Part II : Radiative transfer and Spectropolarimetry

Hands-on exercise: Estimating magnetic field from the observations

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Quick Summary

- The emerging polarized radiation is governed by the polarized radiative transfer equation
- Emission is unpolarized
- Absorption is now 4x4 matrix that transforms Stokes components into each other.
- Elements of that matrix are given by Zeeman effect acting on the absorbing atoms.
- Today we are studying simplified solutions.

Polarized RTE

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - B_v(T) \\ Q \\ U \\ V \end{pmatrix}$$

Where, for example:

$$\eta_I = \frac{\eta_0}{2}\left\{\phi_0 \sin^2\theta + \frac{1}{2}\left[\phi_{+1} + \phi_{-1}\right]\left(1 + \cos^2\theta\right)\right\}$$

$$\eta_V = \frac{\eta_0}{2}\left[\phi_{-1} - \phi_{+1}\right]\cos\theta$$

$\eta_0 = \chi\lambda/\chi_{\mathrm{referent}}$

We are often interested in the circular polarization the most

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & 0 & 0 & \eta_V \\ 0 & \eta_I & 0 & 0 \\ 0 & 0 & \eta_I & 0 \\ \eta_V & 0 & 0 & \eta_I \end{pmatrix} \begin{pmatrix} I - B_\nu(T) \\ Q \\ U \\ V \end{pmatrix}$$

And that turns into:

$$\frac{dI}{d\tau} = \eta_I I - \eta_I B + \eta_V V$$

$$\frac{dV}{d\tau} = \eta_V I - \eta_V B + \eta_I V$$

Do the good old, add, subtract thingie:

We are often interested in the circular polarization the most

$$\frac{d(I \pm V)}{d\tau} = (\eta_I \pm \eta_V)(I \pm V - B)$$

$$\eta_I + \eta_V = \eta_0 \phi_r$$

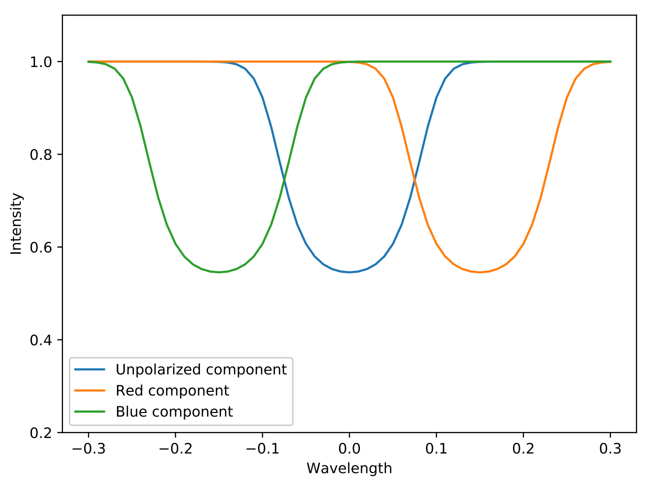
$$\eta_I - \eta_V = \eta_0 \phi_b$$

Let's say $I_r = I + V$, $I_b = I - V$

$$\frac{dI_r}{d\tau} = \eta_0 \phi_r (I_r - B)$$

$$\frac{dI_b}{d\tau} = \eta_0 \phi_b (I_b - B)$$

After solving for the two components



Not that, $I_r(\lambda) = I_0(\lambda - \Delta\lambda_B)$, and $I_b(\lambda) = I_0(\lambda + \Delta\lambda_B)$

Our Stokes profiles would look like this

In the limit of small $\Delta\lambda_B$:

$$I + V = I_r(\lambda) = I_0(\lambda - \Delta\lambda_B) = I_0(\lambda) - I_0'(\lambda)\Delta\lambda_B$$

$$I - V = I_b(\lambda) = I_0(\lambda + \Delta\lambda_B) = I_0(\lambda) + I_0'(\lambda)\Delta\lambda_B$$

So

$$V = -\frac{dI_0}{d\lambda}\Delta\lambda_B$$

Where:

$$\Delta\lambda_B = 4.67 \times 10^{-13}\,\mathrm{\mathring{A}}^{-1}\mathrm{G}^{-1}\,\mathrm{g}\,\lambda_0^2\,\mathrm{B}$$

$$V \propto -\frac{dI}{d\lambda} \times B$$

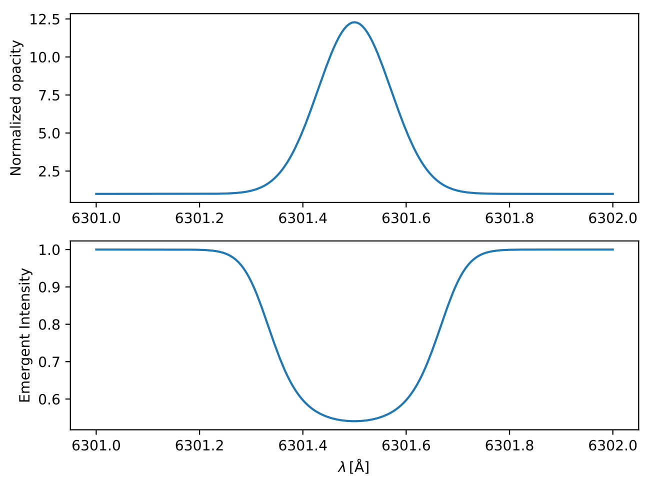
Weak field approximation

The simplest, yet rather reliable way to deduce the mean magnetic field in the observed pixel. We will now use this approximation to estimate magnetic field from a set of observations.

- Line center wavelenghts are 6301.5 and 6302.5 Å(check NIST).
- Lande factors of the lines are: 1.67 and 2.49 (At BASS2000 solar spectrum atlas website there is a pdf atlas with all the Lande factors)
- That is all we need, let's start coding!

Milne Eddington Approximation

$$I_\lambda = a + b/k\phi_\lambda$$



Milne Eddington for polarized radiation

The formulas are now somewhat more complicated, but we are again parametrizing our atmosphere with only a handful of parameters:

- a, b describe source function variation
- k describes strength of the line
- a describes amount of damping
- v describes velocity for Doppler broadening
- v_{los} describes line-of-sight velocity
- $\vec{B}$ three numbers describing magnetic field vector
- And, in the implementation I suggested, there are μ.

Milne Eddington fitting

- Set-up line parameters
- Using Milne forward module. Define your merit function (usually χ^2).
- Minimize merit function w.r.t to model parameters (previous slide).
- Repeat for all the pixels.
- You just did an inversion.