COLLAGE 2019

Magnetic Field Structures over a Solar Cycle

Xudong Sun (UH/IfA)

Apr 04 2019

Outline

- PFSS Model for coronal field
- Solar cycle
- Babcock-Leighton mechanism & surface flux transport

PFSS Model for Coronal Field

PFSS Model

- Assuming lower corona is current free,
  B can be expressed as a scaler potential

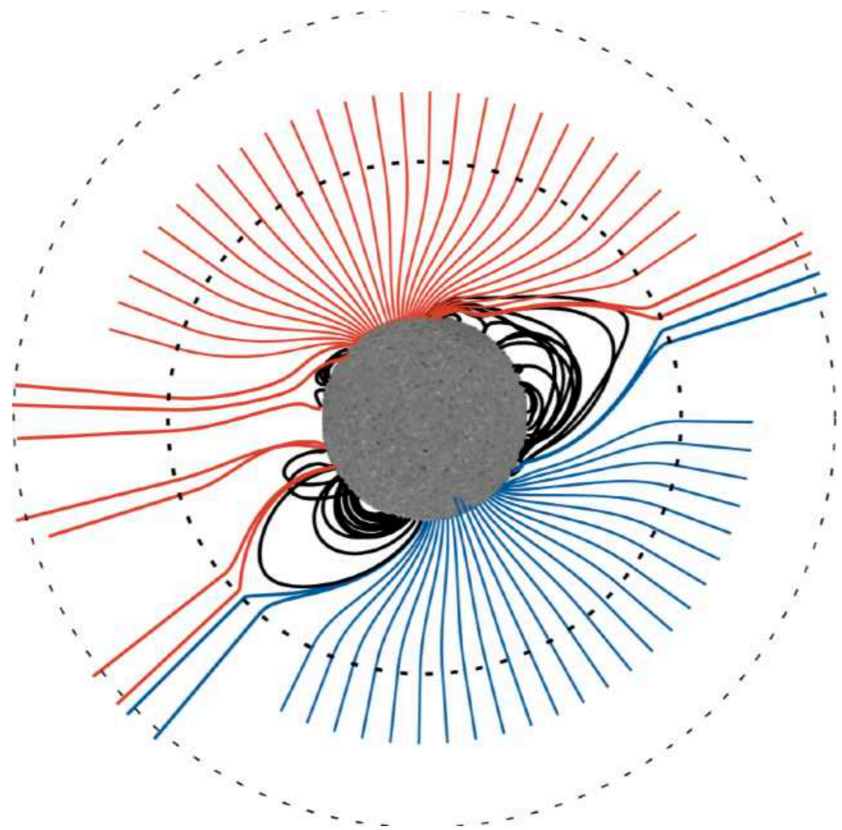
$$\boldsymbol{B} = -\nabla\Psi$$

- ***B*** vector in spherical coord

$$B_r = -\frac{\partial\Psi}{\partial r}$$

$$B_\theta = -\frac{1}{r}\frac{\partial\Psi}{\partial\theta}$$

$$B_\phi = -\frac{1}{r\sin\theta}\frac{\partial\Psi}{\partial\phi}$$



Sun (2012)

- Coronal field governed by Laplace eq.

$$\nabla^2\Psi = 0$$

PFSS Model

- Inner boundary: photosphere radial field

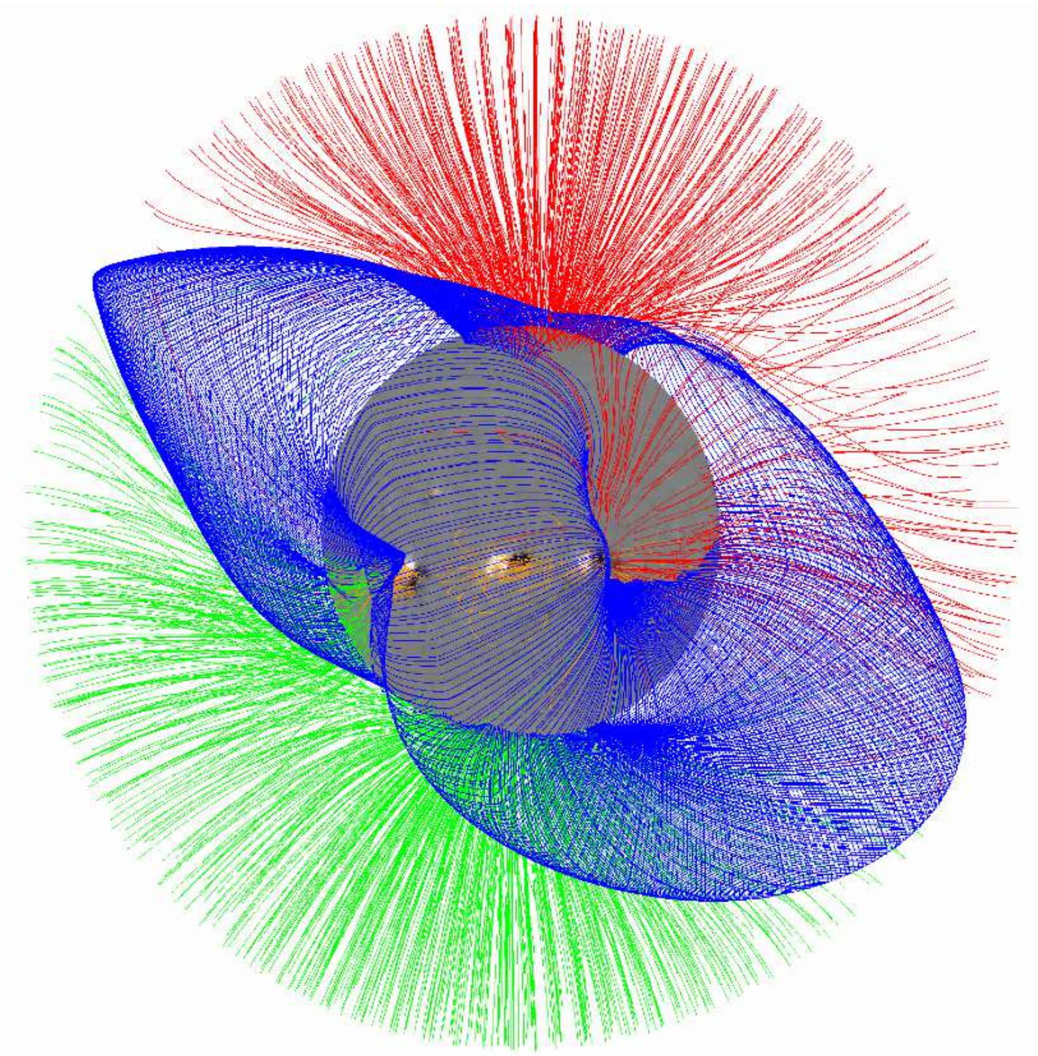
$$-\frac{\partial \Psi}{\partial r}\bigg|_{r=R_\odot} = B_r(\text{obs})$$

- Outer boundary: source surface where field becomes radial and open

$$\Psi\big|_{r=R_s} = 0$$

$$R_s = 2.5R_\odot$$

- General solution of Laplace equation:



NSO/*GONG*

$$\Psi = \sum_{l=0}^{\infty} r^{-(l+1)} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi) + \sum_{l=0}^{\infty} r^l \sum_{m=-l}^{l} b_{lm} Y_l^m(\theta, \phi)$$

PFSS Model Solution

- Matching outer boundary

$$\sum_{l=0}^{\infty} R_s^{-(l+1)} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\phi) + \sum_{l=0}^{\infty} R_s^{l} \sum_{m=-l}^{l} b_{lm} Y_l^m(\theta,\phi) = 0, \quad \text{or,} \quad \boxed{a_{lm} = -R_s^{2l+1} b_{lm}}$$

- Matching inner boundary

$$\sum_{l=0}^{\infty} (l+1) R_\odot^{-(l+2)} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\phi) - \sum_{l=0}^{\infty} l R_\odot^{l-1} \sum_{m=-l}^{l} b_{lm} Y_l^m(\theta,\phi) = B_r(R_\odot,\theta,\phi)$$

- Use normalization properties of spherical harmonics

$$\oint d\Omega \, Y_l^m(\theta,\phi) Y_{l'}^{m'}(\theta,\phi) = \delta_{ll'}\delta_{mm'}$$

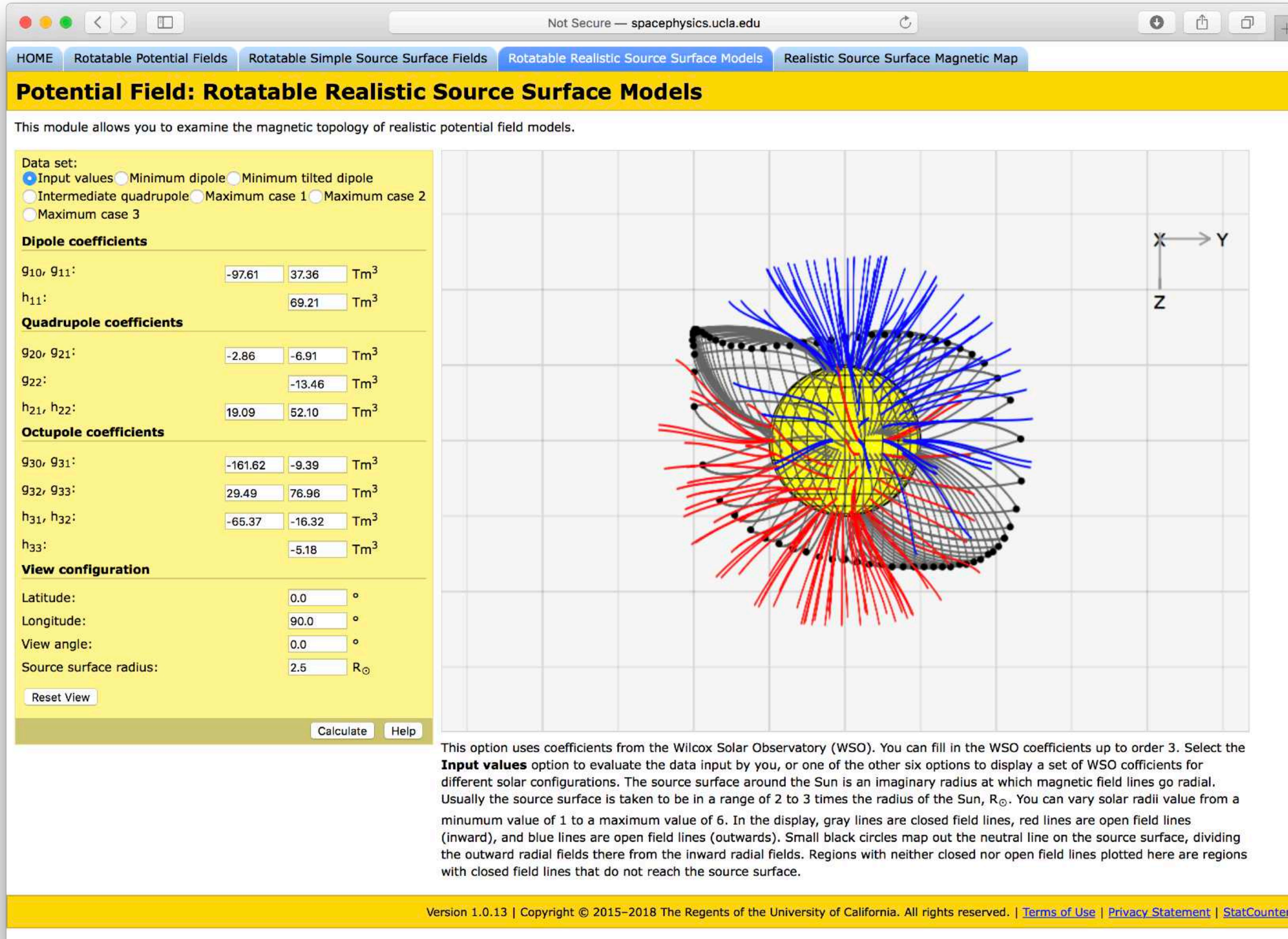
- Surface integral of observed field

$$\boxed{(l+1) R_\odot^{-(l+2)} a_{lm} - l R_\odot^{l-1} b_{lm} = \oint d\Omega B_r(R_\odot,\theta,\phi) Y_l^m(\theta,\phi)}$$

http://wso.stanford.edu/words/pfss.pdf

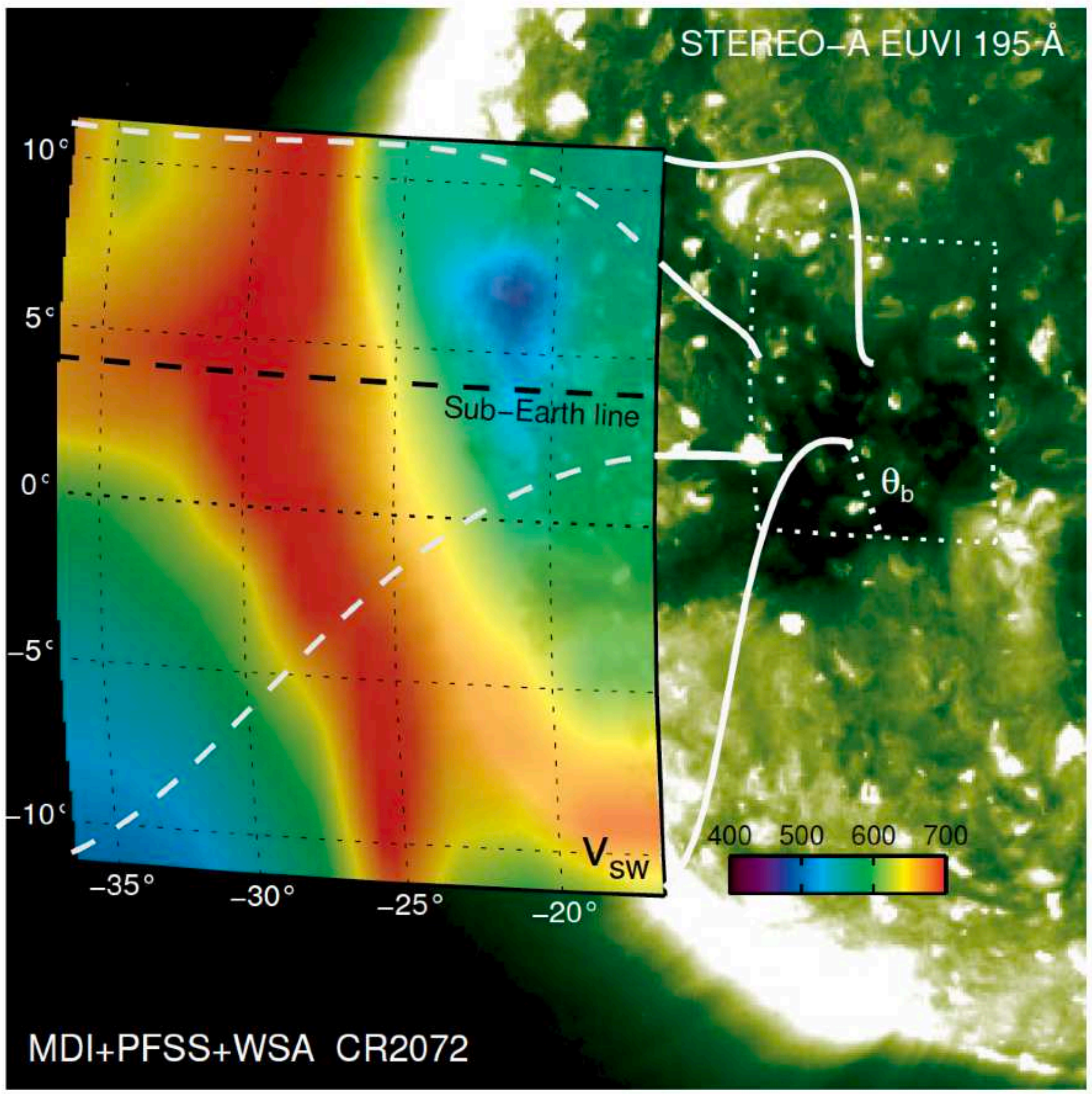
PFSS Widget

https://tinyurl.com/pfss-wid



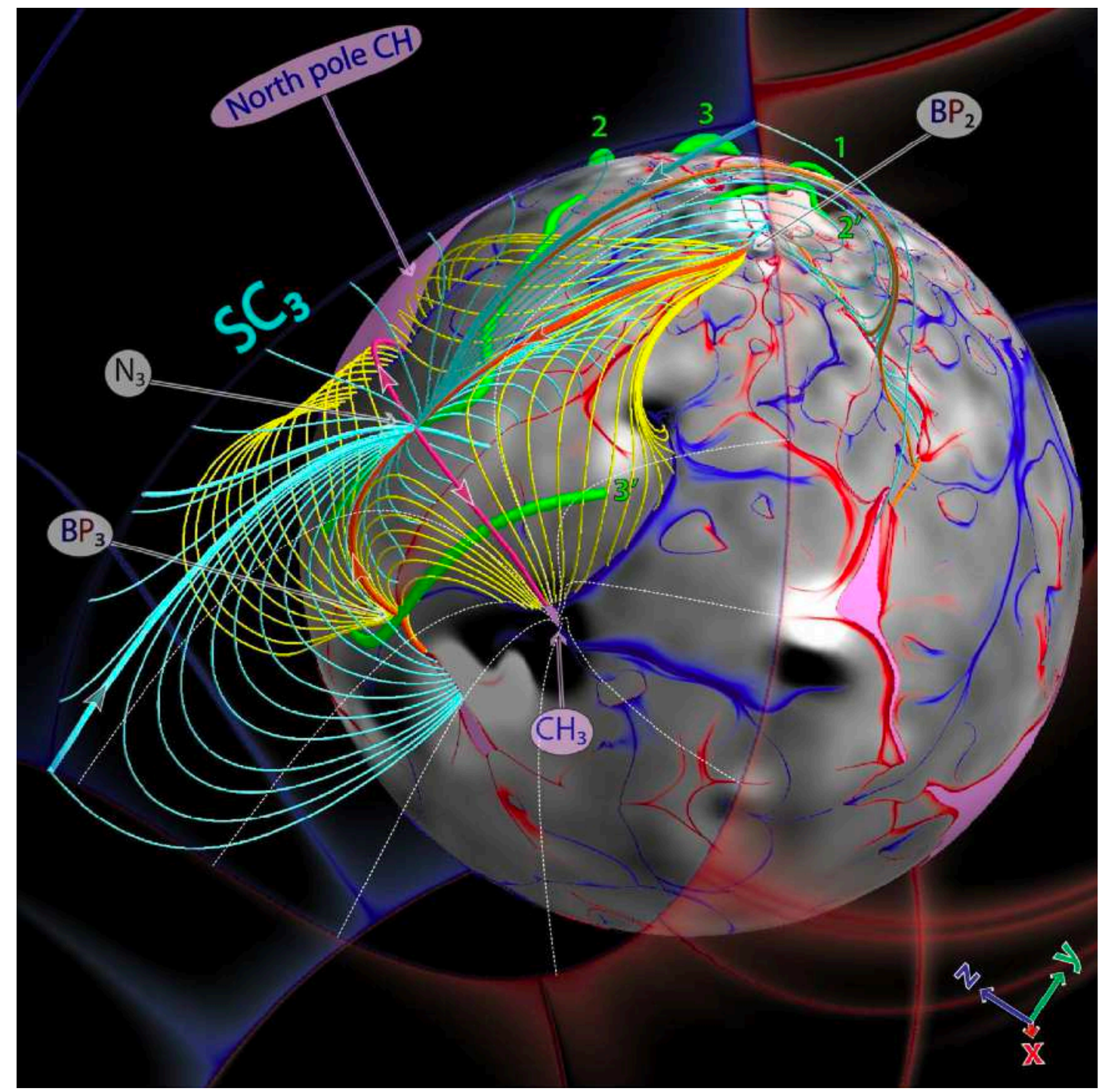
Utility of PFSS Model

Mapping Solar Wind: WSA Model



Sun (2012)

Topology Analysis for Sympathetic Flares

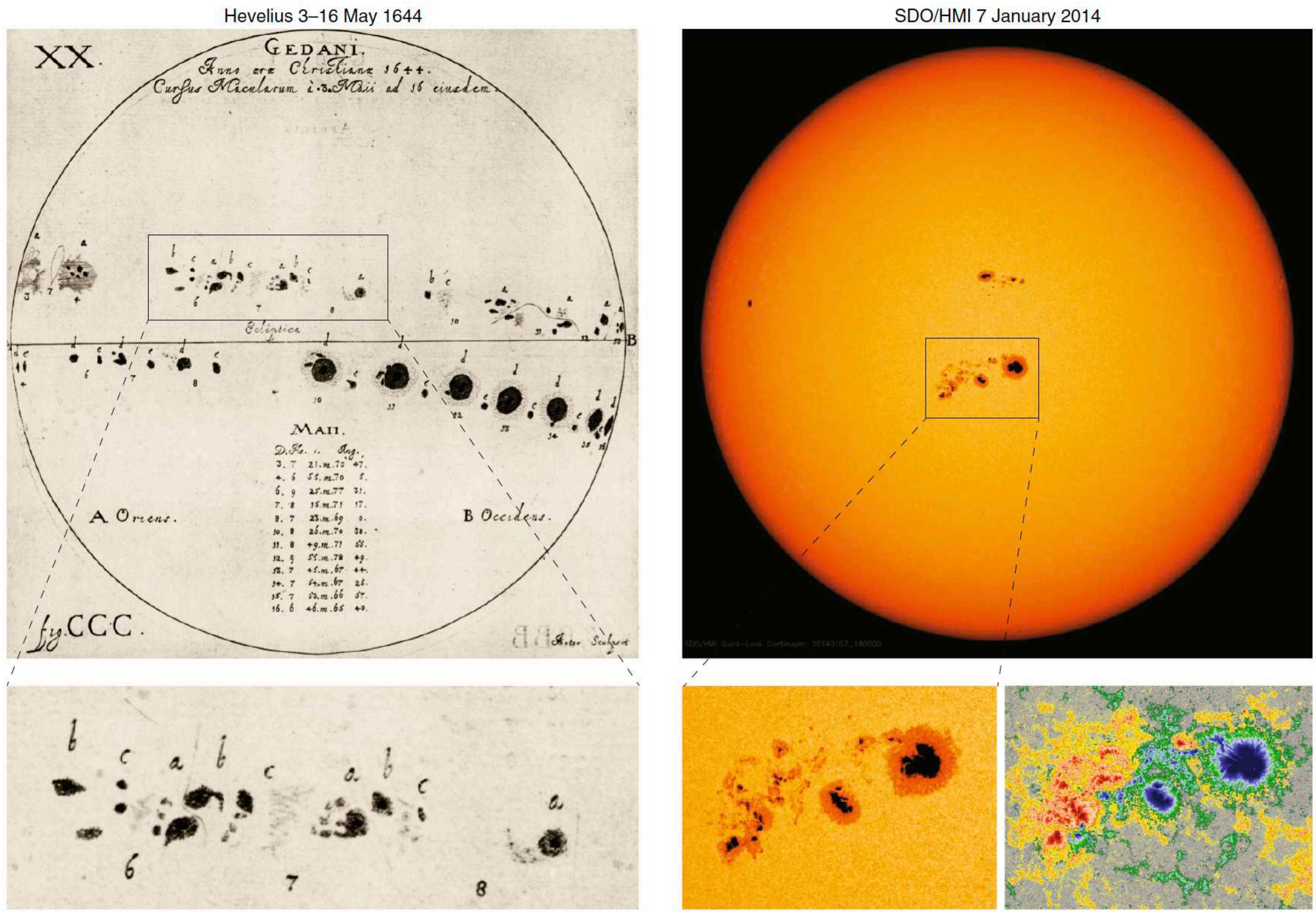


Titov et al. (2012)

$$v \propto f^{-1} = \frac{B_r(R_s)R_s^2}{B_r(R_\odot)R_\odot^2}$$

Solar Cycle

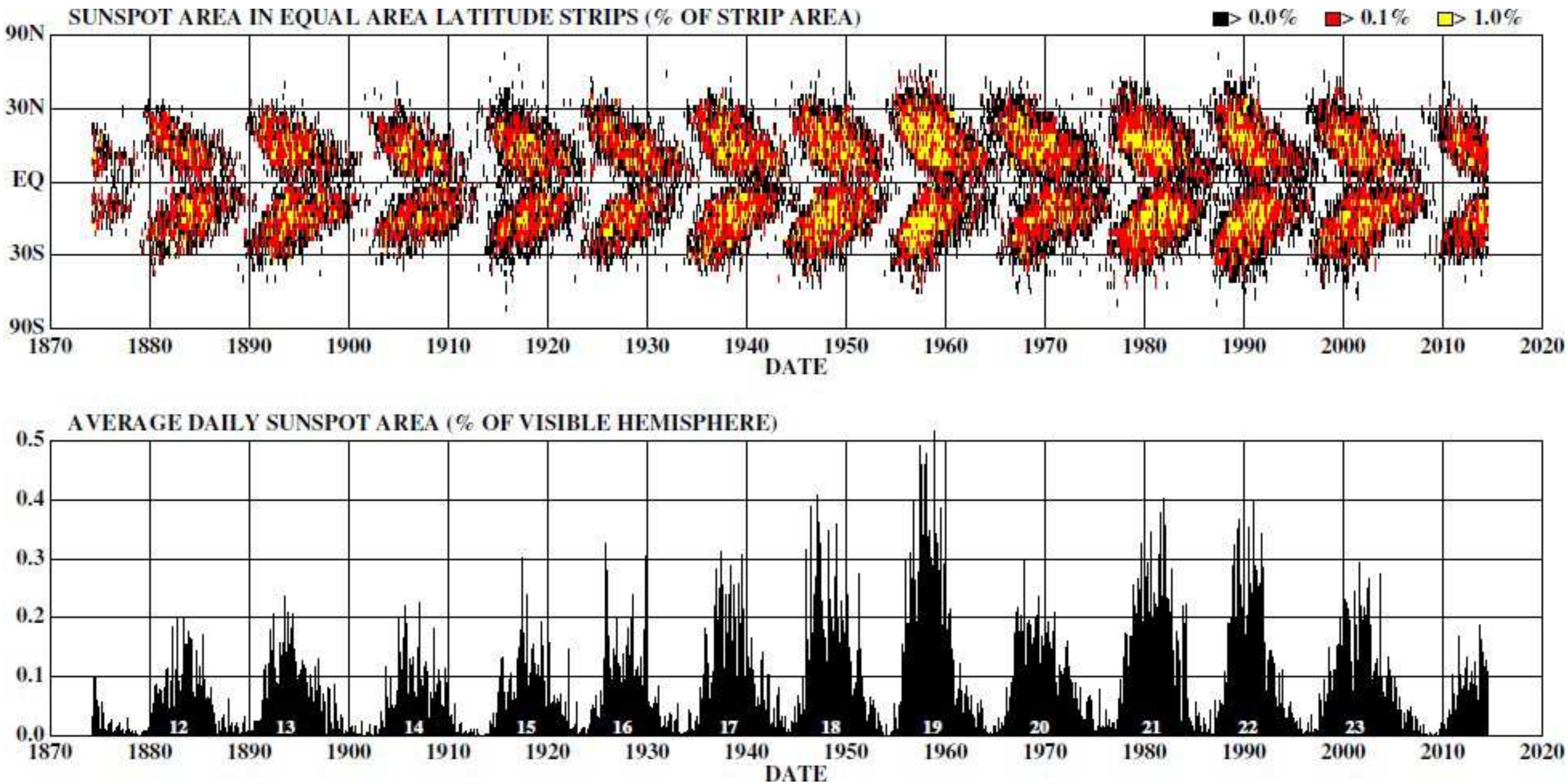
Sunspot Number



Muñoz-Jaramillo & Vaquero (2018)

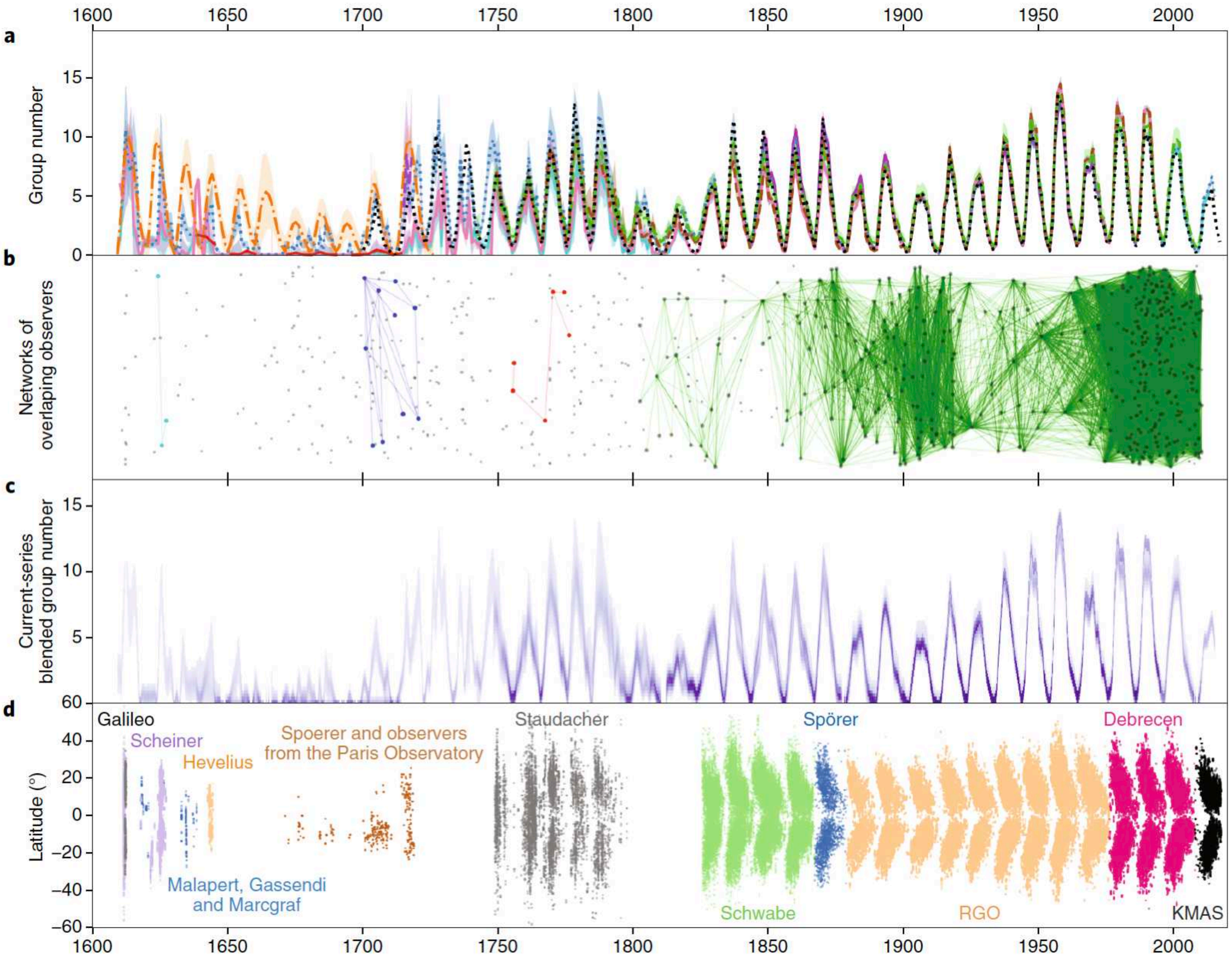
Butterfly Diagram

Defining cycle minimum/maximum



Hathaway (2015)

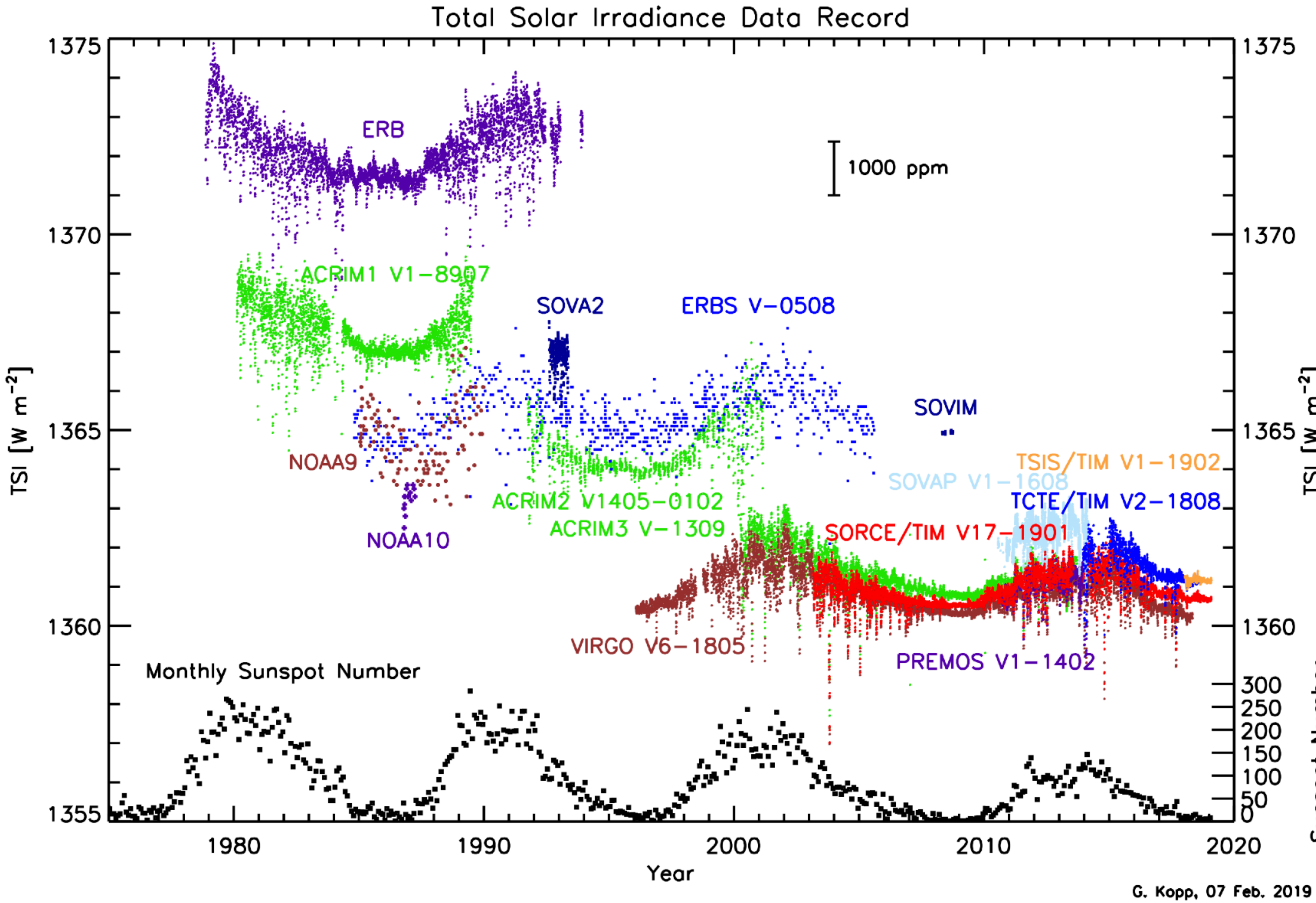
Historic View



Muñoz-Jaramillo & Vaquero (2018)

Total Solar Irradiance

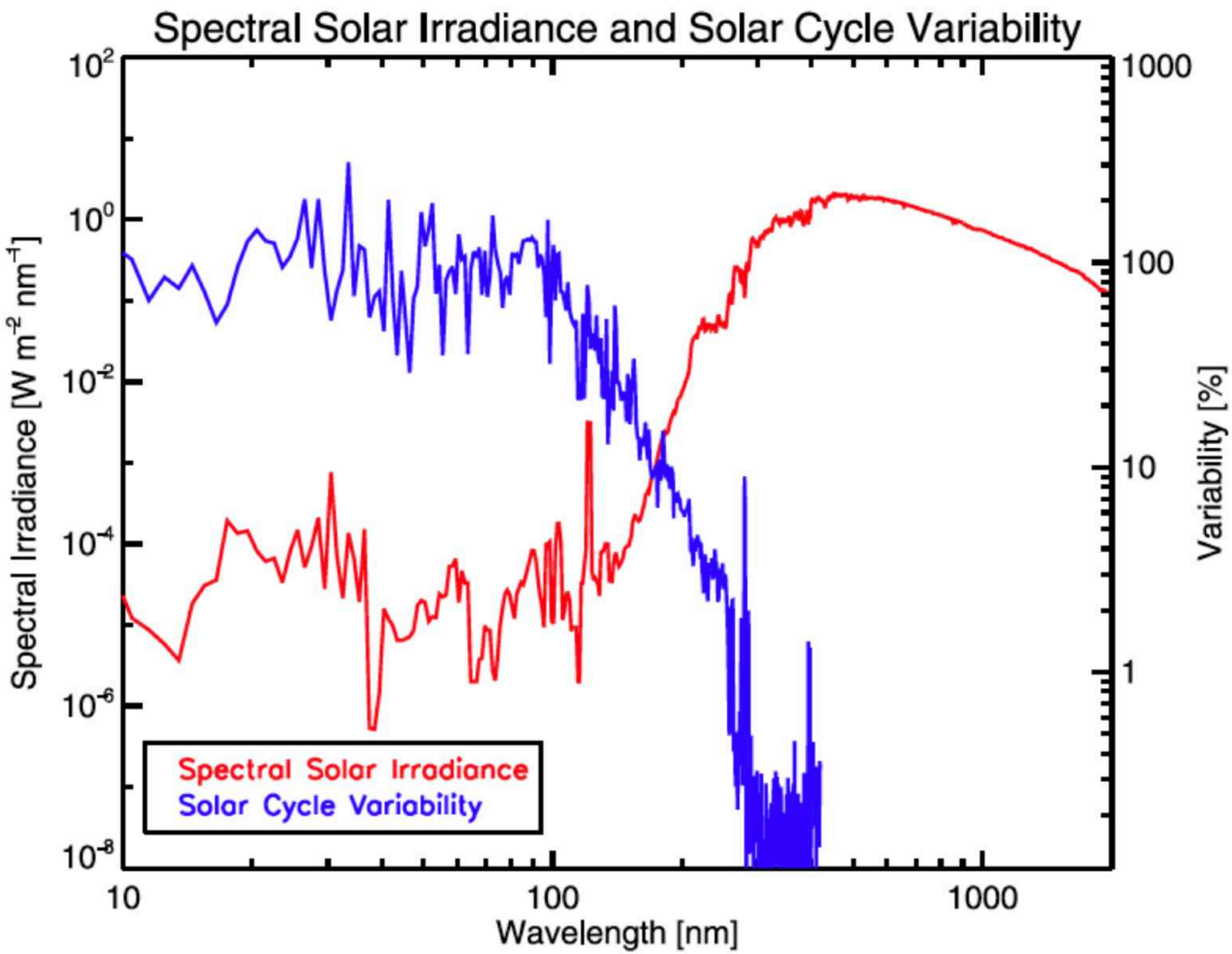
Variability?



Courtesy G. Kopp

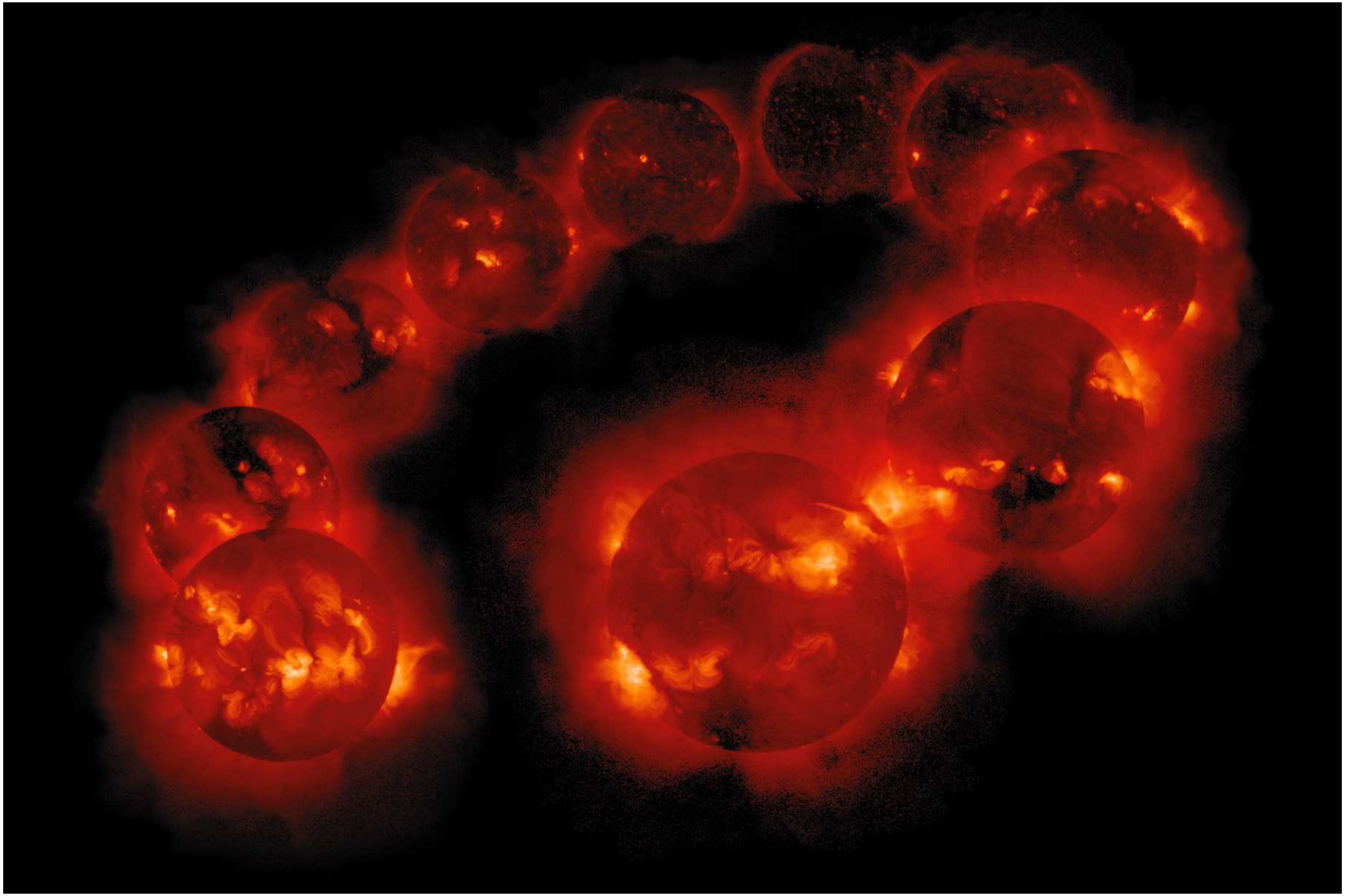
Spectral Irradiance Variability

What features determine the variability in different wavelengths?



Domingo et al. (2009)

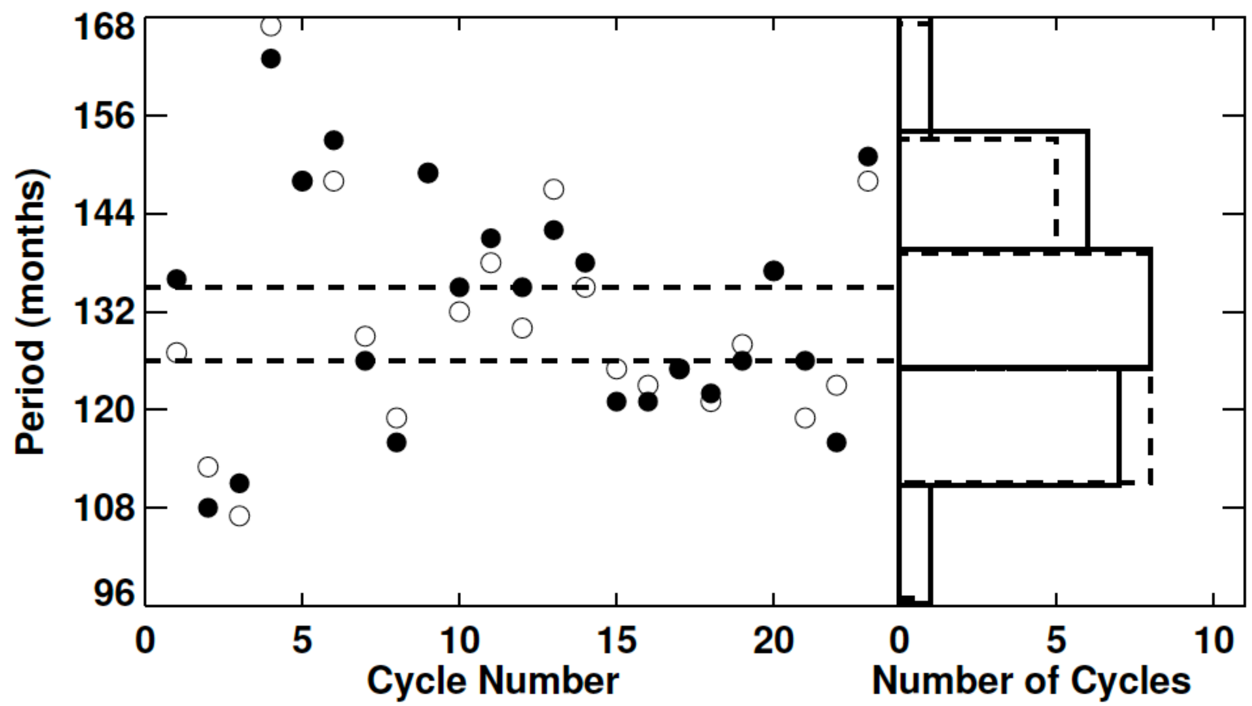
X-Ray View Over a Cycle



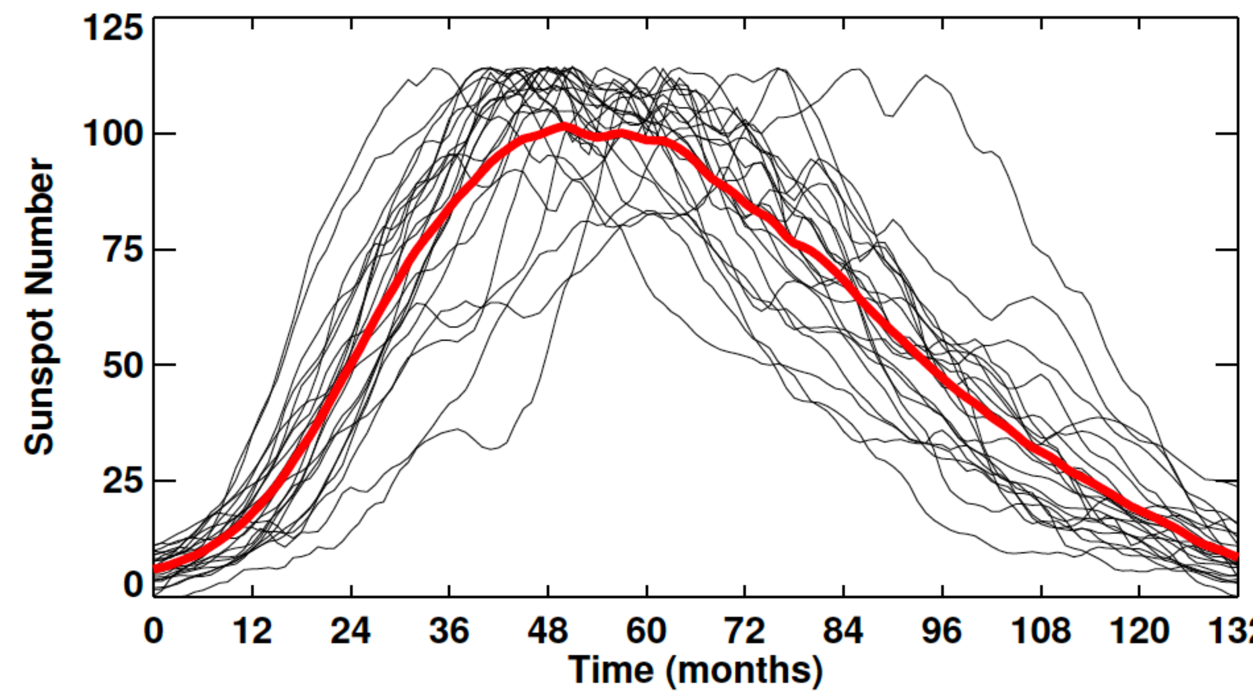
Credit: Yohkoh/ISAS/LMSAL/NAOJ/U. Tokyo/NASA

Cycle Parameters

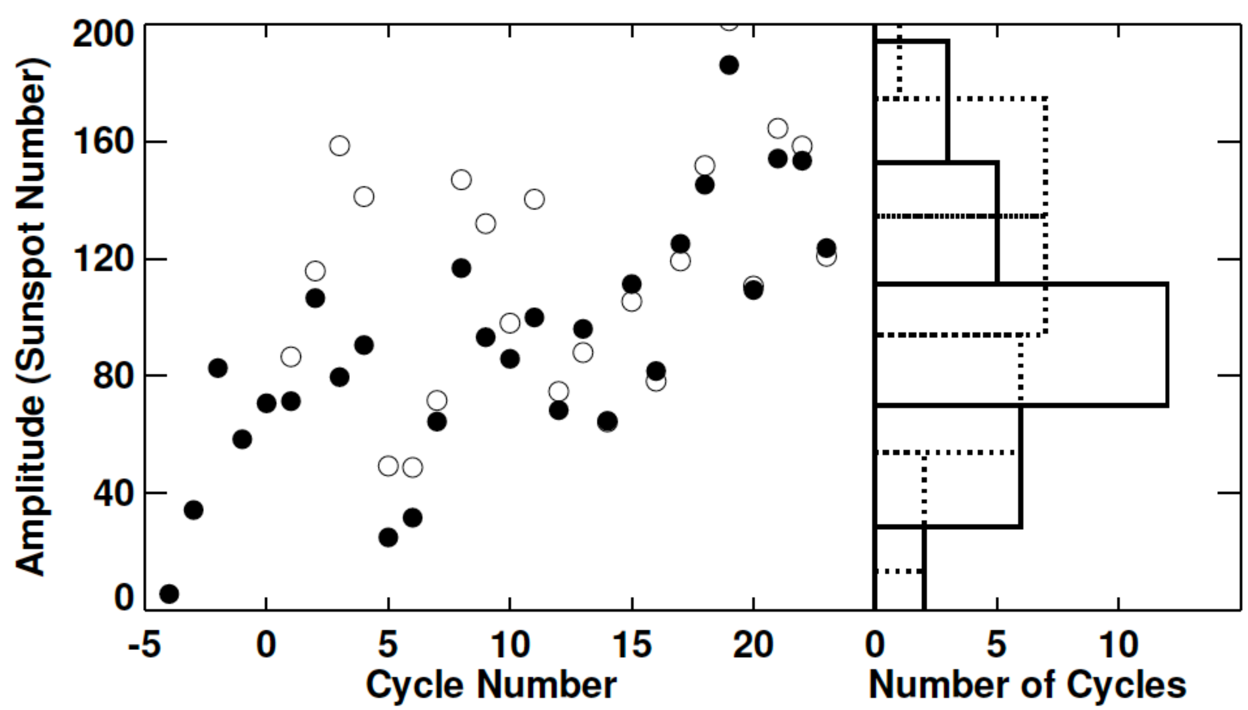
Period



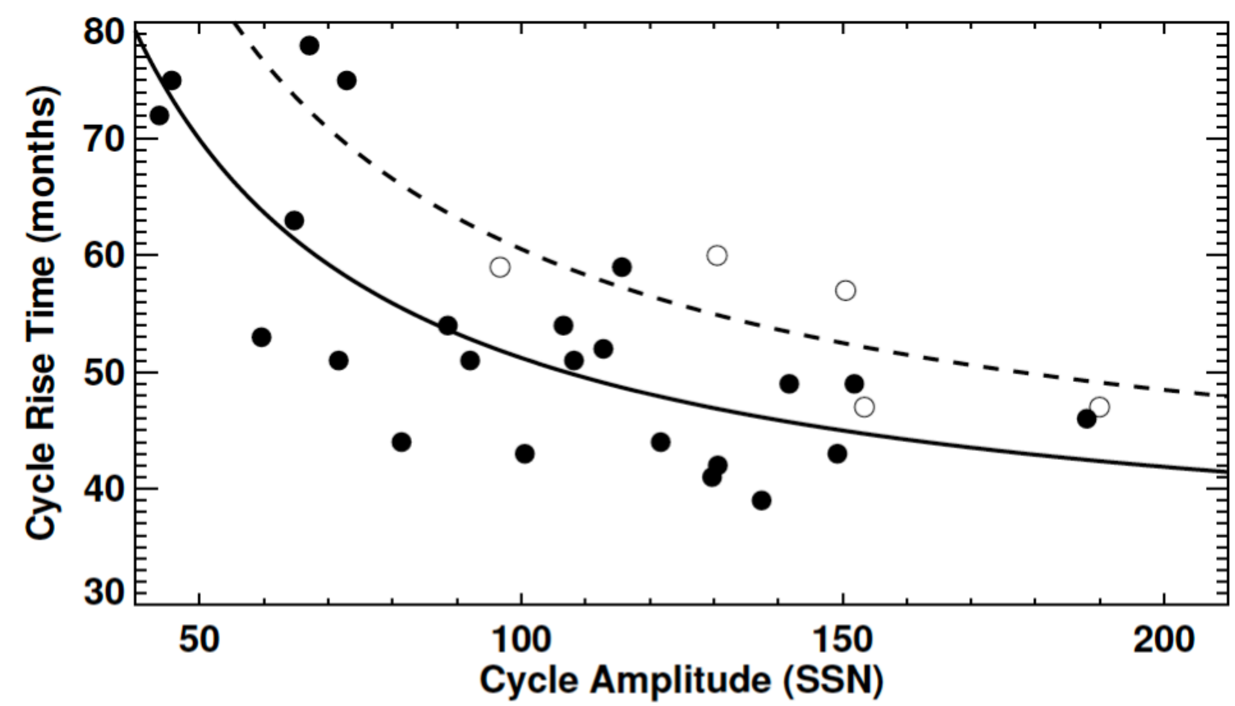
Gnevyshev Gap



Amplitude



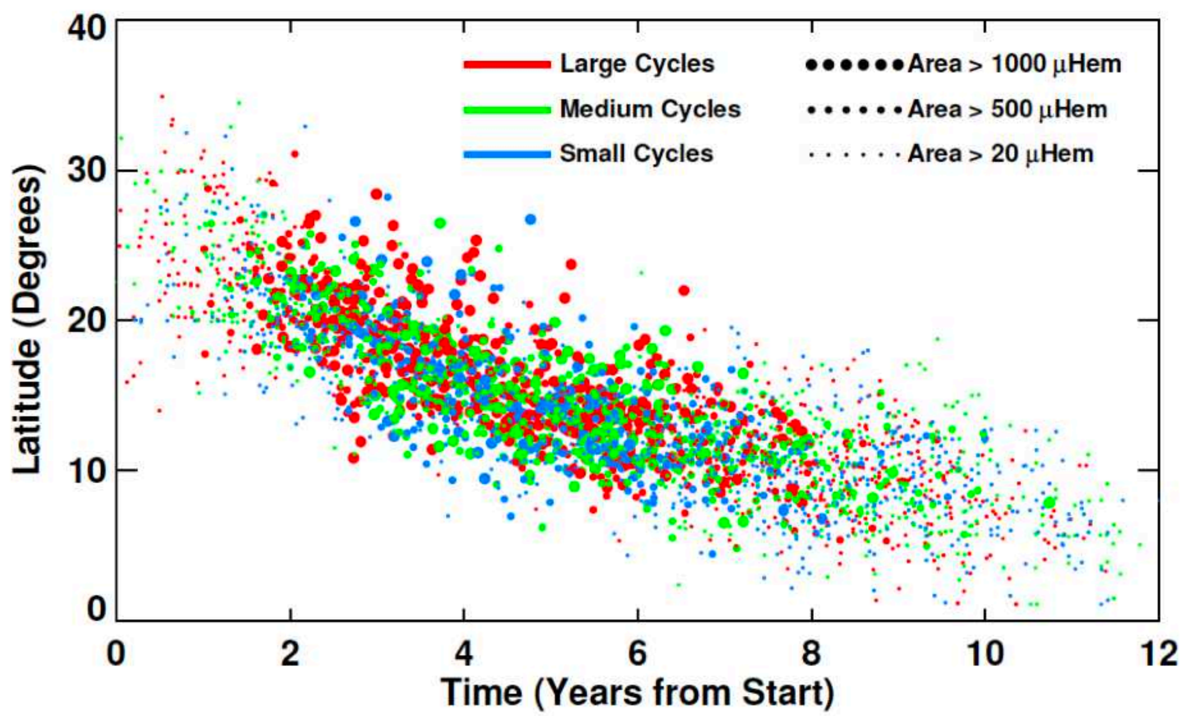
Waldmeier Effect



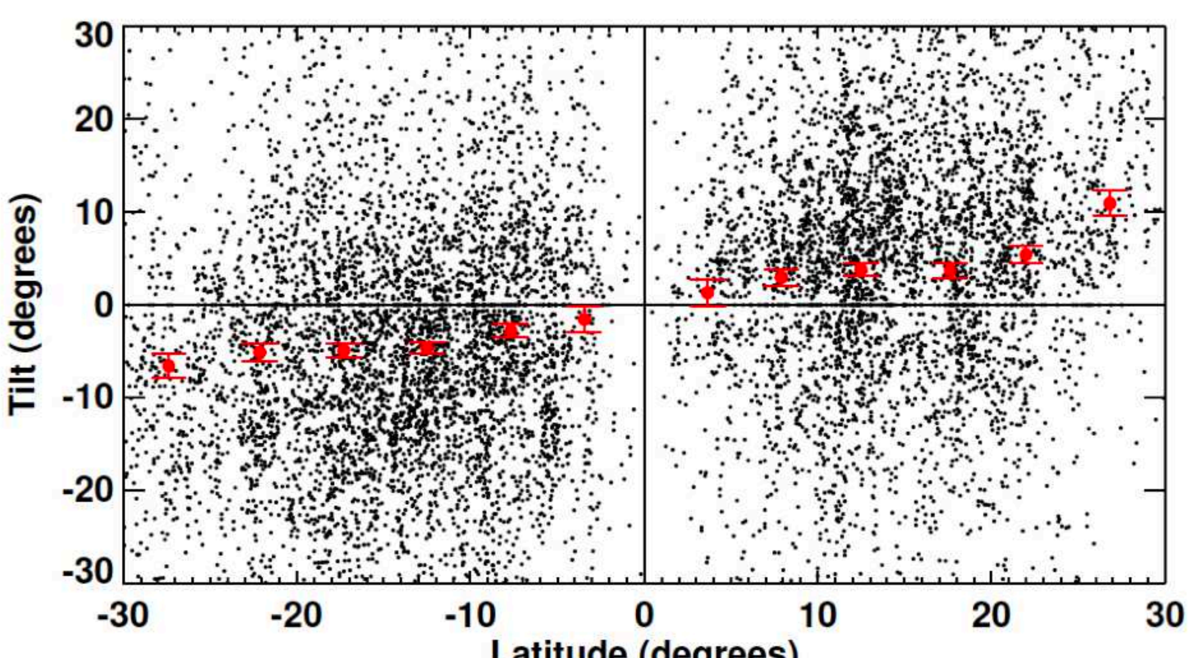
Hathaway (2015)

Spörer's, Hale's, & Joy's Law

Spörer's Law

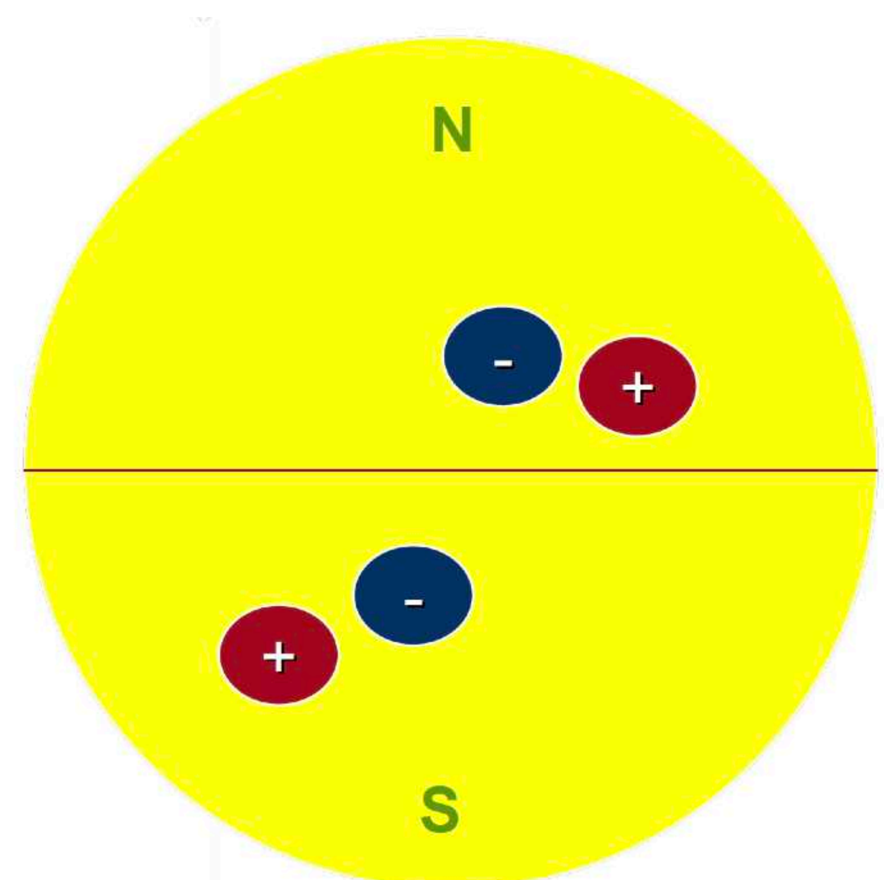


Joy's Law



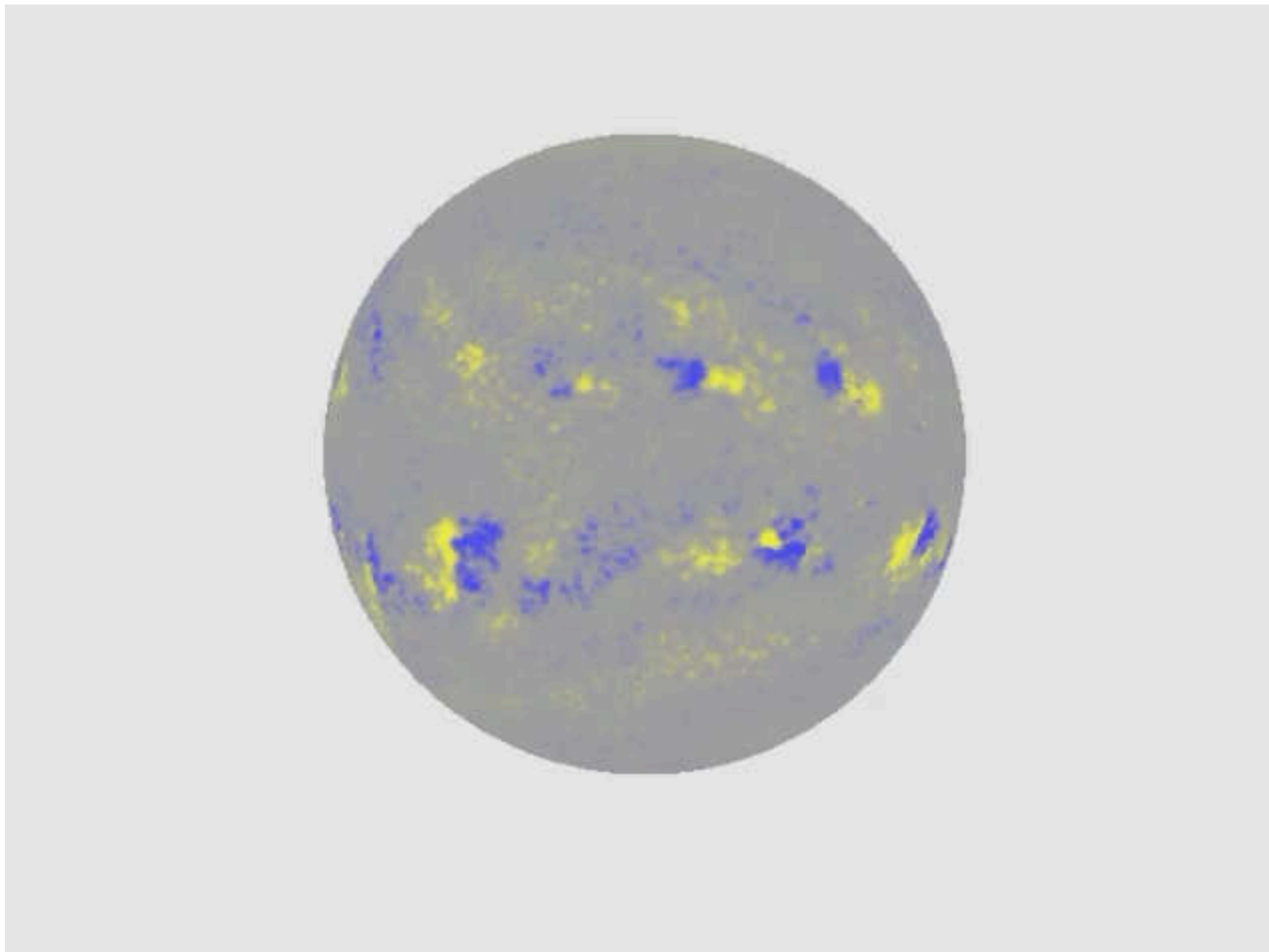
Hathaway (2015)

Hale's Law



Courtesy S. Solanki

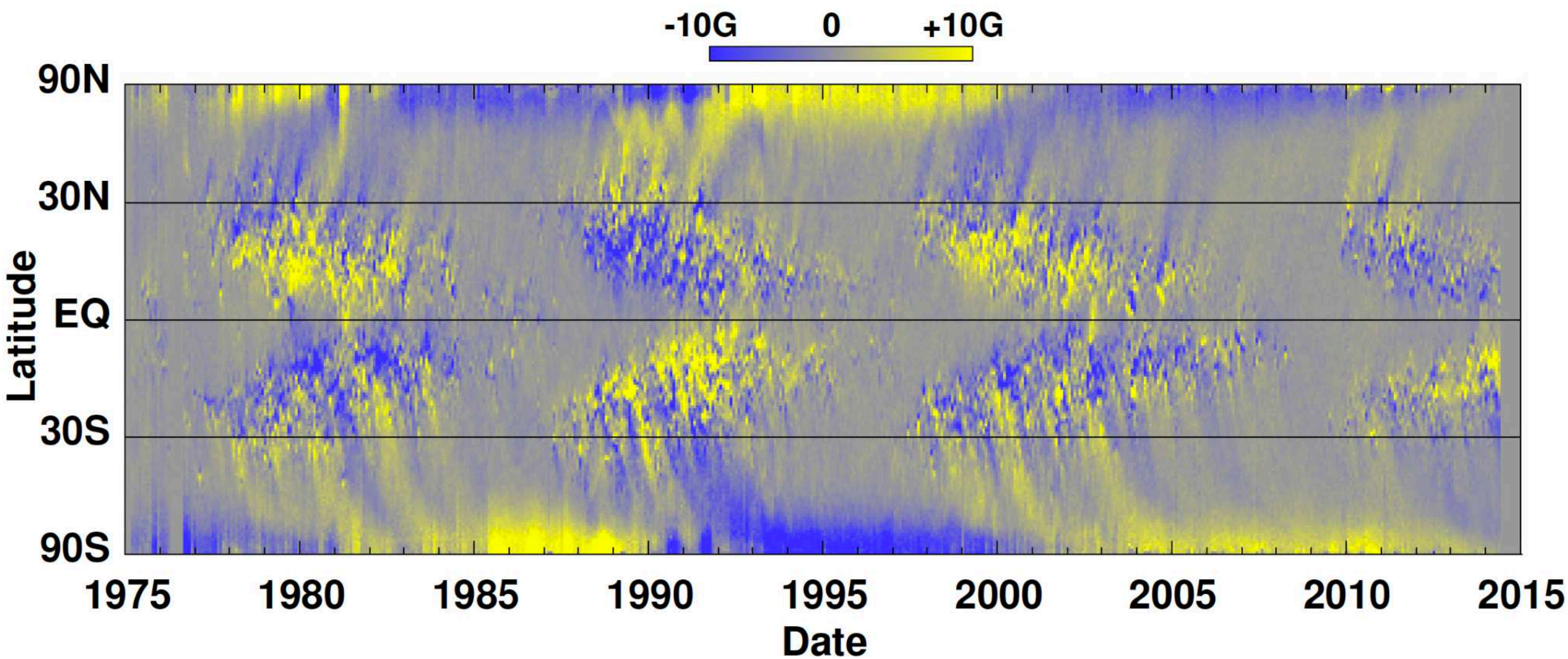
Evolution of Magnetic Field



Hathaway (2015)

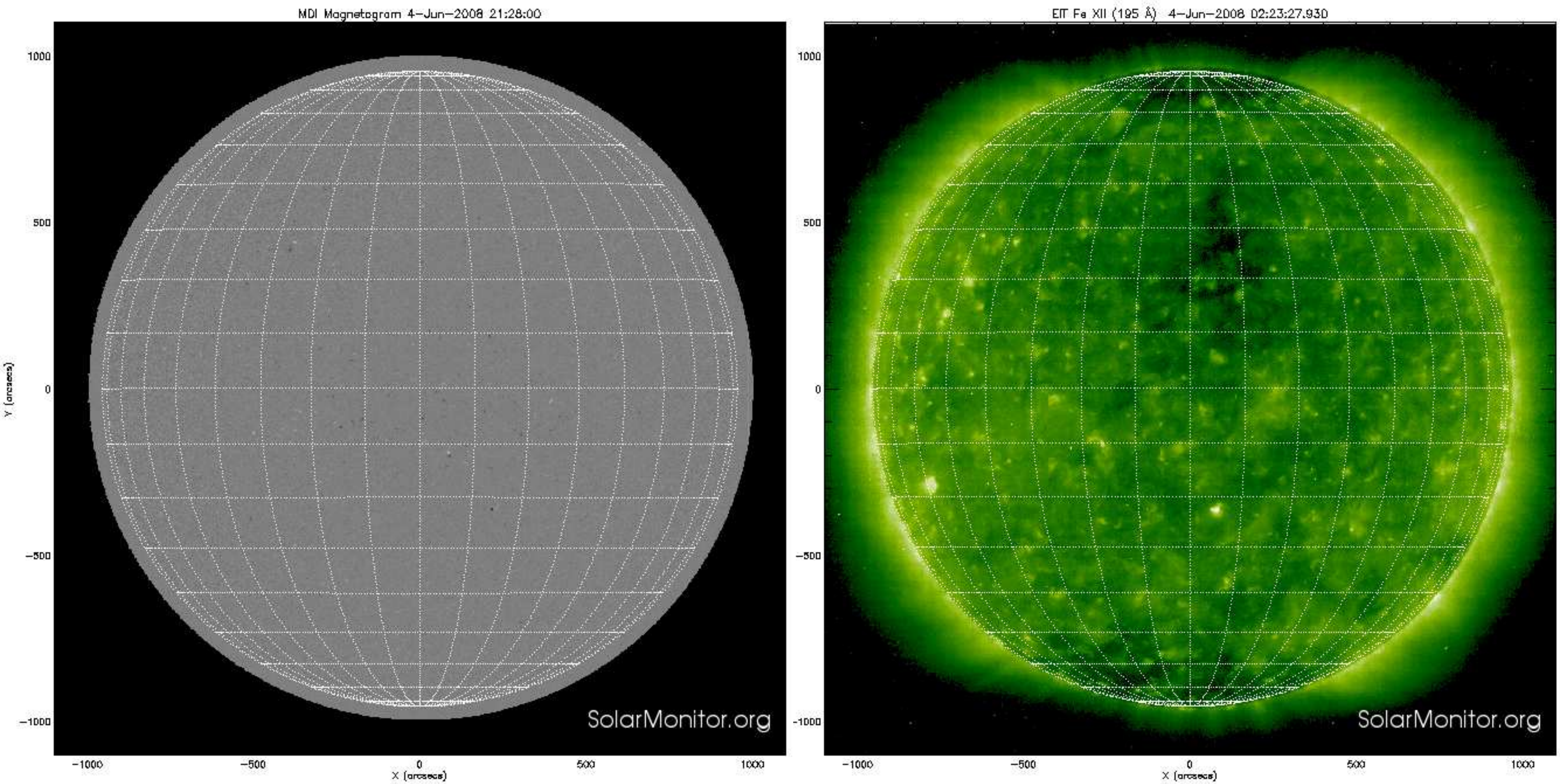
Magnetic Butterfly Diagram

Spörer's Law; N/S asymmetry; polarity reversal



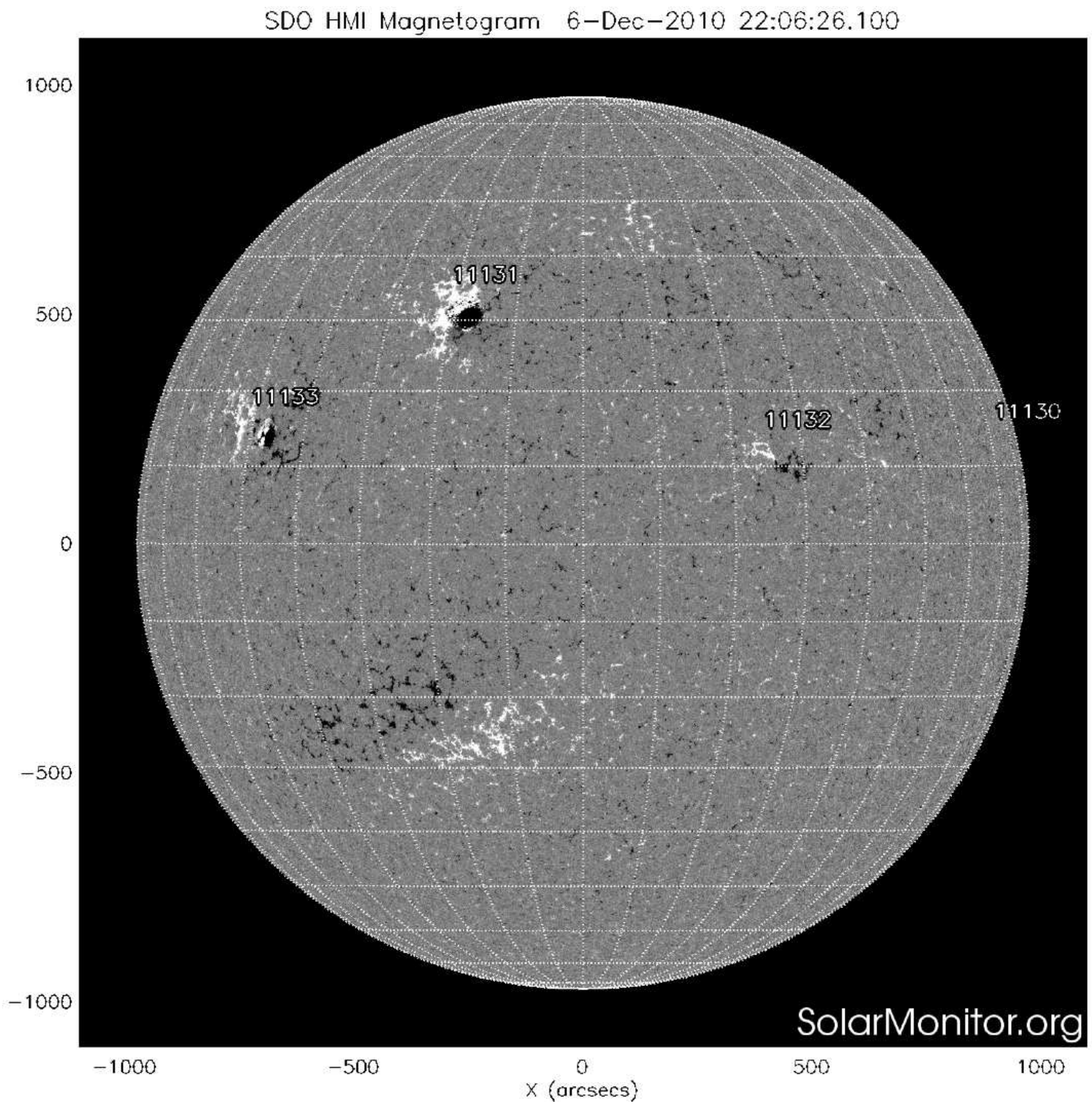
Hathaway (2015)

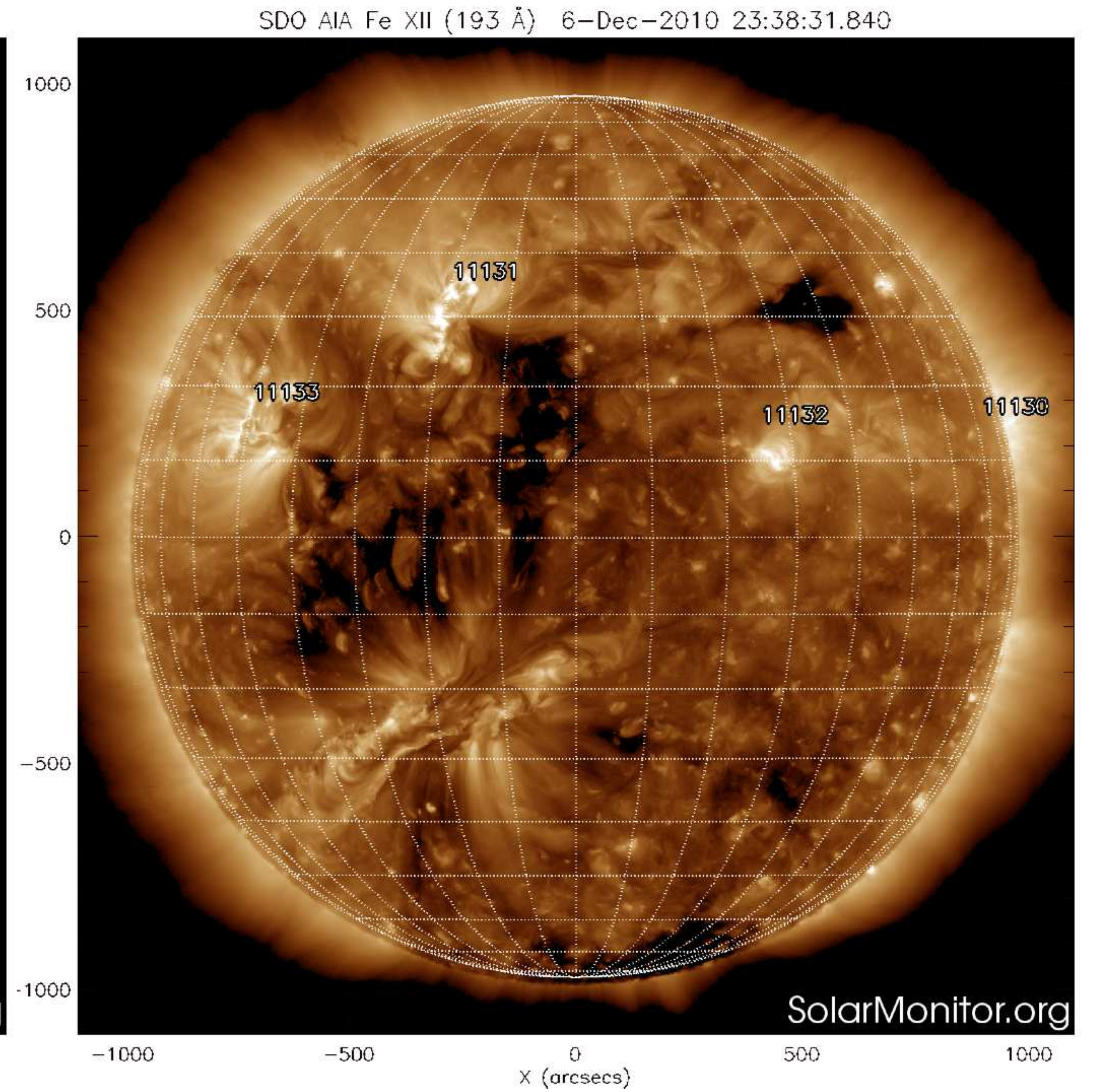
Minimum Phase



SoHO/MDI, EIT 19.5 nm

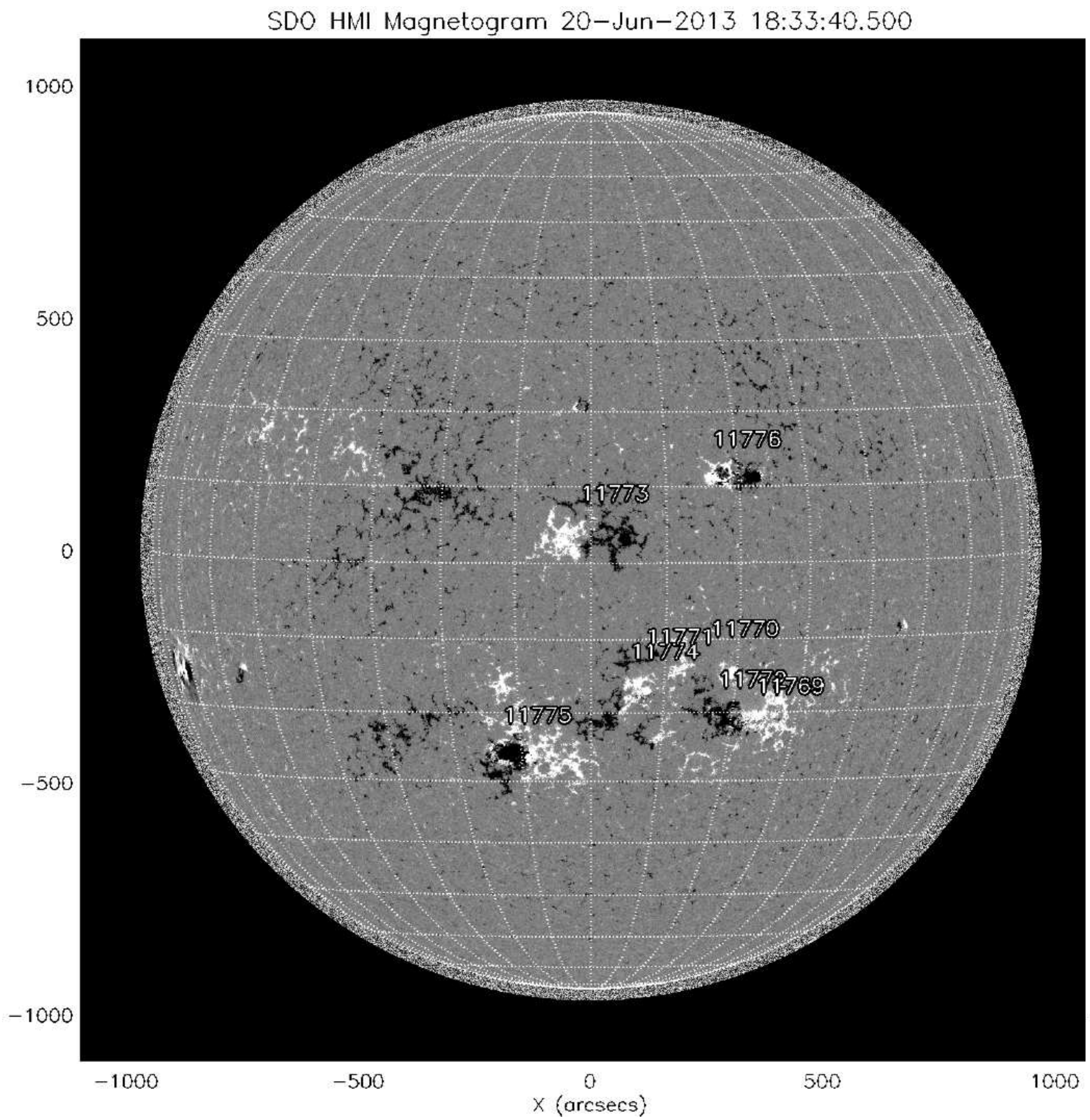
Ascending Phase

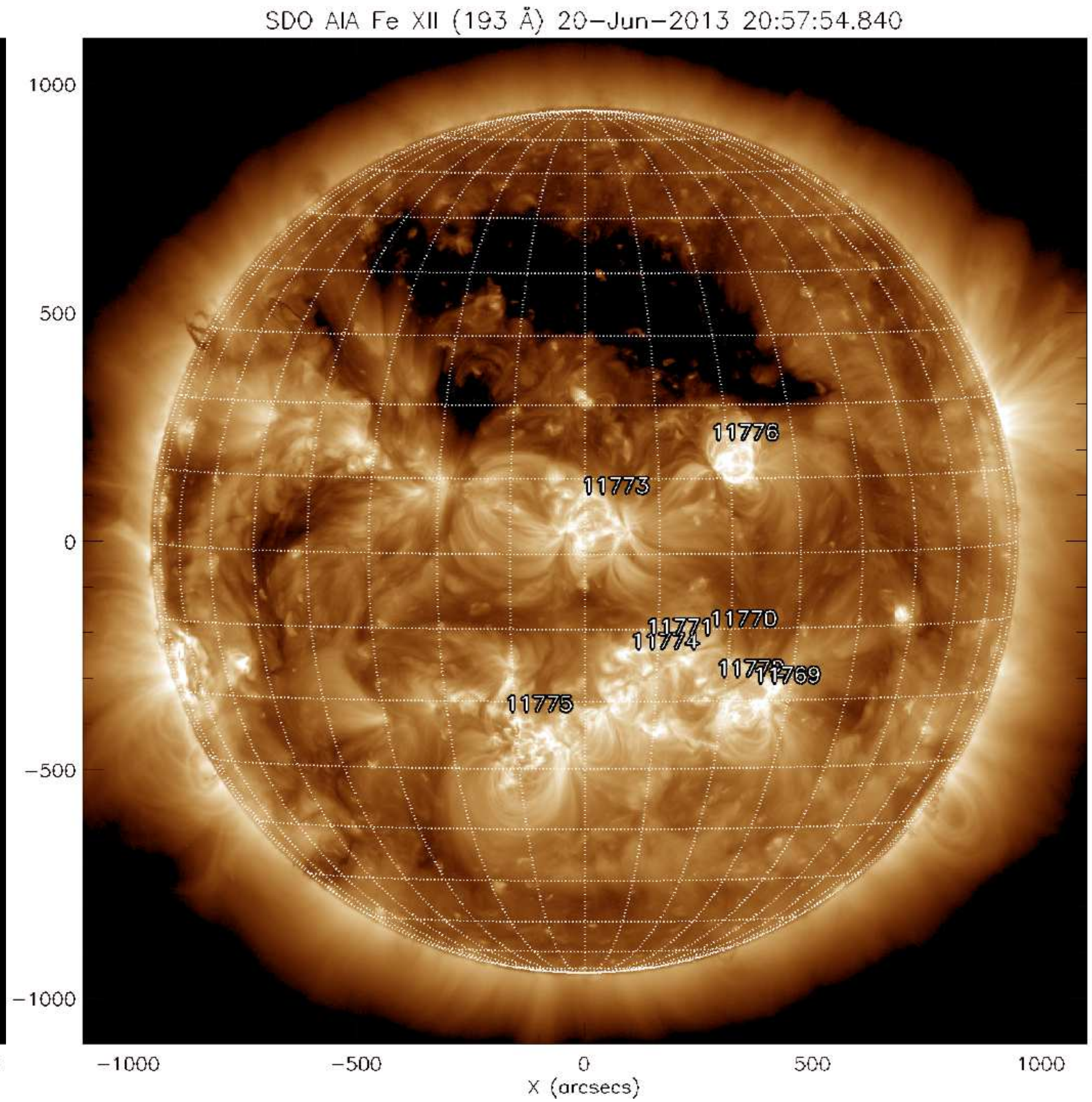




SDO/HMI, AIA 19.3 nm

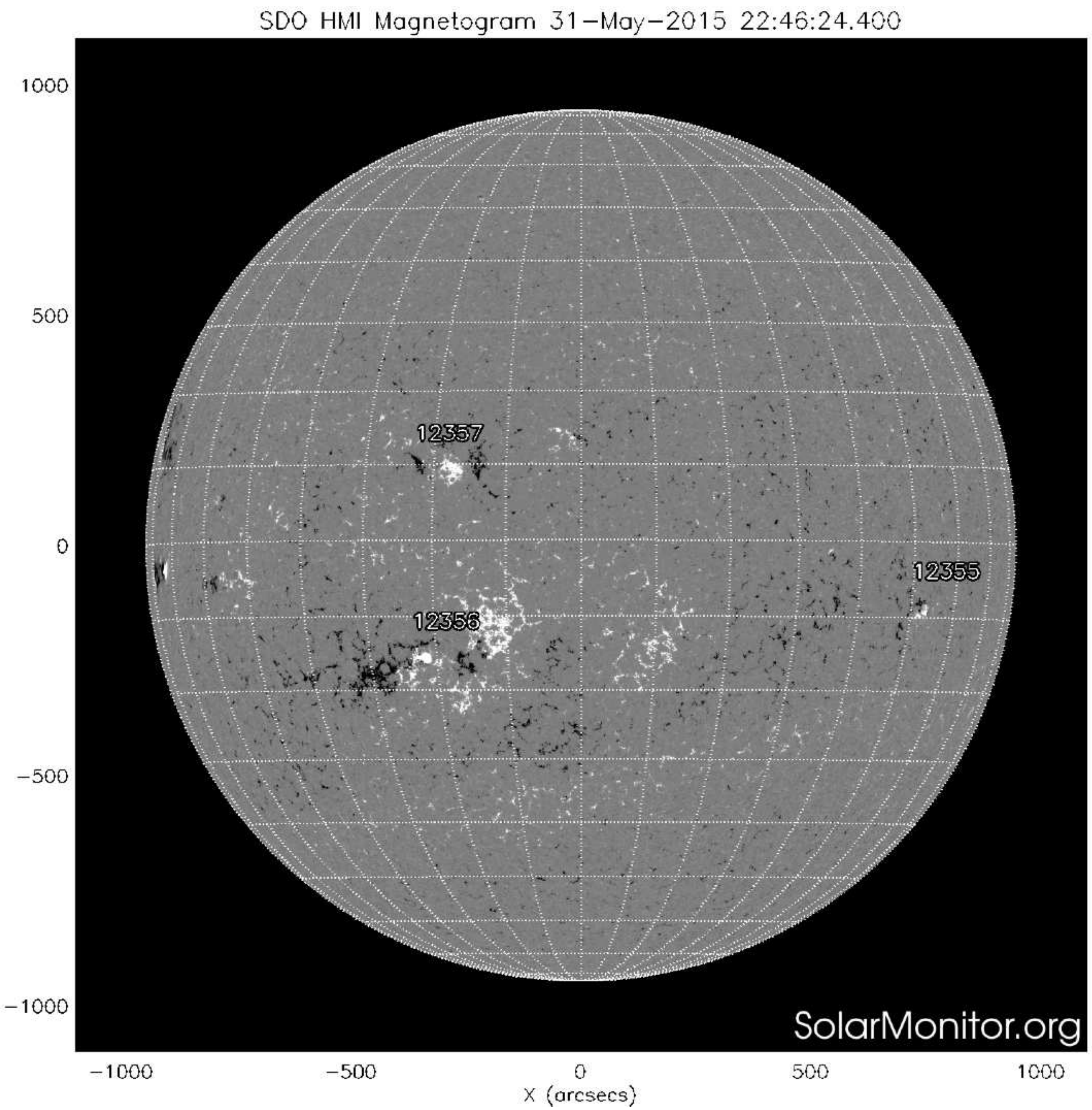
Maximum Phase

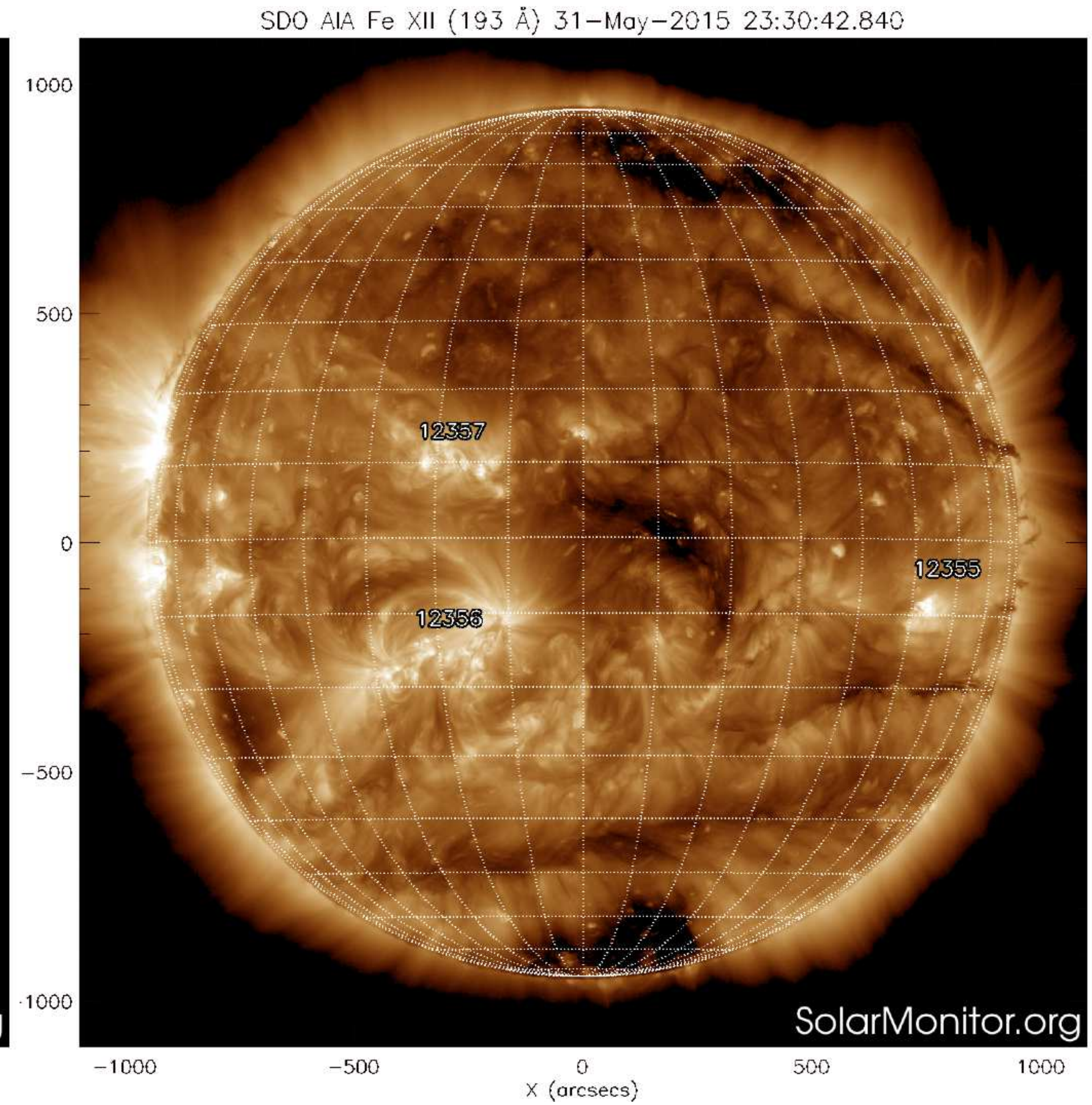




SDO/HMI, AIA 19.3 nm

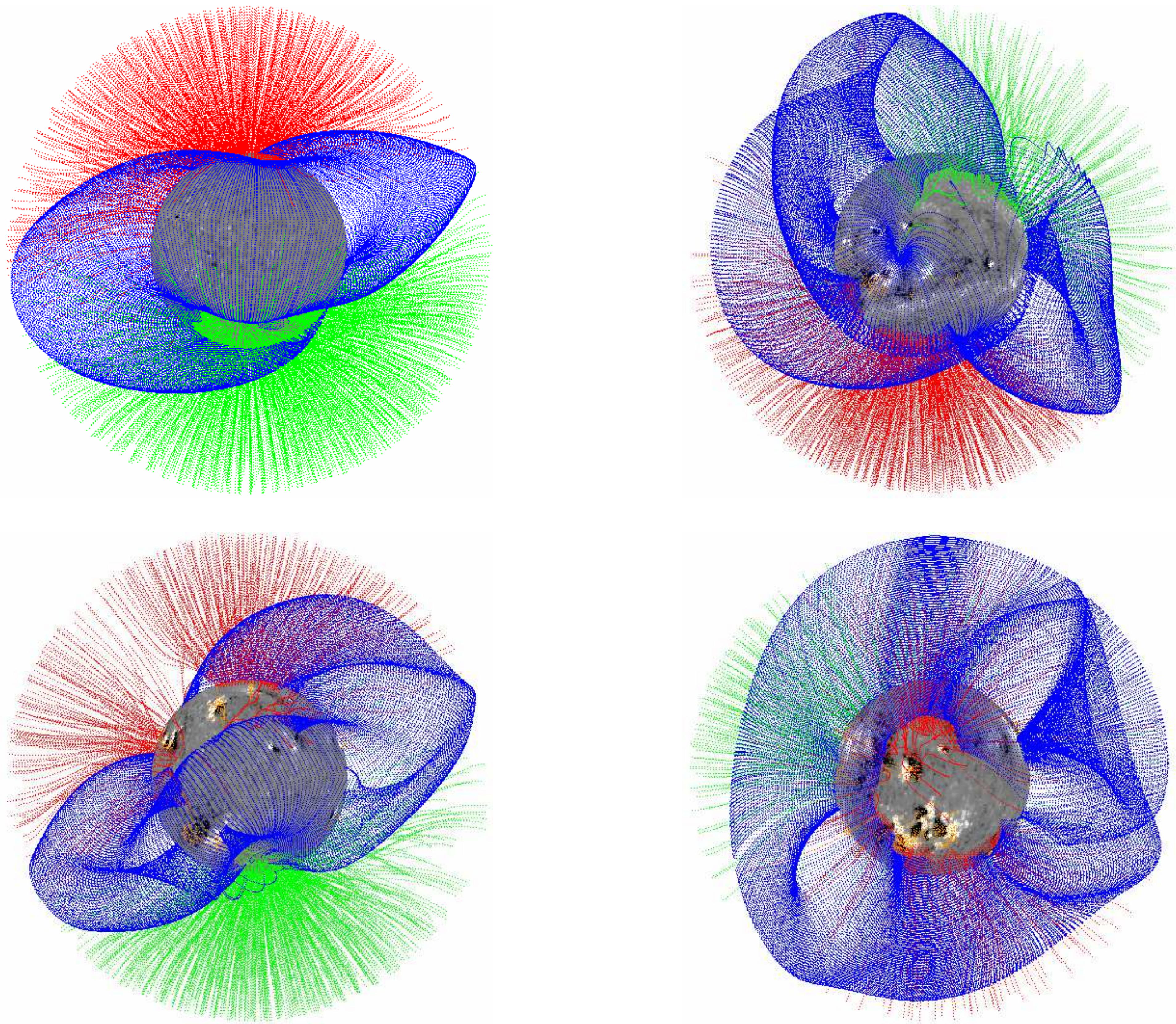
Declining Phase





SDO/HMI, AIA 19.3 nm

Magnetic Topology

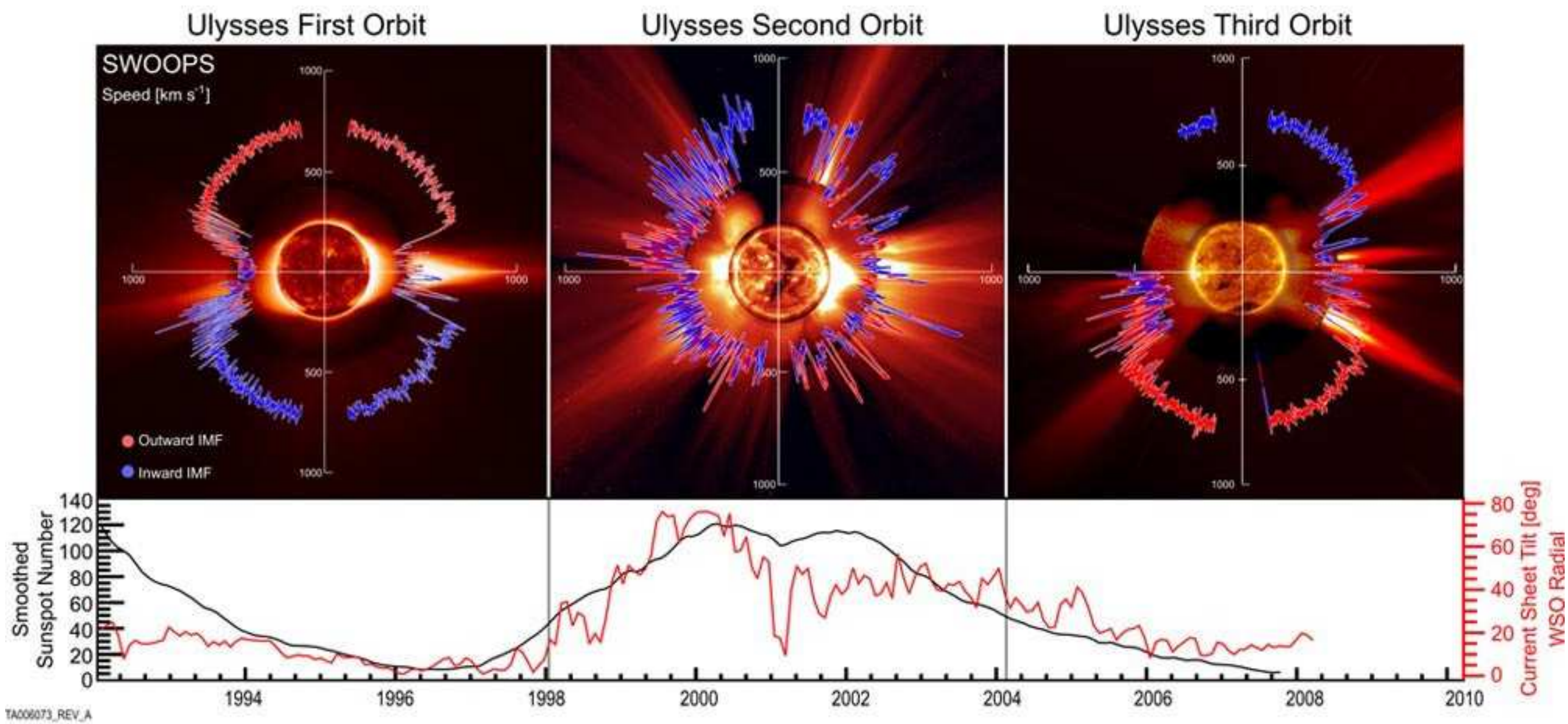


NSO/GONG

Total Solar Eclipse Images

http://www.zam.fme.vutbr.cz/~druck/Eclipse/

Solar Wind Over a Cycle



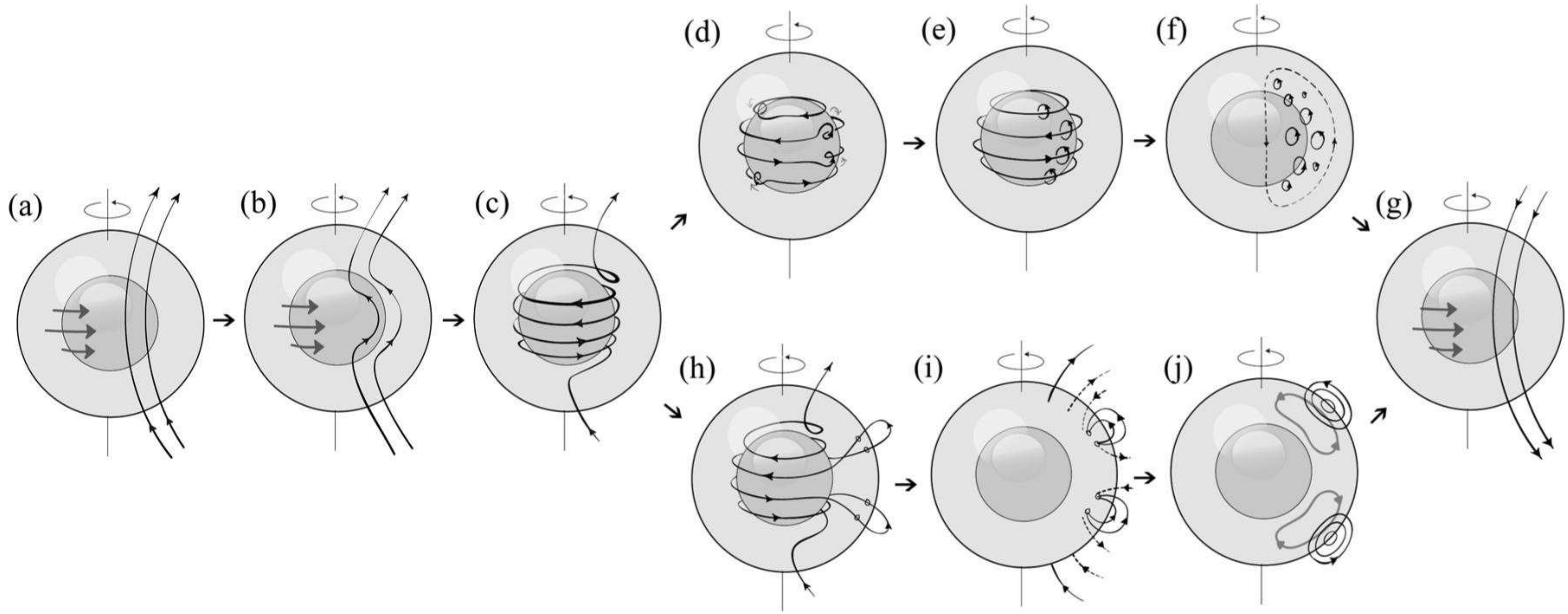
$$\tan\theta \sim |k_{11}|/|k_{10}|$$

McComas et al. (2008)

Babcock-Leighton Mechanism & Surface Flux Transport

Babcock-Leighton Mechanism

α-effect



B-L mechanism

Sanchez et al. (2014)

Surface Flux Transport

- B_r evolution within a thin spherical shell near surface
- Induction equation with source & diffusion

$$\frac{\partial B_r}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})_r + \kappa \nabla^2 B_r + S(\theta, \phi, t)$$

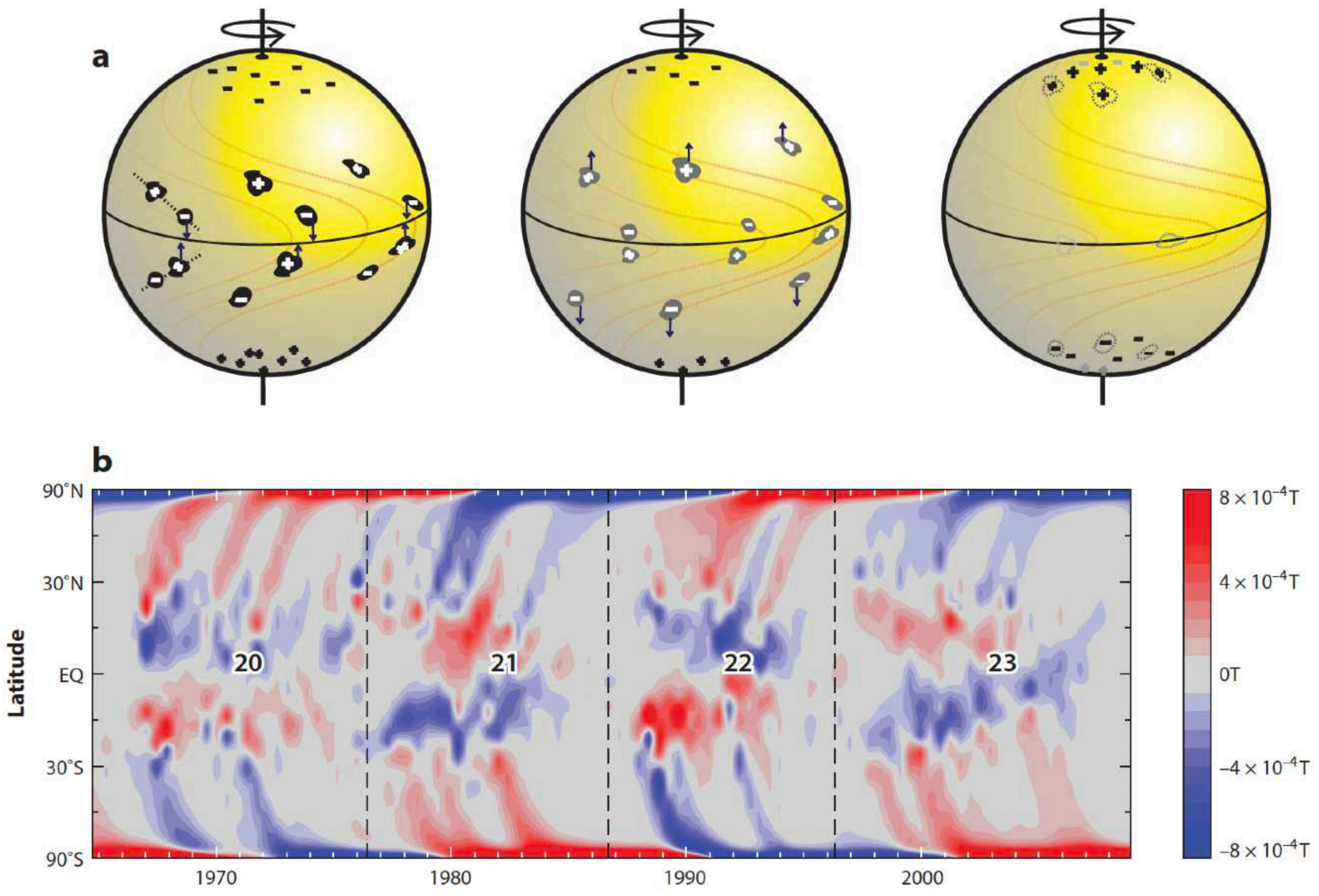
or,

$$\frac{\partial B_r}{\partial t} = -\,\omega(\theta)\frac{\partial B_r}{\partial \phi} - \frac{1}{R_\odot \sin\theta}\frac{\partial}{\partial \theta}[\sin\theta\, v(\theta) B_r] + \frac{\kappa}{R_\odot^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial B_r}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 B_r}{\partial\phi^2}\right] + S(\theta, \phi, t)$$

- Surface flow: differential rotation & meridional flow; measurable
- Diffusion: supergranular dispersion, etc.; empirically determined
- Source: flux emergence; measurable

Surface Flux Transport

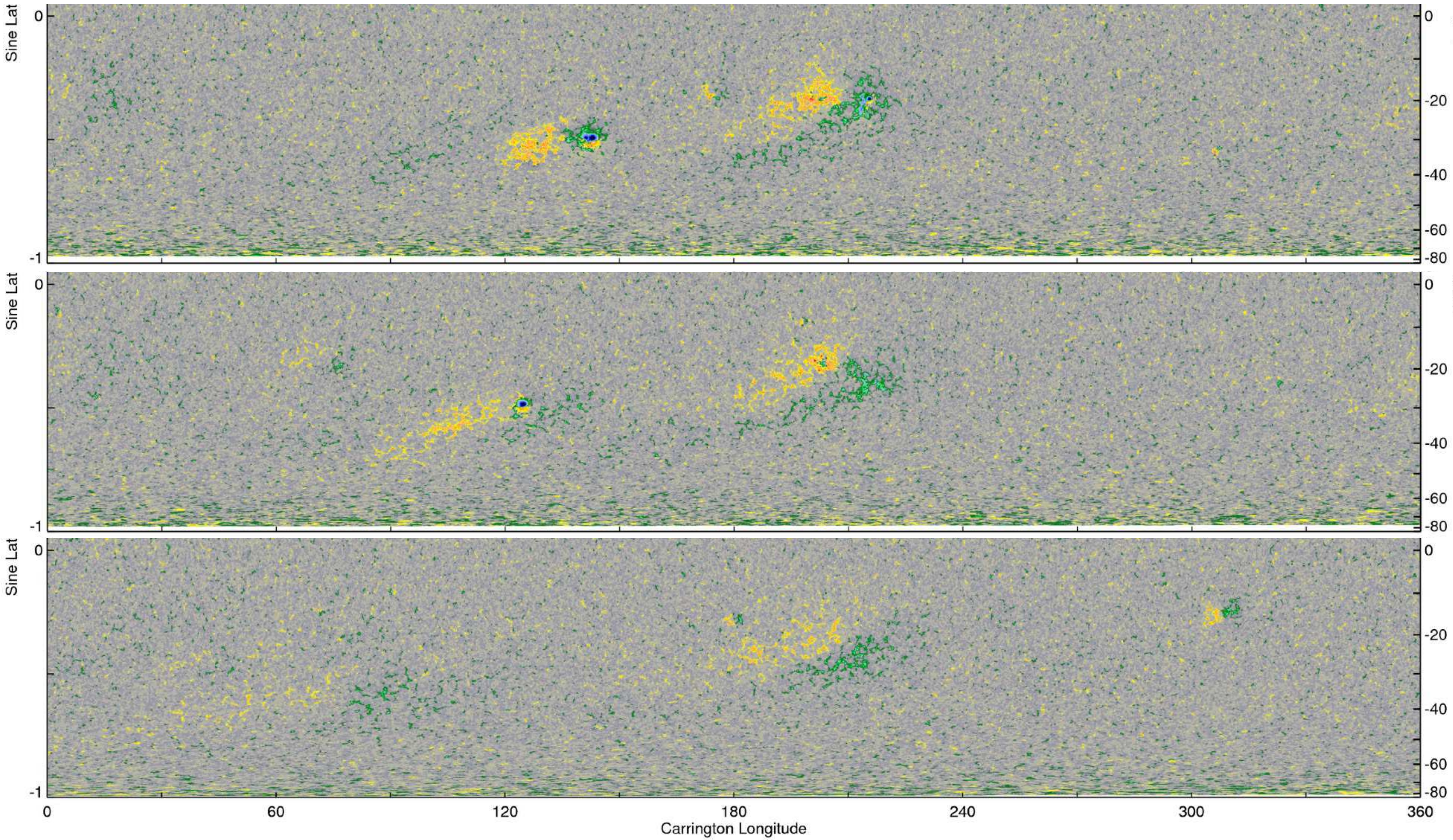
- Toroidal field (sunspot) conversion to poloidal field (dipole)
- Essential ingredient: Hale's law, Joy's law, & cross equatorial cancellation



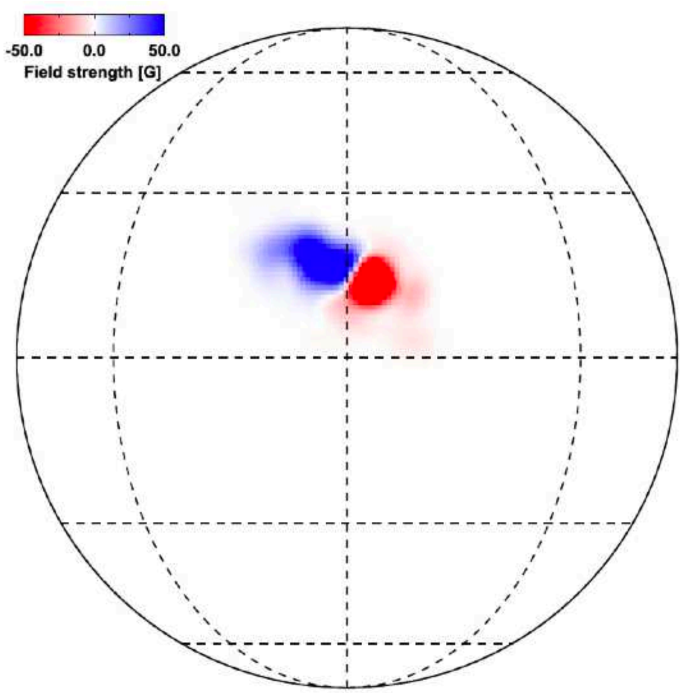
Charbonneau (2014)

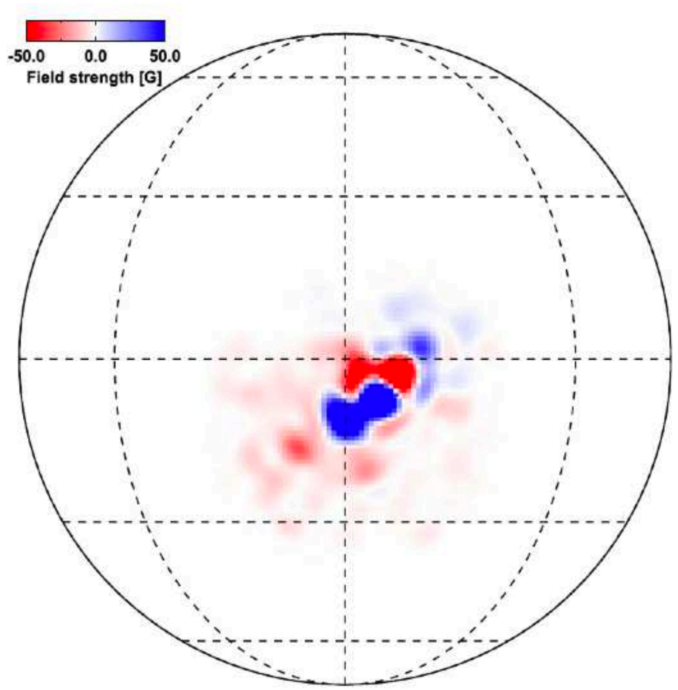
Active Region Flux Dispersal

HMI CR 2102-2104

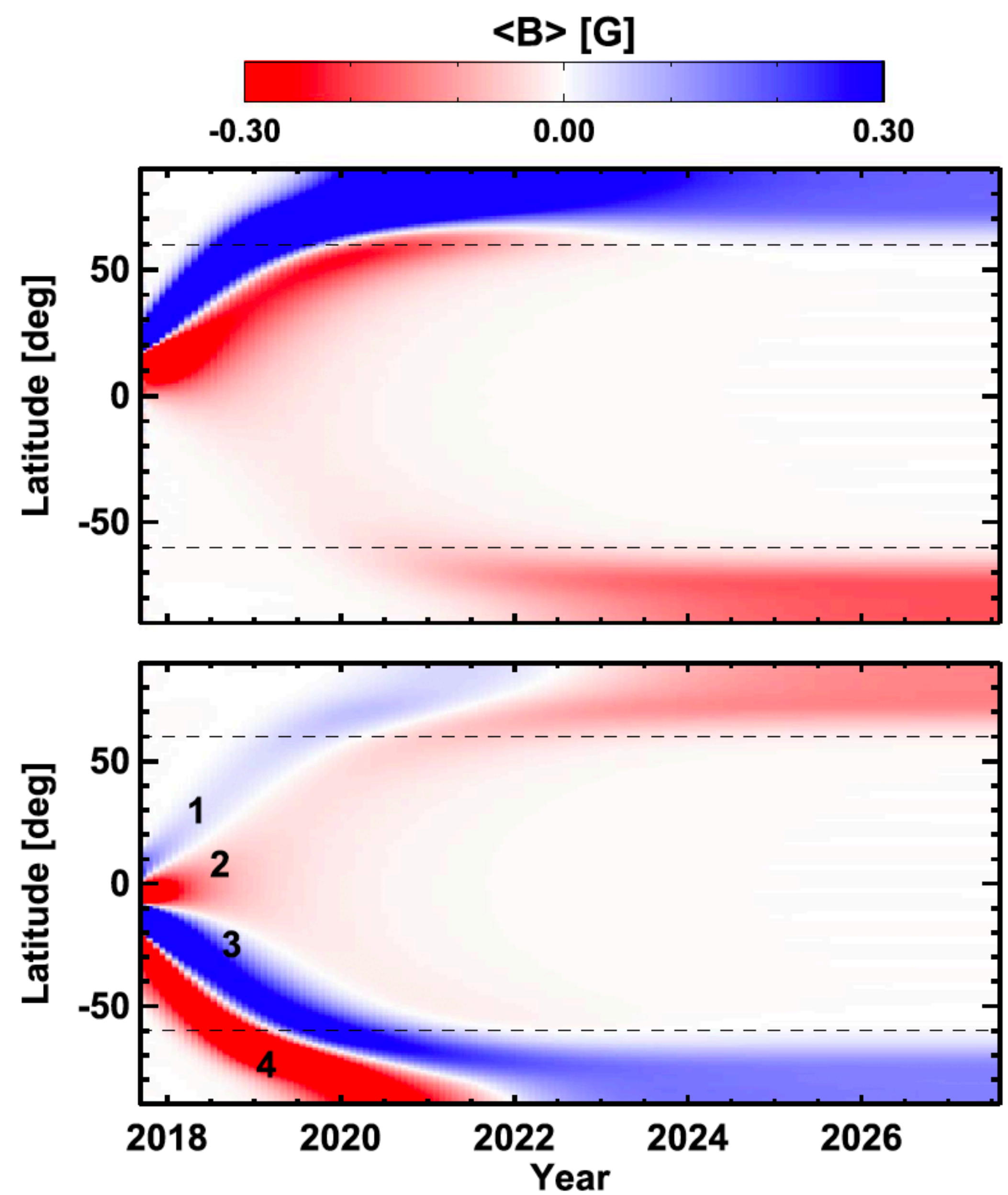


Active Region Flux Dispersal



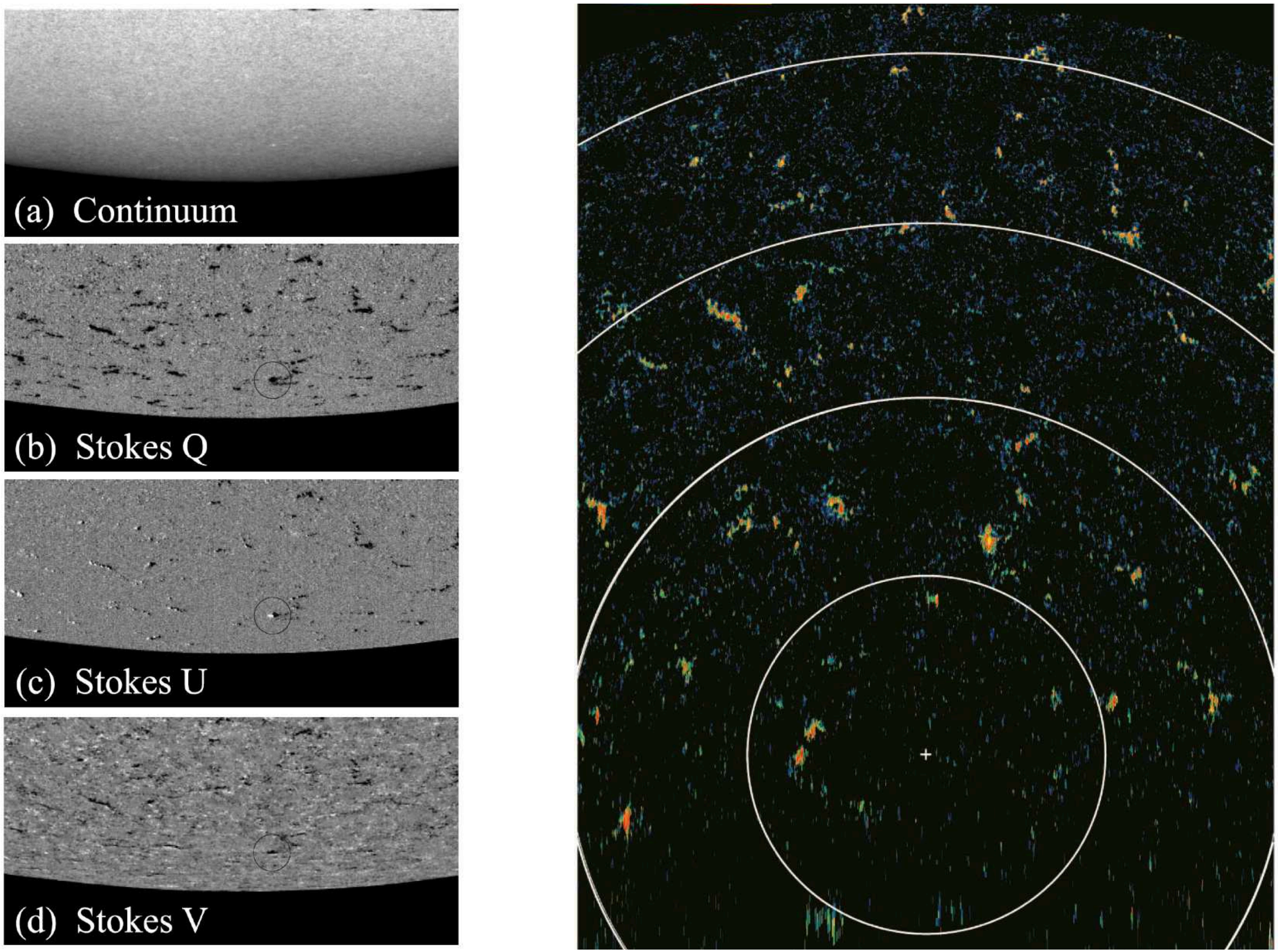


SFT Model of Dipole Contribution



Jiang et al. (2019)

Polar Field

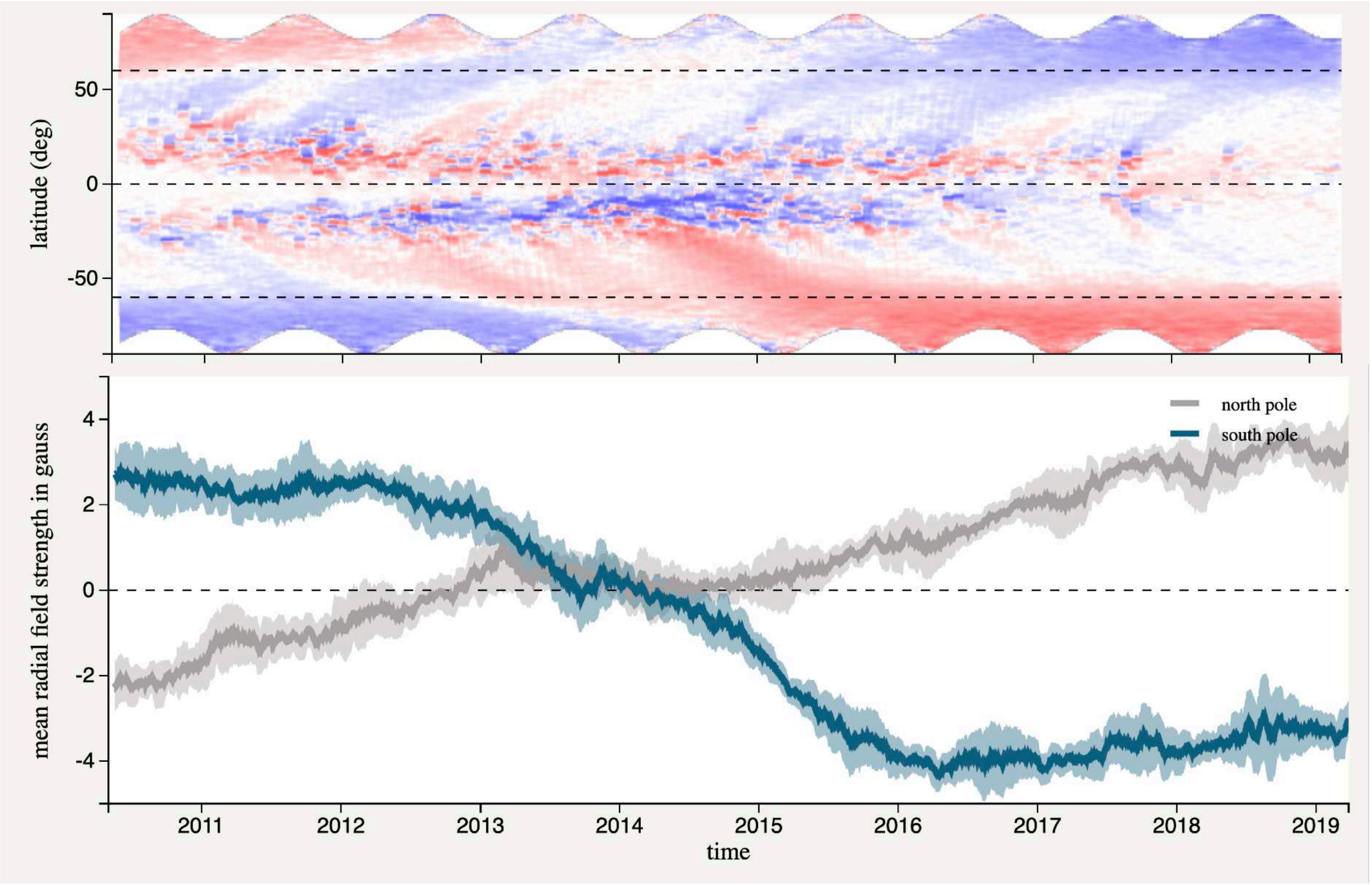


Hinode/SP

Tsuneta et al. (2008)

Polar Field Reversal

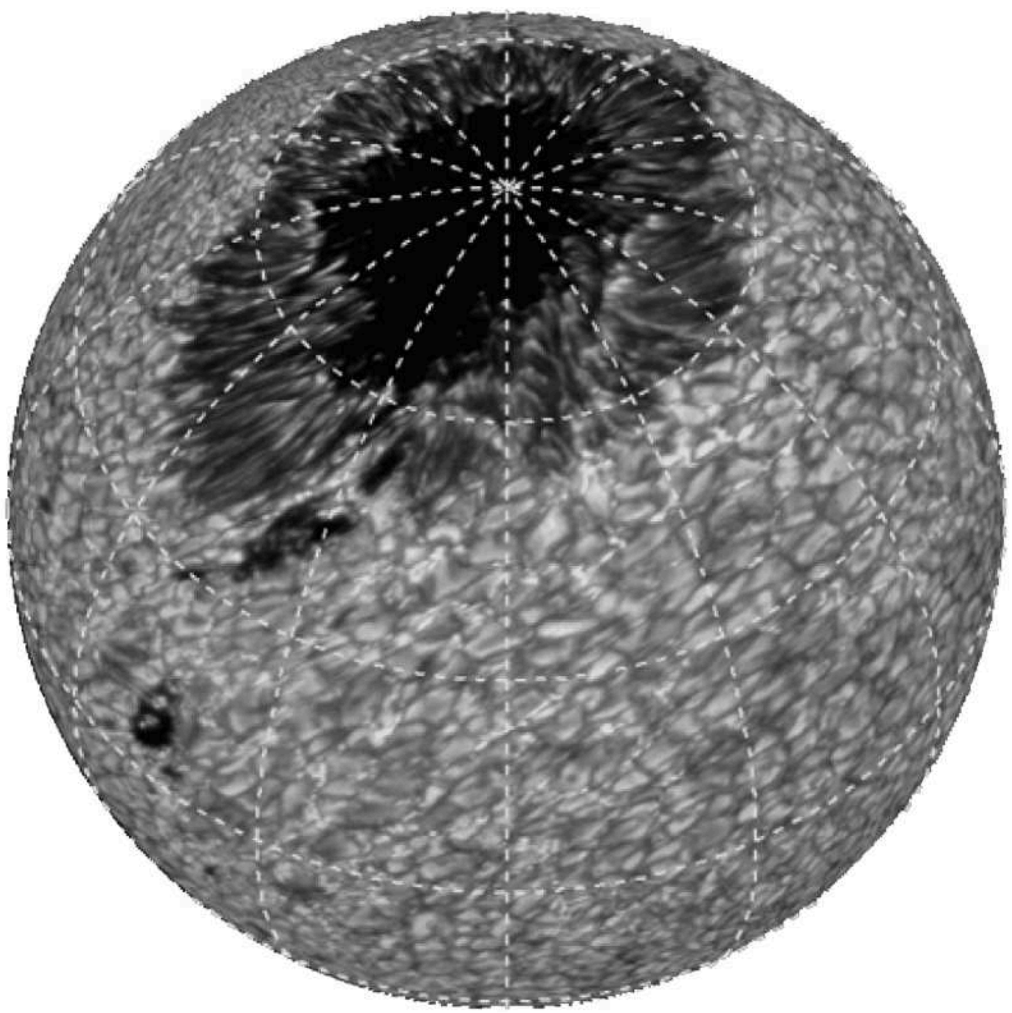
https://github.com/mbobra/plotting-polar-field



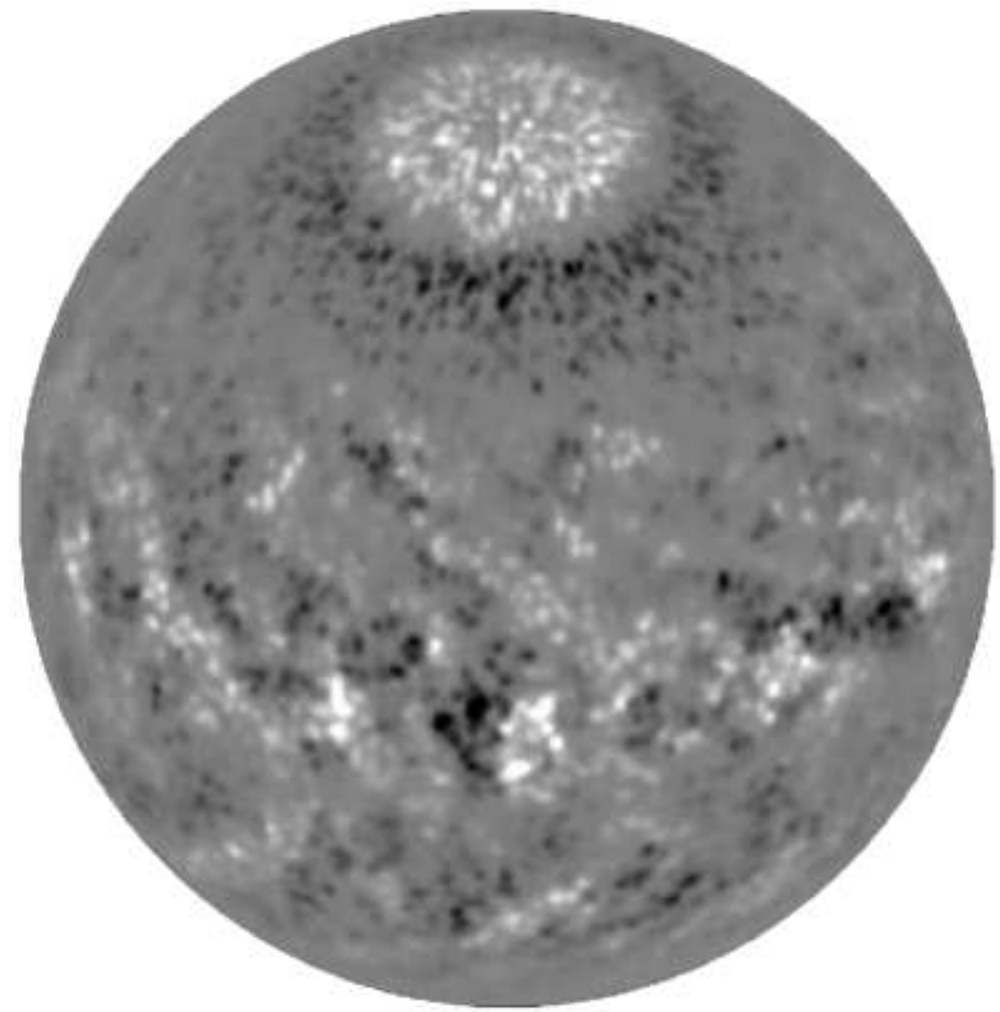
X. Sun & M. Bobra

Application on Active Stars

Emergence rate 10 x + rotation period 6 d = polar field 30 x



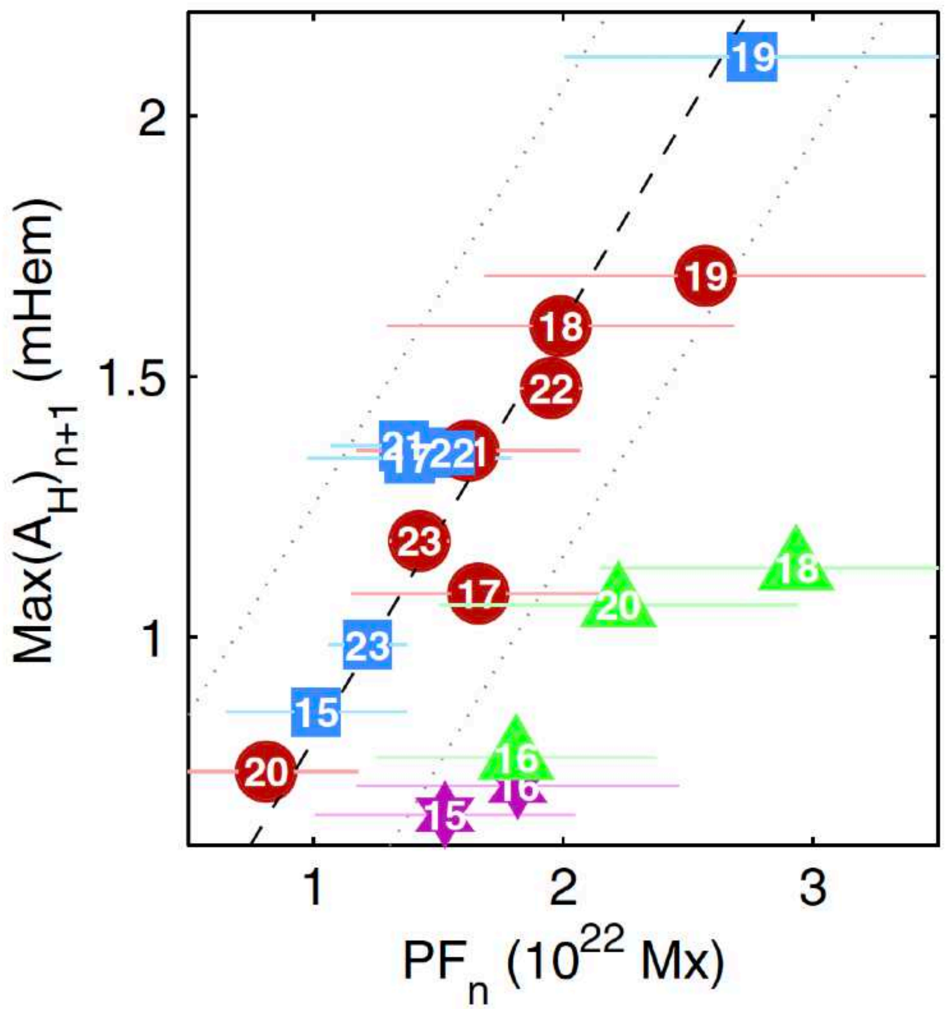
Courtesy S. Solanki



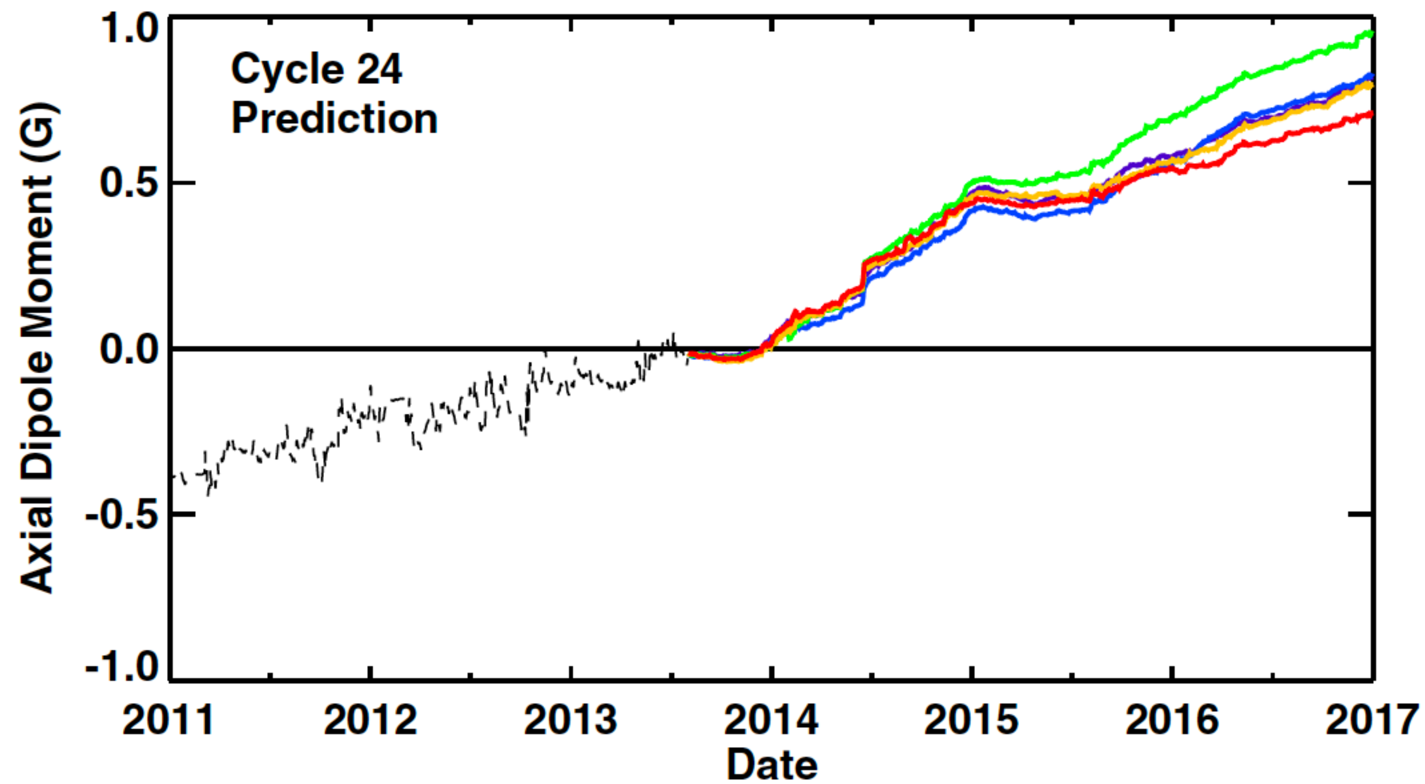
Schrijver & Title (2001)

Cycle Prediction Based on B-L

Poloidal field (polar field at min) of Cycle N ➢ Toroidal Field (SSN) of Cycle N+1



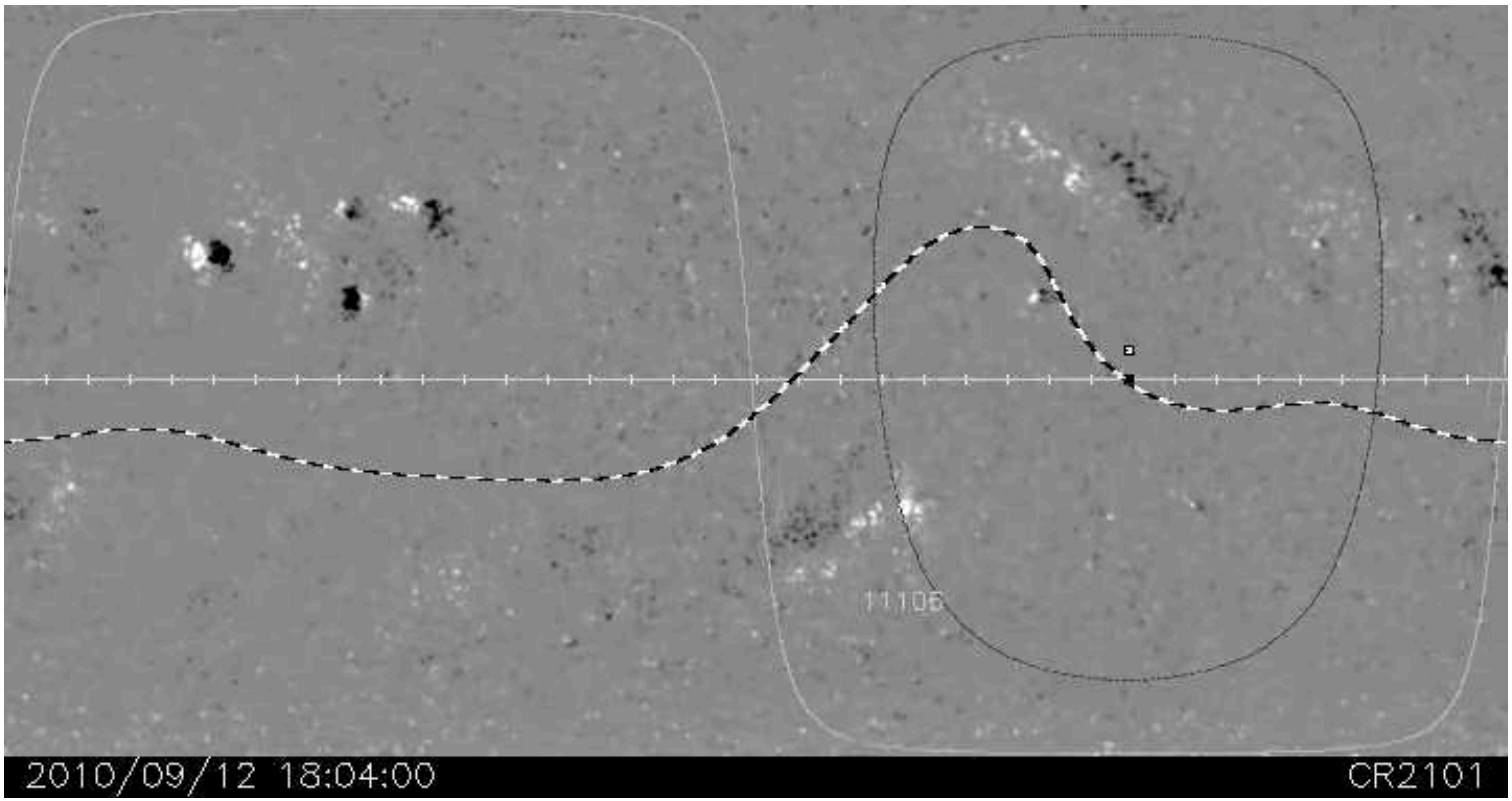
Muñoz-Jaramillo et al. (2013)



Upton & Hathaway (2014)

Constructing Better Synoptic Maps

Near-Real-Time Data Assimilation + Surface Flux Transport



Courtesy M. DeRosa