Hale COLLAGE (NJIT Phys-780)
Topics in Solar Observation Techniques

Lecture 05: Grating Spectrograph

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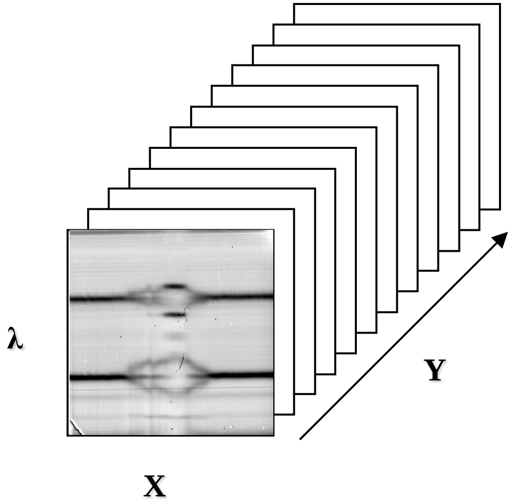
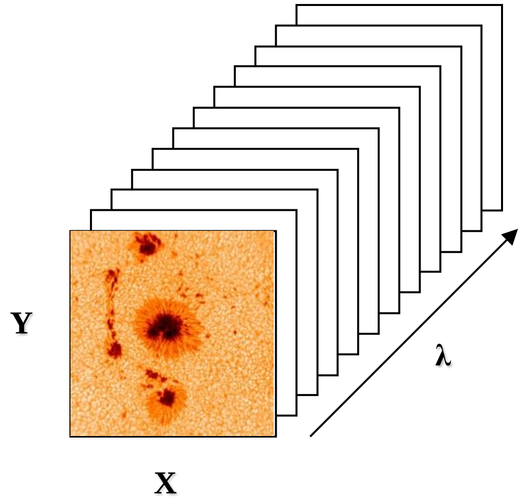
"No single tool has contributed more to the progress of modern physics than the spectrograph …"
-- G. R. Harrison

1. Imaging Spectroscopy

- *Spectrographs vs. Filters*

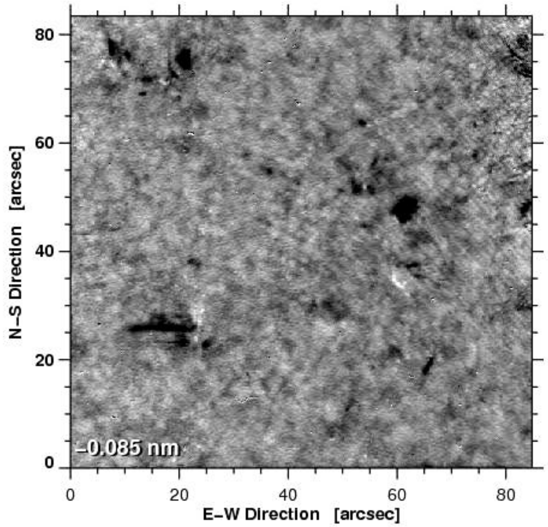


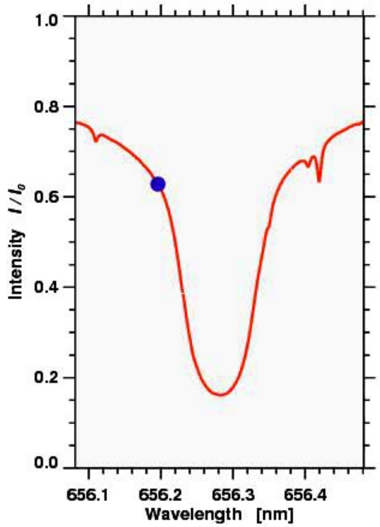


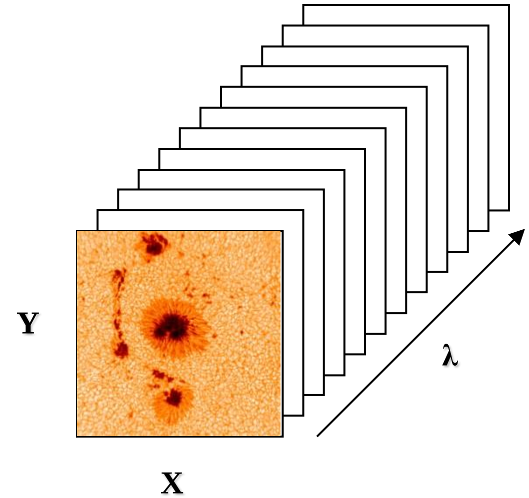
Need x, y, λ observations, can observe 2 simultaneously

Spectrograph observes x, λ
Filter observes y, x

Filter Spectroscopy



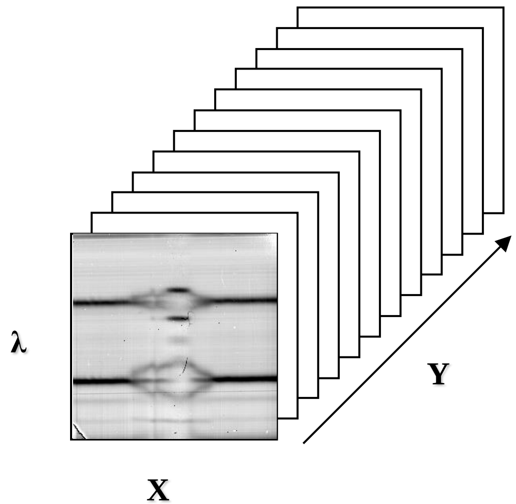


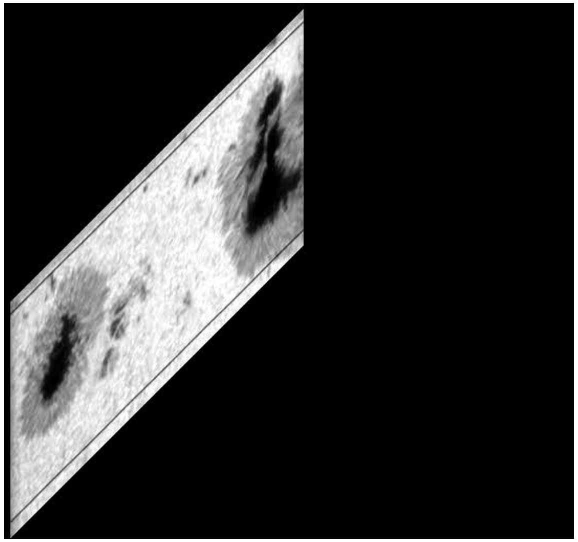


Need x, y, λ observations, can observe 2 simultaneously

Spectrograph observes x, λ
Filter observes y, x

Spectrograph Spectroscopy

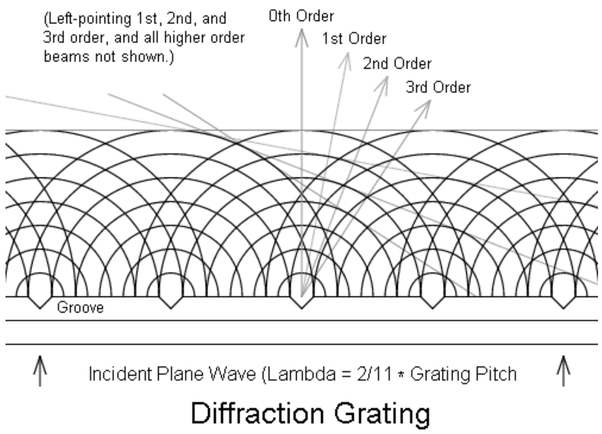


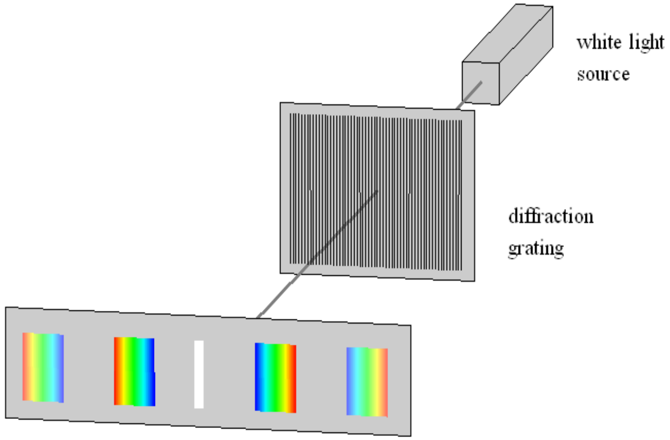


Need *x*, *y*, *λ* observations, can observe 2 simultaneously

Spectrograph observes *x*, *λ*
Filter observes *y*, *x*

2. Diffraction Grating





- ❑ ***Fraunhofer Diffraction Theory***
  - ❑ *Single Slit Diffraction*
  - ❑ *Double Slit Diffraction*
  - ❑ *Diffraction by Many Slits*
- ❑ ***Diffraction Grating Theory***
  - ❑ *Grating Equation*
  - ❑ *Transmission and Reflection Phase Grating*
  - ❑ *Blazing Angle*
  - ❑ *Angular Dispersion*
  - ❑ *Spectral Power*
  - ❑ *Free Spectral Range*
- ❑ ***Grating Spectrographs***
  - ❑ *Reflection Grating Spectrograph*
  - ❑ *Echelle Grating Spectrograph*

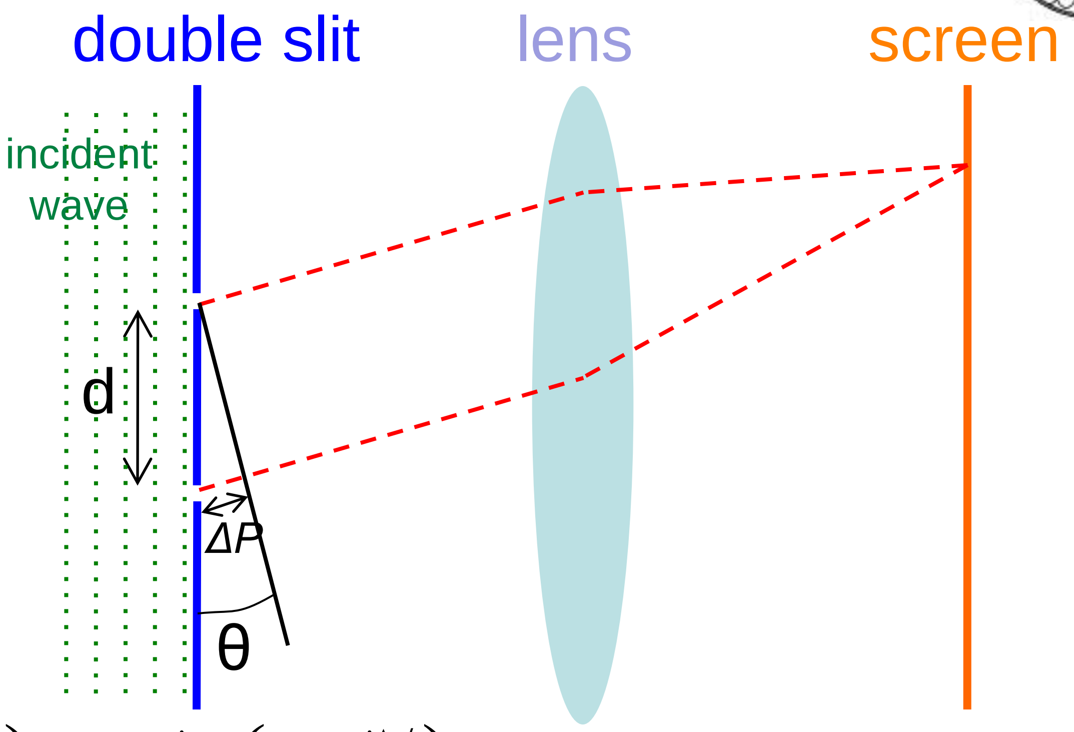
Young's Double Slit Experiment

Optical path difference:

$$\Delta P = d \sin\theta$$

Phase difference:

$$\Delta\phi = 2\pi \frac{\Delta P}{\lambda}$$



Add the two waves:

$$E(t) = E_1\left(e^{i\omega t} + e^{i(\omega t+\Delta\phi)}\right) = E_1 e^{i\omega t}\left(1 + e^{i\Delta\phi}\right)$$

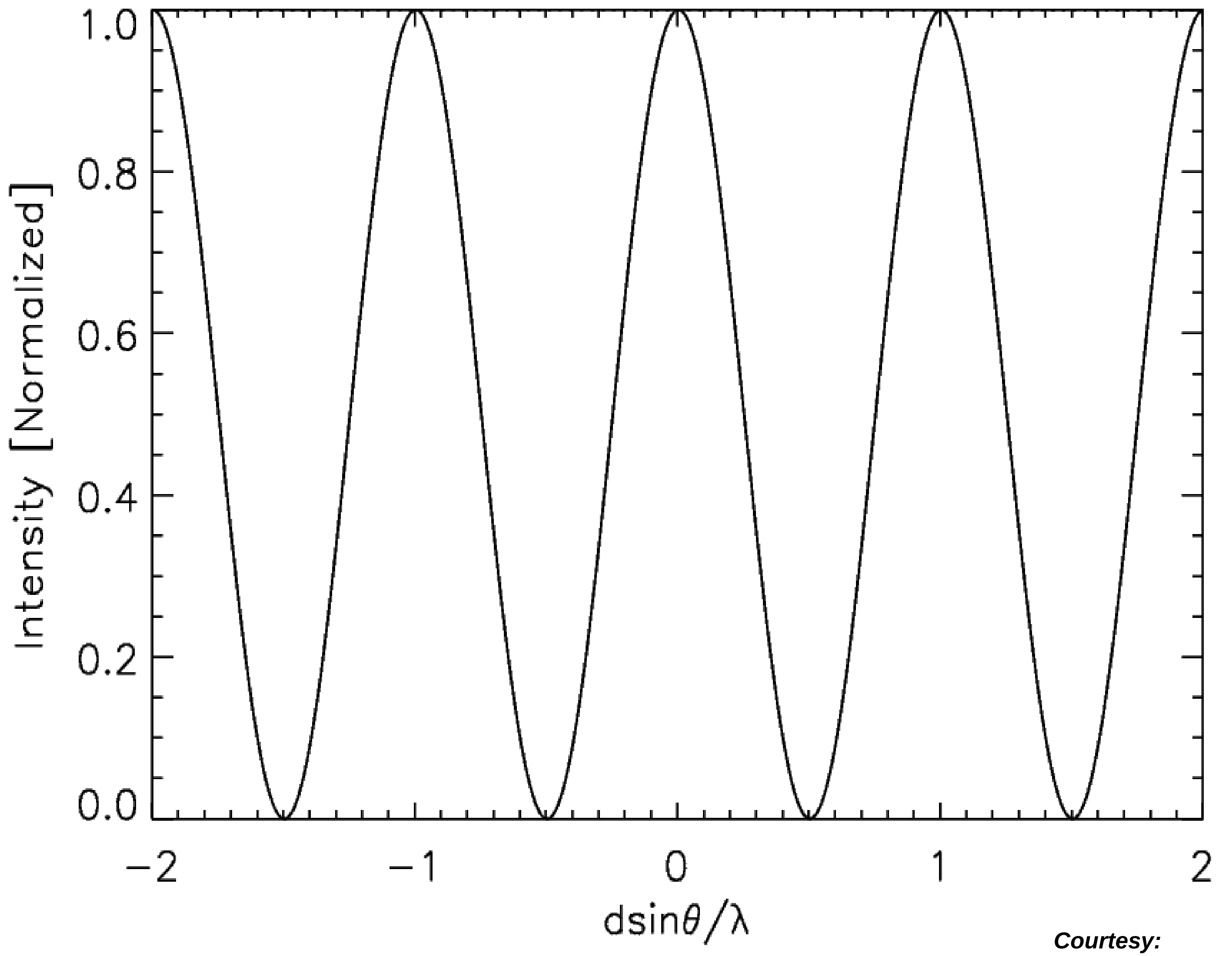
Intensity is amplitude-squared:

$$I = E_1^2\left(1 + e^{i\Delta\phi}\right)\left(1 + e^{-i\Delta\phi}\right) = E_1^2(2 + 2\cos\Delta\phi)$$

$$= 4E_1^2 \cos^2(\Delta\phi/2) = 4E_1^2 \cos^2(\pi d \sin\theta/\lambda)$$

Courtesy:
Roy van Boekel & Kees Dullemond

N=2



Courtesy:
Roy van Boekel & Kees Dullemond

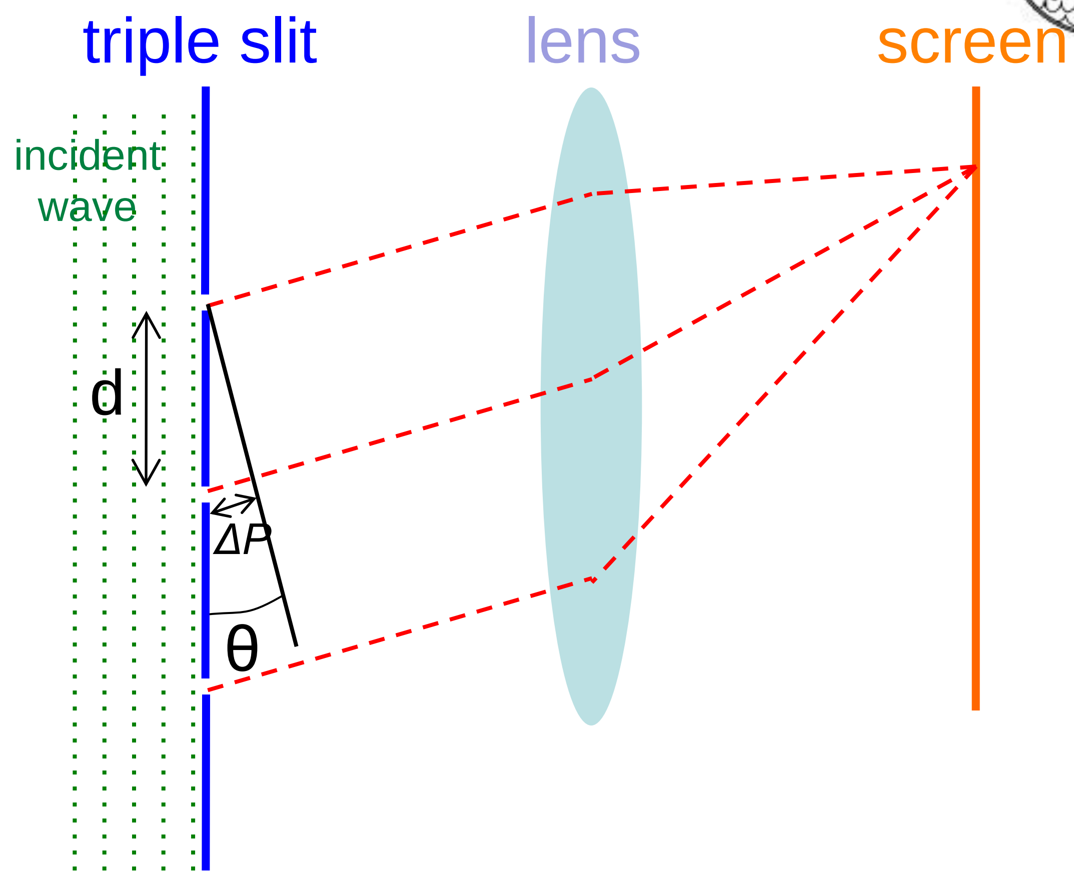
Now a Triple Slit Experiment

Optical path difference:

$$\Delta P = d \sin\theta$$

Phase difference:

$$\Delta\phi = 2\pi \frac{\Delta P}{\lambda}$$

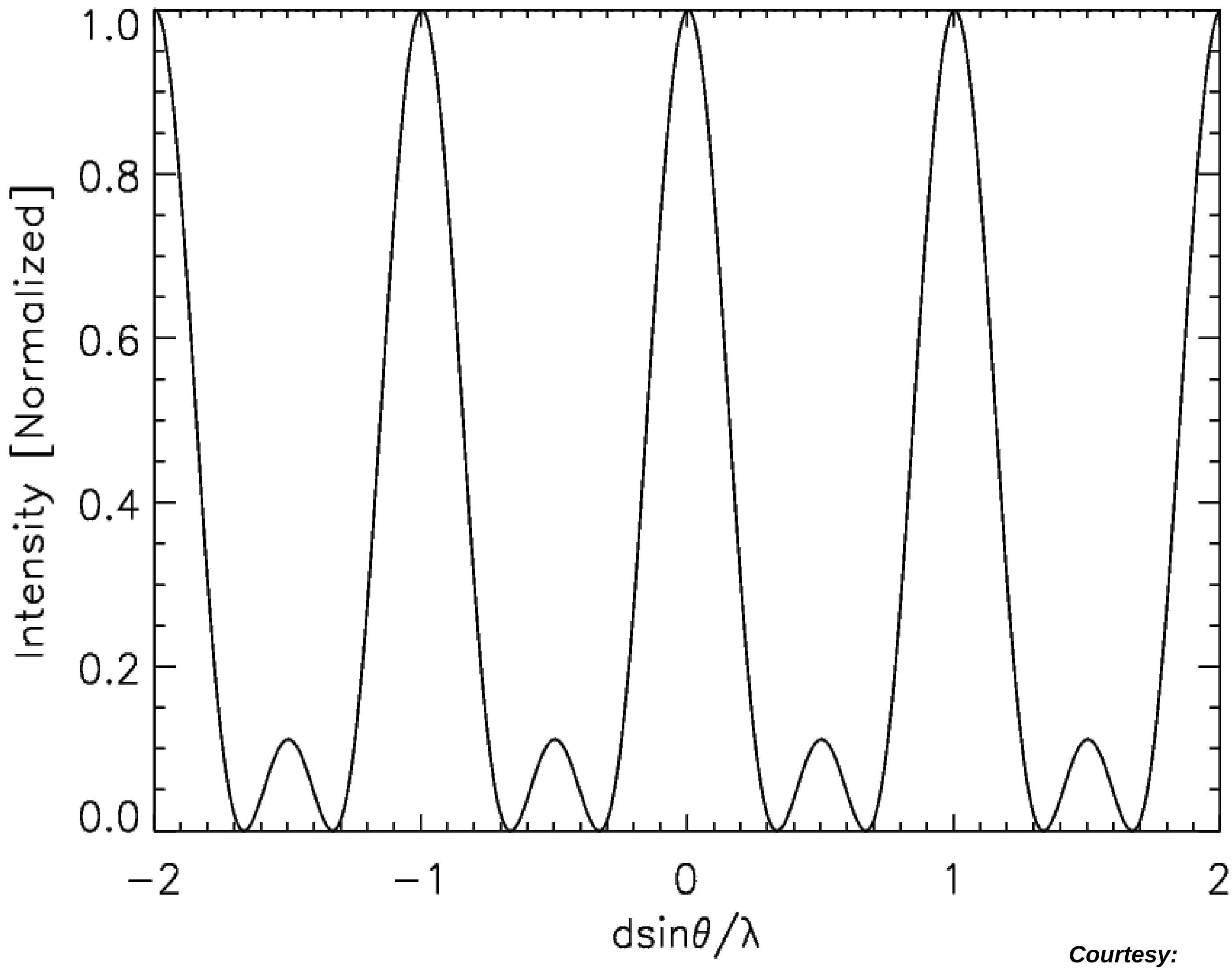


Add the three waves, and take the norm:

$$I = E_1^2 \left(1 + e^{i\Delta\phi} + e^{2i\Delta\phi}\right)\left(1 + e^{-i\Delta\phi} + e^{-2i\Delta\phi}\right)$$

Courtesy:
Roy van Boekel & Kees Dullemond

N=3



Courtesy:
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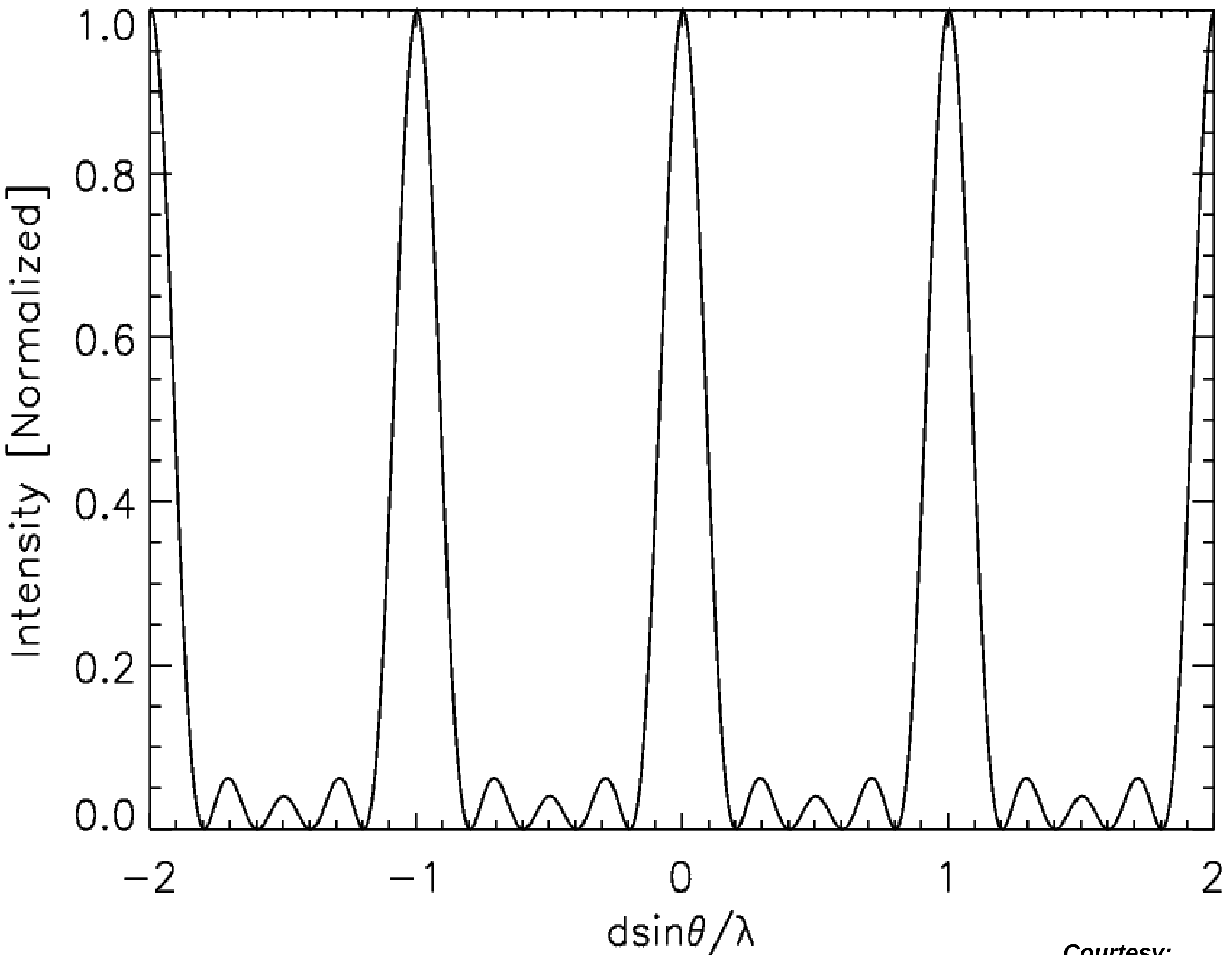
N=5



Courtesy:
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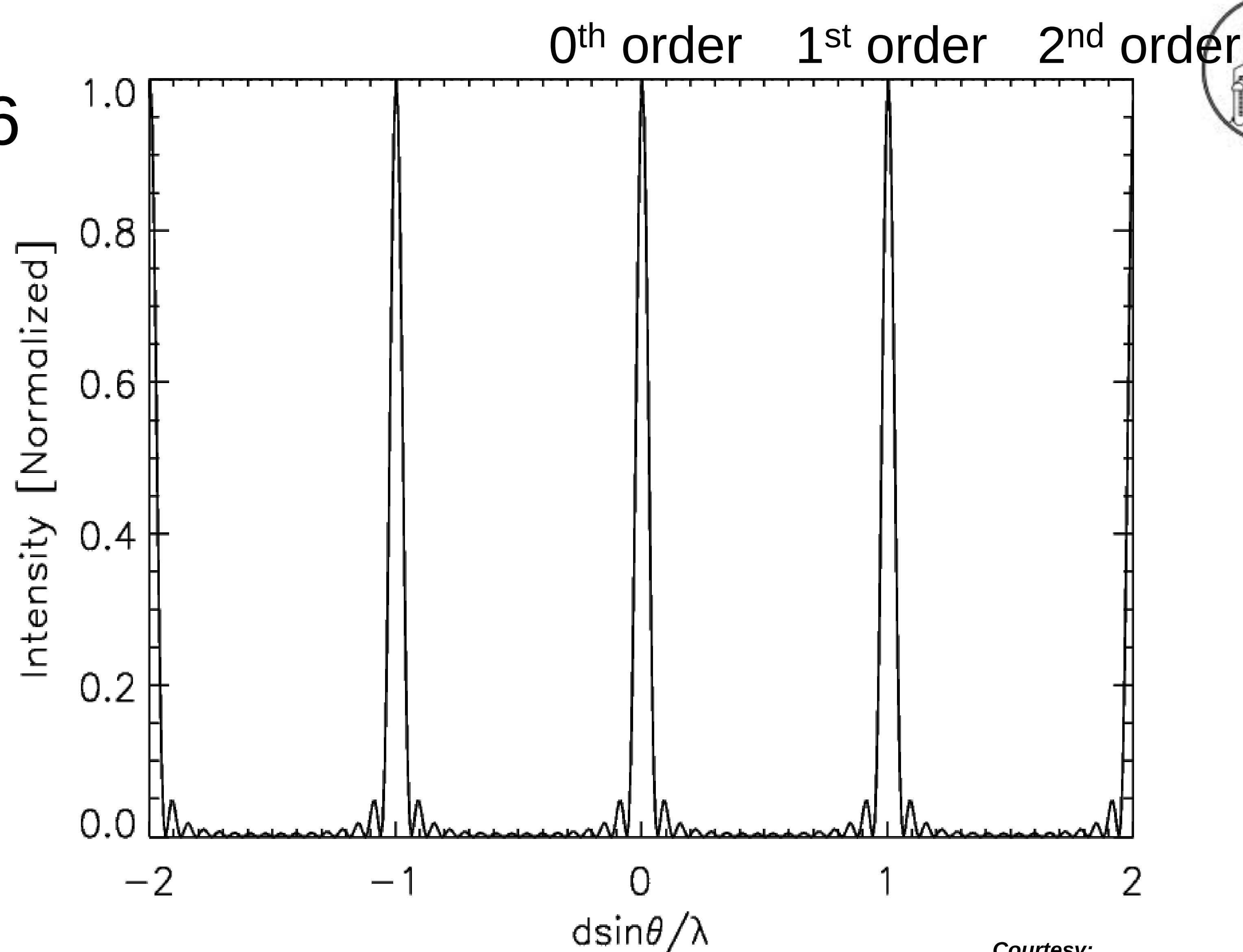
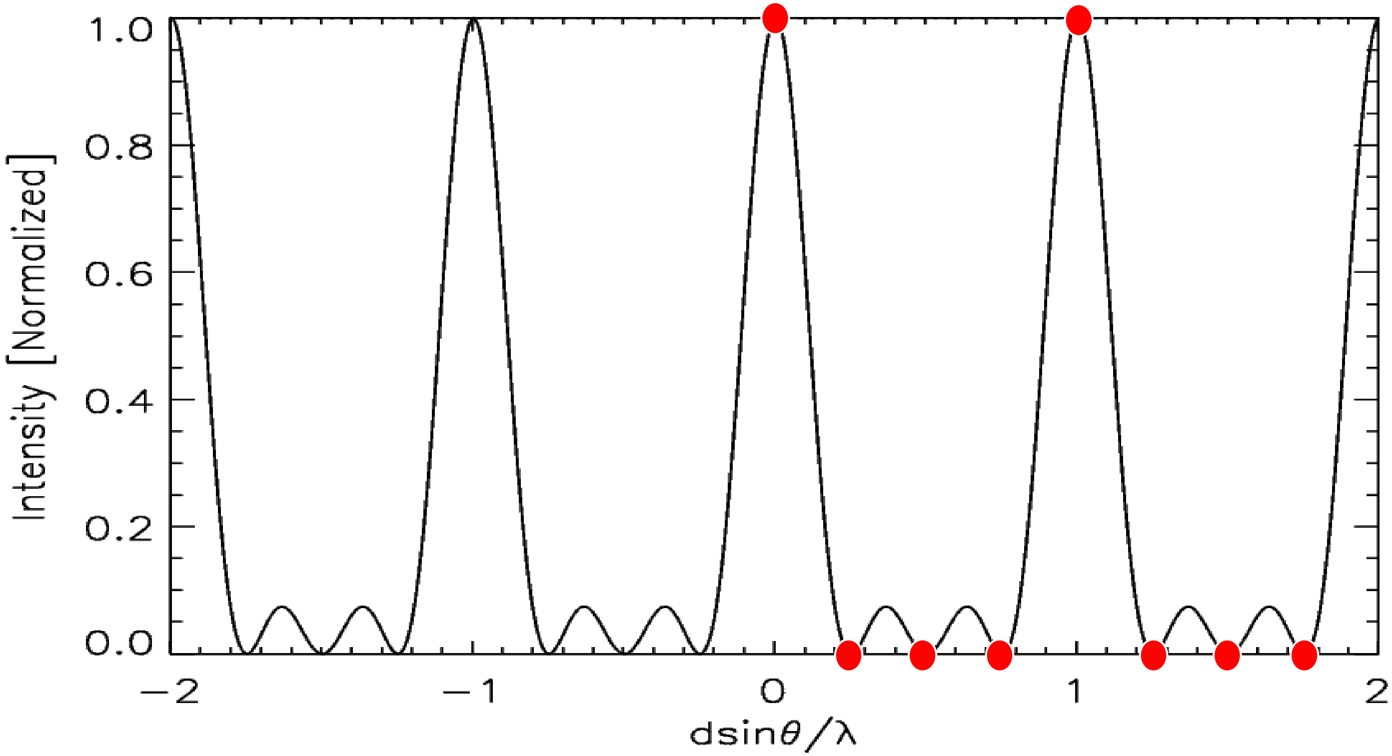
N=16



Courtesy:
Roy van Boekel & Kees Dullemond

N=4



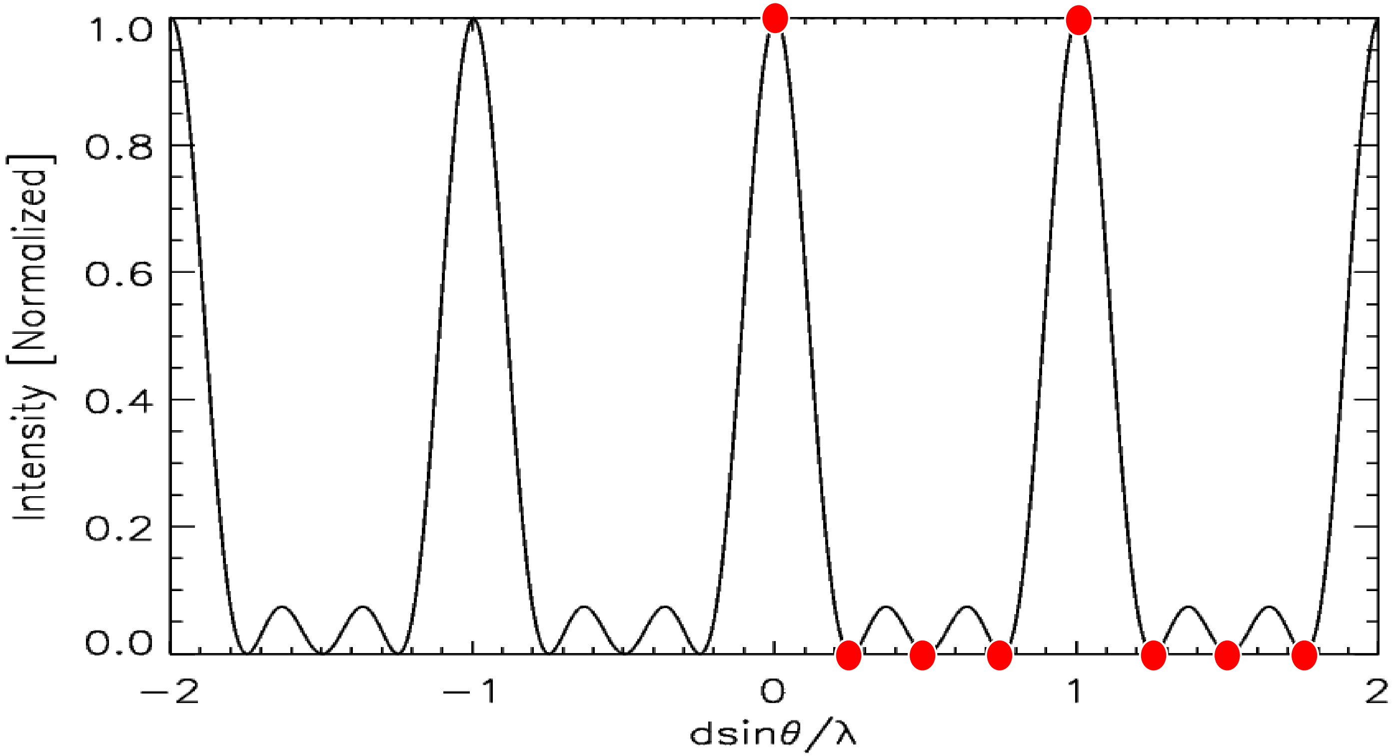
$$I(\theta) = \frac{I(0)}{N^2}\left[\frac{\sin^2\left(\frac{N\pi d\sin\theta}{\lambda}\right)}{\sin^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}\right]$$

For $\frac{d\sin\theta}{\lambda} = \frac{n}{N}$ with $1 \le n < N$

one has $I(\theta) = 0$

Courtesy:
Roy van Boekel & Kees Dullemond

N=4



Peak width is therefore:

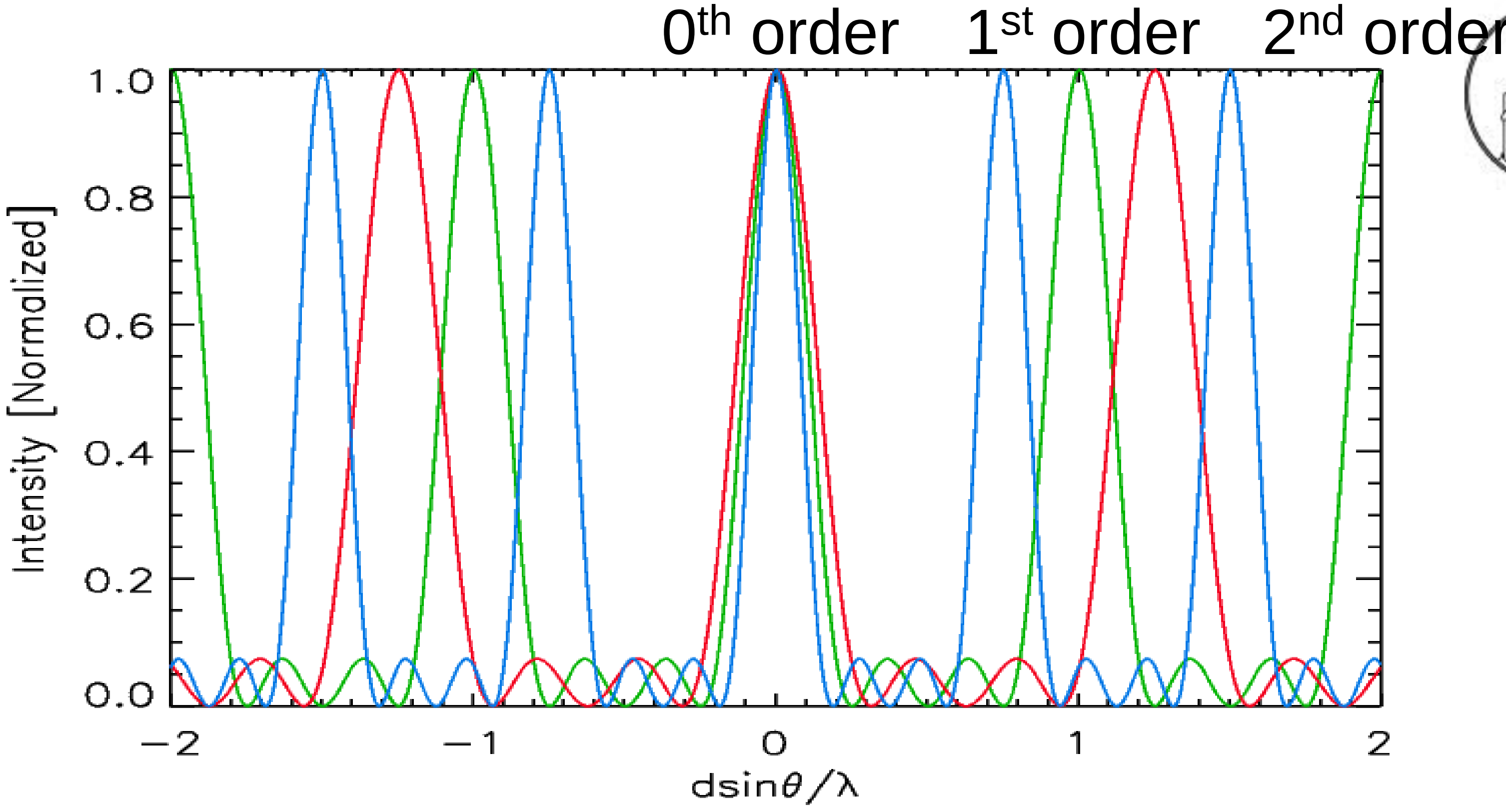
For $\frac{d\sin\theta}{\lambda} = \frac{n}{N}$ with $1 \leq n < N$

$$\Delta \sin\theta = \frac{\lambda}{Nd}$$

one has $I(\theta) = 0$

(Later: Relevance for spectral resolution)

N=4



Green is here the reference wavelength λ.

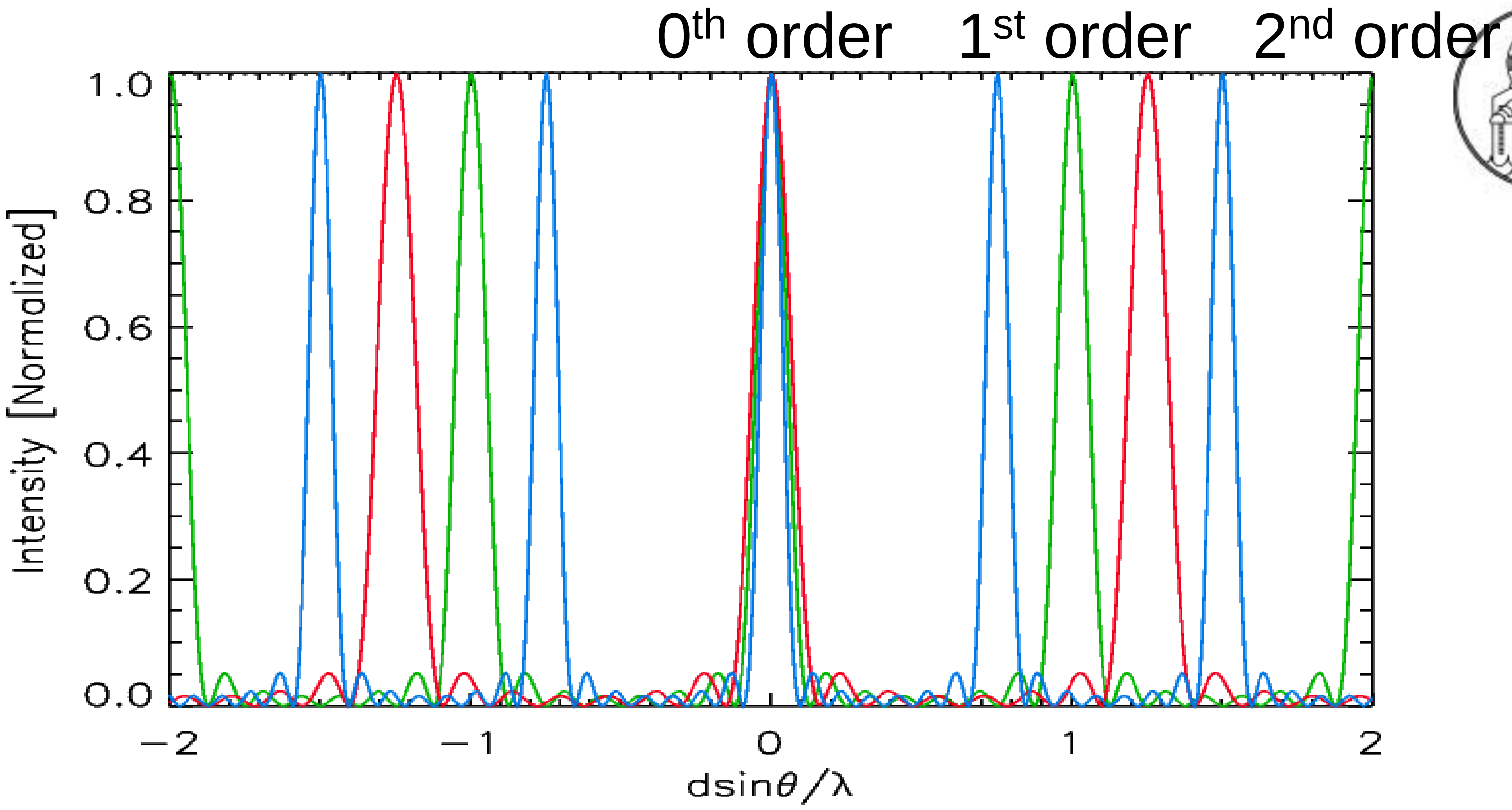
Blue/red is chosen such that its 1st order peak lies in green's first null on the left/right of the 1st order.

$$\lambda_{blue} = \lambda_{green}\left(1 - \frac{1}{N}\right) \qquad \lambda_{red} = \lambda_{green}\left(1 + \frac{1}{N}\right)$$

Spectral resolution:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{N}$$

N=8



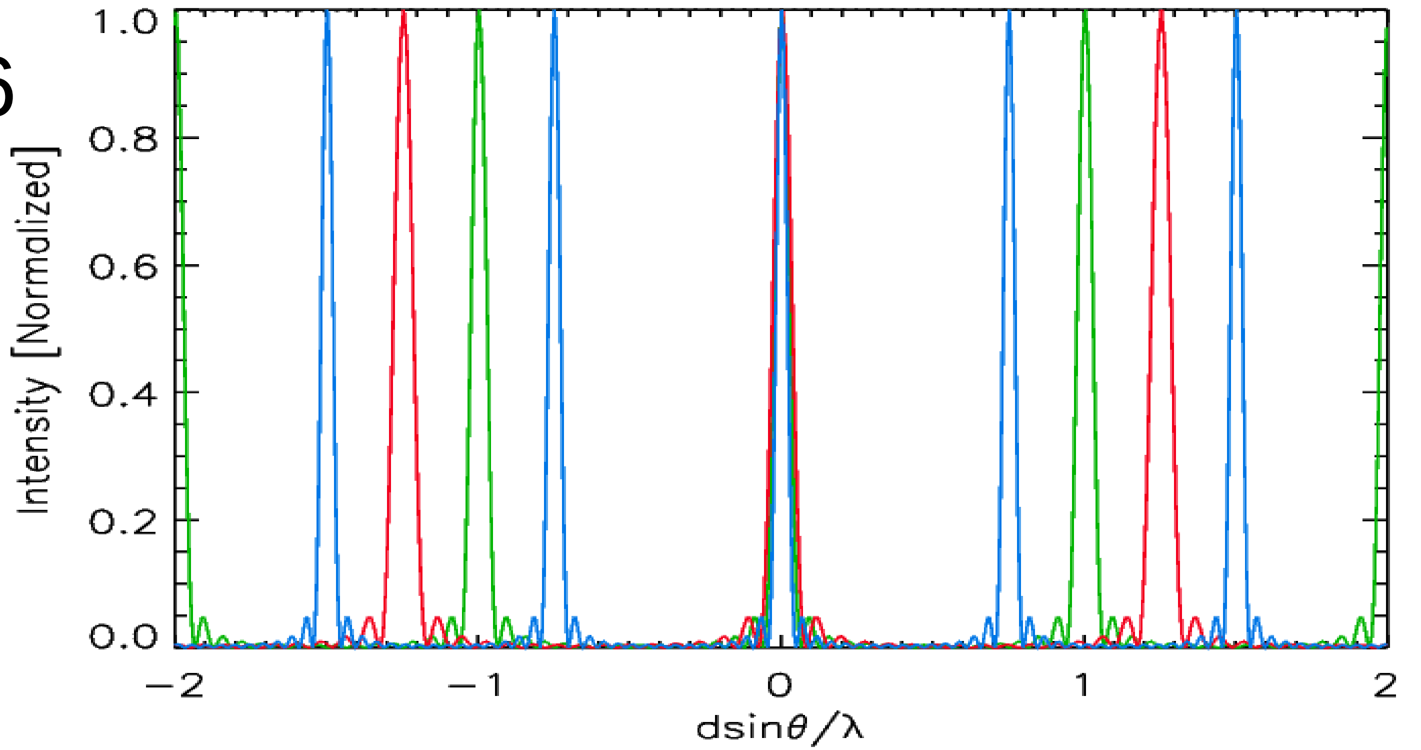
Keeping 3 wavelengths fixed, but increasing N

Spectral resolution:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{N}$$

Courtesy:
Roy van Boekel & Kees Dullemond

N=16



Keeping 3 wavelengths fixed, but increasing N

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Courtesy:
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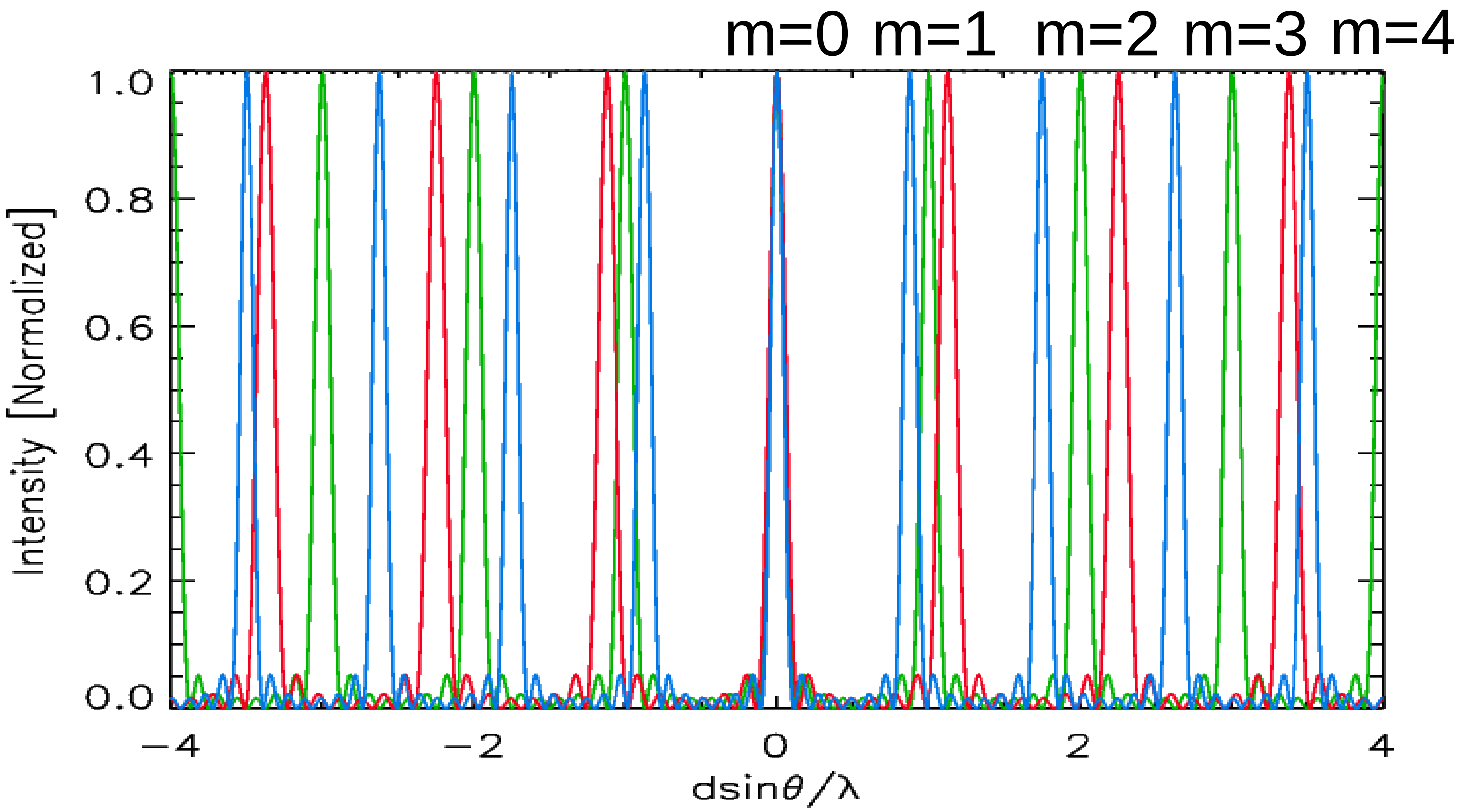
N=8



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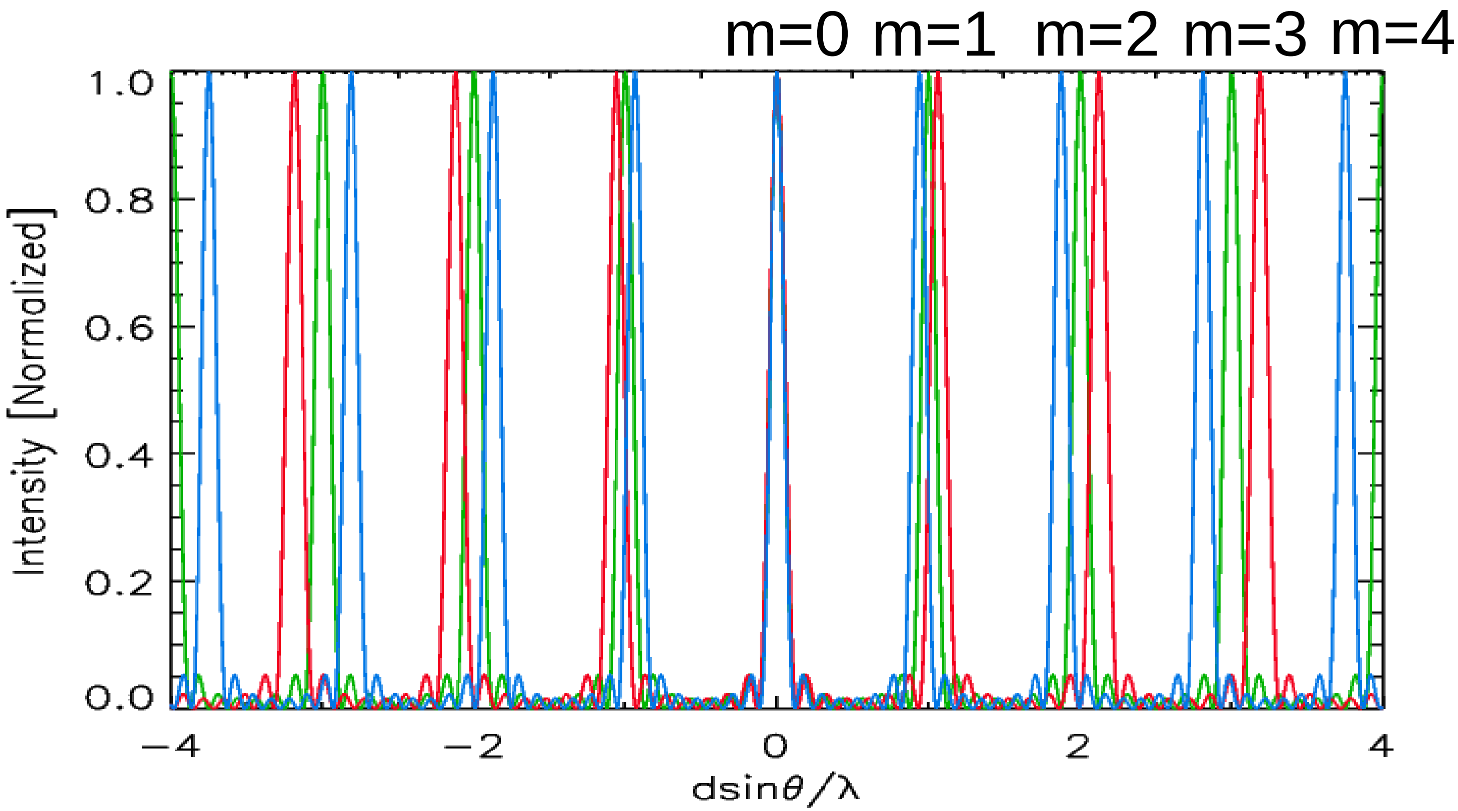
Blue/red is chosen such that its 1st order peak lies in green's first null on the left/right of the 1st order.

$$\lambda_{\text{blue}} = \lambda_{\text{green}}\left(1 - \frac{1}{N}\right) \qquad \lambda_{\text{red}} = \lambda_{\text{green}}\left(1 + \frac{1}{N}\right)$$

Spectral resolution:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{N}$$

N=8



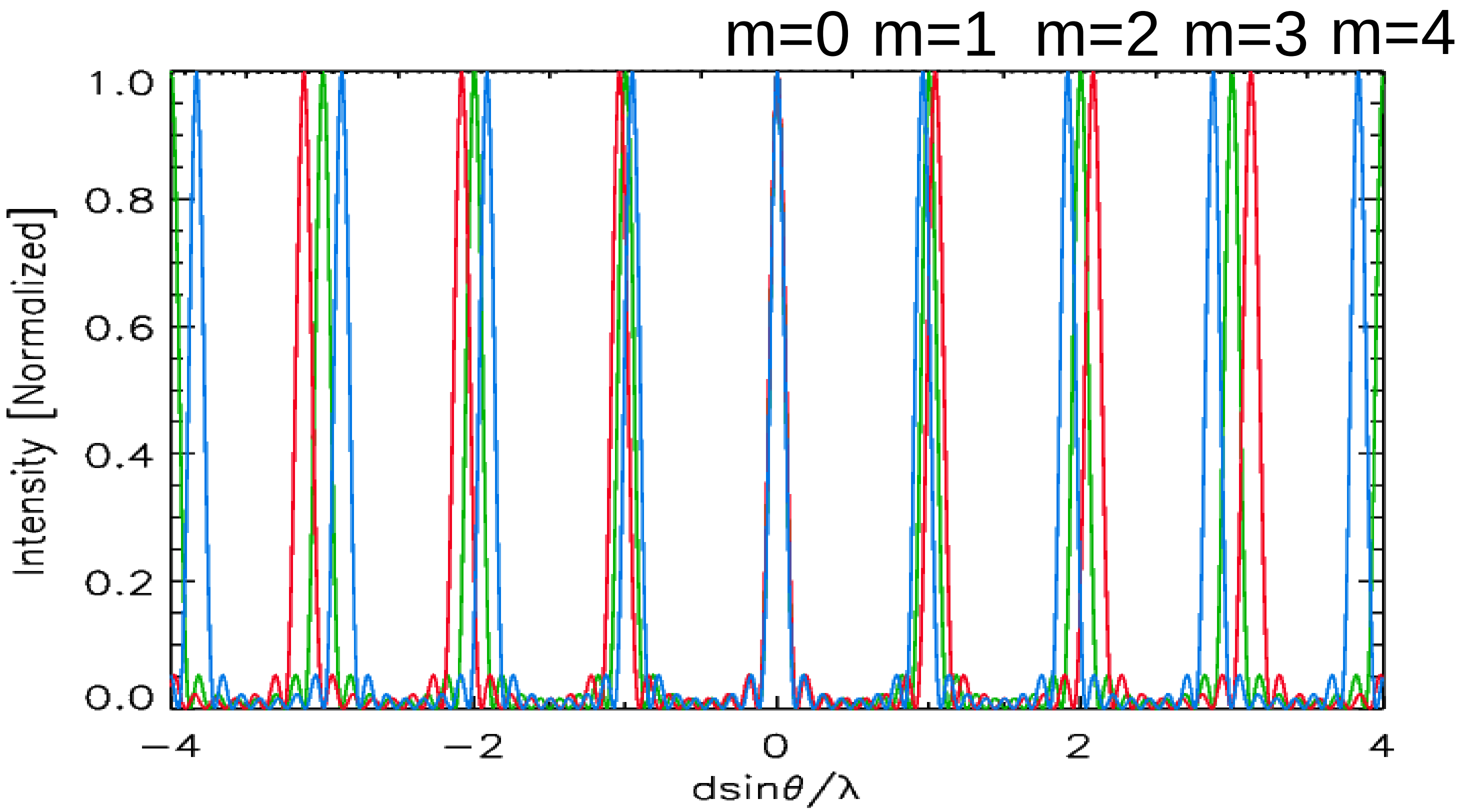
Green is here the reference wavelength λ.

Blue/red is chosen such that its 2nd order peak lies in green's first null on the left/right of the 2nd order.

Spectral resolution:

$$\lambda_{\text{blue}} = \lambda_{\text{green}}\left(1 - \frac{1}{2N}\right) \qquad \lambda_{\text{red}} = \lambda_{\text{green}}\left(1 + \frac{1}{2N}\right) \qquad \frac{\Delta\lambda}{\lambda} \approx \frac{1}{2N}$$

N=8



Green is here the reference wavelength λ.

Blue/red is chosen such that its 3rd order peak lies in green's first null on the left/right of the 3rd order.

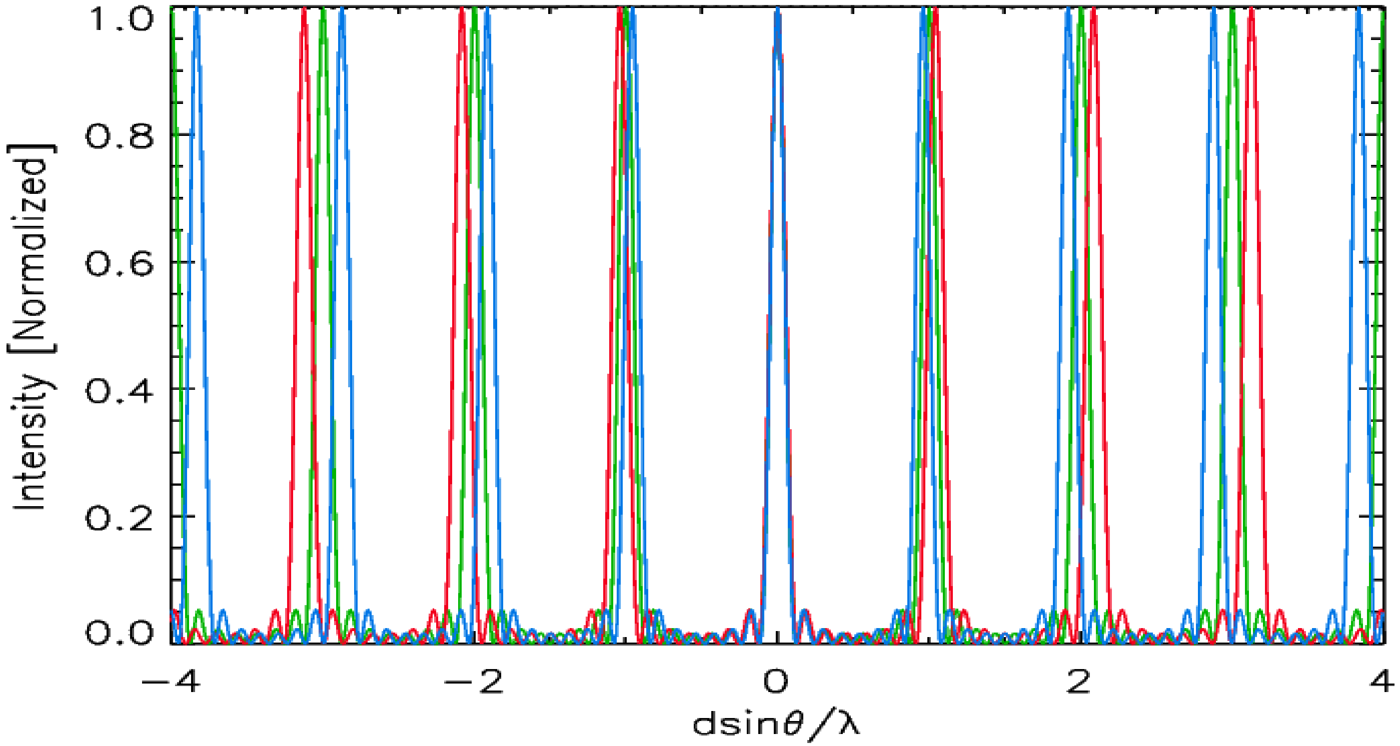
$$\lambda_{\text{blue}} = \lambda_{\text{green}}\left(1 - \frac{1}{3N}\right) \qquad \lambda_{\text{red}} = \lambda_{\text{green}}\left(1 + \frac{1}{3N}\right)$$

Spectral resolution:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{3N}$$

m=0 m=1 m=2 m=3 m=4

N=8



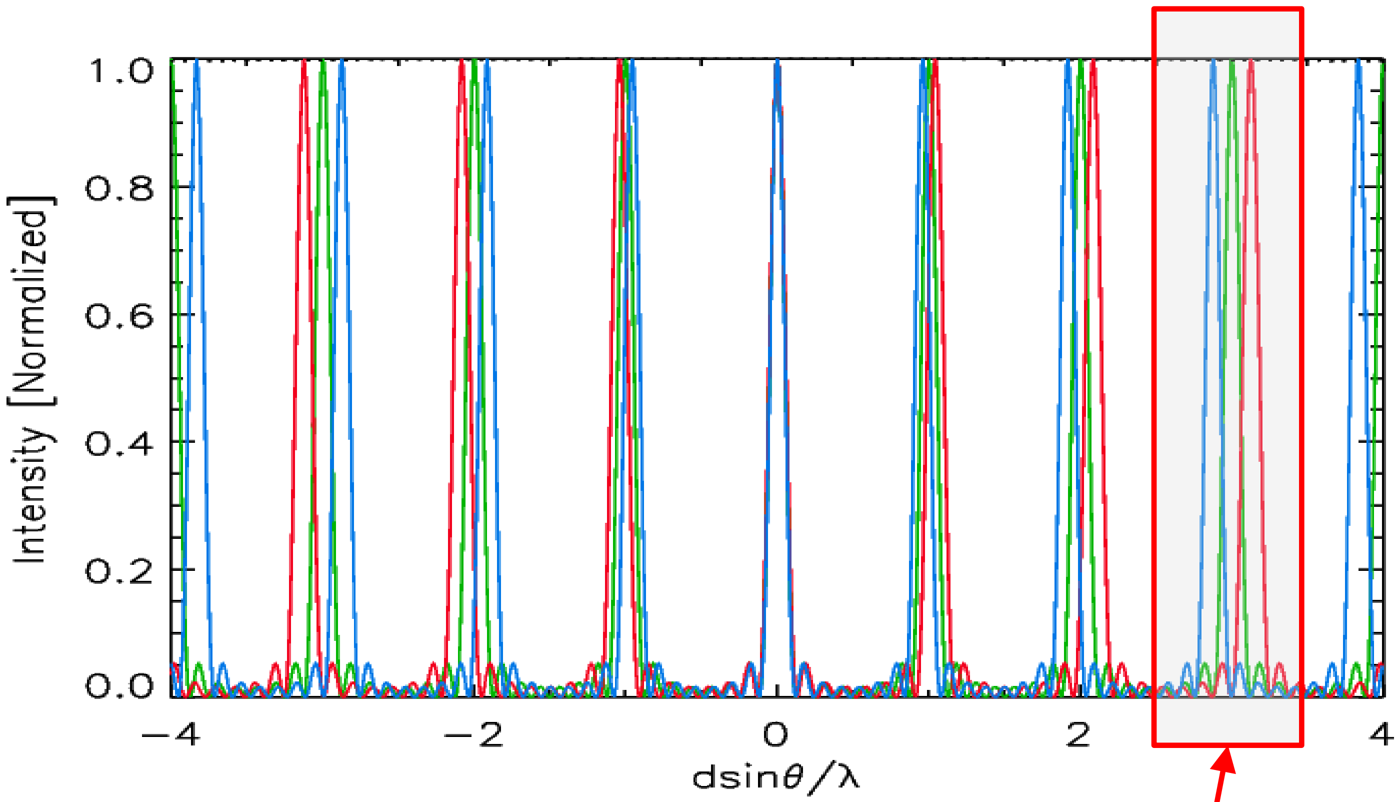
Green is here the reference wavelength λ.

Blue/red is chosen such that its m[th] order peak lies in green's first null on the left/right of the m[th] order.

$$\lambda_{\text{blue}} = \lambda_{\text{green}}\left(1 - \frac{1}{mN}\right) \qquad \lambda_{\text{red}} = \lambda_{\text{green}}\left(1 + \frac{1}{mN}\right)$$

Spectral resolution:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{1}{mN}$$



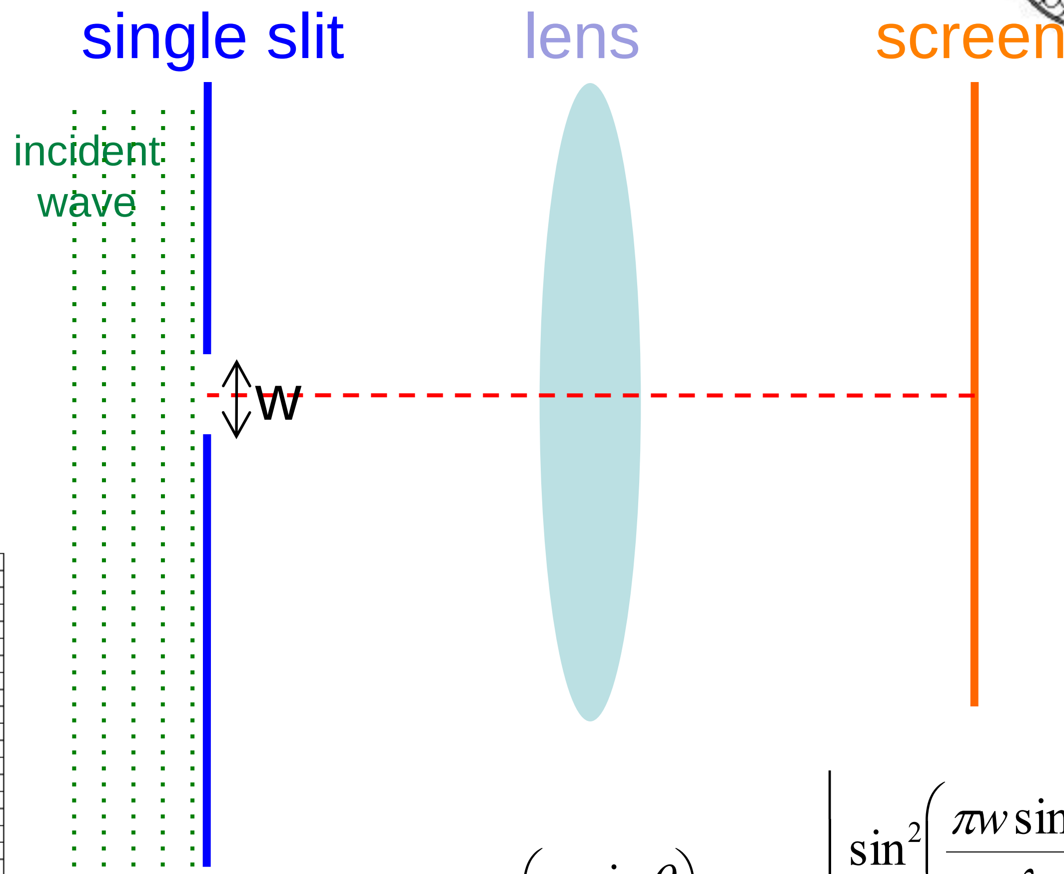
Place a CCD chip here

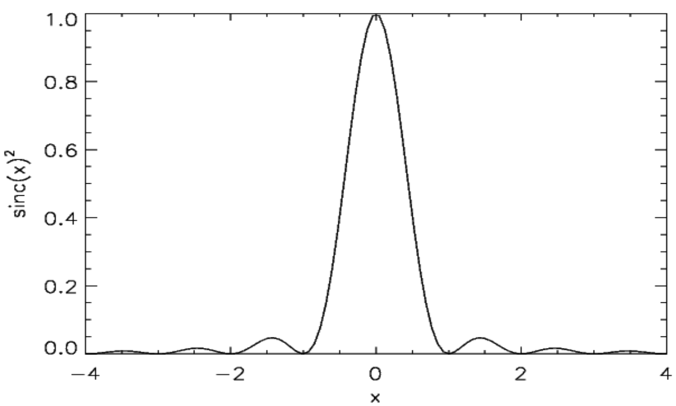
Make sure to have small enough pixel size to resolve the individual peaks.

Courtesy:
Roy van Boekel & Kees Dullemond

Effect of Slit Width

As we know from the chapter on diffraction: This gives the sinc function squared:





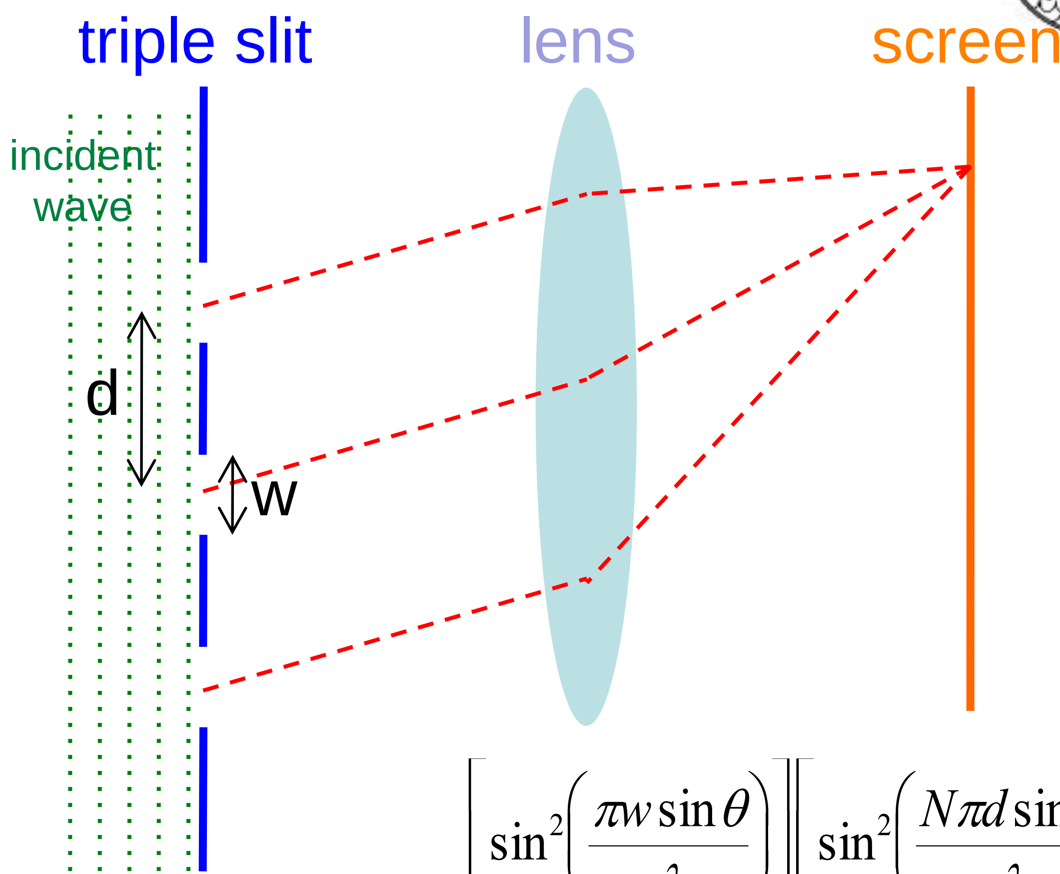
$$I(\theta) = I(0)\,\mathrm{sinc}\left(\frac{w\sin\theta}{\lambda}\right) = I(0)\left[\frac{\sin^2\left(\frac{\pi w\sin\theta}{\lambda}\right)}{\left(\frac{\pi w\sin\theta}{\lambda}\right)^2}\right]$$

Courtesy:
Roy van Boekel & Kees Dullemond

Effect of slit width

At slit-plane:
Convolution of N-slit and finite slit width.

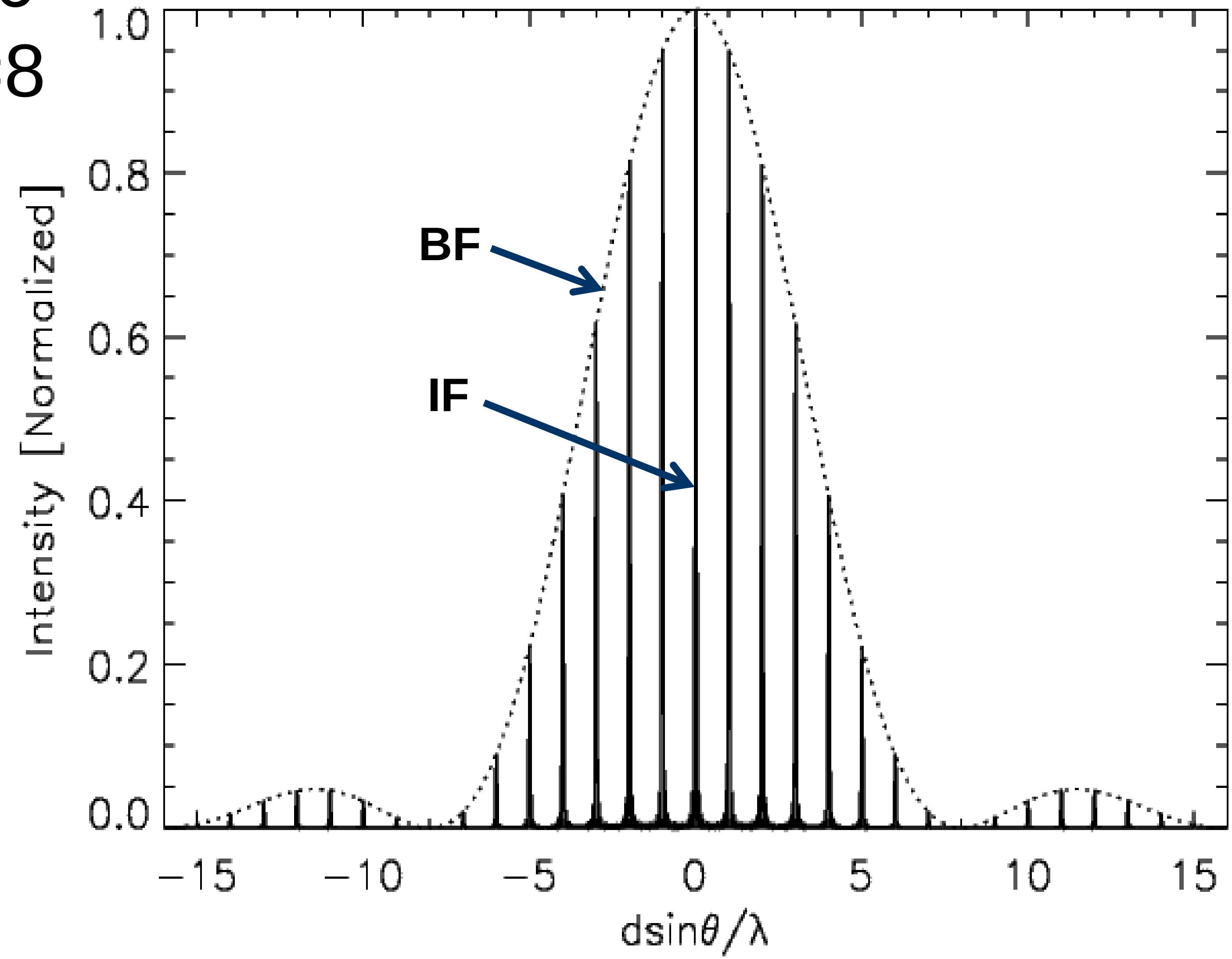
At image plane:
Fourier transform of convolution = multiplication



$$I(\theta) = I(0)\left[\frac{\sin^2\left(\frac{\pi w \sin\theta}{\lambda}\right)}{\left(\frac{\pi w \sin\theta}{\lambda}\right)^2}\right]\left[\frac{\sin^2\left(\frac{N\pi d \sin\theta}{\lambda}\right)}{\sin^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}\right]$$

Courtesy:
Roy van Boekel & Kees Dullemond

N=16
d/w=8



Courtesy:
Roy van Boekel & Kees Dullemond

Diffraction by Many Slits

- *The flux-density distribution function is:*

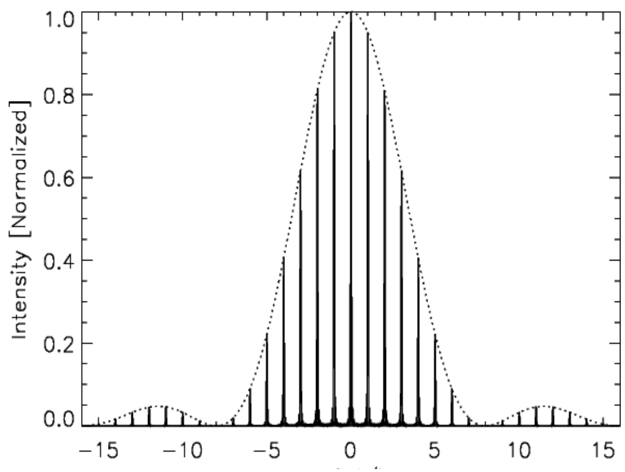
$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2$$

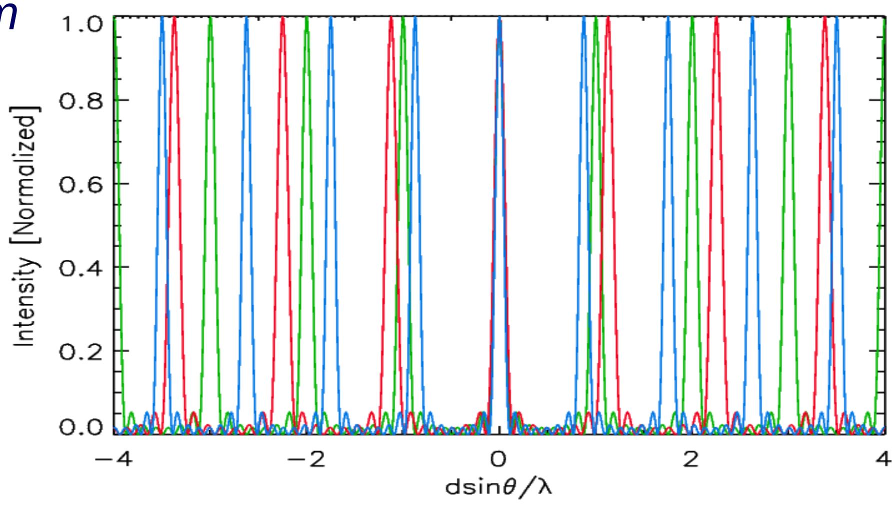
- *Each slit by itself generate precisely the same flux-density distribution.*
- *Superposed, the various contributions yield a multiple-wave interference system modulated by the single-slit diffraction.*
- *Principal maxima occur when*

$$\alpha = m\pi = 0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots \text{ or}$$
$$d \sin \theta_{\max} = m\lambda$$

- *Minima exist when*

$$\alpha = \pm\frac{\pi}{N}, \pm\frac{2\pi}{N}, \pm\frac{3\pi}{N} \ldots$$

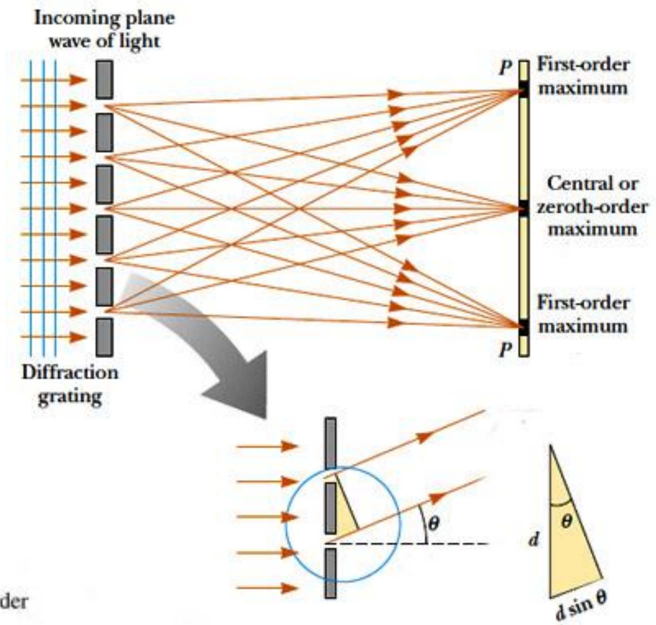


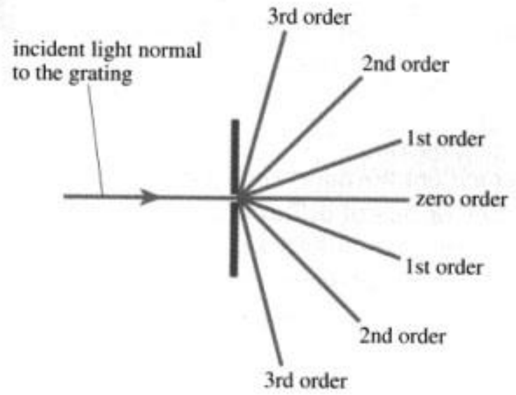


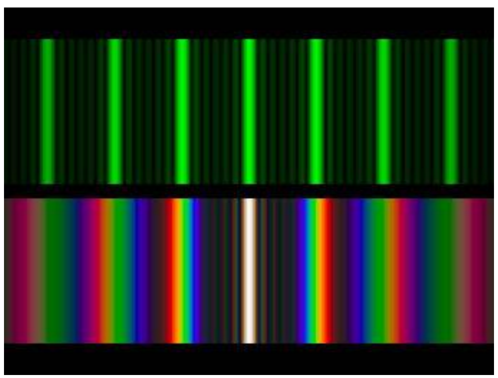
Diffraction Grating

- *A repetitive array of diffraction elements, either apertures or obstacles, that has the effect of producing periodic alterations in the phase, amplitude, or both of an emergent wave.*
- *Principal maxima occur when*

$$\alpha = m\pi = 0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots \text{ or}$$
$$d \sin\theta_{\max} = m\lambda$$
$$m = 0, \pm 1, \pm 2, \pm 3, \pm 4 \ldots$$

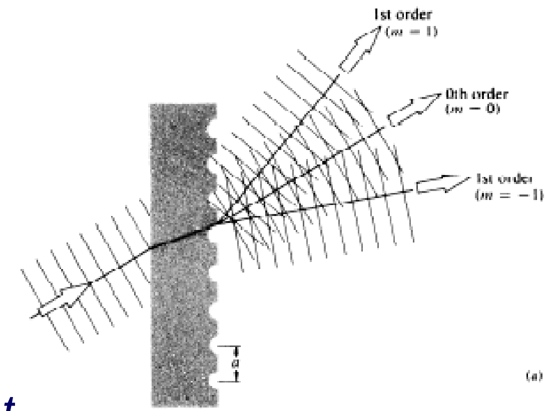


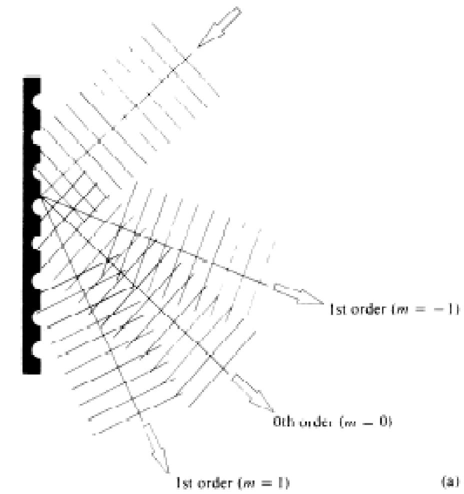


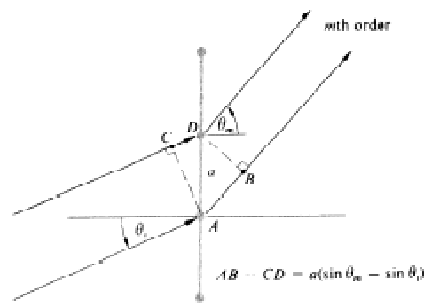


Grating Type

- Many parallel “slits” called “grooves”.
- Transmission grating: the diffracted rays lie on the opposite side of the grating from the incident ray.
- Reflection grating: the incident and diffracted rays lie on the same side of the grating.

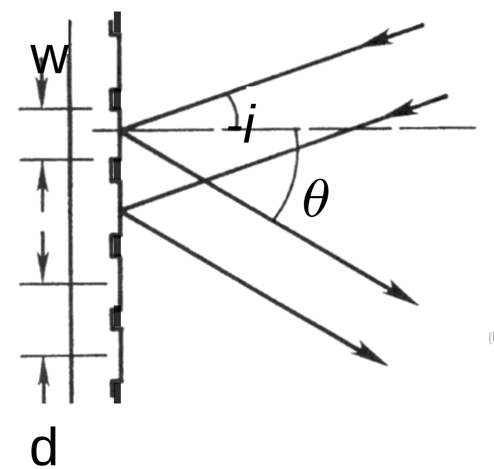






- Gratings are often tilted with respect to beam. Slightly different expression for positions of interference maxima:

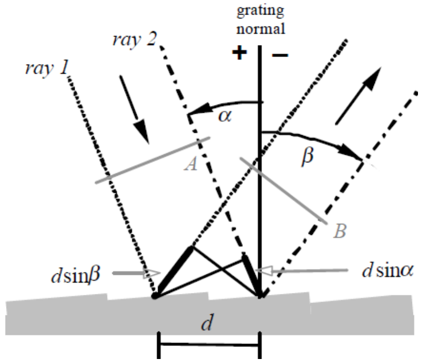
$$\theta = \operatorname{asin}\left(\frac{m\lambda}{d} - \sin i\right)$$

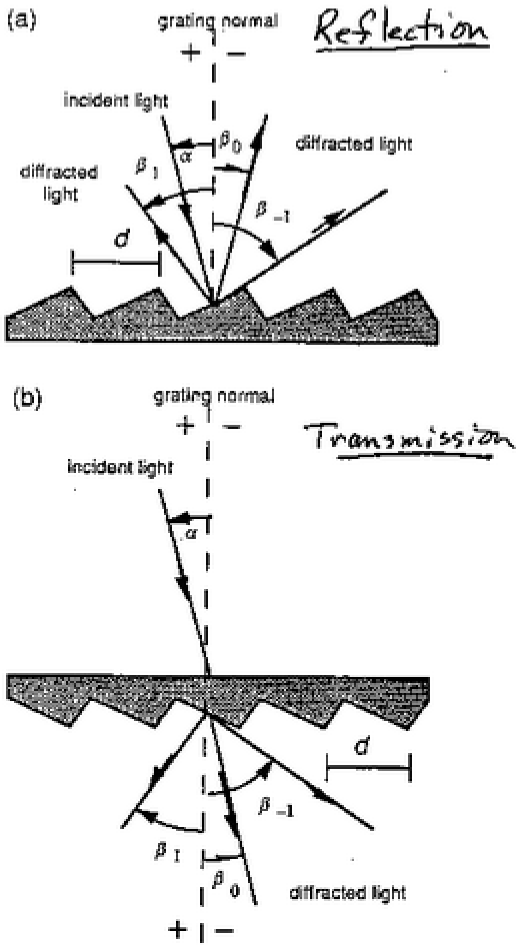


Grating Equation

$$m\lambda = d(\sin\alpha + \sin\beta)$$

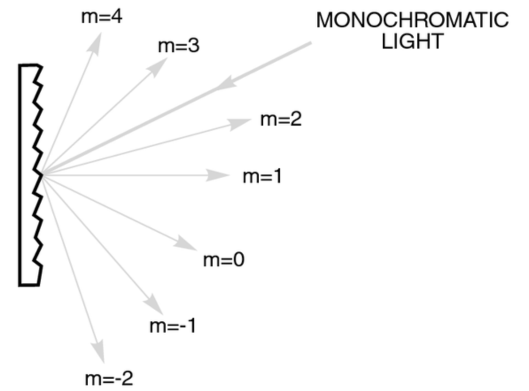
- Angles are measured from the grating normal, which is perpendicular to the grating surface.
- Sign convention for angles depends on if the light is diffracted on the same side or opposite side of the grating as the incident light.
- Angle convention for reflection grating:
  - Positive when measured CCW from the normal.
  - Negative when measured CC from the normal.





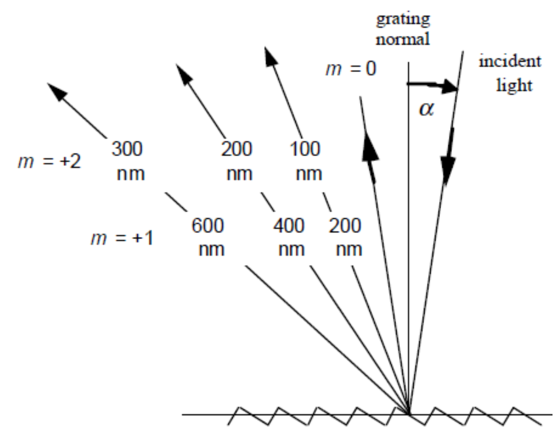
Diffraction Orders

$$m\lambda = d(\sin\alpha + \sin\beta)$$



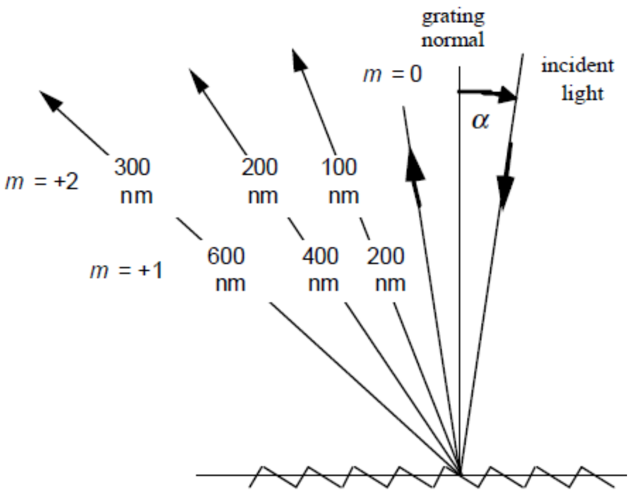
- Only those spectral orders can exist for which $|m\lambda / d| < 2$
- Spectra of all orders m exist for which $-2d < m\lambda < 2d$
- Sign convention for m:
  - $\beta > -\alpha$ for positive orders (m>0): if diffracted ray lies to the left of zero order
  - $\beta < -\alpha$ for negative orders (m<0): if diffracted ray lies to the left of zero order
  - $\beta = -\alpha$ for specular reflection (m=0)

- Overlapping of diffracted spectra lead to ambiguous spectroscopic data, must be prevented by suitable filtering.



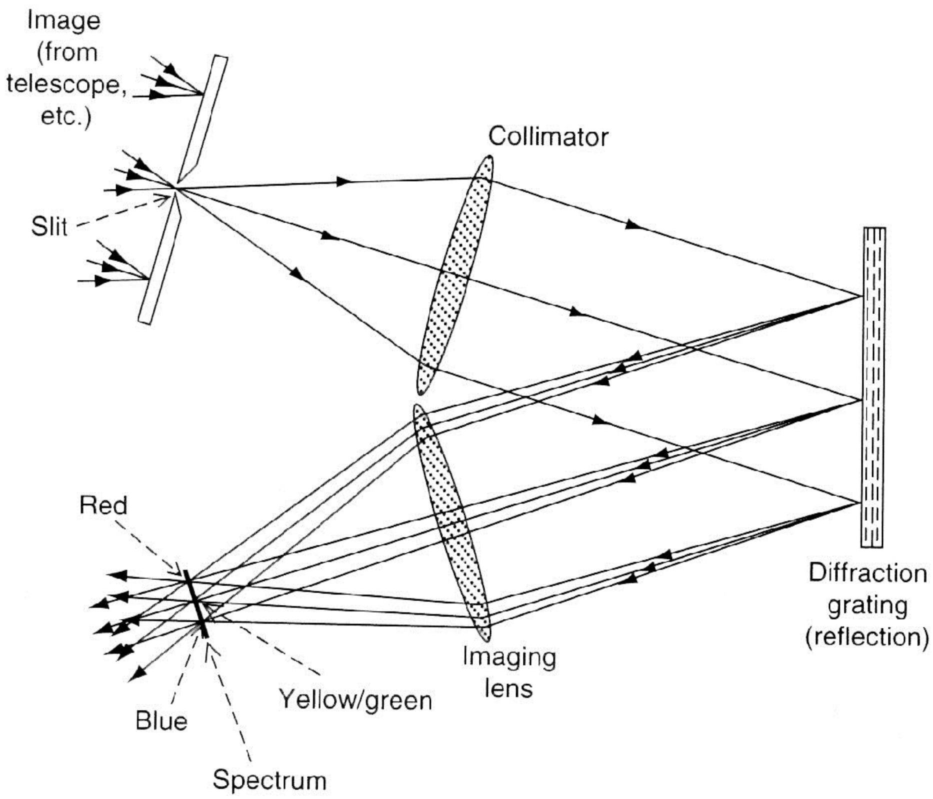
Order Overlap in Grating

- Each order gives its own spectrum. These can overlap in the focal plane: at a given pixel on the detector we can get light from several orders (with different λ)
- We must reject light from the unwanted orders. Solution:
  - For low orders m (low spectral resolution, large free spectral range) one can use a filter that blocks light from the other orders
  - For high orders m (the free spectral range is very small), use "cross disperser".



Grating Spectrograph Layout

- ***Spectrometer:*** *refers to any spectroscopic instrument, regardless of whether it scans wavelengths individually or entire spectra simultaneously.*
- ***Spectrograph:*** *a spectrometer that images a range of wavelengths simultaneously, either onto a series of detector elements; An entire section of spectrum is recorded at once.*
- *A spectrograph typically consists of entrance slit, collimating, grating, imaging optics and detector.*



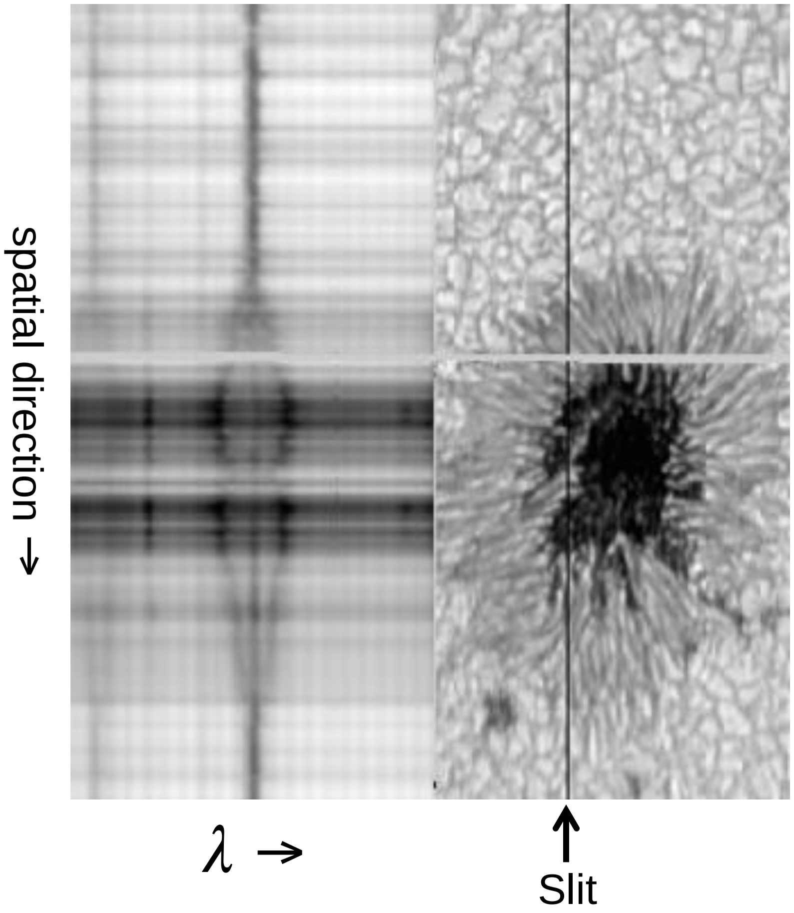
Credit: C.R. Kitchin "Astrophysical techniques"
CRC Press, ISBN 13: 978-1-4200-8243-2

Slit and Spectrum

- Very basic setup: entrance slit in focal plane, with dispersive element oriented parallel to slit (e.g. grooves of grating aligned with slit)
- 1 spatial dimension (along slit) and 1 spectral dimension (perpendicular to slit) on the detector
- Spectral resolution set by dispersive element, e.g. Nm for grating.
- Spectrum can be regarded as infinite number of monochromatic images of entrance slit
- Projected width of entrance slit on detector must be smaller than projected size of resolution element on detector, e.g. for grating:

$$s \leq \frac{\lambda f_1}{Nd\cos\theta}$$

where s is the physical slit width and f_1 is the collimator focal length

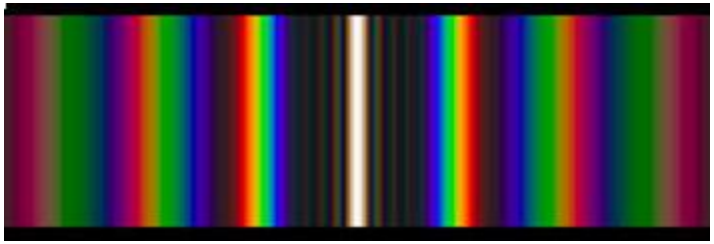


Dispersion

$$D = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\beta} = Gm\sec\beta$$

- *Angular dispersion: the difference in angular position corresponding to a difference in wavelength.*
  - *Unit: rad/nm or rad/angstrom*
  - *D is proportional to order m.*
  - *D is proportional to groove density G = 1/d.*
  - *Once diffraction angle has been determined,*

    the choice must be made whether a fine-pitch (small d) should be used in a low order, or a coarse-pitch (large d, such as echelle grating) should be used in a high order.



- *Linear dispersion (unit: mm/Å) and reciprocal linear dispersion P (unit: Å/mm)*

$$\frac{dl}{d\lambda} = \frac{d\beta}{d\lambda} f = \frac{m}{d\cos\beta} f$$

$$P = \frac{d\lambda}{dl} = \frac{d\cos\beta}{mf}$$

Resolving Power

- Resolving power R is a measure of its ability to separate adjacent spectral lines of average wavelength λ

$$R = \frac{\lambda}{\Delta\lambda_{\min}} = mN$$

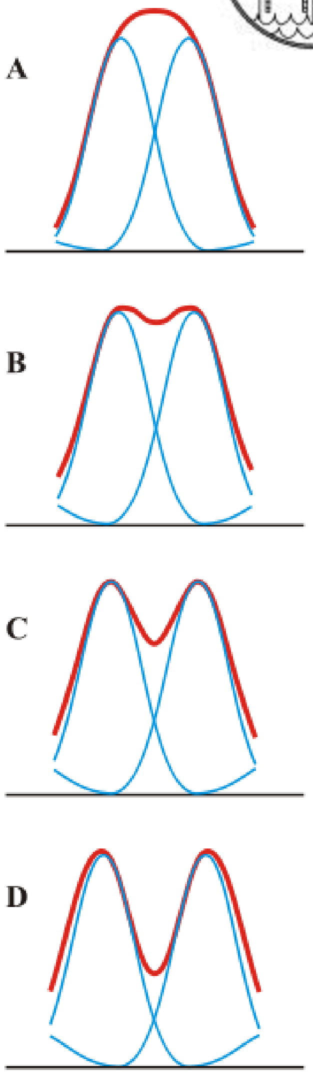
  - m is the diffraction order
  - N is the total number of grooves illuminated on the surface of the grating

- A more meaningful expression for R

$$R = \frac{Nd(\sin\alpha + \sin\beta)}{\lambda} \qquad R_{\max} = \frac{2Nd}{\lambda}$$

- Real R also depends on optical quality of grating surface , uniformity of groove spacing, the quality of the associated optics in the system, the width of the slits …

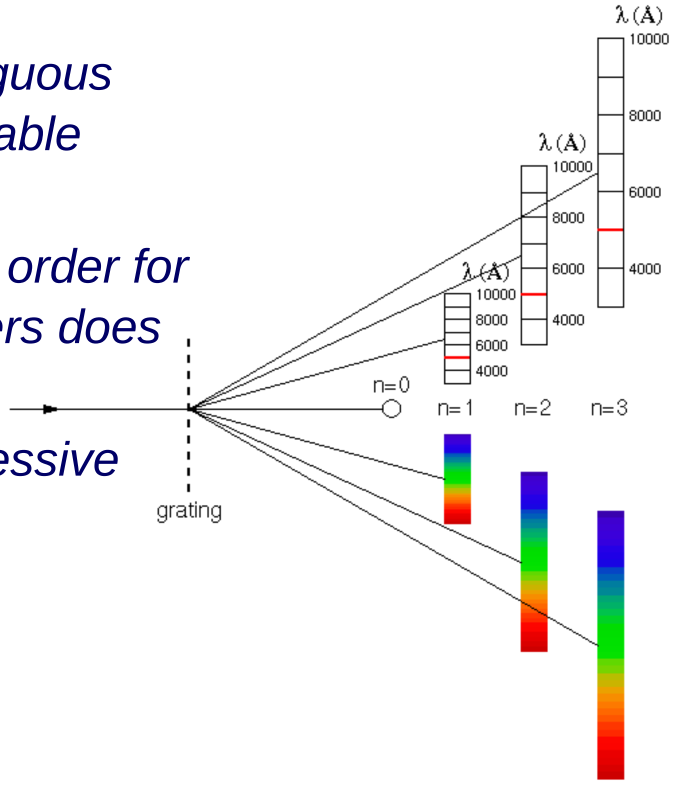
$$R_{real} = \frac{\lambda}{\Delta\lambda_{\min} + \Delta\lambda_{slit} + \Delta\lambda_{stray} + \ldots}$$



Free Spectral Range (FSR)

$$m\lambda = d(\sin\alpha + \sin\beta)$$

- *Overlapping of diffracted spectra lead to ambiguous spectroscopic data, must be prevented by suitable filtering.* $\lambda_1 = 2\lambda_2 = 3\lambda_3 = ... = m\lambda_m$
- *FSR: range of wavelengths in a given spectral order for which superposition of light from adjacent orders does not occur.*
- *If two lines of wavelength λ and $\lambda + \Delta\lambda$ in successive orders (m+1) and m just coincide, then*

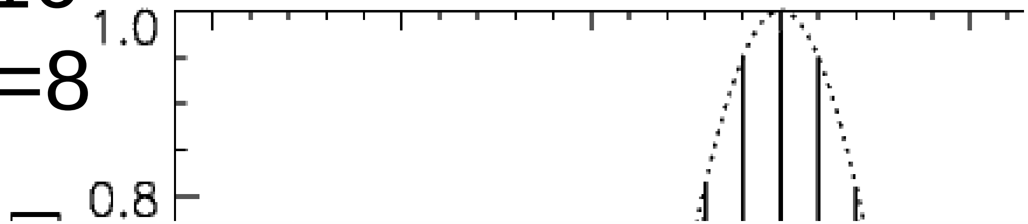


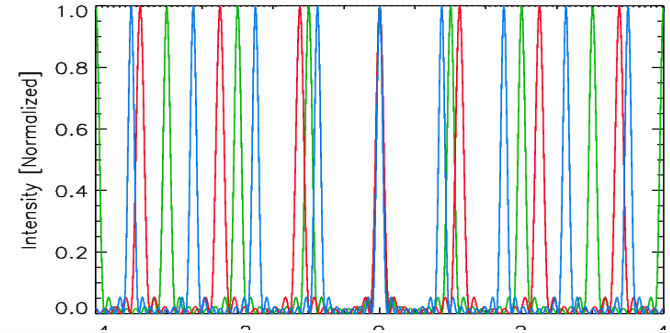
$$(m+1)\lambda = m(\lambda + \Delta\lambda) = d(\sin\alpha + \sin\beta)$$

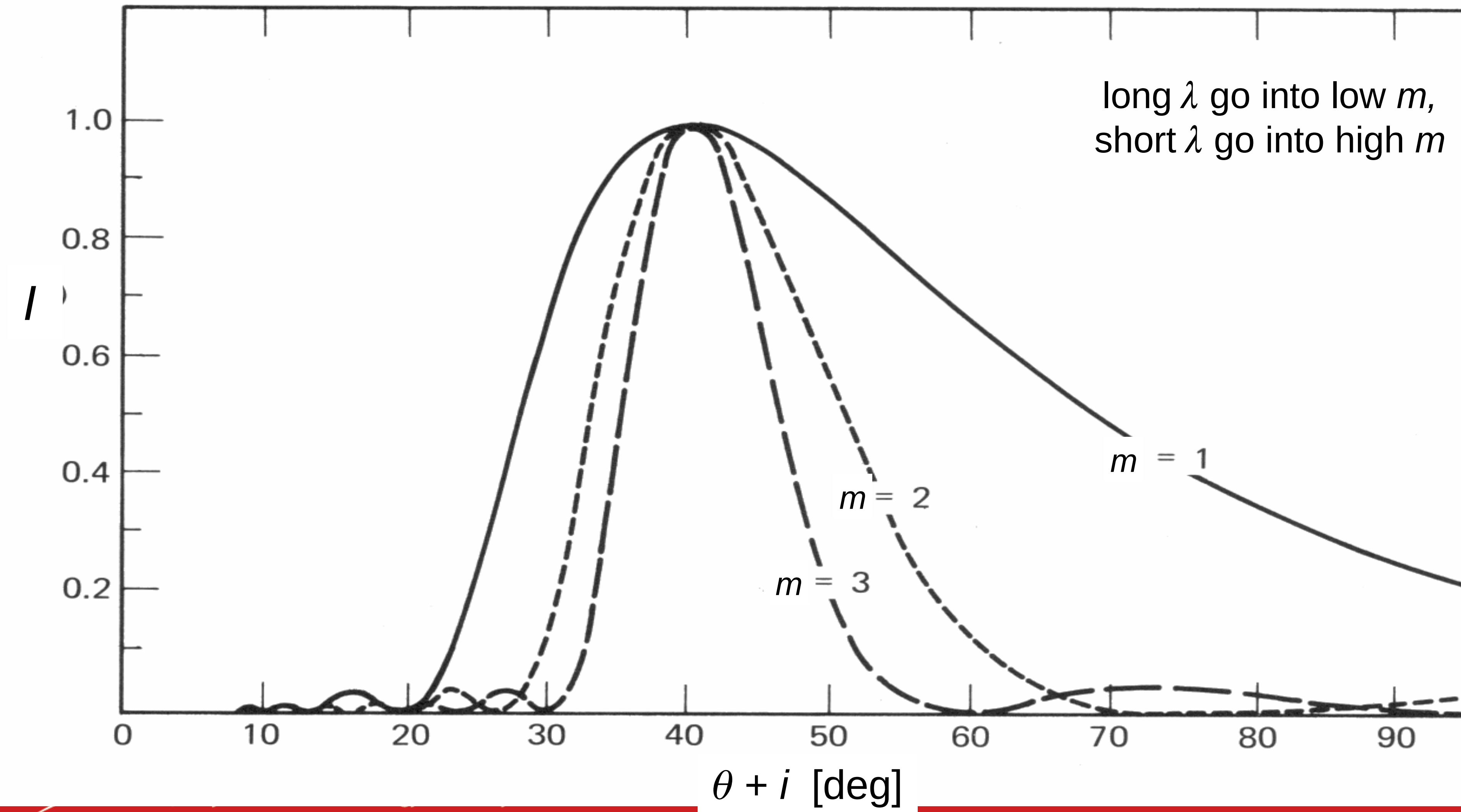
$$\Delta\lambda_{FSR} = \frac{\lambda}{m}$$

- *It is particularly important in the case of echelles because they operate in high orders with small FSR.*

N=16
d/w=8

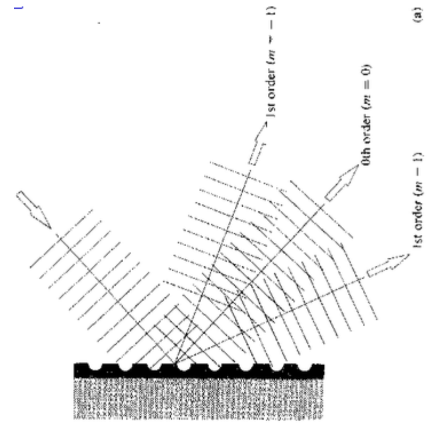

1.0
0.8

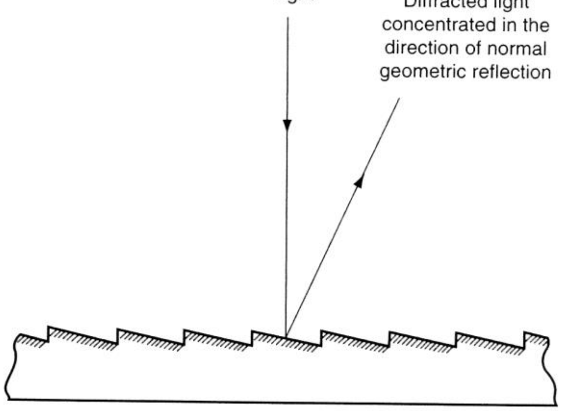

Intensity [Normalized]
1.0
0.8
0.6
0.4
0.2
0.0


long λ go into low m,
short λ go into high m
1.0
0.8
0.6
0.4
0.2
I
m = 1
m = 2
m = 3
0
10
20
30
40
50
60
70
80
90
θ + i [deg]

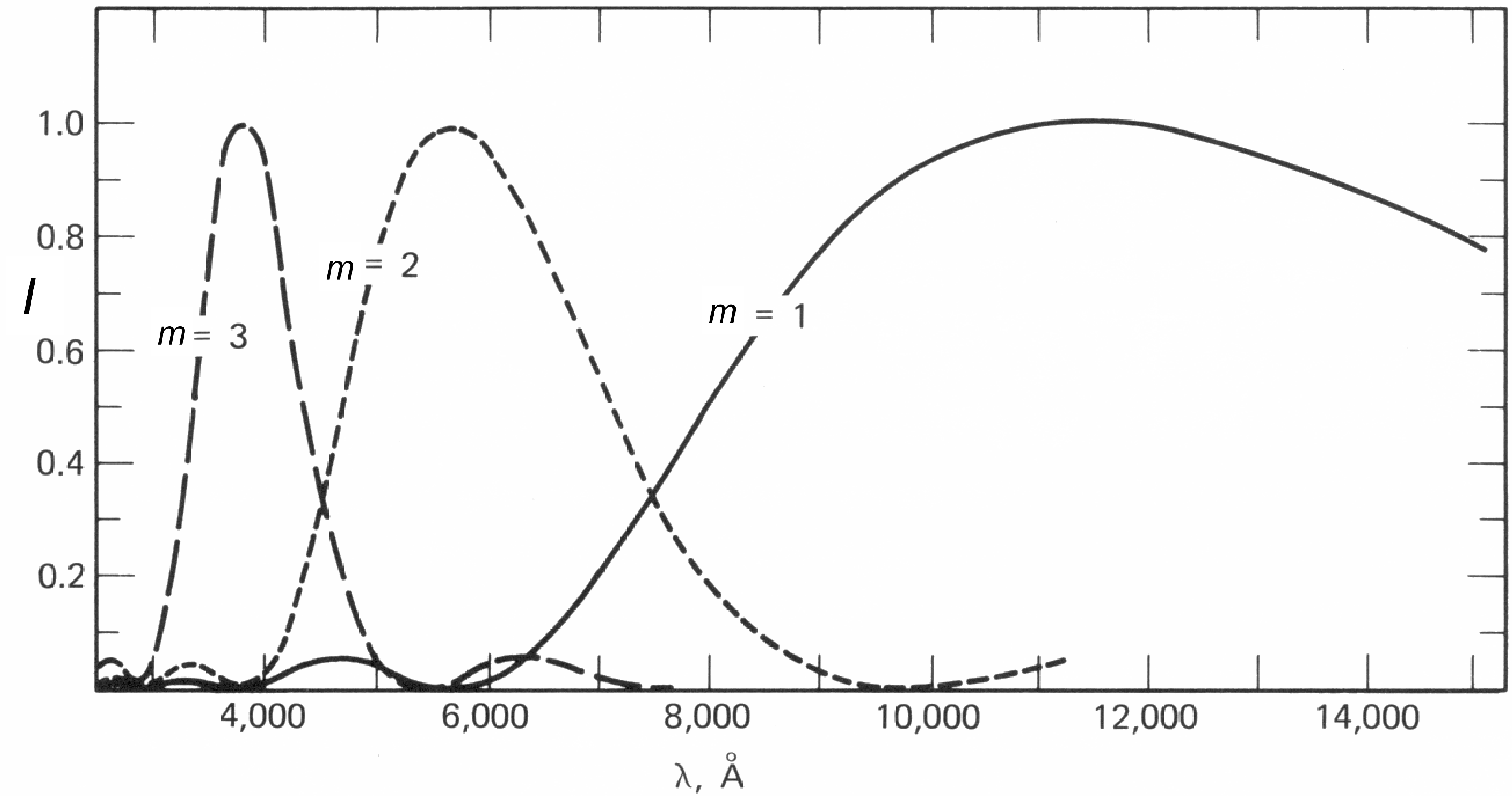
Blazed Grating

- To concentrate light away from zero order to higher orders, gratings are blazed
- To tilt groove surfaces to concentrate light towards certain direction
- The reflecting surfaces are now oriented at some angle with respect to the surface of the grating, reflecting light preferentially in that direction
- Blaze shifts the peak of the grating efficiency envelope towards higher orders
- An additional advantage is that the whole surface can now be reflecting, since the step where two facets join provides a phase difference to allow diffraction to occur
- But if a grating is blazed to be efficient at a particular wavelength with m=1, then it is also efficient at half that wavelength with m=2, so order overlap can be a problem





BF vs. Wavelength



Blaze Angle

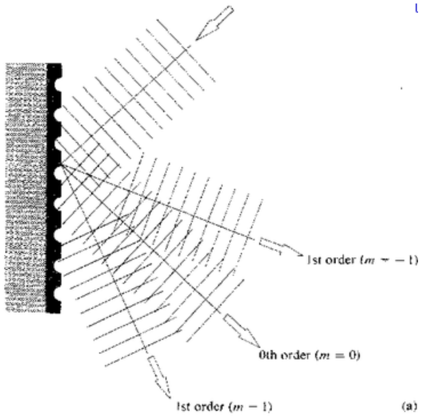
- *Most of energy are concentrated in specular reflected beam.*
- *It's possible to shift energy out of the zeroth order into one of the higher-order spectra by ruling grooves with a controlled shape (blazed).*
- *All angles are measured from the normal to grating plane.*
- *Blaze condition*

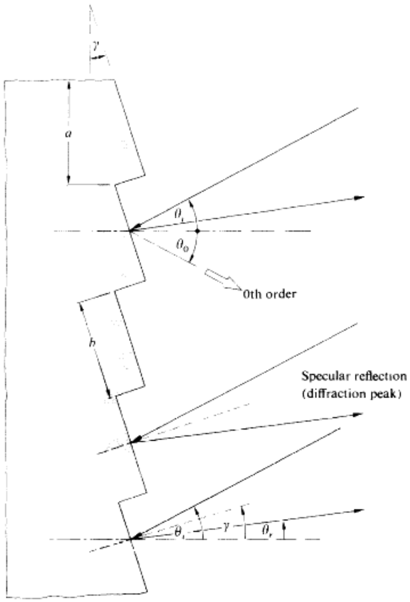
$$\alpha - \beta = 2\theta_B$$

- *Blaze wavelength is the wavelength for which the efficiency is maximal when the grating is used in Littrow configuration*
- *When* $\alpha = \theta_B$ *we have* $\beta = \theta_B$ *so* $m\lambda = 2d\sin\theta_B$
- *Blaze wavelength defined by manufacturer*

$$\lambda_B = \frac{2d\sin\theta_B}{m} = 2d\sin\theta_B \ (\text{for m} = 1)$$

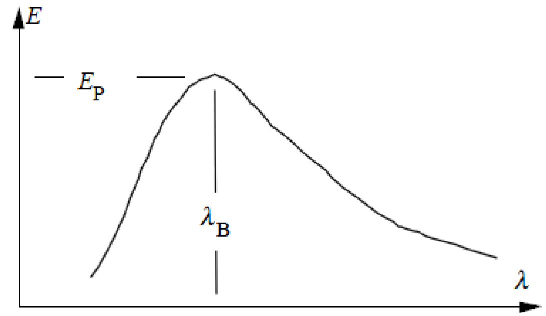
- *How does blaze wavelength change when* $\alpha \neq \beta \neq \theta_B$



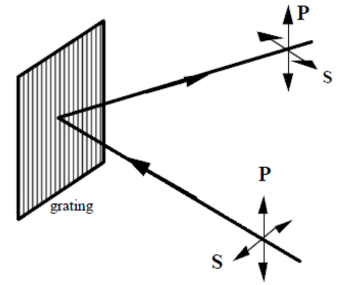


Energy Distribution

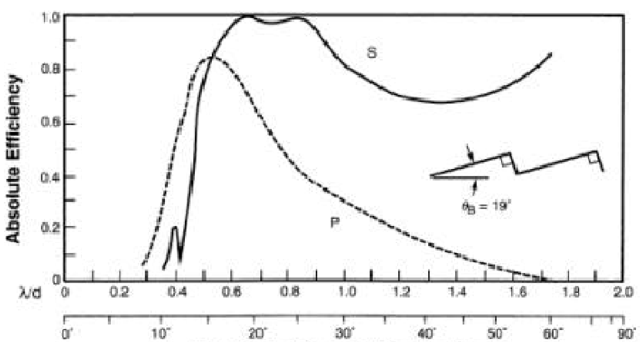
$$BF = \frac{\sin^2 \gamma}{\gamma^2} = \operatorname{sinc}^2\left(\frac{\pi d \cos\theta_B}{\lambda}[\sin(\alpha-\theta_B)+\sin(\beta-\theta_B)]\right)$$



- *Blaze wavelength is the wavelength for which the efficiency is maximal when the grating is used in Littrow configuration*



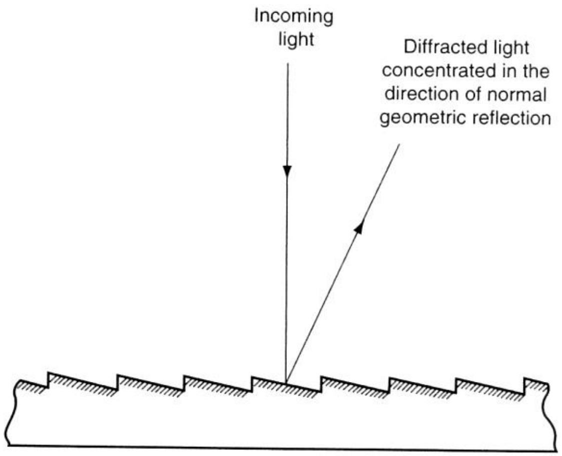
- *Energy distribution also depends on many parameters, including the power and polarization of the incident light, the angles of incidence and diffraction, the index of refraction of material at the surface of grating, and the groove spacing.*



- *A complete treatment of grating efficiency requires the vector formulation of electromagnetic theory.*

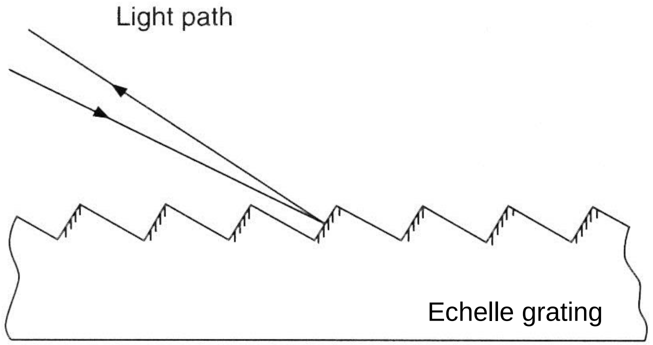
Gratings: Characteristics

- *Light dispersed. If d ~ w most light goes into 1 or 2 orders at given λ. Light of (sufficiently) different λ gets mostly sent to different orders*
- *Light from different orders may overlap (bad, need to deal with that!)*
- *Spectral resolution scales with fringe order m and is nearly constant within a fringe order → ~linear dispersion (in contrast to prism!)*
- *"Blazing": tilt groove surfaces to concentrate light towards certain direction → controls in which order m light of given λ gets concentrated*
- *Groove density > 1000 grooves/mm*



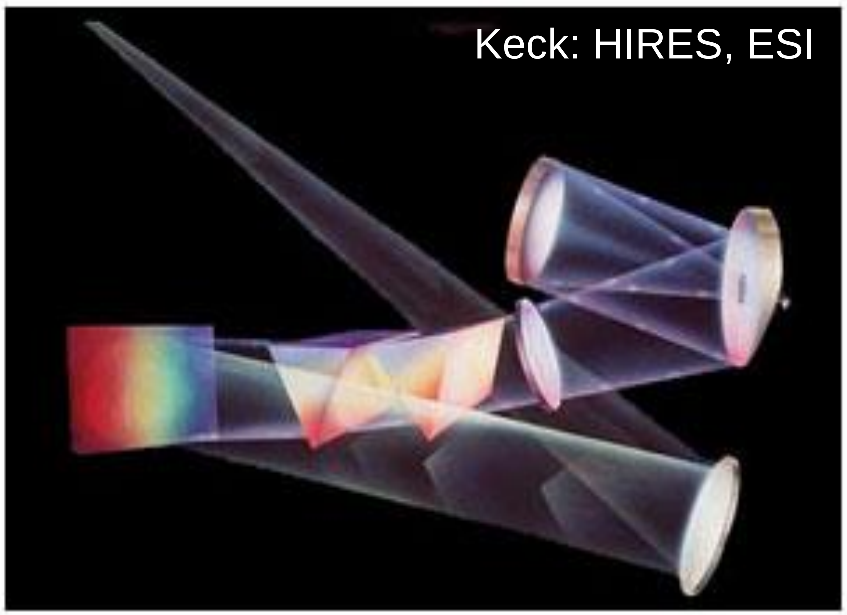
Echelle Grating

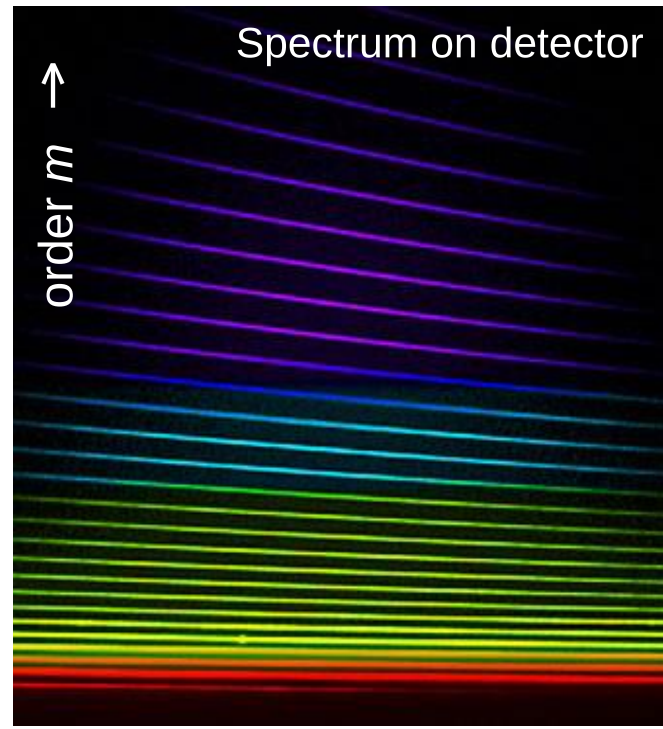
- *Relatively large groove spacing (few grooves/mm), but very high blazing angle. Concentrate light in high orders (m – could be tens or even hundreds) to achieve a high spectral resolution.*
- *Order overlap is much worse, because adjacent orders differ in wavelength by small amounts (e.g. order 6 @500 nm is coincident with order 5 @ 600 nm, order 7 @ 429 nm, order 8 @ 375 nm, etc).*
- *Must separate these orders by cross-dispersion, usually dispersing with a prism at right angles to the grating dispersion.*
- *Echelle spectrum consists of a number of spectral orders arranged side by side on the detector.*



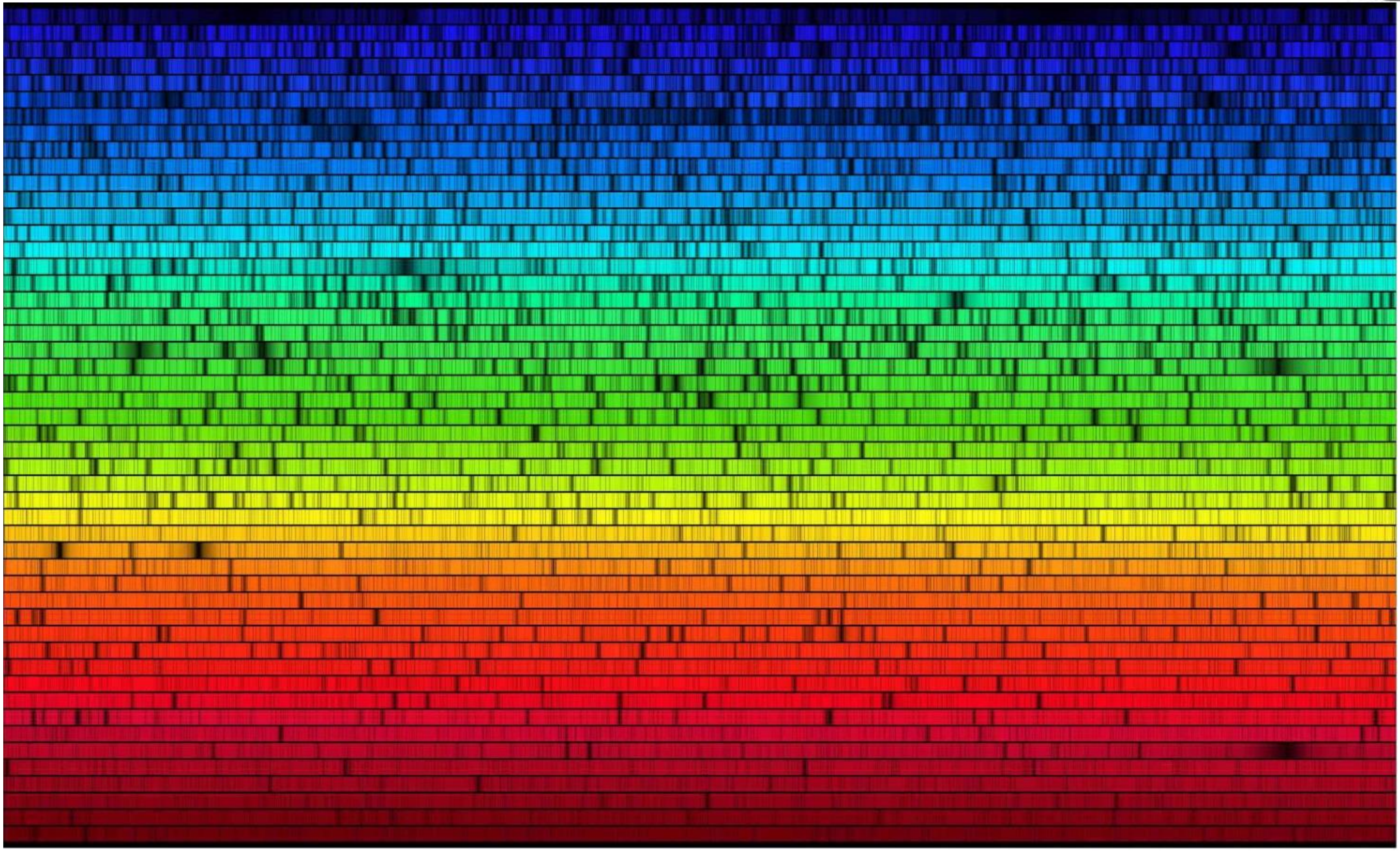
Echelle Spectrograph

- Cross dispersion with prism placed before grating
- High blaze angle, grating used in very high orders (up to $m \sim 200$)
- Coarse groove spacing (~ 20 to ~ 100 mm^{-1}) at optical wavelengths → $w >$ few λ most light concentrated in 1 direction → at given λ most light in 1 order
- Each order covers small λ range, but many orders can be recorded simultaneously





Solar Spectrum with an Echelle



Multiple Object Spectroscopy

- *Often you want spectra of many/extended objects in the same region*
- *Doing them one by one with a long-slit is very time consuming*
- *When putting a slit on a source in the focal plane, the photons from all other sources are blocked and thus "wasted"*
- *Wish to take spectra of many sources simultaneously!*
- *Solution: "multiple object spectrograph". Constructed to guide the light of >>1 objects through the dispersive optics and onto the detector(s), using:*
  - *Multislit: a small slit over each source ("slitlets")*
  - *Fiber-fed: a glass fiber positioned on each source*
  - *IFUs: Integral Field Unit"*