Part II : Radiative transfer and Spectropolarimetry

More on model atmospheres and spectra calculation

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Quick Summary

- We try to model specific monochromatic intensity emerging from the atmosphere. For that we solve RTE.
- For that, in turn, we need boundary conditions, opacity and emissivity throughout the atmosphere.
- Opacity and emissivity depend on all the atmospheric parameters, either directly or indirectly.
- The tricky part are the level populations.

Refresher: How do physical parameters influence spectra?

- Temperature: ionization, excitation, line broadening, collisions (weakly)
- Pressure/density: ionization, collisions (strongly), total number of particles
- Velocity: spectral line shifts
- Microturbulent velocity: ad hoc parameter, influences line width

What is a model atmosphere?

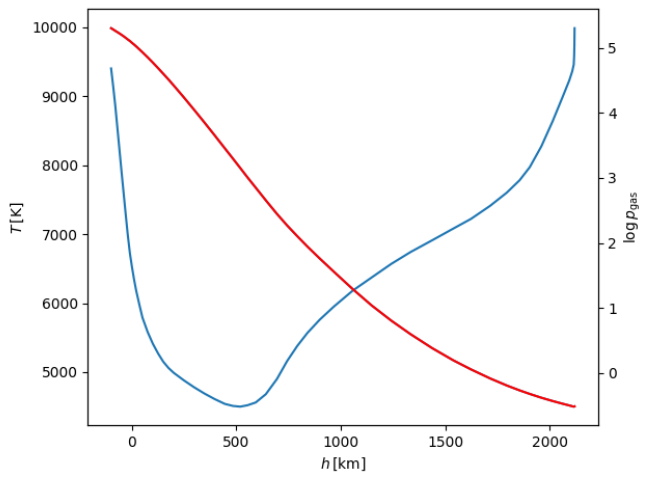

Figure 1: Temperature and gas pressure stratification in FALC model atmosphere (Avrett et al. 1993)

But, frankly, it is this...

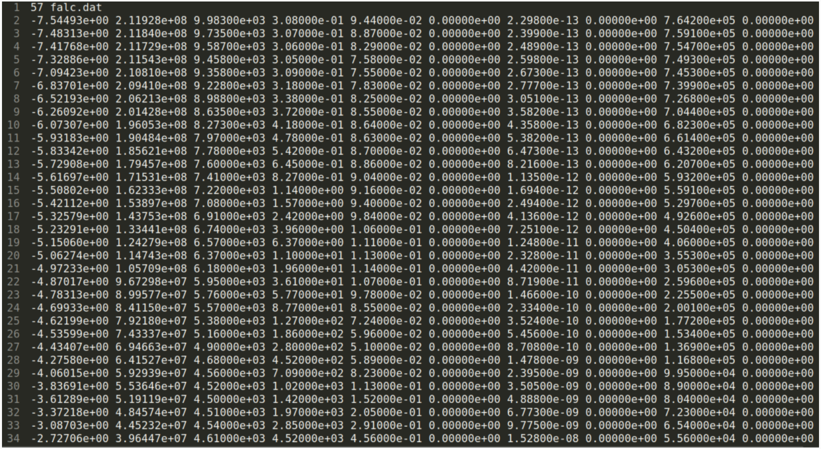

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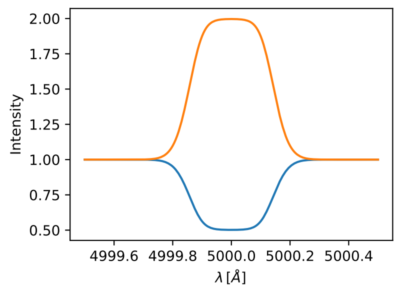
Figure 2: Tabulated values from FALC model atmosphere (Avrett et al. 1993)

And it might turn out that this is enough!

$$I_\lambda = I_{0,\lambda} e^{-\tau_\lambda} + S_\lambda (1 - e^{-\tau_\lambda})$$

Let's set: $\tau_\lambda = \tau \phi_\lambda$, $\tau = 10$, $S = const$.



(You can't always get what you want, but if you try sometime you might find, you get you what you need.)

But let’s go back to the tabulated model

We have temperature, gas pressure (maybe even e^- pressure), microturbulent and line-of-sight velocity for each point in the atmosphere...

If we want to calculate the spectrum, what comes next?

(No, you tell me.)

Modeling the spectra

I want opacity and emissivity, but I have only this tabulated values, what can I do?

Let's try calculating the $H\alpha$ line. Let's try as hard as we can!

You might remember that opacity in the line looks something like:

$$\chi_\lambda = (n_l B_{lu} - n_u B_{ul}) \frac{hc}{4\pi\lambda_0} \phi(\lambda).$$

And the emissivity is:

$$\eta_\lambda = n_u A_{ul} \frac{hc}{4\pi\lambda_0} \phi(\lambda).$$

Modeling the spectra

Ah yes, for $H\,\alpha$, n_u will be n_3, and n_l will be n_2, but before that, where do I find these Einstein coefficients?

A_{ul} is usually given in nice websites with atomic data. I like NIST website.

B_{ul} and B_{lu}? Do your homework! :-)

For $H\alpha$, $A_{32} = 4.41 \times 10^{-7}\mathrm{s}^{-1}$.

Can anyone remind me how to calculate n_i?

$$n_i = N_j \frac{g_i e^{-E_i/kT}}{U_j}$$

where

$$U_j = \sum_i g_i e^{-E_i/kT}$$

So for our levels it seems like $E_2 = 10.2\,\mathrm{eV}$, $E_3 = 12.08\,\mathrm{eV}$, and $g_2 = 8$, $g_3 = 18$.

We can more or less safely assume $U_j = 2$.

Nice, but N_j?

We got total gass pressure up there in the model... what can we do with that? $p_{\mathrm{gas}} = N_t k T$, where N_t is **total number of all massive particles:** atoms,ions,molecules,electrons.

What now? Let's assume we only have neutral, ionized H and electrons (still, do your homework!):

$$\frac{N(H^+)}{N(H)} = \frac{1}{n_e}...$$

$$n_e = N(H^+)$$

$$n_e + N(H^+) + N(H) = N_t$$

Now we solve this system and finally the ingredients are there...

Other sources

Even though we are considering only Hydrogen we need to take into account other opacity/emissivity sources:

- Bound-free and free-free hydrogen opacity.
- Thomson scattering on free electrons.
- Rayleigh scattering on neutral hydrogen
- H^- opacity!

That is a lot of work...

This is why writing a spectra synthesis code is cumbersome. A lot of debugging and optimization too..

Let's say we handled this somehow. Or we used an opacity package (say, famous Uppsala opacity package by Gustafsson, B. 1973).

We now have tabulated opacities and emissivities for our model atmosphere and wavelengths we want.

Now we can do what we did in the hands-on last week, and synthesize the spectrum.

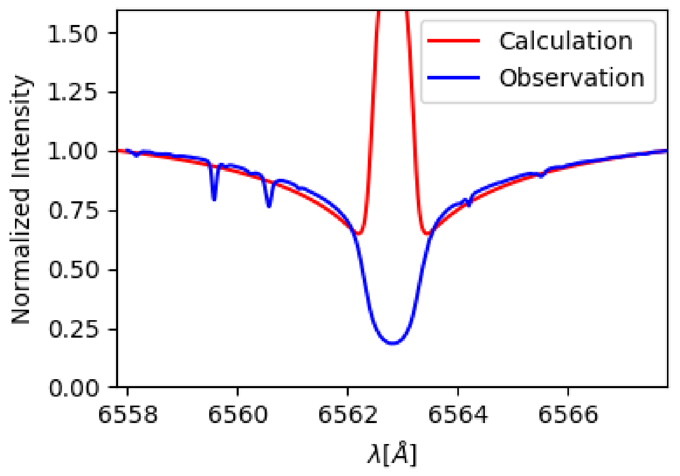
Before we continue, ask yourself...

Am I interpreting spectra? Can I use simpler diagnostics? Examples:

- Line center red/blue shift → velocity.
- Or maybe bisectors?
- Do I want to measure temperature, and if yes how accurately?

Or am I studying the line itself, and want to understand each detail,in that case...

We need to try harder



Put in the magic...

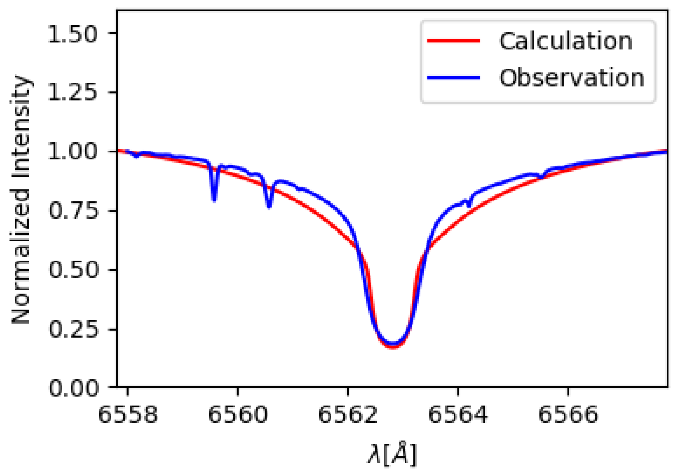


Figure 3: Far from perfect but much better

- Why was the calculated line in the emission, when it is clearly in the absorption in reality?
- What did we do to turn in the absorption?
- What did we do wrong?
- **Nothing, our model was wrong.**

Line formation “heights”

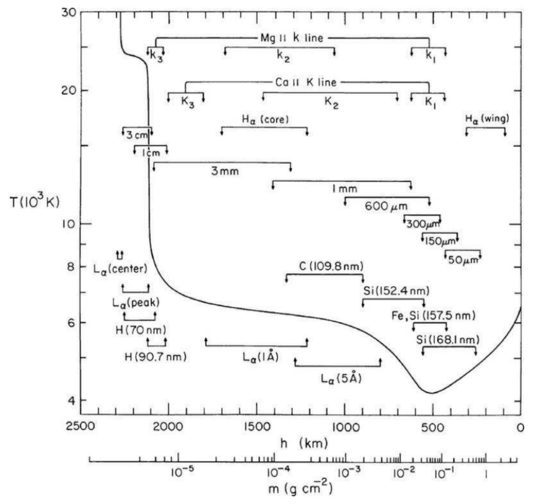


Figure 4: Semi-empirical VALC model atmosphere. From Vernazza et al. 1981

How do we get these?

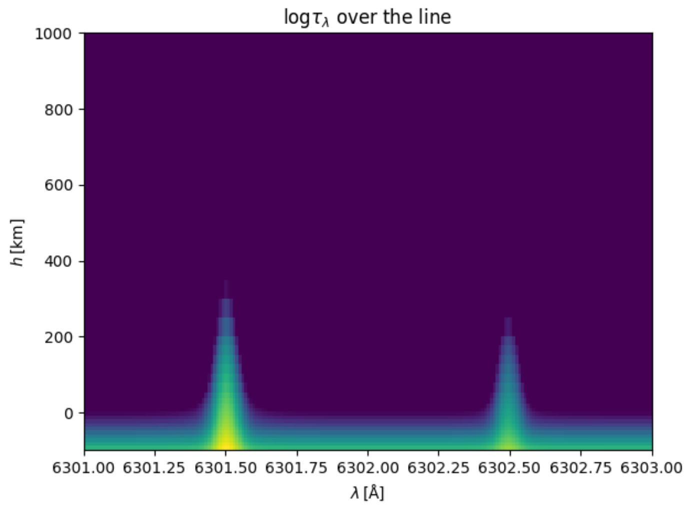


Figure 5: Optical depth in the lines - Fe I 6300

How do we get these?

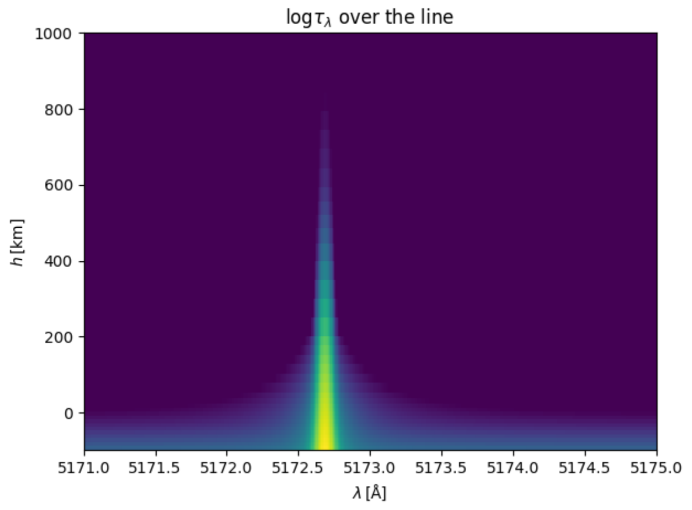


Figure 6: Optical depth in the lines - Mg I 5173

Contribution functions

$$I^{+}\lambda = \int_0^{\infty} S(\tau_\lambda) e^{-\tau_\lambda} d\tau_\lambda$$

Contribution function (let's denote it C_λ) is basically the function under the integral weighted by the integration step ($\propto \tau_\lambda$):

$$C(\tau, \lambda) = S(\tau) e^{-\tau_\lambda} \tau_\lambda$$

These tell us, in a way, where the line "forms." Similar to the optical depth estimation, but also takes the source function into account.

Contribution functions

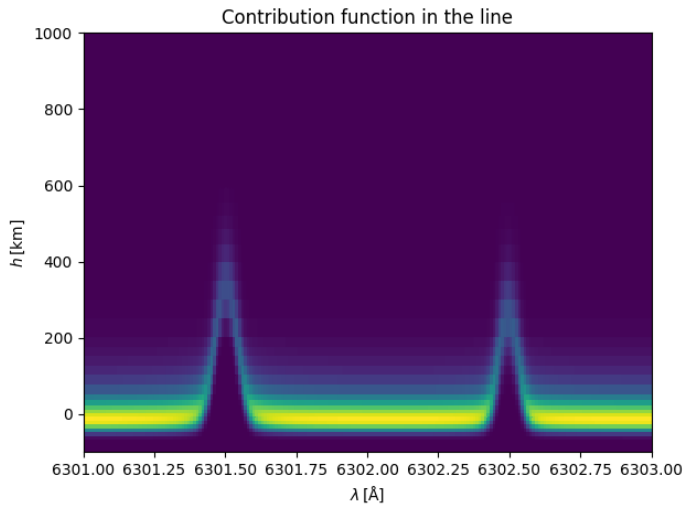


Figure 7: Contribution function - Fe I 6300

Contribution functions

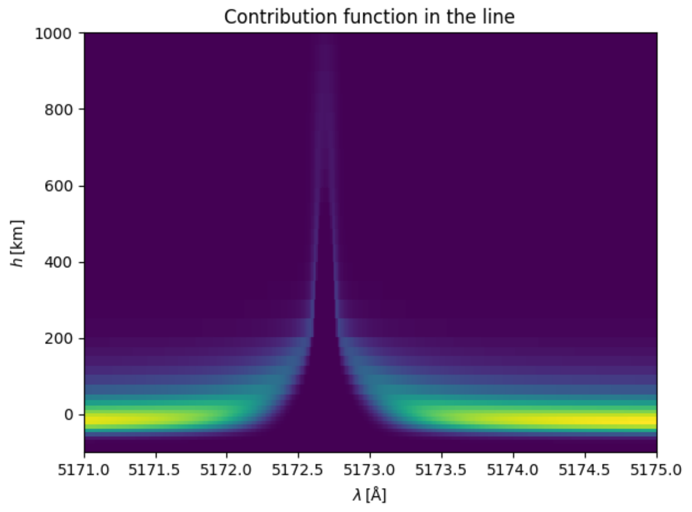


Figure 8: Contribution function - Mg I 5173

Remember the rule of thumb

- $I_\lambda^+ \approx S(\tau_\lambda = 1)$.
- More opaque wavelengths sample the source function higher in the atmosphere.
- $S \propto T$.
- The lines that are “not too opaque” are in absorption because they see the temperature decrease.
- Stronger lines should turn to emission at some point.

And some of them do...

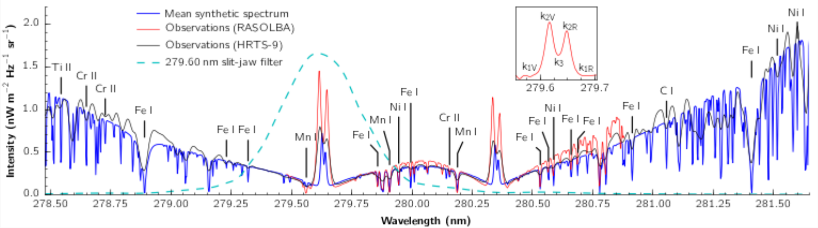


Figure 9: Mg II lines observed by IRIS satellite

But not completely (there is still an absorption in the core!).

What is going on here?

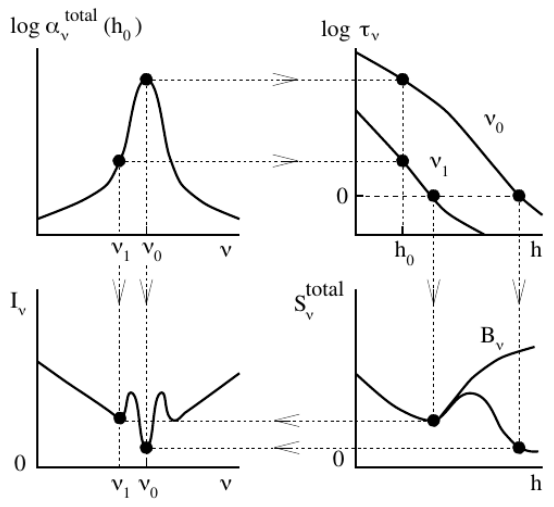


Figure 10: Example from page 20 of Rob Rutten's book.

Line source function decoupling - NLTE

$$S_\lambda \neq B_\lambda$$

To understand this, it is better to discuss seemingly completely different problem, Thomson scattering.

Imagine that the atmosphere consists of electrons that scatter radiation, and some particles that can absorb (and emit) the radiation. Then:

$$\chi = \chi_{\mathrm{abs}} + \chi_{\mathrm{scattering}}$$

Less awkward notation

$$\chi = \alpha + \sigma.$$

What is the emission then? Well the fraction of the particles that absorbs thermally will emit thermally. And the rest will scatter:

$$\eta = \alpha \times B(T) + \sigma \frac{1}{4\pi} \oint I(\hat{\Omega}) d\hat{\Omega}.$$

Divide this with total opacity, to get the source function:

$$S = \varepsilon B + (1 - \varepsilon) J$$

This is a magical formula.

NLTE problem

$$S = \varepsilon B + (1 - \varepsilon) J$$

The source function depends on the mean (angle averaged) intensity.

But to calculate the intensity we need the source function.

This is what people refer to as “NLTE coupling”.

Why do we call it NLTE

To paraphrase Hubeny & Mihalas: "Every state of the matter where there is a departure from the statistical distributions implied by the LTE".

So it could be: ionization state, excitation strate, velocity distribution (or any other energy distribution).

In the case of electron scattering it is not immediately clear what departs. But emissivity definitely won't follow LTE.

NLTE problem in the continuum

$$S = \varepsilon B + (1 - \varepsilon) J$$

swapping this back to the RTE, in 1D:

$$\frac{dI(\tau, \mu)}{\mu d\tau} = I(\mu, \tau) - \varepsilon B(\tau) - (1 - \varepsilon) \oint I(\tau, \mu) d\mu$$

This is an **integro-differential equation** for $I(\tau, \mu)$.

The different depths and the directions are **coupled**.

Some of the commonly encountered statements

“Radiative Transfer is non-local.” - Because RTE couples different regions.

“Non-LTE (scattering)...” - These things are often equalized because scattering implies photons are not destroyed and hence mess up with distributions in far away regions.

“Line forms in...” - This is a dangerous statement in NLTE case because the formation region $\neq$ sensitivity region.

“self-consistent solution” - Usually meaning solution if integro-differential form of RTE in presence of scattering.

Next Lecture

Solving NLTE problem: how to solve the coupling in the continuum scattering problem.

What does the solution imply? How does the solution look like and what changes?

How do we generalize this to spectral lines? What is different? What is more complicated?