Part II : Radiative transfer and Spectropolarimetry

Other opacity and emissivity sources. Features of solar and stellar spectra.

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Summary

So far:

- We saw the RTE allows us to calculate spectra from the distribution of opacity and emissivity.
- In a simple Milne-Eddington atmosphere we have seen that wavelength dependency of opacity translates into wavelength dependency of the spectra:

$$I_\lambda = a + \frac{b}{k_\lambda}$$

- This motivated us to look for ways to calculate opacity and emissivity.

Spectral lines cheat sheet

We started by calculating the emissivity in the line:

$$\eta_\lambda = n_u A_{ul} \frac{hc}{4\pi\lambda} \phi_\lambda$$

where, the Voigt profile is:

$$\phi_\lambda = \phi_0(\lambda) * \phi_{\mathrm{Doppler}}(\lambda) = H(a, \lambda_0, \Delta\lambda_D, \lambda)$$

Where $a = \frac{(\Gamma_C + \Gamma_R)\lambda_0^2}{c\Delta\lambda_D}$ describes damping and $\Delta\lambda_D = \sqrt{\frac{2kT}{m} + v_{\mathrm{turbulent}}^2}$. Similarly to the emissivity, the opacity, with stimulated emission is:

$$\chi_\lambda = (n_l B_{lu} - n_u B_{ul}) \frac{hc}{4\pi\lambda} \phi_\lambda .$$

Where we assumed identical profiles for absorption and emission.

Spectral lines summary

Forget the equations, what is important to remember is:

- Opacity and emissivity in spectral lines change dramatically over small wavelength range → we probe multiple dephts with one spectral line.
- Lines are the spectral features that are sensitive to velocities and magnetic field (broadening/Doppler shift/splitting).
- Always remember that line absorption/emission profile is *local*, and that it eventually leaves imprint in the line profile in the spectrum.
- Sometimes we can gain insight by assuming absorption/emission profile is depth independent, remember:

$$I_\lambda = a + \frac{b}{1 + k_{\mathrm{line}}\phi_\lambda}$$

.

Example: line formation

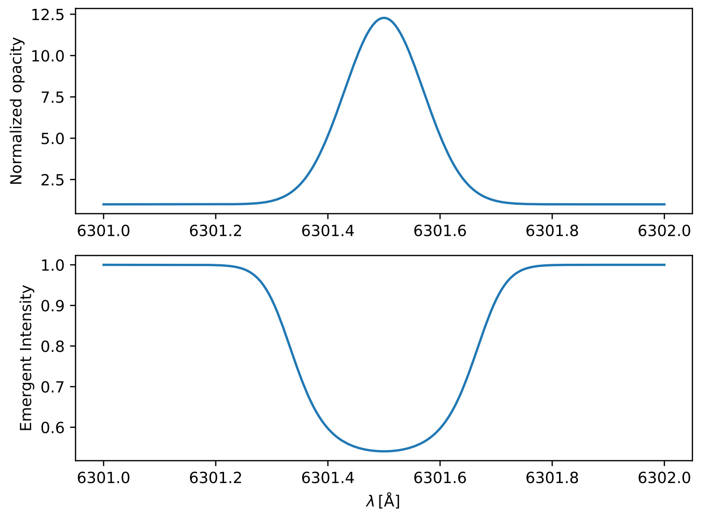


Figure 1: Simplified line formation: Higher opacity samples upper layers where the source function is lower.

Today

- There are other sources of opacity - the Sun is not transparent outside of spectral lines.
- We will cover the most important ones for the Sun specifically, and for the optical wavelenghts.
- We refered to lines as “bound-bound” processes. Similarly we can define “bound-free” and ”free-free” processes.
- Specifically, we will mention H- opacity (famous negative ion of Hydrogen).

Bound-free processes

Akka “photoionization.” Energy of the photon is spent on ionization plus kinetical energy of the electron.

$$\frac{hc}{\lambda} + Z^{i} \rightarrow Z^{i+1} + m_e \frac{v_e^2}{2}$$

Obviously there is a critical wavelength:

$$\lambda_0 = \frac{hc}{E_i}.$$

What happens at lower wavelenths (higher energies)?

Bound-free processes

For hydrogen, they follow Kramer's formula:

$$\sigma_\nu^{\mathrm{bf}} = 2.815 \times 10^{29} \frac{Z^4}{n^5 \nu^3} g_{\mathrm{bf}}.$$

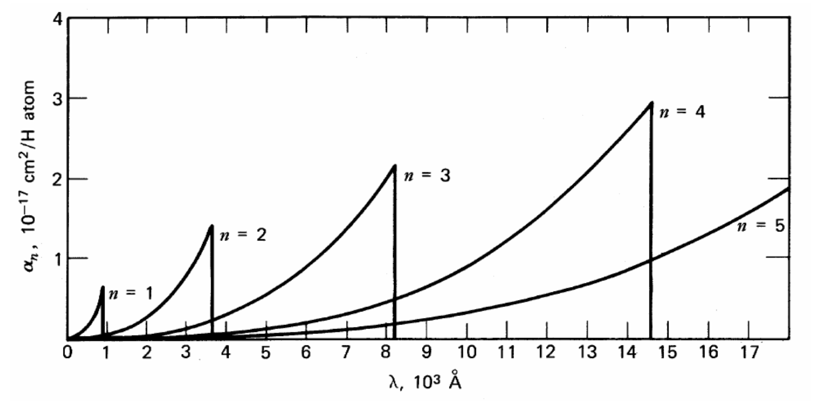


Figure 2: B-F opacity of neutral hydrogen from UV to IR. From Gray (1992)

Some nomenclature

Wavelengths that can ionize H in the ground state, ($\lambda < 912\,\text{Å}$) - Lyman continuum.

... that can ionize H in the first excited state, ($\lambda < 3640\,\text{Å}$) - Balmer continuum.

... that can ionize H in the second excited state, ($\lambda < 8200\,\text{Å}$) - Pashen continuum.

So, for the visible domain we are mostly interested in Pashen continuum opacity, right?

Give us the equations finally!

Absorption should be easy: $\chi_\lambda = n_i \sigma_\lambda$ right?

Believe it or not there is stimulated emission here too:

$$\chi_\lambda^{\mathrm{bf}} = n_i \sigma_\lambda (1 - e^{-hc/\lambda kT})$$

And the (spontaneous) emission (radiative recombination) is:

$$\eta_\lambda^{\mathrm{bf}} \approx \chi_\lambda \times B_\lambda(T)$$

Free-free processes

What are these? Maybe you remember bremsstralung? This would be a free-free emission process. Absorption in this case is (Rybicki & Lightman 1972):

$$\sigma_\lambda^{\mathrm{ff}} = 3.7 \times 10^8 n_{\mathrm{e}} \frac{Z^2}{T^{1/2}(c/\lambda)^3} g_{\mathrm{ff}}$$

Similar scaling as b-f, except there is no cut-off wavelength! Total opacity is:

$$\chi_\lambda^{\mathrm{ff}} = \sigma_\lambda^{\mathrm{ff}} N_{\mathrm{ion}} \left(1 - \mathrm{e}^{-hc/\lambda kT}\right)$$

And the emissivity, again:

$$\eta_\lambda^{\mathrm{bf}} = \chi_\lambda^{\mathrm{ff}} \times B_\lambda(T)$$

Free-free processes

As expected, opacity is proportional to $N_{\mathrm{ion}} n_e$.

Opacity scales with λ^3, but there is no cutoff, it means it is more important in IR than in visible.

Indeed it is. For optical wavelengths you can often safely ignore f-f.

Bound-bound, bound-free and free-free processes are *inelastic* processes.

Thomson scattering

This is an elastic process, photons are preserved. They just change the direction. We have scattering.

Opacity is simple (valid for low energy photons and electrons, that is not for X and γ wavelengths):

$$\chi_\lambda^{\mathrm{T}} = n_e \sigma^{\mathrm{T}} = n_e \times 6.65 \times 10^{-25} cm^2.$$

What about emissivity though?

This time it does not come from thermal pool of the gas.

Thomson scattering

Total emission = total absorption.

$$\eta_\lambda^{\mathrm{T}} = \frac{1}{4\pi} \oint \chi_\lambda^{\mathrm{T}} I(\hat{\Omega}) d\hat{\Omega}.$$

That is:

$$S_\lambda^{\mathrm{T}} = J_\lambda.$$

Source function is equal to the *mean intensity*.

Remember this. This gets tricky.

Rayleigh scattering

Similar treatment as Thomson scattering except this one scales λ.

$$\chi_\lambda^{\mathrm{R}} = N_{\mathrm{scatterers}}\,\sigma(\lambda_0)(\frac{\lambda_0}{\lambda})^4$$

This time λ_0 is some referent wavelength. Scatterers are usually neutral atoms and molecules. We tend to account for H and H_2.

Thomson and Rayleigh scattering are usually not crucial for spectra formation but can be useful when we want to model *scattering polarization*.

They also might prove critical for atmospheres of stars of other spectral classes.

Calculating level populations

What we did not discuss is: How to calculate $n_u, n_l, n_e, N_{\rm ion}$?

Let's assume we know the total number density of a given ion.
Population of the level under consideration:

$$n_i = N_{\rm ion} \frac{g_i e^{-E_i/kT}}{U_{\rm ion}}.$$

This means we have assumed Bolztmann distribution. What establishes Boltzmann distribution? Collisions. So valid in LTE (remember laser!).

g_i is statistical weight, U is the partition function. (Nice polynomial expressions for these are given in, e.g. Barklem & Collet 2016).

Calculating level populations

But how to calculate N_{ion} (let's denote it with N_j from now)? If we assume LTE again, Saha equation is valid:

$$\frac{N_{j+1}}{N_j} = \frac{1}{n_e} \frac{2U_{j+1}}{U_j} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-E_j/kT}$$

So for a an atom a, of the ionization state j:

$$N_{a,j} + N_{a,j+1} n_e f(T) = 0.$$

If we write this for every possible a and j, we are going to get a system of equations which is complemented by:

$$n_e = \sum_a \sum_0 (j+1) N_{a,j+1}.$$

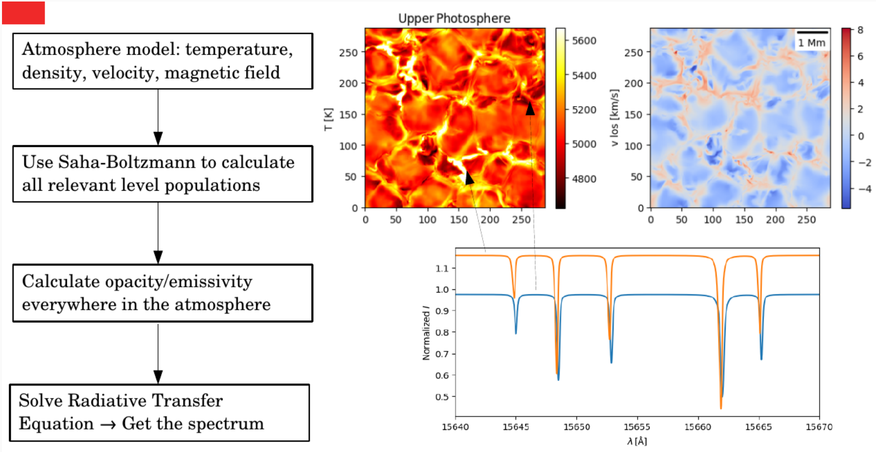
$$\sum_0 Na, j = N_j^{\mathrm{total}}.$$

Chemical equilibrium

This problem is known as solving for chemical (ionization) equilibrium. (Because molecules also influence that, through chemical equilibrium equations). It is a non-linear system, which we usually solve using something like Newton-Rapson iteration.

Actually, this is one of the more cumbersome parts of spectra modeling. This is why most of the people use one and the same opacity package (famous Uppsala opacity package).

LTE line synthesis flowchart



Atmosphere model: temperature, density, velocity, magnetic field

Use Saha-Boltzmann to calculate all relevant level populations

Calculate opacity/emissivity everywhere in the atmosphere

Solve Radiative Transfer Equation → Get the spectrum

MURAM quiet Sun simulation, courtesy of T. Riethmüller

H- opacity

If you calculated some spectra now you would see something is missing.

H- a very special particle, of great importance for opacity in the solar atmosphere.

What is H-? Hydrogen with one extra electron. Its ionization energy is only 0.75 eV, $\lambda = 1.65\,\mu\mathrm{m}$.

H- absorbs in b-f and f-f processes.

H- opacity

How much H- do we really have? Following exercise from Rob Rutten's book (originally Novotny 1962):

Assuming $\log p_e = 1.3,\ T = 6000\,\mathrm{K}$:

$$\log \frac{N(\mathrm{HI})}{N\left(\mathrm{H}^{-}\right)} = -0.1761 - \log P_{\mathrm{e}} + \log \frac{U(\mathrm{HI})}{U\left(\mathrm{H}^{-}\right)} + 2.5 \log T_{\mathrm{e}} - \theta\chi = 7.64$$

$$\frac{N(\mathrm{HI})}{N(\mathrm{H}^{-})} = 4 \times 10^{7}$$

Surely such a small fraction of H- cannot influence opacity!

H- opacity

What does H- compete with?

Remember that in visible domain we want Paschen continuum (ionization from $n = 3$, $E_3 = 12.09\,\mathrm{eV}$):

$$\frac{n_3}{n_1} \approx \frac{n_3}{N(H)} = \frac{g_3 e^{-E_3/kT}}{U} = 6.2 \times 10^{-10}.$$

40 times more H- than neutral hydrogen in $n = 3$!

Example opacity

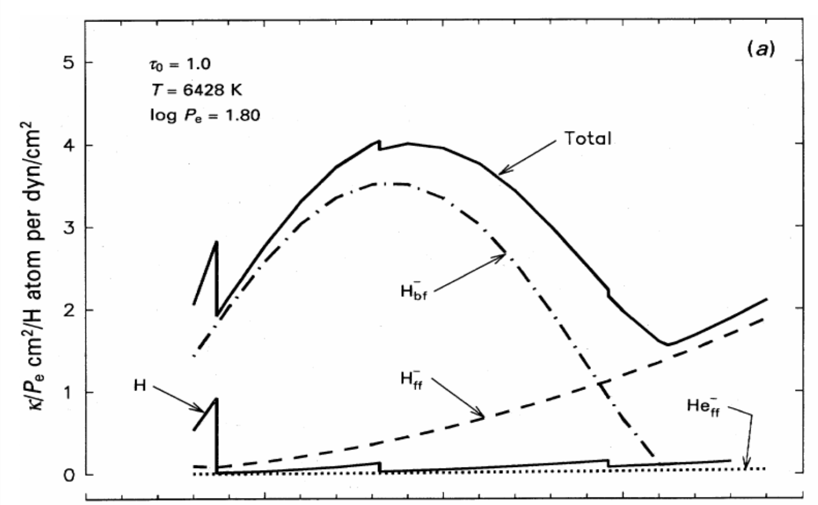


Figure 3: Sources of opacity in the solar atmosphere (Gray 1992).

Example opacity

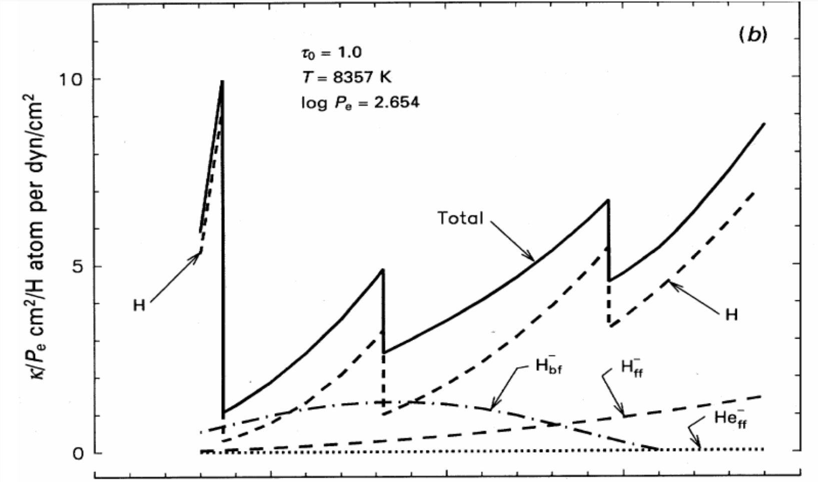


Figure 4: Sources of opacity for a late A dwarf star (Gray 1992).

Example opacity

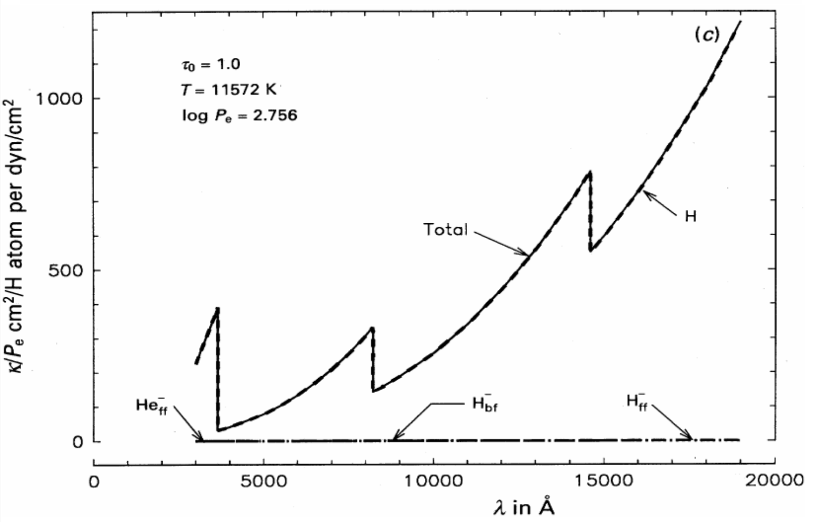


Figure 5: Sources of opacity for a late B dwarf star (Gray 1992).

Example opacity

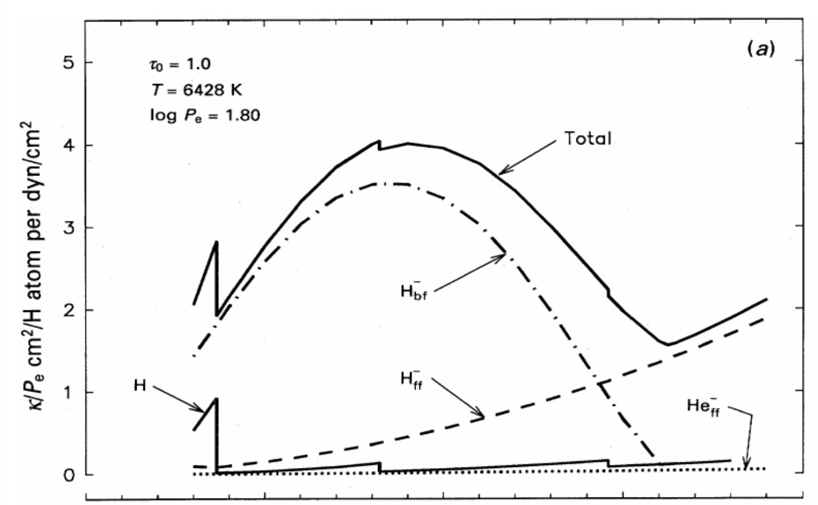
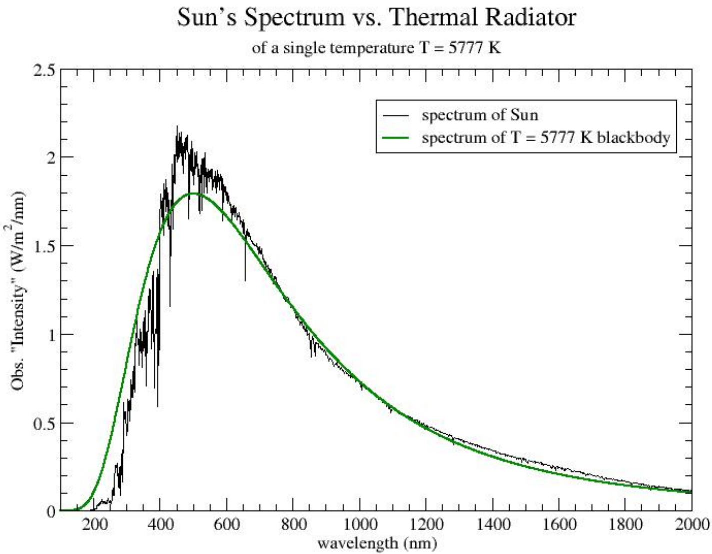


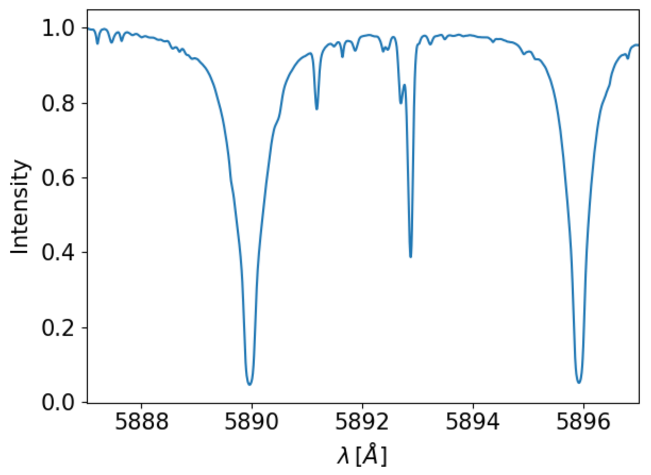
Figure 6: Sources of opacity in the solar atmosphere (Gray 1992).

Solar spectra



Solar spectra - Sodium D lines

Why are these lines so deep?



Solar spectra - Mg h&k lines

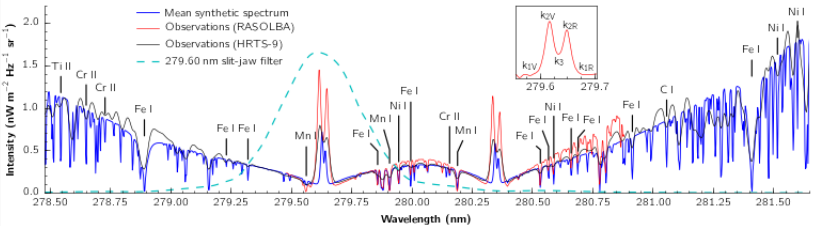


Figure 7: Spectral region around Mg II line observed by IRIS telescope.

Why is this so?

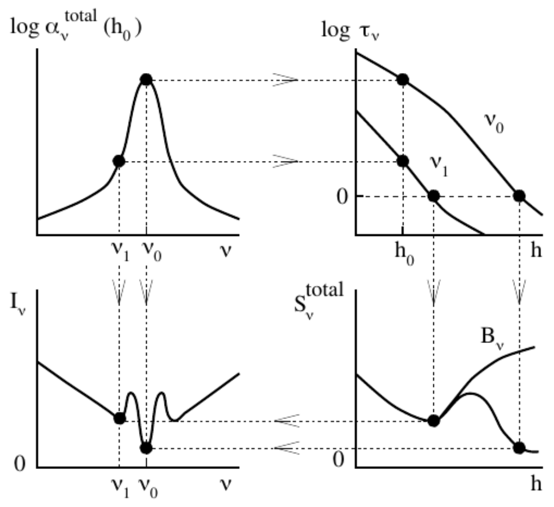


Figure 8: Example from page 20 of Rob Rutten's book.

Conclusions

- We have covered mechanisms of opacity/emissivity that dominate in the solar atmosphere.
- Special attention is on the calculation of ionization and excitation.
- The things are not hard, but are cumbersome. There is a lot of atomic constants to find.
- As rule of a thumb, for LTE lines and continuum: higher the opacity, lower the intensity.
- For strong, NLTE lines, this is more complicated as the source function can have complicated shape.

To-do for the hands-on

In the hands-on we will take a set of calculated opacities/emissivities and calculate the spectrum of a spectral line.

We will start with simple numerical solution of RTE and then synthesize some spectra.

Download python with matplotlib, astropy and numpy.

Data for the exercise is also online.