Part II : Radiative transfer and Spectropolarimetry

Polarized radiation and Zeeman effect

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Quick Summary

- So far we modeled specific intensity, i.e. total amount of photons emitted/transmitted/absorbed/detected
- We are now going to study the light *polarization*, that is, the phase, and the orientation of the electric (magnetic) field vector.
- We can approach the problem from classical or from quantum aspect.

But before that, why?

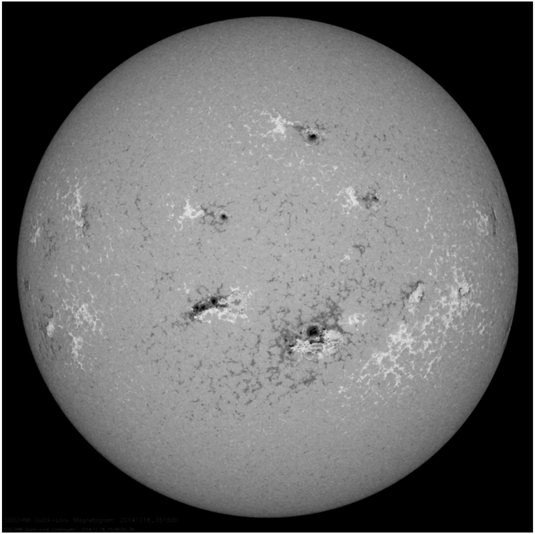


Figure 1: An HMI magnetogram

Classical approach

- Spectral line is treated as a harmonic oscillator.
- Magnetic field interacts with the harmonic oscillator through the Lorenz force.
- Solving differential equation for electron behavior, we obtain both spectral distribution of the light as well as absorption and dispersion coefficients.
- It requires knowledge of classical electrodynamics
- Referent book: "Introduction to spectropolarimetry" by Jose Carlos del Toro Iniesta.

Quantum approach

- We analyze behavior of the Hamiltonian of the atom in the presence of the magnetic field.
- This explains Zeeman splitting.
- Then we can employ quantum electrodynamics to understand photon-atom interaction and the absorption/emission of polarized light.
- Able to explain more exotic effects (scattering polarization, Hanle effect, orientation to alignment conversion).
- Referent book: "Polarization in spectral lines" by Egidio Landi Degl'Innocenti

Plan for upcoming four lectures

- We can't fully go through either of these approaches: time issues, focus on more observable aspects, I don't know QED.
- We will phenomenologically discuss Zeeman effect and why it causes polarization in spectral lines.
- We will see how to calculate *polarized spectrum* from a given model atmosphere.
- We will then apply some methods to the observational data.
- Then we will try to solve so called inverse problem and see what spectropolarimetric inversion is.
- And finally have a review of important results.

Polarization of EM waves

$$E_{x,y}(z,t) = E^0_{x,y} e^{i(kz-\omega t+\delta_{x,y})}$$

Monochromatic wave. Always 100% polarized.

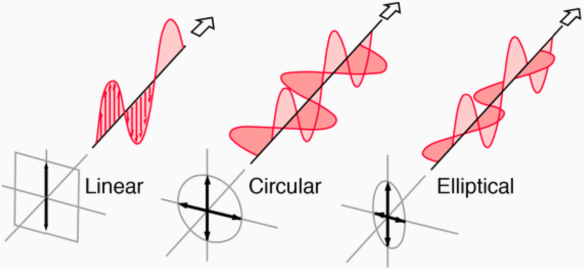


Figure 2: taken from http://hyperphysics.phy-astr.gsu.edu

Stokes Components

$$E_{x,y}(z,t) = E^0_{x,y} e^{i(kz - \omega t + \delta_{x,y})}$$

Monochromatic wave. Always 100% polarized.

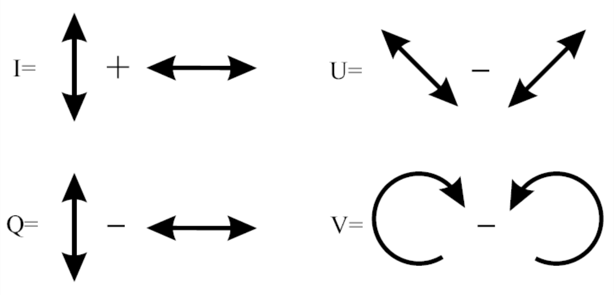


Figure 3: taken from "Introduction to Spectropolarimetry"

What does this mean?

Stokes components can be understood as "instrumental definition." How do we measure the polarization in practice?

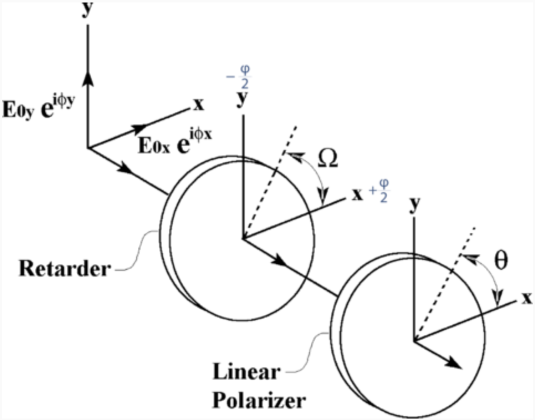


Figure 4: taken from www.fiberoptics4sale.com

Measuring Stokes components

Let's recast the original expression using complex amplitudes. And, let's consider only one point, so we can get rid of kz factor in the exponent.

$$E_x(t) = E_{0,x} e^{i\delta_x} e^{i\omega t}$$

$$E_y(t) = E_{0,y} e^{i\delta_y} e^{i\omega t}$$

Now what are the elements doing. Retarder (quarter wave plate) changes the phase:

$$E_{x,y} \rightarrow E_x e^{i\varphi/2}, \; E_y e^{-i\varphi/2}$$

Linear polarizer "projects" on that axis:

$$E = E_x \cos\theta + E_y \sin\theta$$

So if we have a retarder, followed by linear polarizer

:

$$E(\phi, \theta) = E_x e^{i\frac{\varphi}{2}} \cos\theta + E_y e^{-i\frac{\varphi}{2}} \sin\theta$$

What is the intensity? $E\,E^*$. So:

$$I(\varphi, \theta) = E_x E_x^* \cos^2\theta + E_y E_y^* \sin^2\theta + E_x^* E_y e^{-i\varphi} \sin\theta\cos\theta + E_x E_y^* e^{i\varphi} \sin\theta\cos\theta$$

Or, using some trigonometric identities that we are too old to remember:

$$I(\varphi, \theta) = \frac{1}{2}[\left(E_x E_x^* + E_y E_y^*\right) + \left(E_x E_x^* - E_y E_y^*\right) \cos 2\theta$$
$$+ \left(E_x E_y^* + E_y E_x^*\right) \cos\varphi \sin 2\theta + i\left(E_x E_y^* - E_y E_x^*\right) \sin\varphi \sin 2\theta]$$

We will call these combinations in the parenthesis *Stokes parameters*, (I, Q, U, V)

Stokes parameters

:

$$I = E_x E_x^* + E_y E_y^* \,;\; Q = E_x E_x^* - E_y E_y^* \tag{1}$$

$$U = E_x E_y^* + E_y E_x^* \,;\; V = i\left(E_x E_y^* - E_y E_x^*\right) \tag{2}$$

To measure I, Q, U, set the retarder to $\phi = 0$, and the polarizer to $0^o, 45^o, 90^o$, respectivelly. What do we get?

$$I_1 = I(0,0) = \frac{1}{2}(I + Q)$$

$$I_2 = I(0,45^o) = \frac{1}{2}(I + U)$$

$$I_3 = I(0,90^o) = \frac{1}{2}(I - Q)$$

So, $I = I_3 + I_1,\ Q = I_1 - I_3,\ U = 2I_2 - I_1 - I_3$

Stokes parameters

:

$$I = E_x E_x^* + E_y E_y^* \; ; \; Q = E_x E_x^* - E_y E_y^* \tag{3}$$

$$U = E_x E_y^* + E_y E_x^* \; ; \; V = i \left(E_x E_y^* - E_y E_x^* \right) \tag{4}$$

To measure V, set the retarder to $\phi = \pi/2$ (quarter wave plate), and the polarizer to 45^o, respectivelly. What do we get?

$$I_1 = I(0,0) = \frac{1}{2}(I + Q)$$

$$I_2 = I(0, 45^o) = \frac{1}{2}(I + U)$$

$$I_3 = I(0, 90^o) = \frac{1}{2}(I - Q)$$

$$I_4 = (90^o, 45^o) = \frac{1}{2}(I + V)$$

So, $I = I_3 + I_1, \; Q = I_1 - I_3, \; U = 2I_2 - I_1 - I_3, \; V = 2I_4 - I_1 - I_3$

In practice

You usually independently measure $I - U$ and $I - V$ the same way you measure $I - Q$, so you can reconstruct separately each of the Stokes parameters.

This is called *modulation*. Modulation needs to be fast because you want to be sure you are seeing one and the same scene in the Sun.

For ground-based observations, modulation has to be even faster, because you want to avoid *seeing effects*.

Spectral line polarization

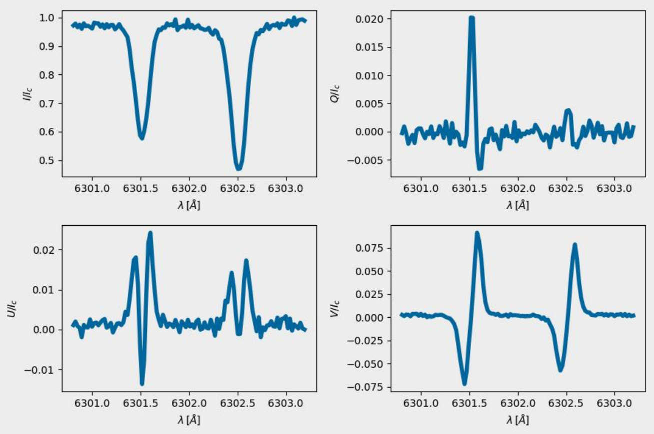


Figure 5: Spectral line polarization in Fe I 6300 Å lines, observed by HINODE/SOT.

How do we model these?

The polarization is always a consequence of some sort of anisotropy.

In this case, that anisotropy comes from the magnetic field.

At the lower boundary, we can assume light is unpolarized. We want to model how the intensity and the polarization change as the light is transported outward.

Mueller matrices

Influence of any medium (e.g. slab of gas, optical element, or so) on polarized rady can be represented by a Mueller matrix, which is, naturally, 4x4 matrix.

$$\boldsymbol{I}' = \hat{M}\boldsymbol{I}$$

For example, horizontal linear polarizer has:

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

What does this mean?

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \frac{1}{2}\begin{pmatrix} I+Q \\ I+Q \\ 0 \\ 0 \end{pmatrix}$$

The radiation truly is completely linearly polarized! Nice!

For practice, tell me how would purely absorbing slab look like?

Solar atmosphere

In the solar atmosphere we will call Mueller matrix defined per unit length *absorption matrix* or *absorption tensor*. Then:

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = -\begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

The coefficients η and ρ follow from the nature of the microscopic processes involved in the polarization, in this case selective absorption and dispersion of the light.

NOTE: η was emission before, now it is absorption, sorry for this!

Why do off-diagonal elements exist - Zeeman effect

Magnetic fields are to astrophysics what sex is to psychology.
(van der Hulst)

Magnetic field changes the Hamiltonian of the atom by (Most of the following material is from the Introduction to Spectropolarimetry by J.C. del Toro Iniesta):

$$\mathbf{H}_B = \boldsymbol{\mu} \cdot \boldsymbol{B} + O\left(B^2\right)$$

where:

$$\boldsymbol{\mu} = \mu_0(\boldsymbol{J} + \boldsymbol{S})$$

And then the diagonal, which gives us possible energy values is:

$$\langle lsjm | \mathbf{H}_B | \mathrm{sj}\, m \rangle = mg\mu_0 B = mgh\nu_{\mathrm{L}}$$

Where g is so called Landé factor, that describes the sensitivity of the level to the magnetic field (can be calculated for some simpler cases, such as LS coupling).

And then the levels get split!

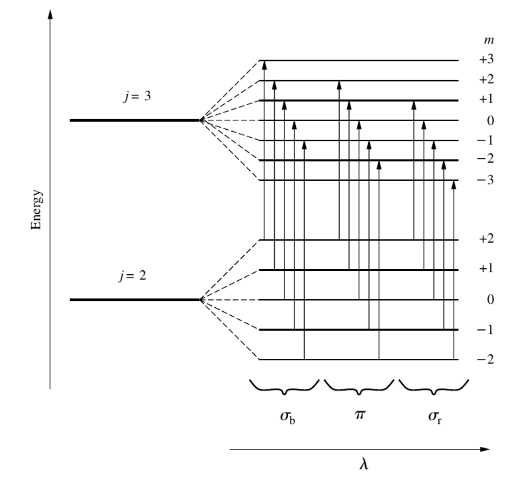


Figure 6: Zeeman splitting where the upper level has J=3 and lower J=2. There are total of 15 sub-transitions.

Ok, and the polarization?

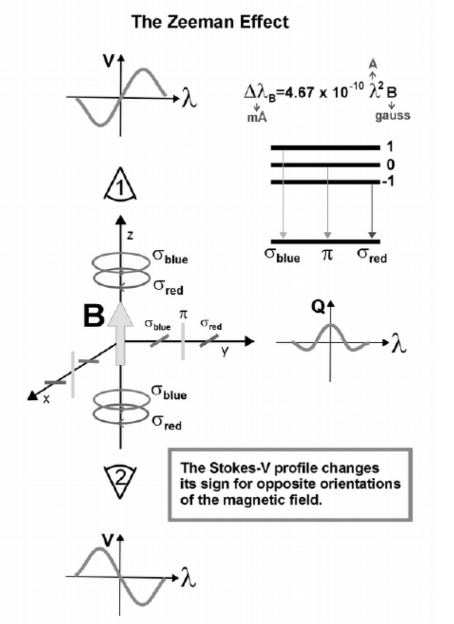
Some reminders:

- Photons have spin one and to possible states, -1 and 1.
- Quantization axis for angular momentum is the direction of the magnetic field.
- Different Δm transitions correspond to different changes of the angular momentum. Only allowed $\Delta m = 0, \pm 1$

Angular momentum has to be conserved. Photon going in the direction of $\boldsymbol{B}$ has unit angular momentum (think of the angular momentum as projection to the axis).

Perpendicular to $\boldsymbol{B}$ - zero. So all three transitions are possible.

The simplest example - Zeeman triplet



Now, the whole atmosphere

These were the things happening in the micro-world, to get the spectrum we solve polarized radiative transfer equation:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - B_v(T) \\ Q \\ U \\ V \end{pmatrix}$$

Where, for example:

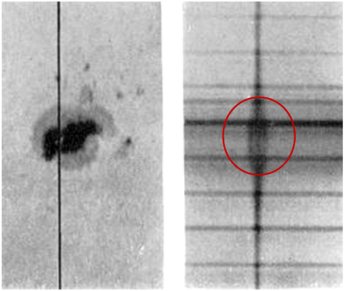
$$\eta_I = \frac{\eta_0}{2}\left\{\phi_0 \sin^2\theta + \frac{1}{2}\left[\phi_{+1} + \phi_{-1}\right]\left(1 + \cos^2\theta\right)\right\}$$

$$\eta_V = \frac{\eta_0}{2}\left[\phi_{-1} - \phi_{+1}\right]\cos\theta$$

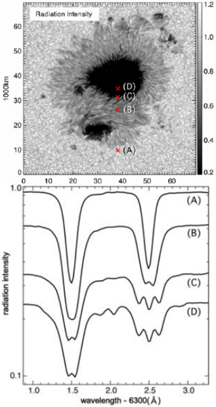
$\eta_0 = \chi\lambda / \chi_{\mathrm{referent}}$

Simplest - neglect polarization

The diagonal and the splitting still remains.



G.E. Hale, F. Ellerman, S.B. Nicholson, and A.H. Joy (ApJ, 1919)



Credits: Yukio Katsukawa

We are often interested in the circular polarization the most

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & 0 & 0 & \eta_V \\ 0 & \eta_I & 0 & 0 \\ 0 & 0 & \eta_I & 0 \\ \eta_V & 0 & 0 & \eta_I \end{pmatrix} \begin{pmatrix} I - B_\nu(T) \\ Q \\ U \\ V \end{pmatrix}$$

And that turns into:

$$\frac{dI}{d\tau} = \eta_I I - \eta_I B + \eta_V V$$

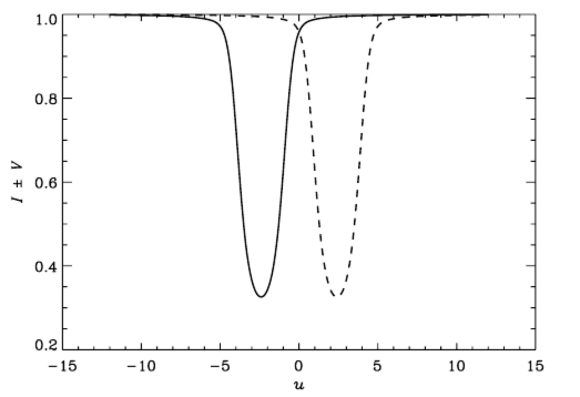
$$\frac{dV}{d\tau} = \eta_V I - \eta_V B + \eta_I V$$

Do the good old, add, subtract thingie:

We are often interested in the circular polarization the most

$$\frac{d(I \pm V)}{d\tau} = (\eta_I \pm \eta_V)(I \pm V - B)$$

These are two RTE that we can solve separately. (Boundary contidion $I = B$, $V = 0$), and we get something like this:



Our Stokes profiles would look like this

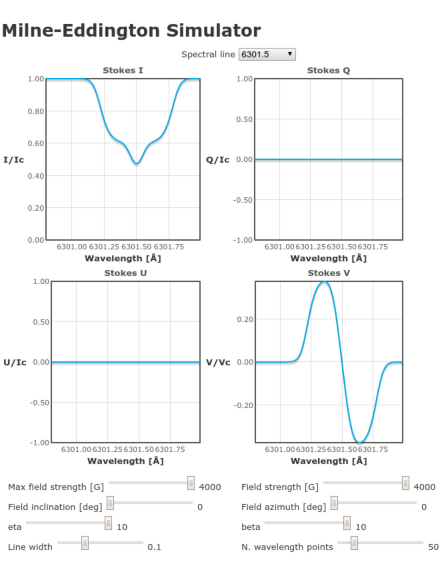


Figure 7: M-E simulator written by Andrés Asensio Ramos.

Milne Eddington Approximation

We again assume that the thermal, unpolarized, source function behaves like $S = S0 + S1\tau$.

$$
\begin{aligned}
I(0) &= S_0 + \Delta^{-1}\eta_I\left(\eta_I^2 + \rho_Q^2 + \rho_U^2 + \rho_V^2\right)S_1 \\
Q(0) &= -\Delta^{-1}\left[\eta_I^2\eta_Q + \eta_I\left(\eta_V\rho_U - \eta_U\rho_V\right) + \rho_Q\left(\eta_Q\rho_Q + \eta_U\rho_U + \eta_V\rho_V\right)\right]S_1 \\
U(0) &= -\Delta^{-1}\left[\eta_I^2\eta_U + \eta_I\left(\eta_Q\rho_V - \eta_V\rho_Q\right) + \rho_U\left(\eta_Q\rho_Q + \eta_U\rho_U + \eta_V\rho_V\right)\right]S_1 \\
V(0) &= -\Delta^{-1}\left[\eta_I^2\eta_V + \eta_I\left(\eta_U\rho_Q - \eta_Q\rho_U\right) + \rho_V\left(\eta_Q\rho_Q + \eta_U\rho_U + \eta_V\rho_V\right)\right]S_1
\end{aligned}
$$

Where:

$$\Delta = \eta_I^2\left(\eta_I^2 - \eta_O^2 - \eta_U^2 - \eta_V^2 + \rho_Q^2 + \rho_U^2 + \rho_V^2\right) - \left(\eta_Q\rho_Q + \eta_U\rho_U + \eta_V\rho_V\right)^2$$

All the elements of absorption matrix, for reference

$$\eta_I = 1 + \frac{\eta_0}{2}\left\{\phi_\mathrm{p}\sin^2\theta + \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\left(1 + \cos^2\theta\right)\right\}$$

$$\eta_Q = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

$$\eta_U = \frac{\eta_0}{2}\left\{\phi_\mathrm{p} - \frac{1}{2}\left[\phi_\mathrm{b} + \phi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

$$\eta_V = \frac{\eta_0}{2}\left[\phi_\mathrm{r} - \phi_\mathrm{b}\right]\cos\theta$$

$$\rho_Q = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\cos 2\varphi$$

$$\rho_U = \frac{\eta_0}{2}\left\{\psi_\mathrm{p} - \frac{1}{2}\left[\psi_\mathrm{b} + \psi_\mathrm{r}\right]\right\}\sin^2\theta\sin 2\varphi$$

$$\rho_V = \frac{\eta_0}{2}\left[\psi_\mathrm{r} - \psi_\mathrm{b}\right]\cos\theta$$

Take away messages

- We describe the polarization with the Stokes vectors.
- Matter can influence the Stokes vector, by selectively absorbing/retarding.
- Zeeman effect causes this in the solar atmosphere.
- Individual Δm transitions have different polarizations, breaking energy degeneracy causes them to show up.
- Some simple ways to model polarization include focusing on LOS magnetic field, assuming Milne-Eddington approximation, etc.
- Next lecture we will work with the data and employ some of these.