Part II : Radiative transfer and Spectropolarimetry

Hands-on exercise: calculate depth of formation and the spectra of spectral lines

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Up to now

- We studied relationship between depth-dependent opacity/emissivity and emergent spectra - RTE
- We saw how to calculate, opacities and emissivites in spectral lines.
- And how to calculate the opacity and emissivity in the continuum.

How do physical parameters influence spectra?

- Temperature: ionization, excitation, line broadening, collisions (weakly)
- Pressure/density: ionization, collisions (strongly), total number of particles
- Velocity: spectral line shifts
- Microturbulent velocity: ad hoc parameter, influences line width
- Magnetic field: line splitting, polarization (still not there)

Today

- We are going to start from a pre-computed opacities and emissivities on a height mesh.
- We are going to turn them into optical depths and the source functions.
- We will write our own formal solver and use it to calculate the emergent spectra.
- Then we will compare the emergent intensities to source functions at $\tau_\lambda \approx 1$

Make sure

- You can read fits files
- You have a handy way of quickly plotting things. My favorite is something like:

```
import matplotlib.pyplot as plt
plt.ion()
plt.clf()
```

- Then whatever you plt.plot() shows up immediately.
- You can perform basic operations (exponent, log, power).

Calculating optical depths

We have χ_λ for different h (z).

We need τ_λ. We can assume that the top of the atmosphere is 0 at each wavelength, what then?

Simplest would be to say:

$$\tau_i = \tau_{i-1} + \frac{\chi_i + \chi_{i-1}}{2} \times (h_i - h_{i-1}).$$

Maybe better would be linear interpolation in $\log \chi$ (why?).

$$\tau_i = \tau_{i-1} + \sqrt{\chi_i \times \chi_{i-1}} \times (h_i - h_{i-1}).$$

Now we have optical depths at all the wavelenghts

Now we can already use:

$$I_\lambda \approx S_\lambda(\tau_\lambda = 1)$$

to estimate spectrum.

Now let's go do the real thing:

Integrating RTE

Now we are going to integrate RTE, outward (starting from the bottom of the atmosphere). Recall:

$$I_i = I_{i+1} e^{-\Delta} + \int_0^{\Delta} S(t) e^{-t} dt.$$

But, we don't know the behavior of S(t) between the two points!

We can assume it is linear $a + bt$. Let's solve this analytically:

$$\int_0^{\Delta} a e^{-t} dt = -a\, e^{-t}|_0^{\Delta}. = a(1 - e^{-\Delta}).$$

Integrating RTE

Second part (integrating by parts):

$$\int_0^{\Delta} bte^{-t}dt = -b\,t\,e^{-t}|_0^{\Delta} + \int_0^{\Delta} be^{-t}dt = \ldots$$

$$-b\,\Delta\,e^{-\Delta} + b(1 - e^{-\Delta})$$

But what are a and b? Well, we do know S_{i+1} and S_i.. $a = S_i$, $b = \frac{S_{i+1} - S_i}{\Delta}$.

Integrating RTE

$$I_i = I_{i+1} e^{-\Delta} + w_{i,i+1} S_{i+1} + w_{i,i} S_i$$

where

$$w_{i,i+1} = -e^{-\Delta} + \frac{1}{\Delta}(1 - e^{-\Delta})$$

$$w_{i,i} = 1 - \frac{1}{\Delta}(1 - e^{-\Delta})$$

Admire these two for a while...

Scheme

- Using opacity and height, calculate optical depth scale for each wavelength. Note that CGS units do not matter because τ_λ is dimensionless.
- Write a function that, given Δ, S_i, S_{i+1} and I_{i+1} caculates I_i (short characteristics formal solver).
- Agree on the boundary condition.
- Starting from the bottom of the atmosphere, use the function for the formal solution to, layer-by-layer, wavelength-by-wavelength, calculate intensity everywhere.
- Plot emergent intensity and discuss.
- If there is time, calculate contribution functions (next slide).

Contribution functions

Contribution function (let's denote it C_λ) is basically the function under the integral weighted by the integration step ($\propto \tau_\lambda$):

$$C(\tau, \lambda) = S(\tau) e^{-\tau_\lambda} \tau_\lambda$$

These tell us, in a way, where the line "forms."