Homework: Magnetic Field Observation and Modeling

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1 Analyzing Vector Magnetograms

The systematics in spectropolarization observations can be subtle. Consider the following examples.

a) When smalls-scale, mixed polarity flux tubes co-exist in a same resolution element, the circular polarization is reduced. Spatial filling factor f may be introduced for each flux tube to describe the relative areas they cover ($0 \le \sum f \le 1$). The longitudinal *flux density* is then a linear combination of the longitudinal fields weighted by f.

Extend this concept to linear polarizations and consider a single flux tube. In the weak field regime, the Stokes parameters follow

$$Q \propto f B^2 \sin^2\gamma \cos 2\phi,$$
$$U \propto f B^2 \sin^2\gamma \sin 2\phi,$$
$$V \propto f B \cos\gamma,$$

where B is the intrinsic field strength, γ is the inclination angle with respect to the sight line, and ϕ is the azimuth angle. For a set of observed Stokes parameters (Q, U, V), discuss the dependence of (B, γ, ϕ) on f. Is the result biased if f is held constant at 1, for sunspot umbra, plage, and quite Sun respectively?

b) An isolated, well-formed sunspot is known to have a single polarity. Consider the longitudinal field B_l maps of AR 11084 at two instances (Figure 1). What is the origin of the apparent polarity inversion line (PIL) across the sunspot at the second instance, given what you know about the sunspot photospheric field structure? Estimate the Heliographic longitude limb-ward of which such PIL is likely to occur.

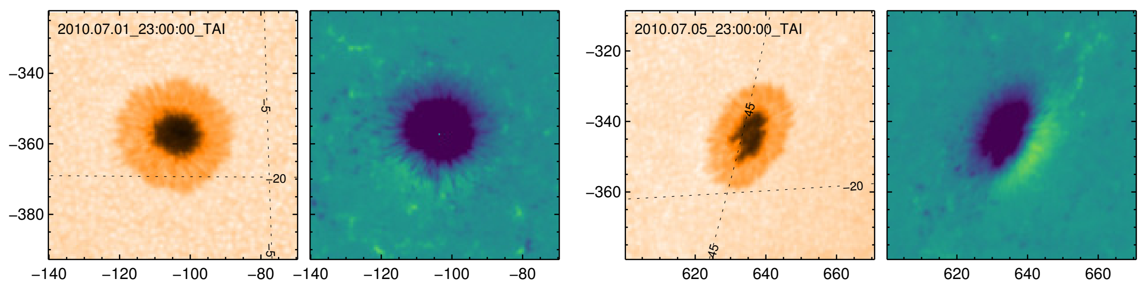

Figure 1: Two HMI observations of continuum (I_c) and longitudinal field (B_l) for AR 11084, separated by 4 days. Dotted lines in I_c maps show Heliographic longitude and latitude.

2 Force-Free Fields

Plasma $\beta = 8\pi p / B^2$ (cgs unit) determines the relative importance of gas and magnetic pressure. In the low β limit, the magnetic field dictates the plasma evolution. For a system in equilibrium, the Lorentz force needs to balance itself: $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} = 0$. The field is force-free.

a) Solar active regions (ARs) accumulate excess magnetic energy in the low corona (~10 Mm). Estimate the order of magnitude of β.

b) For a force-free field $\boldsymbol{B}$ described as $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$, show that α is constant along individual field lines.

c) A linear force-free (LFF) field has constant α everywhere. Show that LFF satisfies

$$\nabla^2 \boldsymbol{B} + \alpha^2 \boldsymbol{B} = 0.$$

d) The *Woltjer's Theorem* states that LFF magnetic fields have the minimum magnetic energy for a closed system assuming fixed boundary values and constant magnetic helicity. This limits the amount of magnetic energy accessible during a flare.

   Using vector potential $\nabla \times \boldsymbol{A} = \boldsymbol{B}$, a system's energy $\mathscr{E}$ and helicity $\mathscr{H}$ can be written as

$$\mathscr{E} = \frac{1}{8\pi} \int_V (\nabla \times \boldsymbol{A})^2 \, \mathrm{d}V,$$

$$\mathscr{H} = \int_V \boldsymbol{A} \cdot (\nabla \times \boldsymbol{A}) \, \mathrm{d}V.$$

   For the following function with a constant Lagrangian multiplier α

$$\mathscr{W} = \mathscr{E} - \frac{\alpha}{8\pi} \mathscr{H} = \frac{1}{8\pi} \int_V (\nabla \times \boldsymbol{A})^2 - \alpha \boldsymbol{A} \cdot (\nabla \times \boldsymbol{A}) \, \mathrm{d}V,$$

   evaluate the variation of this function $\delta \mathscr{W}$ in terms of $\boldsymbol{A}$ and $\delta \boldsymbol{A}$. For minimal $\mathscr{E}$, there must be $\delta \mathscr{W} = 0$ under the two aforementioned assumptions. Show that $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$, i.e. LFF fields, lead to $\delta \mathscr{W} = 0$.

   Hint: Consider *Gauss Theorem*. Integrate by parts.

e) The magnetic *Virial Theorem* states that the energy of a force-free field may be estimated from its lower boundary. Show that

$$\mathscr{E} = \frac{1}{8\pi} \int_V B^2 \, \mathrm{d}V = \frac{1}{8\pi} \oint_S [B^2 \boldsymbol{r} - 2(\boldsymbol{B} \cdot \boldsymbol{r}) \boldsymbol{B}] \cdot \mathrm{d}\boldsymbol{S}.$$

   Write down the expression of $\mathscr{E}$ in a Cartesian domain with lower boundary at $z = 0$, assuming only the lower boundary contributes.

   Hint: Consider *Gauss Theorem*. Use the fact that force $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}$ and torque $(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} \cdot \boldsymbol{r}$ both vanish.

3 Grand Archive of Flare and CME Cartoons

Solar physicists are known for their artistic creativity. Dr. Hugh Hudson has gathered over 200 cartoons from literature regarding flare and CME processes. See http://solarmuri.ssl.berkeley.edu/~hhudson/cartoons/.

Select one cartoon that focuses on magnetic fields, read the relevant paper, and write a short summary in your own words. Additional discussion is encouraged. You do not have to agree with the paper's conclusions.