## Symmetry constraints

$\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$ depend on large scale symmetries of the system defining the symmetry properties of the turbulence (e.g. rotation and stratification). Additional to that the expansion

$$\overline{\boldsymbol{\mathcal{E}}} = \boldsymbol{\alpha}\overline{\boldsymbol{B}} + \boldsymbol{\gamma}\times\overline{\boldsymbol{B}} - \boldsymbol{\beta}\nabla\times\overline{\boldsymbol{B}} - \boldsymbol{\delta}\times\nabla\times\overline{\boldsymbol{B}} + \dots$$

is a relation between polar and axial vectors:

- $\overline{\boldsymbol{\mathcal{E}}}$: polar vector, independent from handedness of coordinate system
- $\boldsymbol{B}$: axial vector, involves handedness of coordinate system in definition (curl operator, cross product)

Handedness of coordinate system pure convention (contains no physics), consistency requires:

- $\boldsymbol{\alpha}$, $\boldsymbol{\delta}$: pseudo tensor
- $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$: true tensors



## Symmetry constraints

Turbulence with rotation and stratification

- true tensors: $\delta_{ij}$, $g_i$, $g_i g_j$, $\Omega_i\Omega_j$, $\Omega_i\varepsilon_{ijk}$
- pseudo tensors: $\varepsilon_{ijk}$, $\Omega_i$, $\Omega_i g_j$, $g_i\varepsilon_{ijk}$

Symmetry constraints allow only certain combinations:

$$\alpha_{ij} = \alpha_0(\boldsymbol{g}\cdot\boldsymbol{\Omega})\delta_{ij} + \alpha_1(g_i\Omega_j + g_j\Omega_i)\ , \quad \gamma_i = \gamma_0 g_i + \gamma_1\varepsilon_{ijk}g_j\Omega_k$$
$$\beta_{ij} = \beta_0\delta_{ij} + \beta_1 g_i g_j + \beta_2\Omega_i\Omega_j\ , \quad \delta_i = \delta_0\Omega_i$$

The scalars $\alpha_0 \dots \delta_0$ depend on quantities of the turbulence such as rms velocity and correlation times scale.

- isotropic turbulence: only $\boldsymbol{\beta}$
- $+$ stratification: $\boldsymbol{\beta}+\boldsymbol{\gamma}$
- $+$ rotation: $\boldsymbol{\beta}+\boldsymbol{\delta}$
- $+$ stratification $+$ rotation: $\boldsymbol{\alpha}$ can exist



## Simplified expressions

Assuming $|\boldsymbol{B}'| \ll |\overline{\boldsymbol{B}}|$ in derivation $+$ additional simplification for (quasi) isotropic, non-mirror symmetric, (weakly) inhomogeneous turbulence (see homework assignment):

$$\overline{v_i' v_j'} \sim \delta_{ij},\ \alpha_{ij} = \alpha\delta_{ij},\ \beta_{ij} = \eta_t\delta_{ij}$$

Leads to:

$$\frac{\partial\overline{\boldsymbol{B}}}{\partial t} = \nabla\times\left[\alpha\overline{\boldsymbol{B}} + (\overline{\boldsymbol{v}}+\gamma)\times\overline{\boldsymbol{B}} - (\eta+\eta_t)\nabla\times\overline{\boldsymbol{B}}\right]$$

with the scalar quantities

$$\alpha = -\frac{1}{3}\tau_c\,\overline{\boldsymbol{v}'\cdot(\nabla\times\boldsymbol{v}')},\qquad \eta_t = \frac{1}{3}\tau_c\,\overline{\boldsymbol{v}'^2}$$

and vector

$$\gamma = -\frac{1}{6}\tau_c\nabla\overline{\boldsymbol{v}'^2} = -\frac{1}{2}\nabla\eta_t$$

Expressions are independent of $\eta$ (in this approximation), indicating fast dynamo action!



## Turbulent diffusivity - destruction of magnetic field

Turbulent diffusivity dominant dissipation process for large scale field in case of large $R_m$:

$$\eta_t = \frac{1}{3}\tau_c\,\overline{\boldsymbol{v}'^2} \sim L\,v_{\rm rms} \sim R_m\eta \gg \eta$$

- Formally $\eta_t$ comes from advection term (transport term, non-dissipative)
- Turbulent cascade transporting magnetic energy from the large scale $L$ to the micro scale $l_m$ (advection $+$ reconnection)

$$\eta j_m^2 \sim \eta_t\overline{\boldsymbol{j}}^2 \longrightarrow \frac{B_m}{l_m} \sim \sqrt{R_m}\frac{\overline{B}}{L}$$

Important: The large scale determines the energy dissipation rate, $l_m$ adjusts to allow for the dissipation on the microscale.
Present for isotropic homogeneous turbulence



## Turbulent diamagnetism, turbulent pumping

Expulsion of flux from regions with larger turbulence intensity 'diamagnetism'

$$\gamma = -\frac{1}{2}\nabla\eta_t$$

Turbulent pumping (stratified convection):

$$\gamma = -\frac{1}{6}\tau_c\nabla\overline{\boldsymbol{v}'^2}$$

- Upflows expand, downflows converge
- Stronger velocity and smaller filling factor of downflows
- Mean induction effect of up- and downflow regions does not cancel
- Downward transport found in numerical simulations

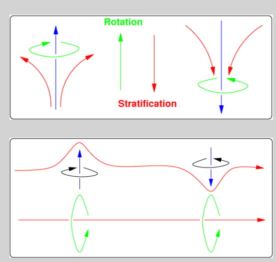
Requires inhomogeneity (stratification)



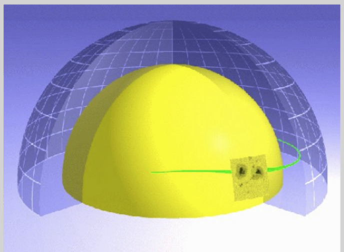
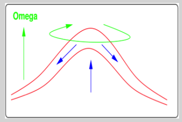
## Kinematic $\alpha$-effect

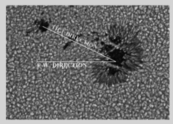
$$\alpha = -\frac{1}{3}\tau_c\,\overline{\boldsymbol{v}'\cdot(\nabla\times\boldsymbol{v}')} \qquad H_k = \overline{\boldsymbol{v}'\cdot(\nabla\times\boldsymbol{v}')}\ \text{ kinetic helicity}$$

Requires rotation $+$ additional preferred direction (stratification)



## Babcock-Leighton $\alpha$-effect



- Similar to kinetic $\alpha$-effect, but driven by magnetic buoyancy
- Leading polarities have larger propability to reconnect across equator with counterpart on other hemisphere
- Polarity of hemisphere = polarity of following sunspots



## Fast or slow dynamo?

Turbulent induction effects require reconnection to operate; however, the expressions

$$\alpha_{ij} = \frac{1}{2}\tau_c \left( \varepsilon_{ikl} \overline{v_k' \frac{\partial v_l'}{\partial x_j}} + \varepsilon_{jkl} \overline{v_k' \frac{\partial v_l'}{\partial x_i}} \right)$$

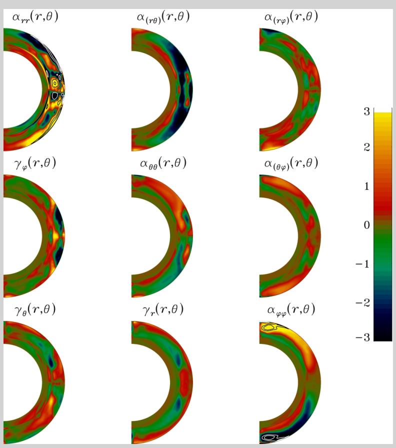
$$\gamma_i = -\frac{1}{2}\tau_c \frac{\partial}{\partial x_k} \overline{v_i' v_k'}$$

$$\beta_{ij} = \frac{1}{2}\tau_c \left( \overline{\boldsymbol{v}'^2} \delta_{ij} - \overline{v_i' v_j'} \right)$$

are independent of $\eta$ (in this approximation), indicating fast dynamo action (no formal proof since we made strong assumptions!)



## How well does this work in practice?

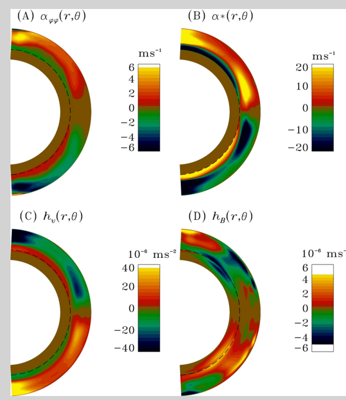



From Racine et al. 2011



## How well does this work in practice?

$\alpha = -\frac{1}{3}\tau_c \overline{\boldsymbol{v}' \cdot (\nabla \times \boldsymbol{v}')}$




From Racine et al. 2011



## Generalized Ohm's law

What is needed to circumvent Cowling's theorem?

- Crucial for Cowling's theorem: Impossibility to drive a current parallel to magnetic field
- Cowling's theorem does not apply to mean field if a mean current can flow parallel to the mean field (since total field non-axisymmetric this is not a contradiction!)

$$\bar{\boldsymbol{j}} = \tilde{\boldsymbol{\sigma}} \left( \overline{E} + \overline{\boldsymbol{v}} \times \overline{\boldsymbol{B}} + \gamma \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} \right)$$

$\tilde{\boldsymbol{\sigma}}$ contains contributions from $\eta$, $\beta$ and $\delta$.
Ways to circumvent Cowling:

- $\alpha$-effect
- anisotropic conductivity (off diagonal elements + $\delta$-effect)



## Meanfield energy equation

$$\frac{d}{dt}\int \frac{\overline{\boldsymbol{B}}^2}{2\mu_0}\, dV = -\mu_0 \int \eta \bar{\boldsymbol{j}}^2\, dV - \int \overline{\boldsymbol{v}} \cdot (\bar{\boldsymbol{j}} \times \overline{\boldsymbol{B}})\, dV + \int \bar{\boldsymbol{j}} \cdot \overline{\mathcal{E}}\, dV$$

- Energy conversion by $\alpha$-effect $\sim \alpha \bar{\boldsymbol{j}} \cdot \overline{\boldsymbol{B}}$
- $\alpha$-effect only pumps energy into meanfield if meanfield is helical (current helicity must have same sign as $\alpha$)!
- Dynamo action does not necessarily require that $\bar{\boldsymbol{j}} \cdot \overline{\mathcal{E}}$ is an energy source. It can be sufficient if $\overline{\mathcal{E}}$ changes field topology to circumvent Cowling, if other energy sources like differential rotation are present (i.e. $\Omega \times \bar{\boldsymbol{j}}$ effect).