Part II : Radiative transfer and Spectropolarimetry

Basic quantities, equations and solutions

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Summary

- Why is the radiation important?
- Technicalities: what quantities and units?
- What equations govern radiative transfer processes?
- Some basic solutions to relate the model and the observable

Why is radiation important?

Another type of particles, photons ($E = hc/\lambda$), which are bosons, with spin one but only two possible spin states $(-1, 1)$.

They interact with the gas (and with the magnetic field).

They carry information about the object of interest:

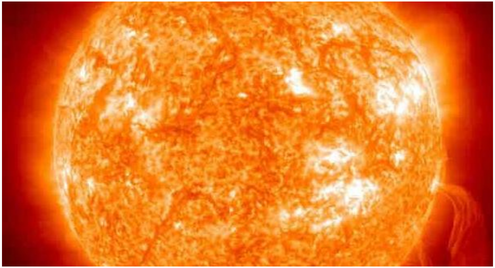
- The spectrum carries information about the temperature and density
- Spectral lines $\rightarrow$ velocity
- From polarized spectra: geometry and the magnetic field...

But, radiation is also plays an active role

- Radiation can exert pressure.
- Radiation can help the medium cool ($E_{\mathrm{thermal}} \rightarrow hc/\lambda$)
- Or heat ($hc/\lambda \rightarrow E_{\mathrm{thermal}}$)
- Or ionize (photoionization), or excite (optical pumping, remember lasers)

"Radiation is a constituent of astrophysical objects, yet the objects are probed only by the radiation." (Hubeny, 2013)

Specific intensity



Let's count the photons that come to us from one pixel of the center of the image:

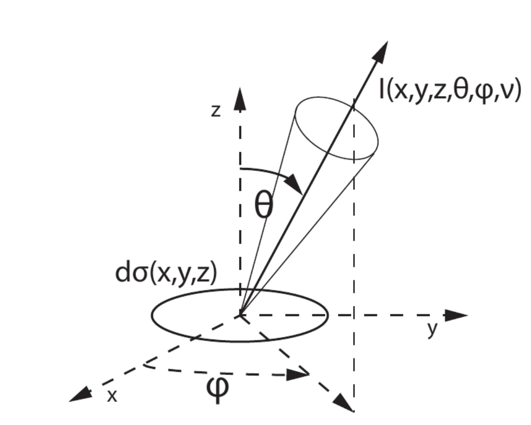
$$N \propto \Delta\sigma\Delta\Omega\Delta\lambda$$

Specific intensity:

$$I(\boldsymbol{s}, \lambda, x, y, t) = \frac{d^4 E}{dt\, d\lambda\, d\hat{\Omega}(\boldsymbol{s})\, \cos\theta\, d\sigma(x, y)}$$

Specific intensity

Iconic figure from every stellar atmosphere book:



$$I(\boldsymbol{s}, \lambda, x, y, t) = \frac{d^4 E}{dt\, d\lambda\, d\hat{\Omega}(\boldsymbol{s})\, \cos\theta d\sigma(x, y)}$$

Some remarks about the specific intensity

- It stays constant 'along the ray' ('ray' is something we will use a lot). Although this is misleading because we can't measure it directly.
- It depends on the time, position and wavelength but also on a direction (photons pass through each other) $\rightarrow$ 7-dimensional quantity in general.
- We, of course, always measure angle- time- space- and wavelength-integrated quantities (energy, number of counts, etc.)

Radiative transfer equation (RTE)

Since we are now in the MHD, conservation equation frame of mind, let's write an equation for photons:

$$\frac{dI(\boldsymbol{r},\hat{\Omega},t,\lambda)}{c\,dt}+\boldsymbol{n}\cdot\nabla I(\boldsymbol{r},\hat{\Omega},t,\lambda)=-\chi(\boldsymbol{r},\hat{\Omega},t,\lambda)\,I(\boldsymbol{r},\hat{\Omega},t,\lambda)+\eta(\boldsymbol{r},\hat{\Omega},t,\lambda)$$

Number of photons and their energy distribution are not constant. There are sources and sinks (emissivity and opacity).

RTE in 1D

Time-independent, one-dimensional case is enough to study many of the problems we will face:

$$\frac{dI(s,\theta,\lambda)}{ds} = -\chi(s,\lambda)I(s,\theta,\lambda) + \eta(s,\lambda)$$

After customary division with $-\chi_\lambda$:

$$\frac{dI_\lambda}{d\tau_\lambda} = S_\lambda - I_\lambda$$

With the 'formal' solution:

$$I(\lambda) = I_0(\lambda)e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t)e^{-t}dt$$

RTE in 1D

To say it in words: Emergent intensity is some weighted sum of the incident one and the contribution by the medium. The weighting factors depend on the (wavelength dependent) opacity of the medium.

Solving RTE is a boundary-value problem. If χ and η are specified everywhere, and we know incident intensity, we can solve RTE.

Usually, situation is more complicated (In some weeks we will study so called NLTE problem).

RTE in 1D

Simplifications we made here:

- Time dependence is taken out. This means light interacts on much smaller timescales than the medium evolves.
- Opacity and emissivity are isotropic. This is good assumption until we reach high chromosphere or consider polarization.
- Physical parameters depend only on z, radiation still depends on z and θ (and λ), so even though the problem is 1D, radiation is three-dimensional.
- This is a good approximation for considering average atmospheric properties, but also for any problem where the vertical radiative transport is dominant.

RTE in 1D

Let's discuss the units a bit:

$$\frac{dI(s,\theta,\lambda)}{ds} = -\chi(s,\lambda)I(s,\theta,\lambda) + \eta(s,\lambda)$$

$$I\left[W\,m^{-2}\,\mathrm{srad}^{-1}\,\mathrm{\mathring{A}}^{-1}\right]$$

$$\chi\left[m^{-}1\right]$$

- inverse mean free path

$$\eta\left[W\,m^{-3}\,\mathrm{srad}^{-1}\,\mathrm{\mathring{A}}^{-1}\right]$$

Emissivity is proportional to the density of the emitters.

Some simple solutions

- No emissivity, $S = 0$, e.g. cold,absorbing slab:

$$I(\lambda) = I_0(\lambda)e^{-\tau_\lambda}$$

.

- No opacity (we have to use the original form), e.g. corona:

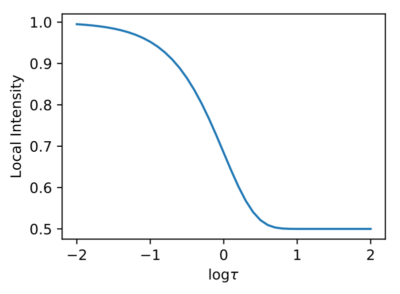
$$I_0(\lambda) = \int_0^s \eta_\lambda(s)ds$$

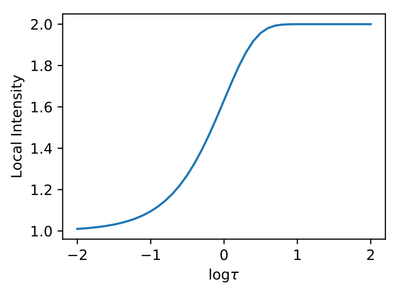
The most interesting are the cases where we have the both, so we have to solve the integral.

Constant source function

$$I = I_0 e^{-\tau} + \int_0^{\tau} S(t) e^{-t} dt$$

$$I = I_0 e^{-\tau} + S(1 - e^{-\tau})$$

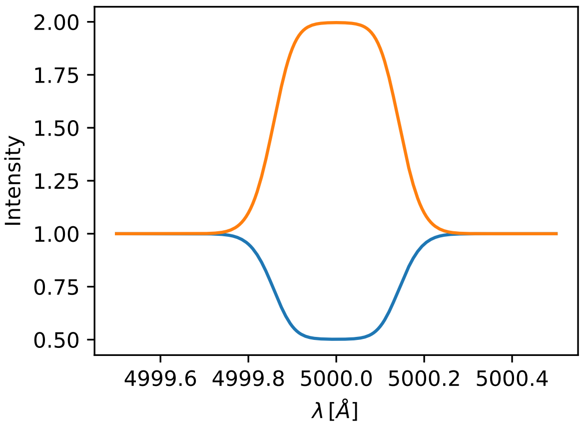




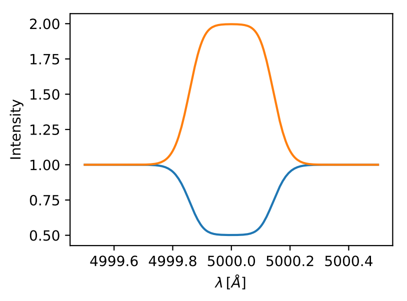
Wavelength-dependent case is more interesting

$$I_\lambda = I_{0,\lambda} e^{-\tau_\lambda} + S_\lambda (1 - e^{-\tau_\lambda})$$

Let's set: $\tau_\lambda = \tau \phi_\lambda$, $\tau = 10$, $S = const$.

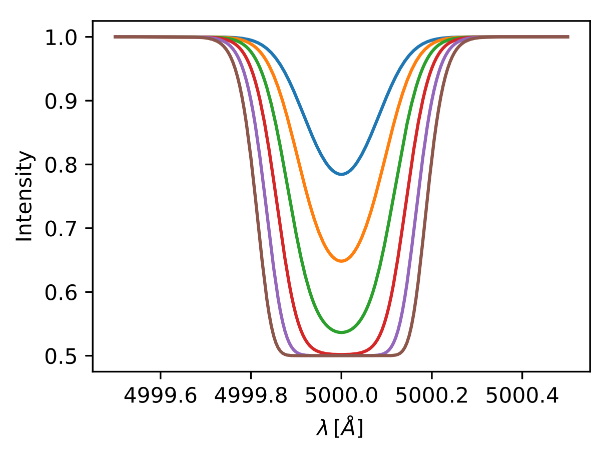


Pay attention!



- The line intensity is set by the value of S
- The continuum is set by the incident intensity
- Line never reaches zero!
- Line 'saturates'

Varying the line strength (total opacity)



In reality, the saturation is rarely so extreme because the source function is never constant. But we leave spectral lines for the next time. Now we can discuss RTE further.

Milne-Eddington atmosphere

Assume that in the region of interest: $S = a\tau + b$. Assume semi-infinite atmosphere:

$$I^+ = \int_0^\infty S(\tau) e^{-\tau} d\tau$$

$$I^+ = \int_0^\infty (a\tau + b) e^{-\tau} d\tau$$

Integrate by parts (we will do this often):

$$I = a + b = S(\tau = 1)$$

. **We see the radiation coming from optical depth unity.**

Solar limb darkening

For the inclined rays: $ds = dz/\cos\theta = ds/\mu$.

$$\frac{1}{\mu}\frac{dI_\lambda(\mu)}{d\tau_\lambda} = S_{\lambda,\mu} - I_{\lambda,\mu}$$

With the formal solution:

$$I^+(\mu) = \int_0^\infty S(\tau) e^{-\tau/\mu} d\tau/\mu$$

$\tau/\mu = t$, solve as previously, we get:

$$I^+(\mu) = a + b\mu$$

. "Eddington-Barbier relation."

Solar limb darkening

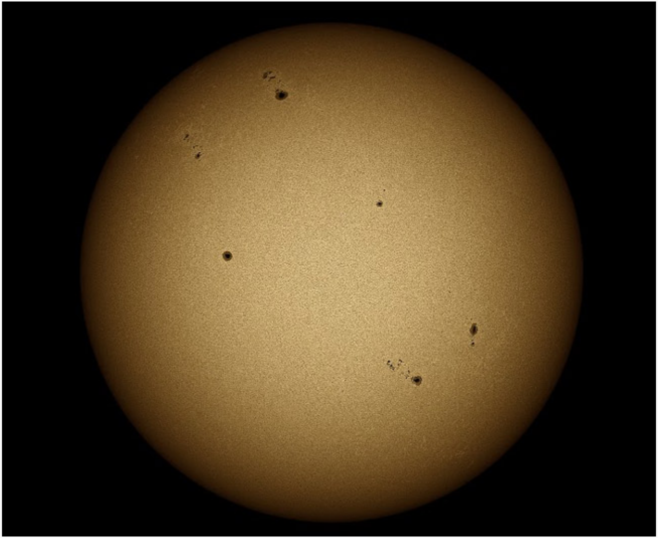


Figure 1: Solar disk in optical continuum. Credits: astronomyconnect.com

What does limb darkening tell us?

If the limb darkening exists, that means b is positive. That is, the source function is decreasing with depth. What is the source function again?

$$S = \frac{\eta}{\chi}$$

Kirchoff's law: For blackbody radiation this is equal to Planck function.

$$S(\lambda) = B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}.$$

For $\lambda = 5000\,\text{Å}$ and $T = 5000\,\mathrm{K}$, exponent is smaller than one (but not by much). Then: $e^{\mathrm{const}/T} - 1 = \mathrm{const}/T$, which means $S \propto T$. Temperature in the solar atmosphere is increasing with depth!

It indeed is, at least in the photosphere

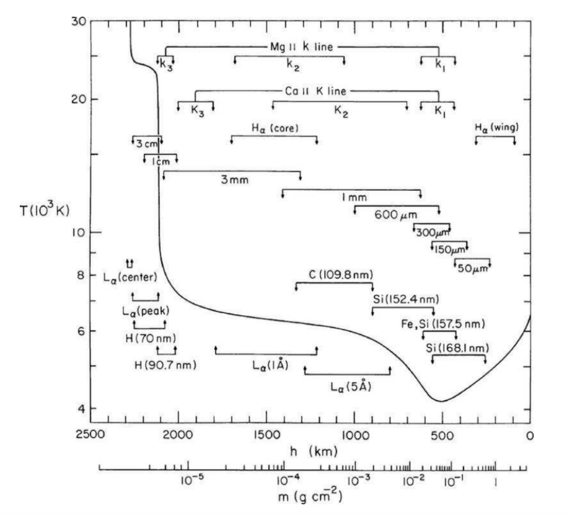


Figure 2: Semi-empirical VALC model atmosphere. From Vernazza et al. 1981

Obviously stars are not blackbodies

To truly be blackbodies, stars would have to be isothermal. There would be no radiation transport outward. Which quantity describes radiation transport? Flux.

$$\mathcal{F} = \oint I(\boldsymbol{n}) \cdot \boldsymbol{n} d\hat{\Omega}$$

. In 1D:

$$\mathcal{F} = 2\pi \int_{-1}^{1} I(\mu)\mu d\mu.$$

At surface the it is enough to integrate from 0 to 1. There is no incident radiation. (Radiation is going out, hence no black body).

RTE deep in the atmosphere

Source function almost constant with τ, and Planckian.
$S(t_\lambda) = S(\tau_\lambda) + S'(\tau_\lambda)$. Remember Eddington-Barbier relationship:

$$I(\tau_\lambda, \mu) = S(\tau_\lambda) + \mu S'(\tau_\lambda).$$

Multply by μ and integrate over μ to get the flux:

$$F_\lambda = \frac{4}{3} B'(\tau_\lambda).$$

Now, assume that opacity is constant over all the relevant wavelenths (grey atmosphere), and integrate over λ...

RTE deep in the atmosphere

$$F = \frac{4}{3}\int_0^\infty B'(\tau_\lambda)$$

$$F = \frac{4}{3}\frac{d}{d\tau}\int_0^\infty B(\lambda)$$

$$F = \frac{4}{3} \times \frac{d}{-\chi dz}\sigma T^4 = -\frac{16}{3}\frac{T^3}{\chi}\frac{dT}{dz}$$

Diffusion approximation! Photons are randomly walking and slowly diffusing toward lower temperatures. The energy is transported outward, proportional to temperature gradient and inversely proportional to opacity.

But, back to the spectra formation

Let's again assume $S = a + b\tau$, for all the λ. But $\tau_\lambda = \tau \times k_\lambda$:

$$I^+(\lambda) = \int_0^\infty (a + b\tau) e^{-\tau k_\lambda} k_\lambda d\tau$$

$$I^+(\lambda) = a + \frac{b}{k_\lambda}$$

If we again simplify, and say everything but the source function is depth-independent, we see that $k_\lambda = \chi\lambda / \chi_{\mathrm{r}eferent}$.

Example: jumps in the opacity

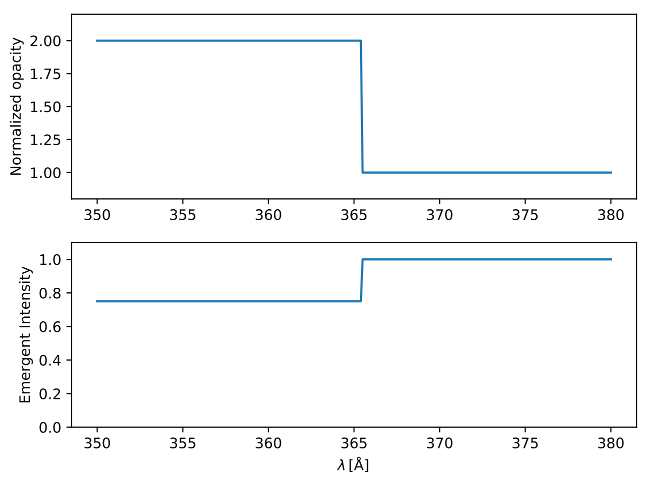


Figure 3: Simplified continuum formation: jump in opacity leads to the jump in intensity, of the opposite sign.

Example: line formation

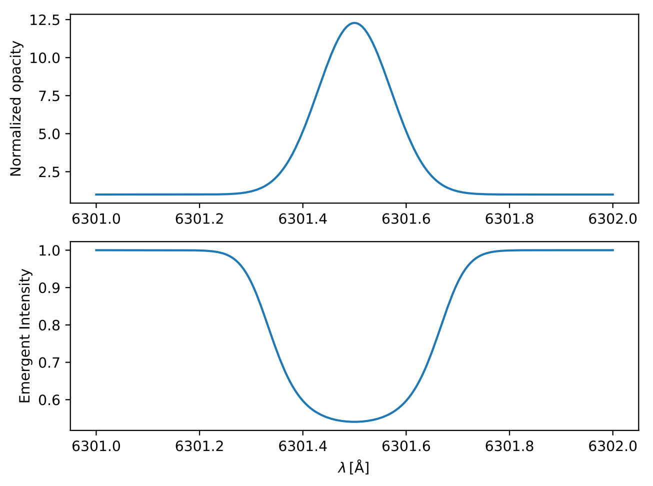


Figure 4: Simplified line formation: Higher opacity samples upper layers where the source function is lower.

Example: line formation

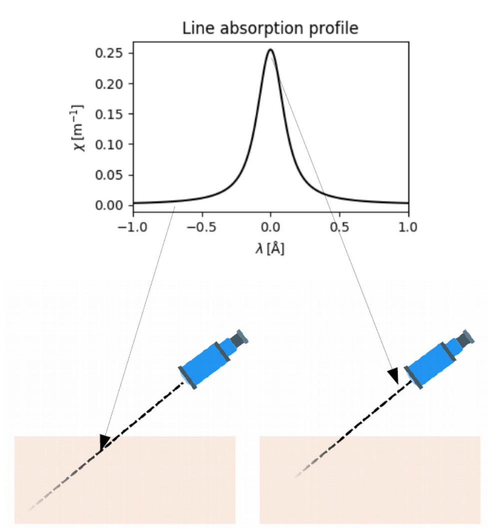


Figure 5: Simplified line formation: Higher opacity samples upper layers where the source function is lower.

Example: line formation

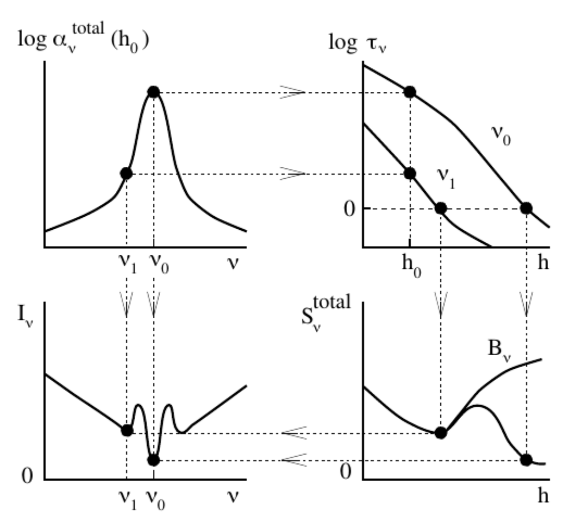


Figure 6: Example from page 20 of Rob Rutten's book.

This appears in the real spectra!

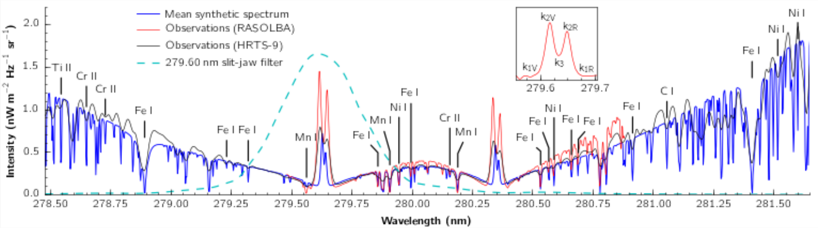
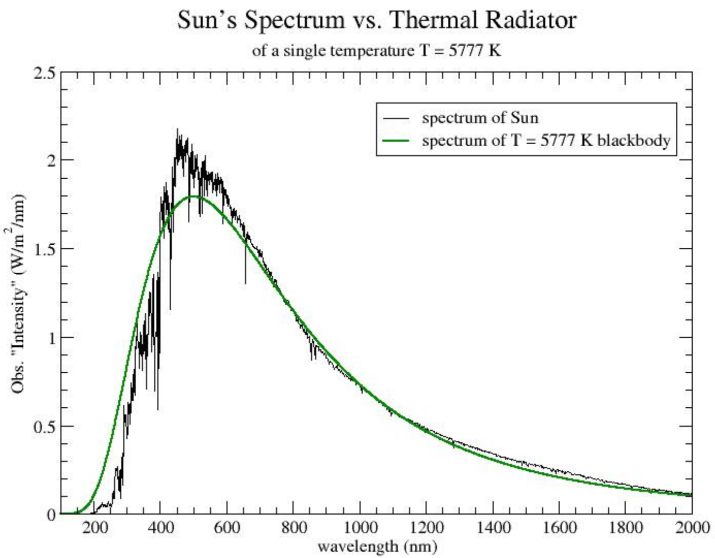


Figure 7: Spectral region around Mg II line observed by IRIS telescope.

Solar Spectra is tricky



Summary

- We are always interested in the intensity
- Intensity is the solution of RTE, which involves knowing opacity and emissivity, and the boundary condition.
- Solving RTE is usually numerical, but some simplifications (e.g. ME atmosphere) can be useful.
- Rule of a thumb: Intensity equals to the source function at optical depth unity, at that wavelenght and direction.
- Different wavelengths probe different depths. This is the reason for existence of spectral lines.