

# Part II : Radiative transfer and Spectropolarimetry

Line opacity and emissivity

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# Summary

- In the previous lecture we have seen what is radiative transfer equation and how it “produces” the spectra we see.
- We will now delve a bit deeper in the microphysics.
- How to calculate, opacity, emissivity and the source function for given model atmosphere?
- What quantities and physical parameters are important for the spectral line formation?

## But before that, some reminders

Our tool for spectra calculation 1D, time independent RTE:

$$\frac{dI(s, \theta, \lambda)}{ds} = -\chi(s, \lambda)I(s, \theta, \lambda) + \eta(s, \lambda),$$

that we usually cast as:

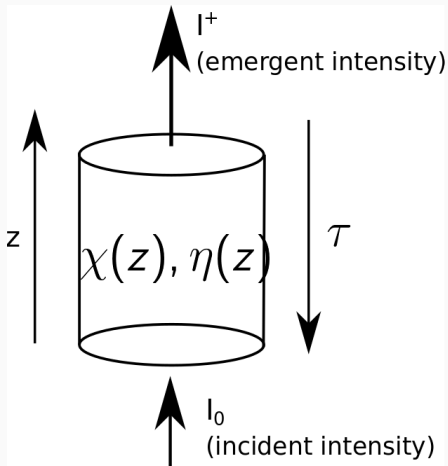
$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda.$$

And the formal solution (once we know opacity and emissivity):

$$I(\lambda) = I_0(\lambda)e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t)e^{-t} dt$$

## Some reminders

$$I^+(\lambda) = I_0(\lambda)e^{-\tau_\lambda} + \int_0^{\tau_\lambda} S(t)e^{-t} dt$$



## Semi-infinite atmosphere

$$I^+(\lambda) = \int_0^{\infty} S_{\lambda}(t_{\lambda}) e^{-t_{\lambda}} dt_{\lambda}.$$

Or:

$$I_{\lambda}^+ = \mathcal{L}_{\lambda}[S_{\lambda}].$$

Given the distribution of the source function on the optical depth scale, intensity is the Laplace transform of the source function. A good question we could already ask (but will solve in a few weeks) is, can we determine

$$S(\tau_{\lambda}) = \mathcal{L}^{-1}[I_{\lambda}]?$$

Short answer: It's hard.

## Semi-infinite atmosphere

So, technically speaking, emergent spectrum is a functional of the opacity and emissivity distributions:

$$I_{\lambda}^{+} = \mathcal{F} [\chi_{\lambda}(z), \eta_{\lambda}(z)].$$

But we already intuitively feel that opacity and emissivity depend on the physical conditions:

$$\chi, \eta = f(T, \rho, n_e, \mathbf{v}, \mathbf{B} \dots).$$

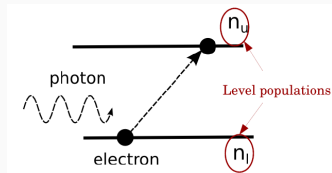
Ok sharpen your pencils and let's deconstruct this.

Contrary from all the other textbooks, we will start with the spectral lines. Why?

- We are interested in the diagnostics, spectral lines excel there.
- Line absorption/emission is easy to understand, as it is a discrete process.
- Expressions are actually simpler and we will relate them more easy to statistical physics.

# Spectral line emission

In line (bound-bound) transitions, photons are created and destroyed in discrete processes.



Say that we have  $n_u$  excited atoms in unit volume, what happens then? How to calculate the emissivity? Remember, the emissivity is:

$$\eta(\hat{\Omega}, \lambda) = \frac{d^5 E^{\text{emitted}}}{d\sigma d\hat{\Omega} dt d\lambda dz}$$



# Spectral line emission

Step by step: Notice that  $d^5 E = d^5 N hc/\lambda_0$  ( $\lambda_0$  because everything happens close to the line core)

$$\frac{d^2 N}{d\sigma dz} = \frac{d^3 N}{dx dy dz} = n_u$$

So:

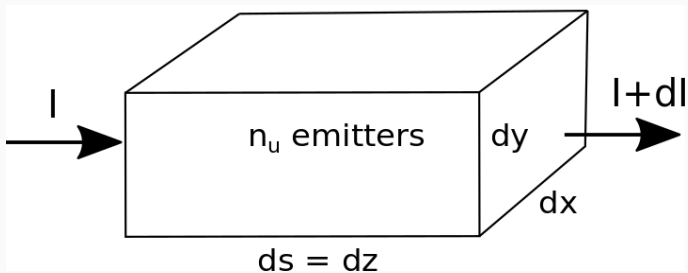
$$\eta \propto n_u$$

. Then, assume that the emission is isotropic:

$$\frac{d}{d\hat{\Omega}} \rightarrow \frac{1}{4\pi}$$

## Spectral line emission

Emissivity is proportional to the density of emitters:



$$\eta = \frac{dI}{ds} \propto \frac{d}{ds} \frac{dN}{dxdy}$$

## Spectral line emission

Now,  $d/dt$  is tricky. Imagine you have an ensemble of excited atoms, that you somehow keep excited (i.e. as soon as one emits a photon, you flip it back).

There should be some quantity that tells us how often, on average, an atom radiatively de-excites. It is called **Einstein coefficient of spontaneous emission**  $A_{ul}$ .

For lines of interest to us  $A_{ul} \approx 10^5 - 10^8 \text{s}^{-1}$ .

So, at the moment:  $\eta \propto n_u \frac{hc}{4\pi\lambda} A_{ul}$ .

# Spectral line emission

Finally, wavelength dependence.

If the line transitions were really discrete, line profiles would be delta functions. So, we would have:

$$\eta(\lambda) = n_u \frac{hc}{4\pi\lambda} A_{ul} \delta(\lambda - \lambda_0).$$

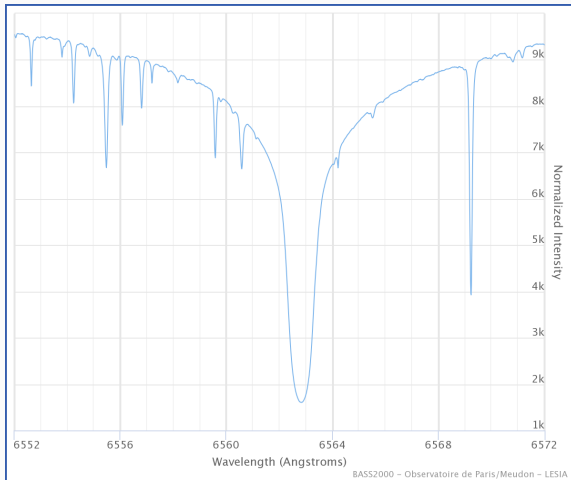
Are the units ok? Well:

$$\int_0^{\infty} \delta(\lambda - \lambda_0) d\lambda = 1.$$

So, the units of the “line profile” have to be  $[\text{\AA}]^{-1}$ .

# Line profile

Apparently, line profiles are not delta functions:



Take a moment to appreciate this fact. Use this useful website.

## Line profile

Let's assume that the emission line profile is a delta function, **in the frame of the atom itself**. Atoms however, are moving, according to the Maxwell distribution:

$$f(v)d^3v = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp(-m|v|^2/2kt)d^3v.$$

What we need is the projection to our line of sight ( $z$  for simplicity):

$$f(v_z) dv_z = \frac{1}{\sqrt{\pi}\Delta v_D} \exp(-v_z^2/\Delta v_D^2).$$

Where **Doppler velocity**,  $\Delta v_D$  is:

$$\Delta v_D = \sqrt{2kT/m}.$$

So, higher the temperature, wider the lines.

Also, heavier the element, more narrow the lines.

## Doppler profile

Now,  $dn(v_z) = n_u f(v_z)$ . Atoms with line-of-sight velocity  $v_z$  will emit at the wavelength  $\lambda' = \lambda(1 + v_z/c)$  (negative velocity is toward the observer). To get the complete profile:

$$\phi(\lambda) = \int_{-\infty}^{\infty} \delta(\lambda' - \lambda_0) n_u f(v_z) (1 - v_z/c) dv_z.$$

In this situation delta function integrates out. So, we get simply:

$$\phi(\lambda) = 1/(\sqrt{\pi}\Delta\lambda_D) \exp(-(\lambda - \lambda_0)^2/\Delta\lambda_D^2)$$

Units check out ( $\text{\AA}^{-1}$ ). Keep in mind that for arbitrary “intrinsic” broadening profile, composite profile, that takes into account the influence of thermal broadening would be:

$$\phi(\lambda) = \phi_0(\lambda) * \phi_{\text{Doppler}}(\lambda)$$

It is customary to define **total** emissivity:

$$\eta = n_u \frac{hc}{4\pi\lambda_0} A_{ul} \frac{1}{\Delta\lambda_D},$$

and then

$$\eta(\lambda) = \eta \phi(\lambda).$$

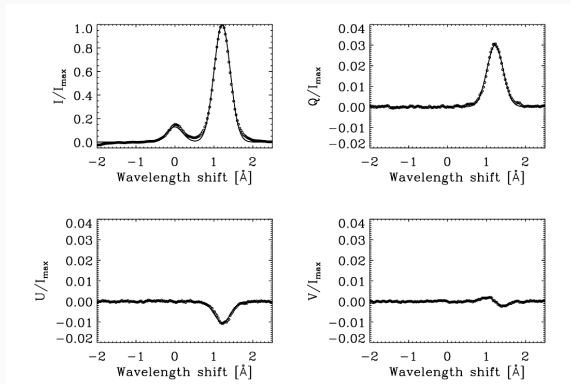
Note that the profile normalizes to unity.

Now you were saying something about an “intrinsic profile”? Before that...



# Additional 'thermal' broadening

In the following example, authors fit helium line at  $10830\text{\AA}$ , and find  $\Delta v_D = 8\text{ km/s}$ .



**Figure 1:** Prominence polarized spectra fitting using HAZEL code. From Asensio Ramos et al.(2008)

## Additional 'thermal' broadening

For Helium  $m \approx 6 \times 10^{-27}$  kg. For this line  $\lambda_0 \approx 10^{-5}$  m.

$$\Delta v_D = \sqrt{\frac{2kT}{m}}$$

$$T = \frac{v_D^2 m}{2k} = \frac{64 \times 10^6 \times 6 \times 10^{-27}}{2 \times 1.38 \times 10^{-23}} = 15000 \text{ K}$$

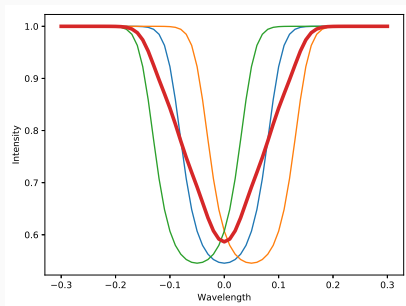
However, we know from other sources that this is too high temperature for a prominence! (For example, all the hydrogen would be completely ionized). What is going on here?

# Mystery of microturbulence

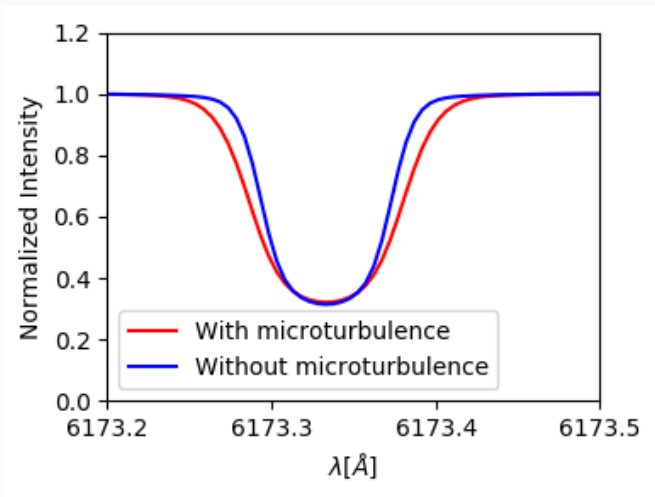
Microturbulence is an ad hoc parameter, added in the early days of spectra modeling.

The idea was to describe the mean width of spectral lines.

Is microturbulence really “turbulence”? It could be, but does not have to be, it could be any unresolved velocity.



## Effects of microturbulence



Why are both the lines identical in the center and differ in the wings?

# Natural broadening

So far, we have assumed that in the frame of the atom, emission is a delta function.

What does this mean? (Keep in mind that we are talking about an ensemble of atoms, no matter how small  $n_u$  is, it has to be large enough so that averaging makes sense).

This means every atom has exactly the same energy of the upper and lower level.

That is not strictly true...  $\Delta E \Delta t \approx \hbar$ .

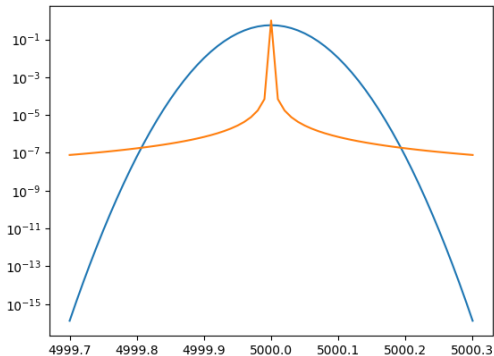
Natural broadening, in frequency units  $\Gamma_R = \frac{1}{\Delta t} = A_{ul}$

(Actually  $\sum_{l' < u} A_{ul'}$ ).

# Natural broadening

Natural broadening has the form of the Lorentz function:

$$\phi(\nu) = \frac{1}{\pi} \frac{\alpha_L}{(\nu - \nu_0)^2 + \alpha_L^2}$$



# Voigt profile

So, in practice, we have the convolution of the natural, Lorentzian shape, and the Doppler profile. It is called Voigt profile:

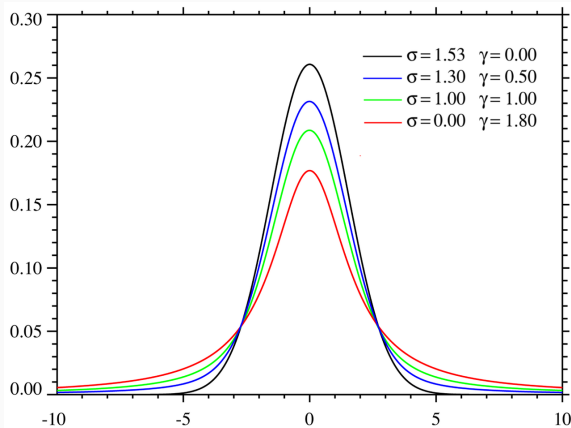


Figure 2: credits: Wikipedia

## Collisional broadening - “damping”

We said that natural broadening appears because averaging over the ensemble (and, well, time!) smears out the photon energy. There is another effect like that.

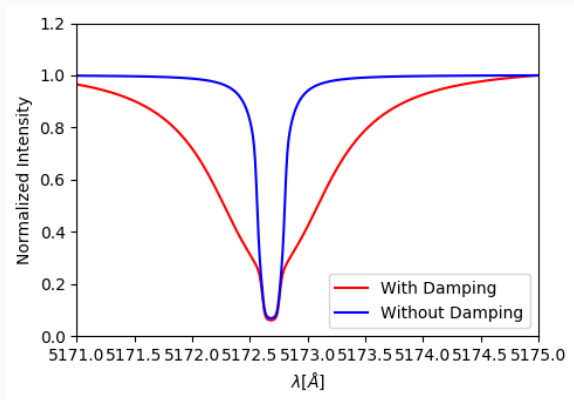
“Collisional” or “pressure” broadening.

Electric fields of the surrounding particles disturb the energy configuration of the atom.

Linear Stark ( $\propto r^{-2}$ ), Quadratic Stark ( $\propto r^{-4}$ ), van der Waals ( $\propto r^{-6}$ ).



# Effect of damping



**Figure 3:** Mg I b2 line calculated from an empirical model atmosphere

## Line opacity

So far, in all the calculations we have seen, there was some opacity involved. How to calculate that?

Classically, we can describe opacity with a cross-section per particle,  $\sigma$  [cm<sup>2</sup>], then:

$$\chi = \sigma \times n_{\text{absorbers}}.$$

But, it would be good to have similar expression to the one for the emissivity. If there only was a way....

$$\chi_{\lambda} = n_l \frac{hc}{4\pi\lambda} B_{lu} \phi(\lambda).$$

$B_{lu}$  is Einstein coefficient of absorption.

# Stimulated emission

We considered radiative excitation (absorption), and spontaneous radiative de-excitation (spontaneous emission). What about stimulated de-excitation (stimulated emission)?

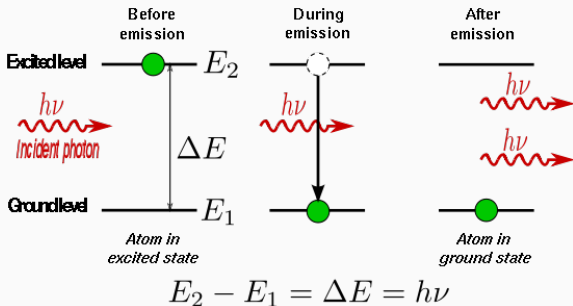


Figure 4: credits: Wikipedia

# Stimulated emission

So in stimulated emission from one photon we get two, in absorption, from one photon we get zero.

Stimulated emission is exactly the same like absorption, just the opposite.

So:

$$\chi_{\lambda} = (n_l B_{lu} - n_u B_{ul}) \frac{hc}{4\pi\lambda} \phi(\lambda).$$

## Some caveats

We have assumed here that the both profiles (absorption and emission) are the same. Why would that be?

Velocity distribution is the same  $\rightarrow$  Doppler broadening is the same.

What happens if the absorption process is followed up by an emission process?

## Source function in the line

Assume the line processes dominate:

$$\frac{\eta(\lambda)}{\chi(\lambda)} = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$$

In LTE, this needs to be equal to the Planck function:

$$\frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$$

From here we can derive relationships between A and B's.

Another important point: For optical domain, usually  $n_l \gg n_u$ , and if  $n_l$  is the ground level, most of the electrons are there. So the value of the source function is set by  $n_u$ . More on this when we talk about NLTE.

# Summary

We “derived” expressions for emissivity and opacity due to the spectral line processes.

Spectral lines are important because they are sensitive to a variety of physical processes and to variety of depths (remember that the opacity changes dramatically).

We now have a vague idea how to calculate the emerging spectrum.

## Next two lectures

We will...

... discuss other opacity/emissivity sources.

... show how to calculate “populations”  $n_u, n_l$ .

... show the signatures the observed solar and stellar spectra.

... calculate a spectrum ourselves.

Please install python with matplotlib, numpy and scipy. I will send instructions via e-mail.