for low Pm regime
- Uppermost 1.5 Mm of convection zone: About >0.3 L<sub>Sun</sub> converted to B