Hale COLLAGE (NJIT Phys-780)

Topics in Solar Observation Techniques

Astronomical Instruments & Observing Techniques

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Astronomical Instruments & Observation Techniques

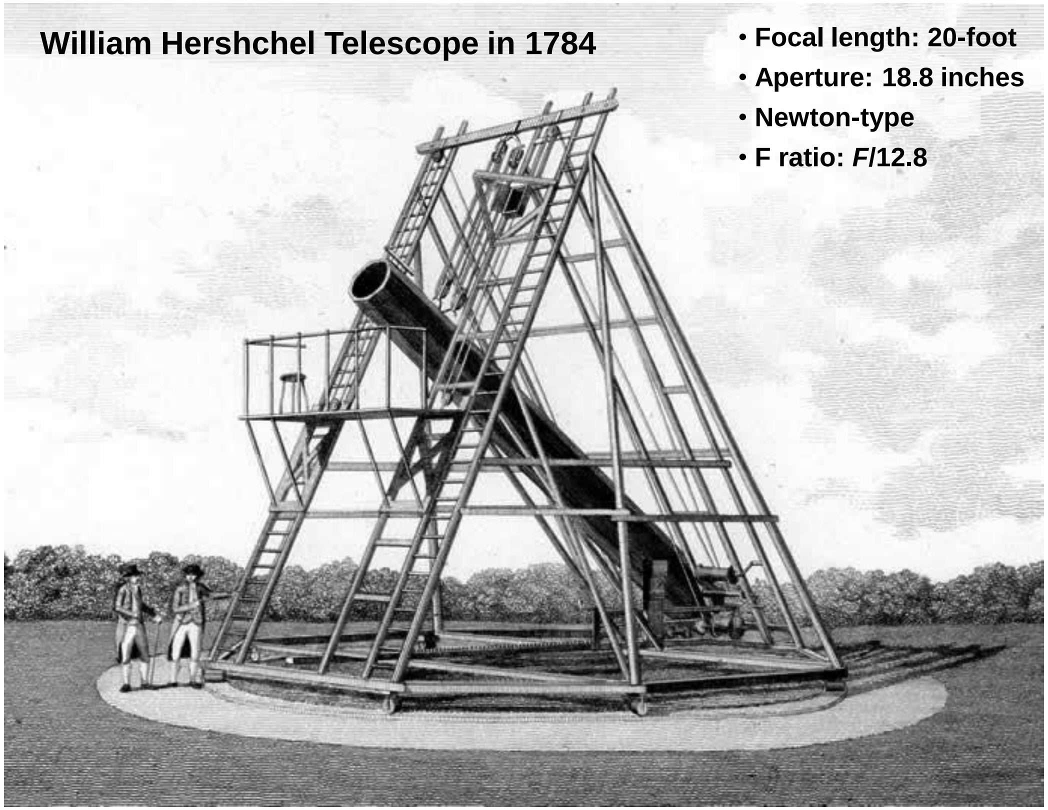
- 03/10: Fundamental optics and optical design
- 03/15: Astronomical telescopes
- 03/17: Astronomical optical/IR detectors: CCD, CMOS and IRFPA
- 03/21 – 03/27 Spring Recess: no classes

- 03/29: Imaging instruments: broad & narrow filters, FP interferometers
- 03/31: Grating-based spectrographs
- 04/05: Turbulent atmosphere and wavefront sensing
- 04/07: Adaptive optics system

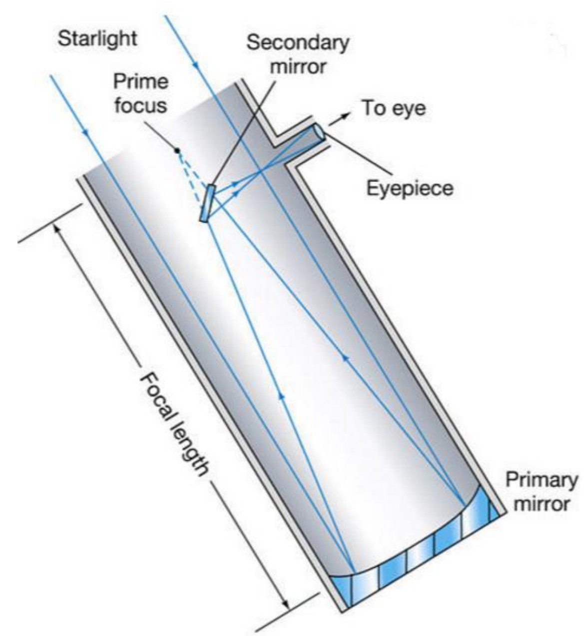
- 04/12: Solar spectropolarimetry I
- 04/14: Solar spectropolarimetry II
- 04/19: Modern solar telescopes

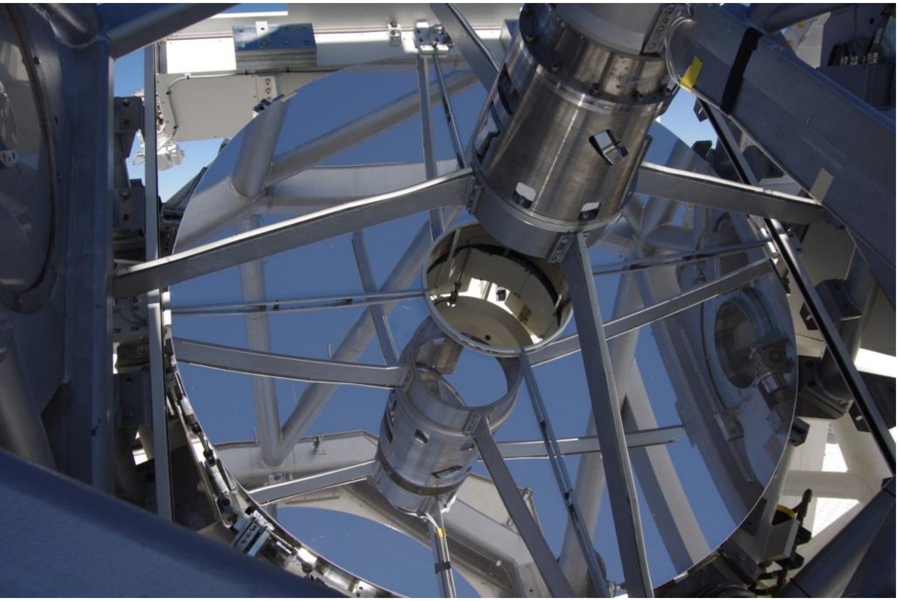
- 04/21, 04/ 26, 04/28: Student project presentations

Lecture 01: Fundamental Optics



- Focal length: 20-foot
- Aperture: 18.8 inches
- Newton-type
- F ratio: *F*/12.8





Outline

- ***Object and Image***
  - *Ray and Wave optics*
  - *Image of Telescopes*
- ***Geometrical Optics***
  - *Paraxial and Gaussian optics*
  - *Lenses*
  - *Stops and Pupils*
  - *Mirrors*
- ***Physical Optics***
  - *Diffraction*
  - *Fourier optics: PSF, OTF & MTF*

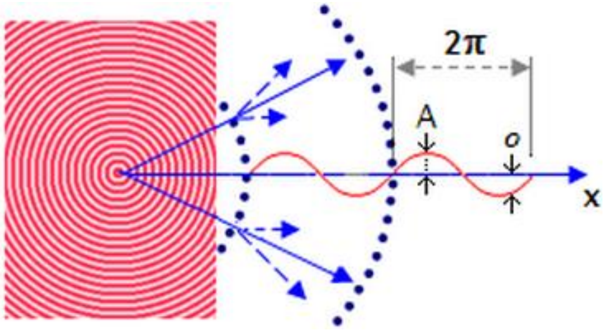
[1] OPTICS, Eugene Hecht, Chap. 5 & 10

[2] Reflecting Telescope Optics I, R. N. Wilson

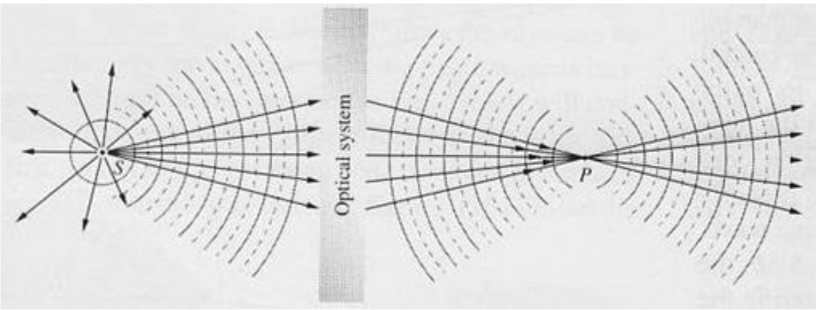
1. Object and Image

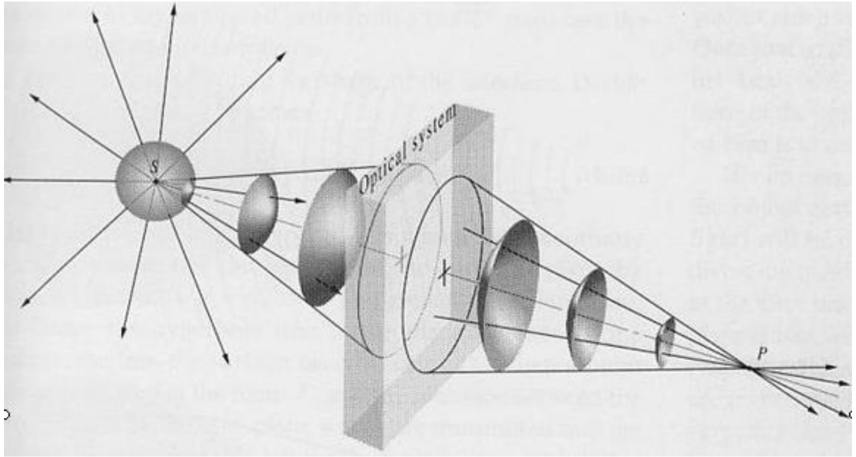
- ***Wavefront*** *is an imaginary surface connecting wave points of* ***identical phase***

$$\Phi = \frac{2\pi}{\lambda} \cdot OPL = \frac{2\pi n x}{\lambda}$$



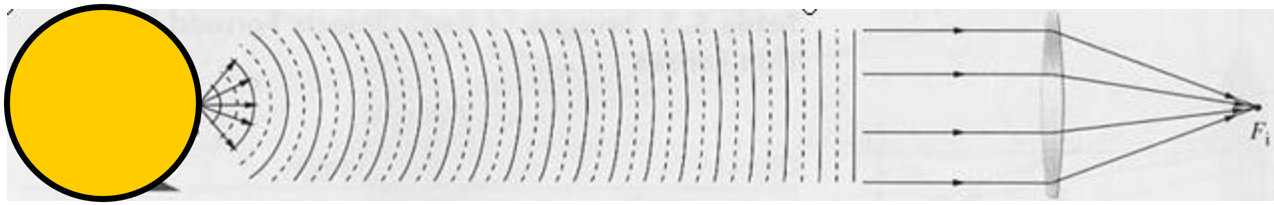
- ***Ray*** *is a straight line presents geometrical aspects of light and indicates light propagating direction*
- ***Ray always remains perpendicular to the wavefront***
- ***Optical Conjugation***

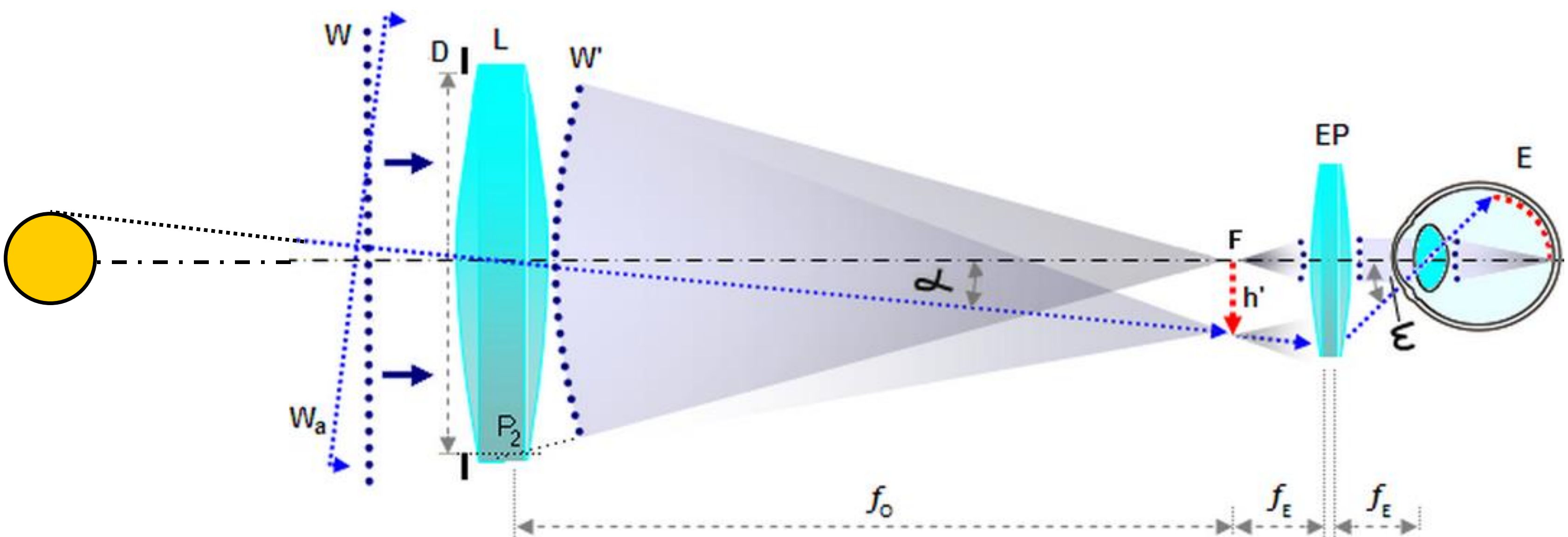




Object and Image

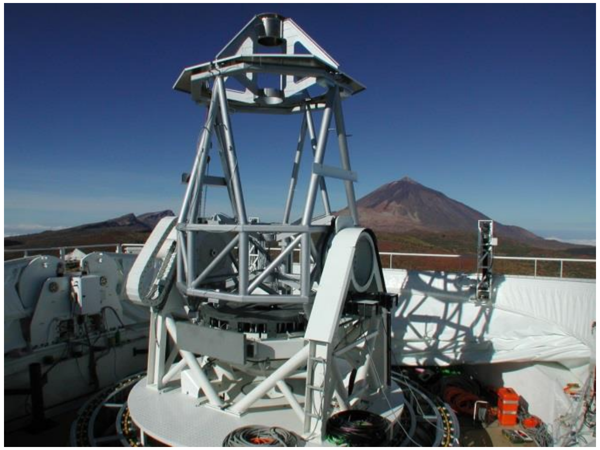
- *Q: Could you imagine what wavefront and light ray of the Sun look like just before a telescope?*





Images of Real Objects

- *Q: Angular diameter of the Sun is about 32' (arc-minute), could you find the physical size of the Sun on prime focus of GREGOR? The focal length of its PM is 2500 mm.*



$$h = f\alpha$$

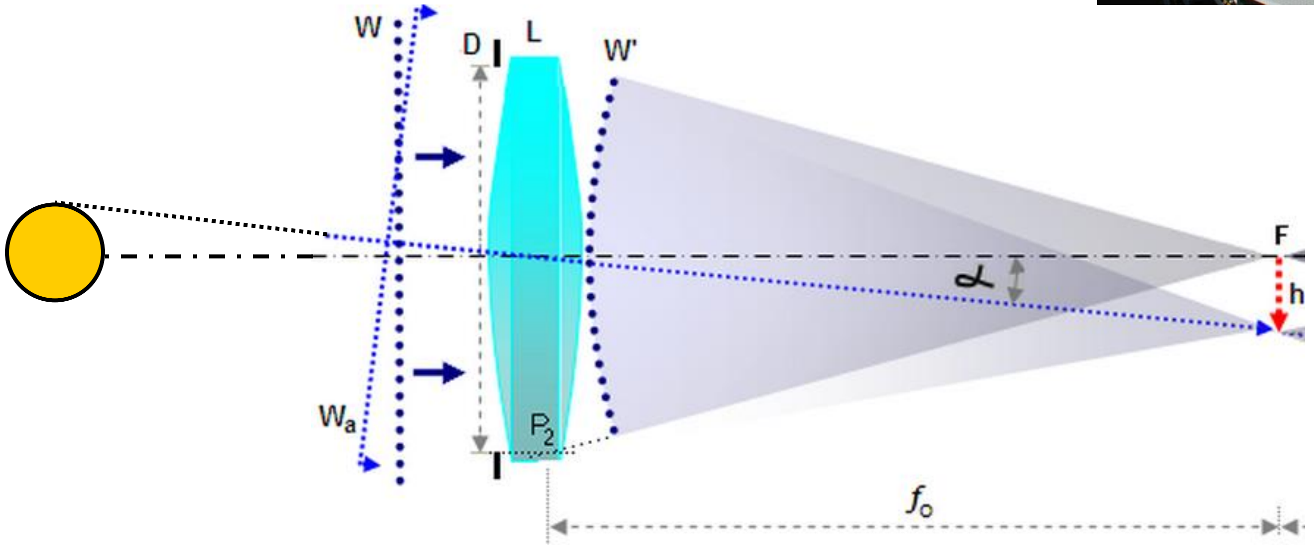


Image Scale of Telescopes

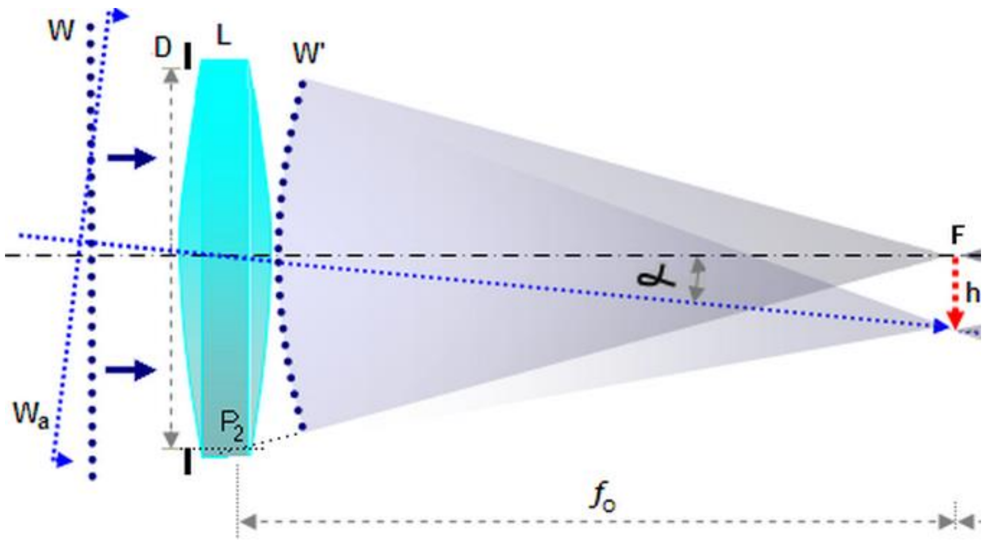
- ***Image scale:*** *a ratio of angular size of an object and its physical size on the focal plane.*

$$S = \frac{\alpha}{h} = \frac{1}{f}\,(\mathrm{rad/mm})$$

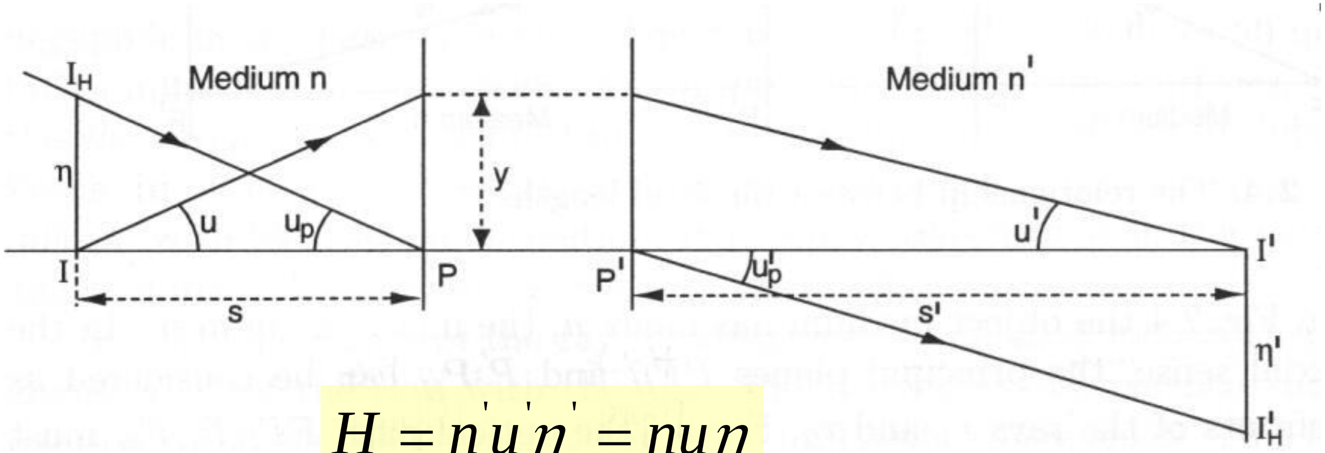
$$S = \frac{206265}{f\,(\mathrm{mm})}\,(\mathrm{arcsec/mm})$$

- ***Inverse image scale***

$$\bar{S} = \frac{f\,(\mathrm{mm})}{206265}\,(\mathrm{mm/arcsec})$$



Lagrange Invariant



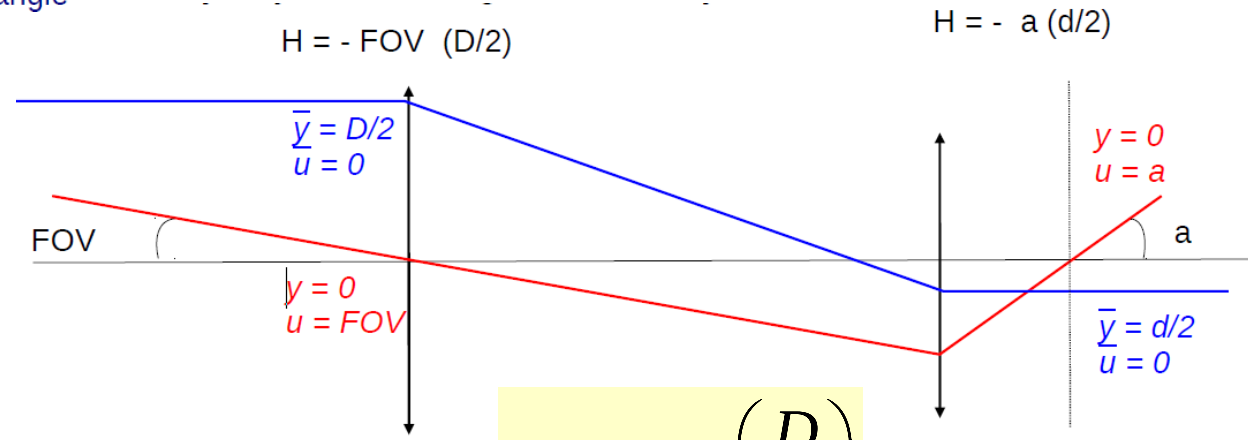
$$H = n'u'\eta' = nu\eta$$

- ***Chief (principle) ray*** *starts at edge of object and crosses center of aperture*
- ***Marginal ray*** *starts center of object and crosses aperture at its edge*
- ***Pupil*** *conjugates to aperture stop where chief ray intersects optical axis*
- ***Focal plane*** *is a plane where marginal ray intersects optical axis*
- *n and n' are ambient refractive index in object space and image space*
- y_p *- chief ray height and* u_p *- chief ray angel*
- *y - marginal ray height and u - marginal ray angel*

$H = n\,\bar{u}\,y - n\,u\,\bar{y}$

n = ambient refractive index
y = chief ray height
u = chief ray angle
$\bar{y}$ = marginal ray height
$\bar{u}$ = marginal ray angle

Afocal Telescope

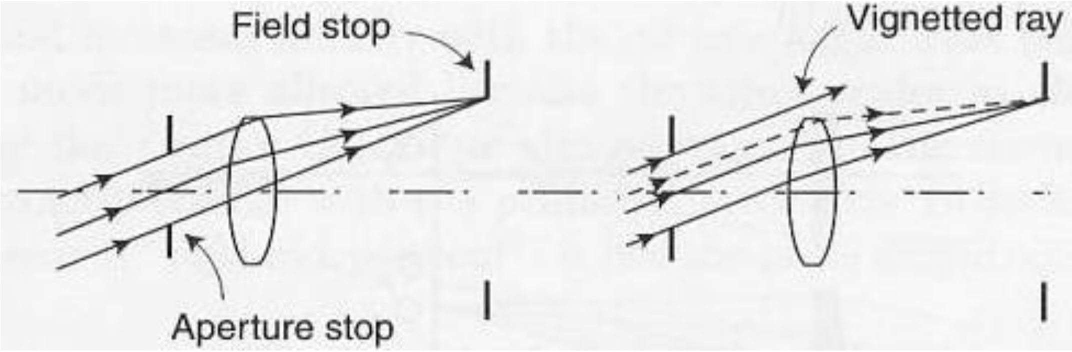


$$a = FOV\left(\frac{D}{d}\right)$$

- ***Pupil*** *conjugates to aperture stop where chief ray intersects optical axis*
- ***Focal plane*** *is a plane where marginal ray intersects optical axis*
- *Compressing the beam by factor x = multiplying angles by factor x*
- *Large FOV and large collecting area requires large optics*
- *Large FOV & large diameter telescopes are challenging to build and have very large optics*

Aperture and Field Stops

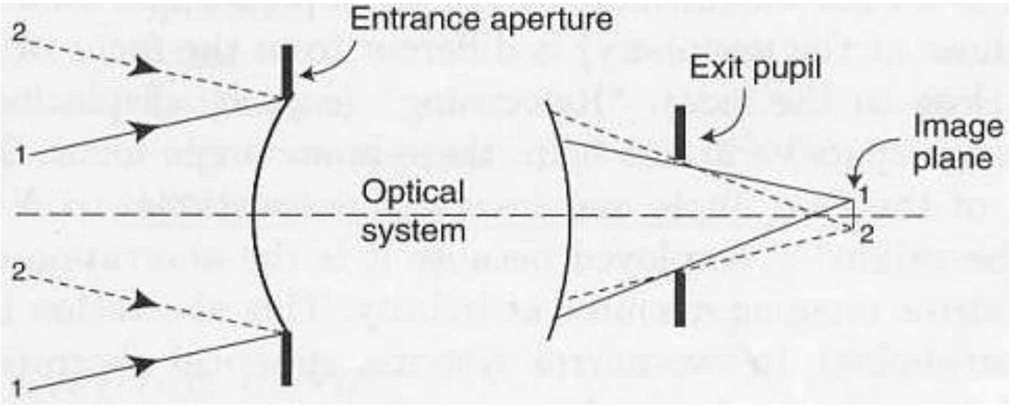
- ***Aperture Stop (AS):*** *determines the ray cone angle. It is typically the edges of the objective lens or mirror (PM) in a telescope.*
- ***Field Stop (FS):*** *limits the size or angular breadth of the object that can be imaged. It defines the field of view of instrument and is typically the size of CCD array.*

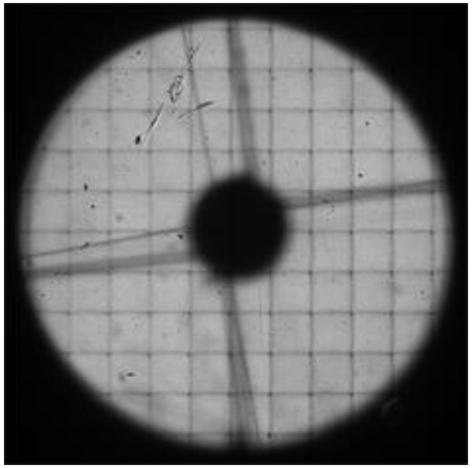


- ***Vignetting effect***

Entrance and Exit Pupils

- ***Object space*** *and* ***Image space***
- ***Pupils:*** *intermediate images of the aperture stop.*
- ***Exit pupil:*** *image of the aperture stop in image space.*
- ***Entrance pupil:*** *image of the aperture stop in object space.*
- ***Characteristic:*** *pupil contains all the rays that will reach the image, whatever the field angle; The ray bundles corresponding to different field angles do not shift over the pupil as a function of the field angle.*
- ***Pupils are good locations for filters, DM, T/T***

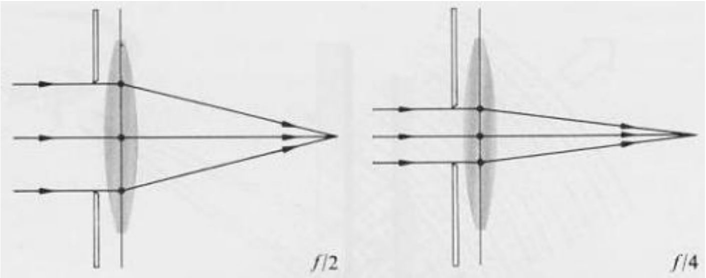




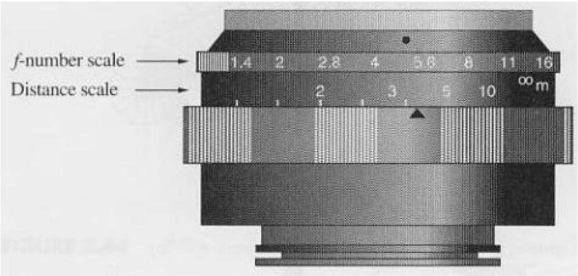
Relative Aperture and f-number

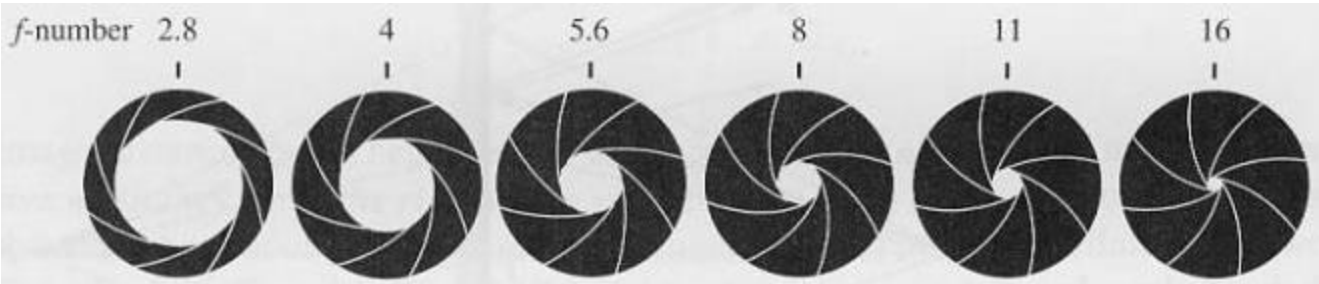
- *Flux density at the image plane varies as $(D/f)^2$ (since image scale varies with f).*
- ***Focal ratio or f-number:***

$$f - number = f/\# = N = \frac{f}{D}$$

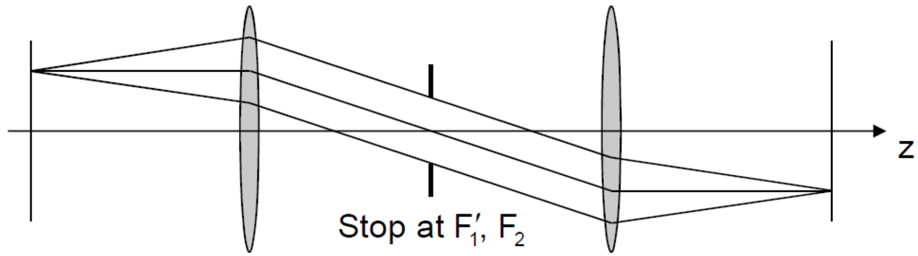


- ***Fast beam:** small f-number, short exposure time*
- ***Slow beam:** large f-number, long exposure time*
- *The photographic exposure time is proportional to the square of the f-number*

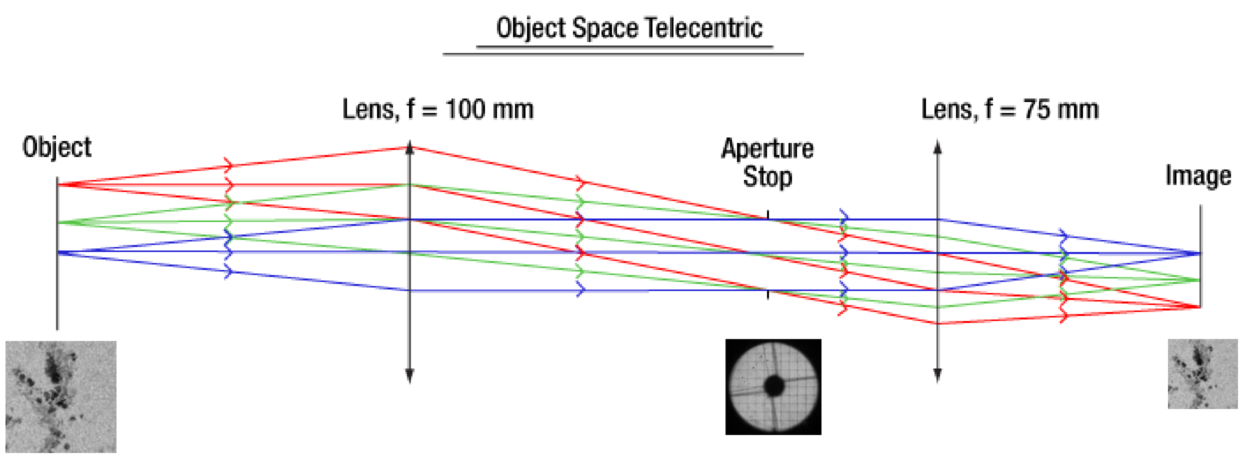




Telecentric Configuration



- When placing the aperture stop at the common focal point. The chief ray is parallel to the axis in object space and image space, and both the EP and the XP are located at infinity.
- On focal plane, telescope pupil is at infinity; on pupil plane, image is at infinity.
- Foci of lenses coincide to each other.
- Easy setup for adjusting image scale and magnification.
- Height of the blur forming image is independent of axial object shifts or image plane shifts.

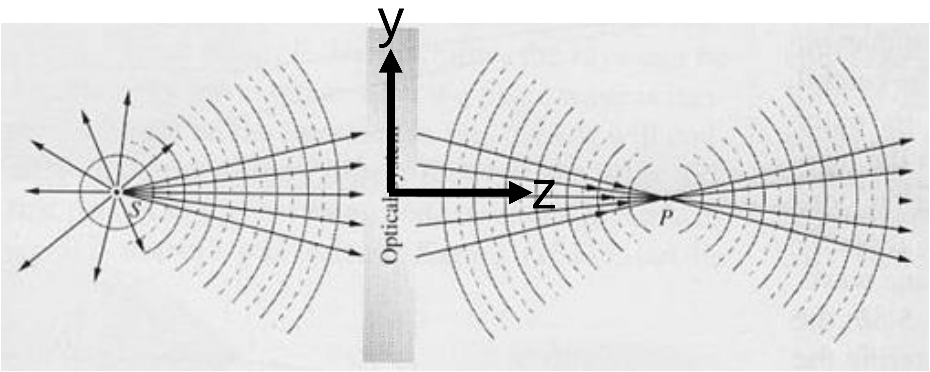


$$M_T \equiv \frac{y_2}{y_1} = \frac{f_2}{f_1}$$

Geometrical and Physical Optics

- ***Geometrical optics (or Ray optics)***
  - describes light propagation in term of “rays” based on Fermat’s principle.
  - provides rules for propagating these rays through an optical system.
  - often simplified by making the *paraxial approximation*
  - offers a convenient way to find object / image positions and magnification
  - fails to account for optical effects such as diffraction and polarization
- ***Physical optics (or Wave optics)***
  - builds on Huygens’s principle and wave superposition to account for the propagation of any wavefront through an optical system, including predicting the wavelength, amplitude and phase of the wave.
  - Offers advanced technique to account for diffraction, interference, and polarization effects and therefore to study image brightness, contrast, resolution and aberration.

2. Geometrical Optics



- Surface of an optical system

$$z = a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 + a_6 y^6 + \cdots$$
$$z = a_2 y^2 + a_4 y^4 + a_6 y^6 + \cdots$$

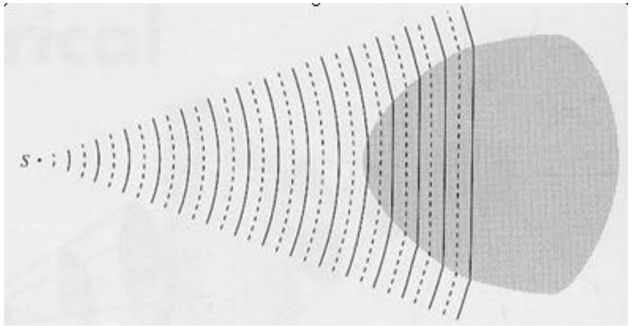
- Wavefronts passing through the system would also be expressed in this way.
- **Gaussian approximation:** only the first term is considered if the height y above the axis is very small compared with the radius of curvature R ($y << R$).

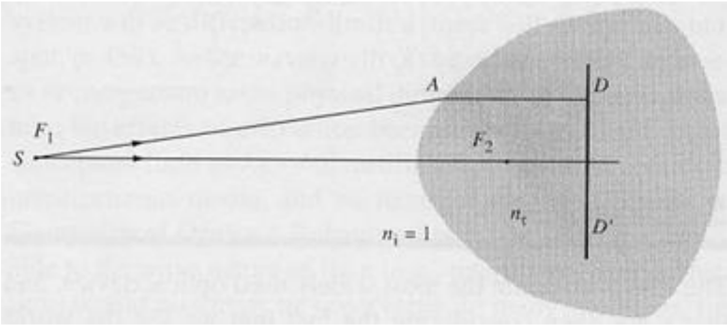
$$z = a_2 y^2 + a_4 y^4 + a_6 y^6 + \cdots \approx a_2 y^2 = \frac{c}{2} y^2 = \frac{1}{2R} y^2$$

- In **paraxial region**, the difference between a parabola and a sphere, or other conic section, is negligible.
- Snell's law for the paraxial region: $n'(i' - \frac{i'^3}{3!} + \frac{i'^5}{5!} + \cdots) = n(i - \frac{i^3}{3!} + \frac{i^5}{5!} + \cdots) \qquad n'i' = ni$

Lenses

- A Lens is a refracting device that reconfigures a transmitted energy distribution.
- Ideal lens surfaces: spherical or aspherical ?
  - *Q: What kind of shape of the interface between air and glass is required to make a diverging spherical wavefront to be planar?*





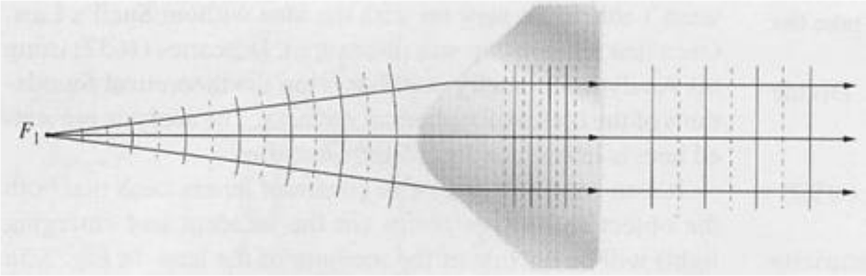
$$n_i(\overline{F_1A}) + n_t(\overline{AD}) = const$$

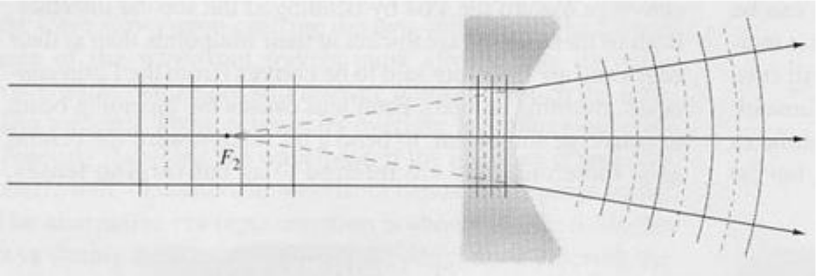
$$\overline{F_1A} + \left(\frac{n_t}{n_i}\right)\overline{AD} = const$$

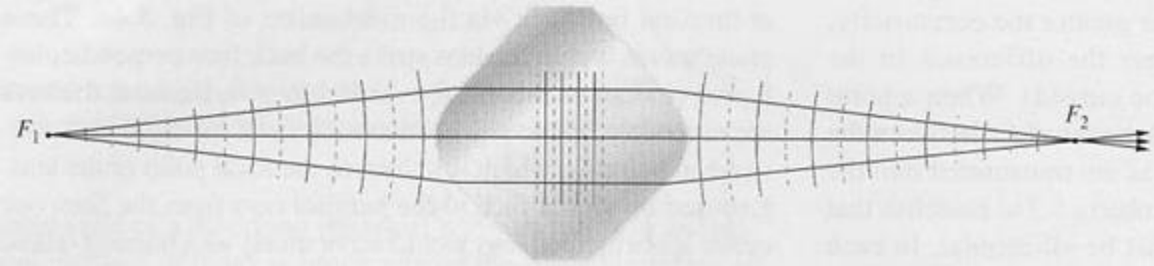
- The interface shape of an ideal lens could be hyperboloidal (n_t / n_i > 0) or ellipsoidal (n_t / n_i < 0), rather than a spherical !

Convex and Concave

- **Convex (converging lens):** is thicker at its midpoints than at its edges and cause the rays to bend a bit more toward the central axis;
- **Concave (diverging lens):** is thinner in the middle than at edges and cause the rays outward away from the central axis;
- When a parallel bundle of rays passes through a convex, the point to which it converges (or when passing through a diverging lens, the point from which it diverges) is a focal point the lens.







Spherical Lens and Paraxial Rays

- **Most lenses in use have spherical surface**
- **Gaussian optics:** first-order theory, or paraxial optics

$$OPL = n_1 l_0 + n_2 l_i$$

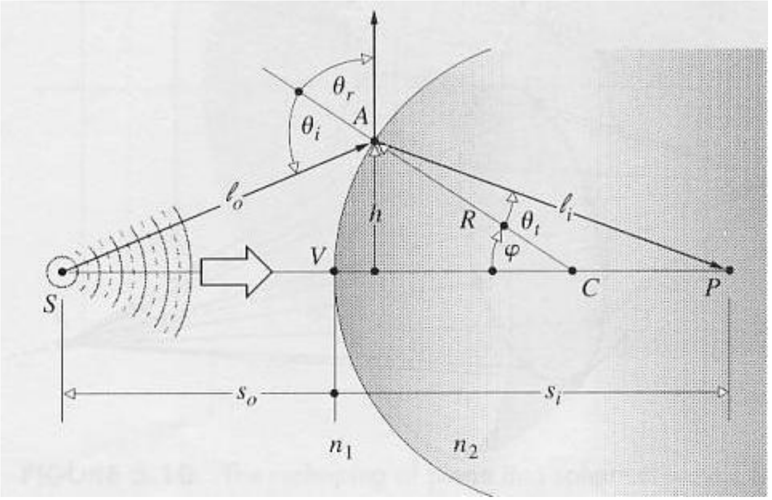
$$l_0 = [R^2 + (s_0 + R)^2 - 2R(s_0 + R)\cos\varphi]^{1/2}$$

$$l_i = [R^2 + (s_i - R)^2 + 2R(s_i - R)\cos\varphi]^{1/2}$$

$$d(OPL)/d\varphi = 0$$

$$\frac{n_1}{l_0} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_0}{l_0}\right)$$

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} + \ldots \qquad \sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} + \ldots$$



- If taking first-order approximation (h <<R): $\cos\varphi \approx 1,\ \sin\varphi \approx \varphi,\ l_0 \approx s_0,\ l_i \approx s_i$

$$\frac{n_1}{s_0} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- In **paraxial region**, the difference between a parabola and a sphere, or other conic section, is negligible.

Thin Lenses

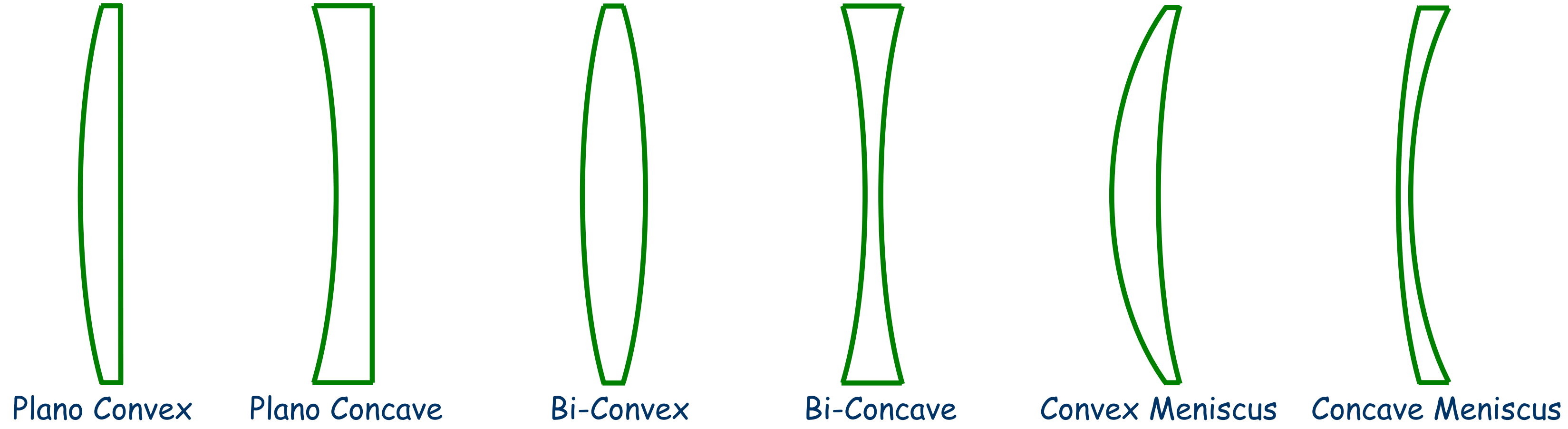
- ***Thin-Lens Equation or Lensmaker's Formula***

$$\frac{1}{s_0}+\frac{1}{s_i}=(n_l-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

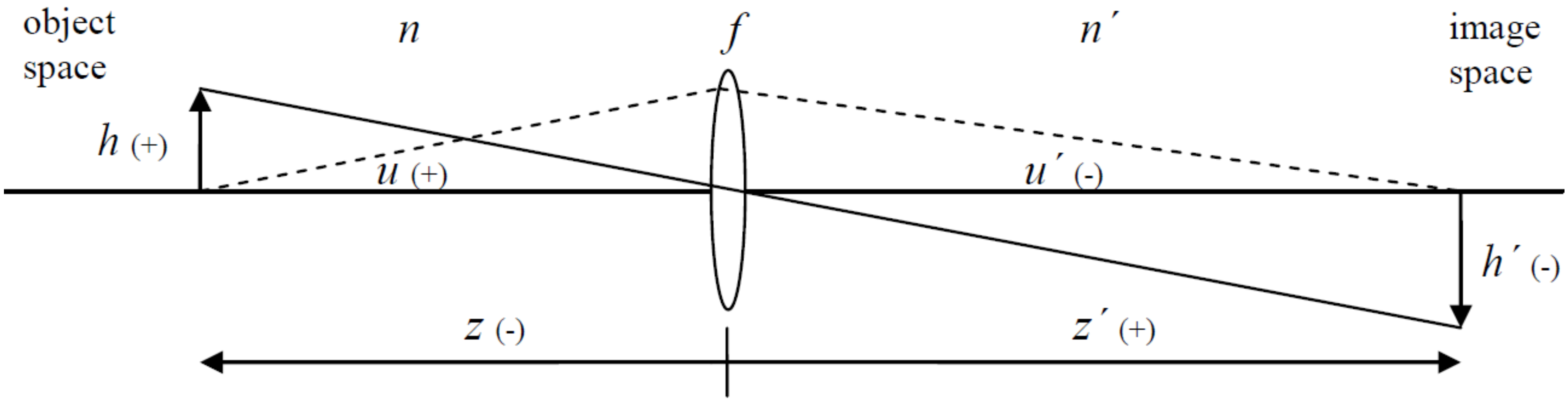
- ***Gaussian Lens Formula***

$$\frac{1}{s_0}+\frac{1}{s_i}=\frac{1}{f}$$

- ***Type of Thin Lens***



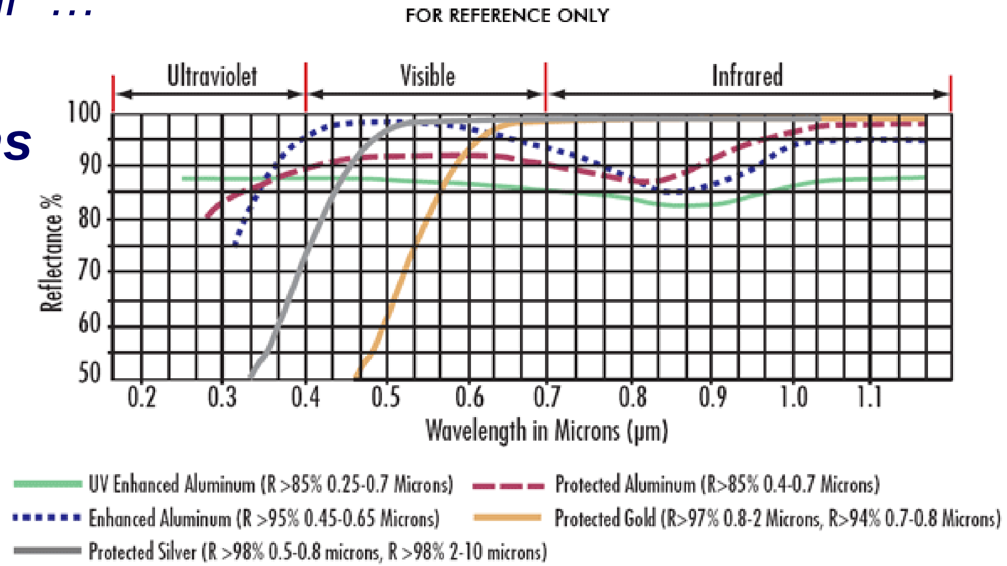
Thin Lens Formula



- *Transverse magnification in air* $$M_T \equiv \frac{h'}{h} = \frac{z'}{z} = \frac{u}{u'}$$
- *Longitudinal magnification in air* $$M_L = \left(\frac{n'}{n}\right) M_T^2$$
- *Image & object locations* $$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

Mirrors

- **_Mirror systems are increasingly being used in large telescope._**
  - *Substrate: glass, metal or silicon …*
  - *Reflective coating: aluminum, silver, gold …*
  - *Protective coating: SiO or MgF …*

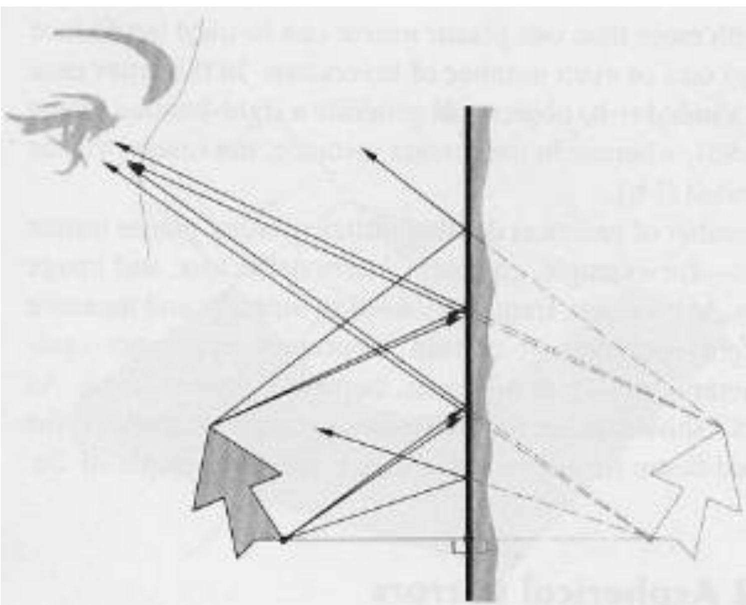


- **_Various mirror configurations_**
  - *Planar mirrors*
  - *Aspherical mirrors*
  - *Spherical mirrors*
  - *On-axis mirrors*
  - *Off-axis mirrors*

- **_NST Primary: 1.7 m off-axis paraboloidal mirror with high-reflectivity aluminum coating_**

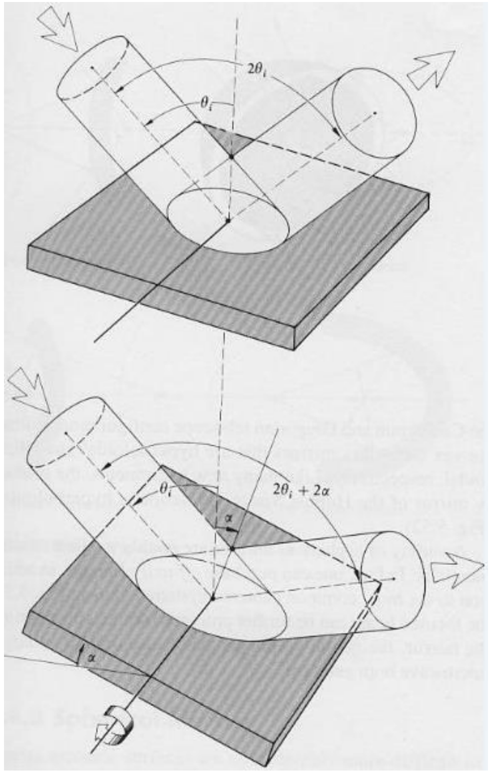
Planar Mirrors

- ***Position:*** *Image and object are equidistant from the mirror surface.*
- ***Magnification:*** *No transverse magnification.*
- ***Rotation:*** *If the mirror rotates through an angle α, the reflected beam or image will move through an angle of 2α.*



$$|s_0| = |s_i|$$

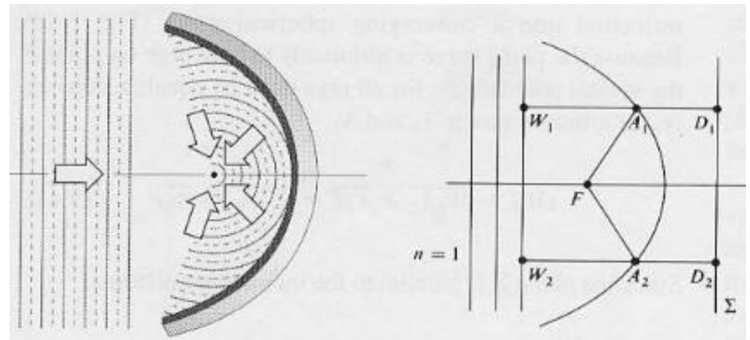
$$M_T = +1$$



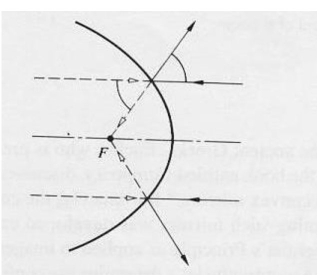
Aspherical Mirrors

- *Ideal curved mirrors have the conic surfaces.*

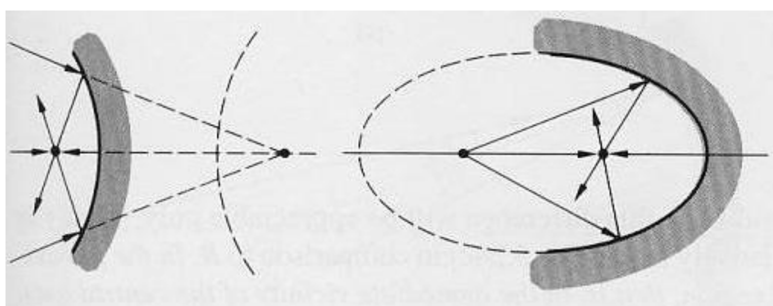
$$
\begin{aligned}
OPL &= \overline{W_1A_1} + \overline{A_1F} \\
&= \overline{W_2A_2} + \overline{A_2F} \\
&= \overline{W_iA_i} + \overline{A_iF} = const
\end{aligned}
$$



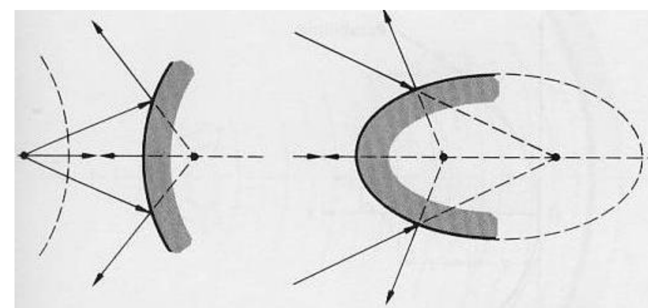
- ***Conic surface:*** *paraboloids(e=1), ellipsoids(e<1), hyperboloids(e>1).*
- ***Conic surface*** *has a pair of conjugate foci.*
- *All optical rays issuing from a source located at one of the foci will converge at the other focus and thus form a perfect image of the source.*



Real and virtual images for a paraboloidal



(c) Concave hyperbolic (d) Concave elliptical



(a) Convex hyperbolic (b) Convex elliptical

Spherical Mirrors

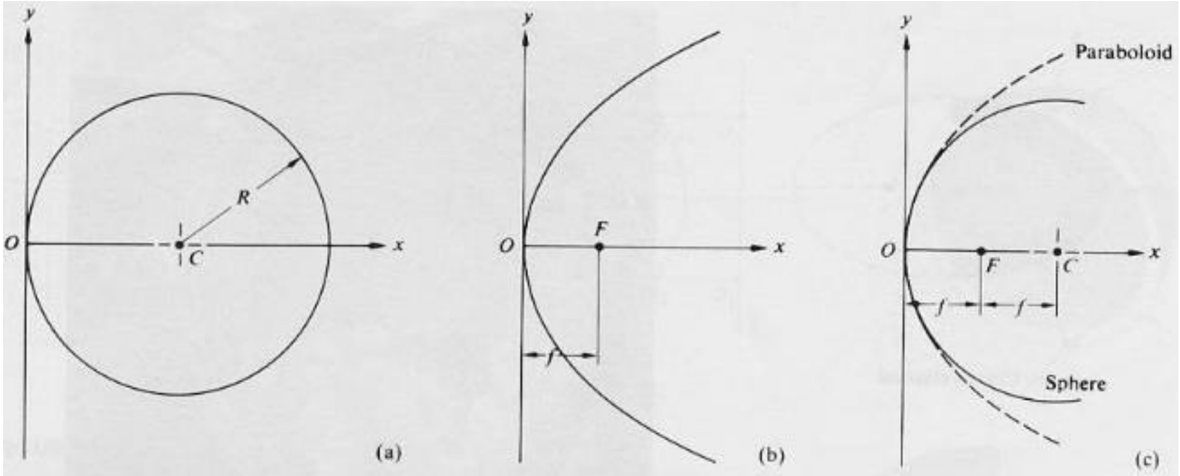
- **_Most mirrors in use have spherical surface_**
- ***Gaussian optics:*** *first-order theory, or paraxial optics*

$$x = R \pm (R^2 - y^2)^{1/2}$$

$$x = \frac{y^2}{2R} + \frac{1y^4}{2^2 2! R^3} + \frac{3y^6}{2^3 3! R^5} \cdots$$

$$y^2 = 4fx \quad \text{or} \quad x = \frac{y^2}{4f}$$

$$\Delta x = \frac{y^4}{8R^3} + \frac{y^6}{16R^5} \cdots$$



- *In the **paraxial region (y<<R)**, spherical and paraboloidal configurations will be essentially indistinguishable.*

Mirror Formula

- *Under paraxial approximation,*

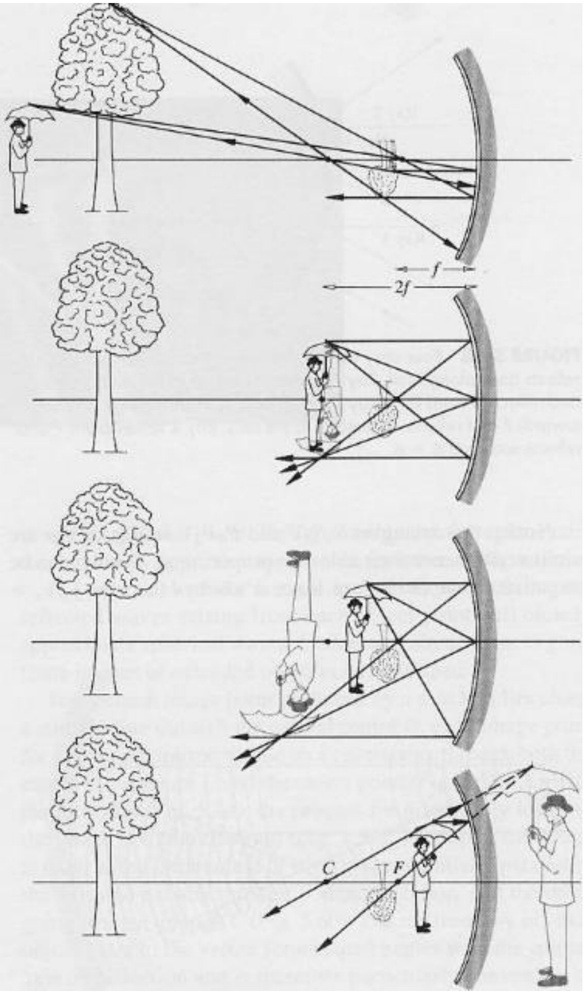
$$f_0 = f_i = -\frac{R}{2}$$

Concave

Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\|s_i\| > s_o$	Erect	Magnified

Convex

Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
Anywhere	Virtual	$\|s_i\| < \|f\|$, $s_o > \|s_i\|$	Erect	Minified

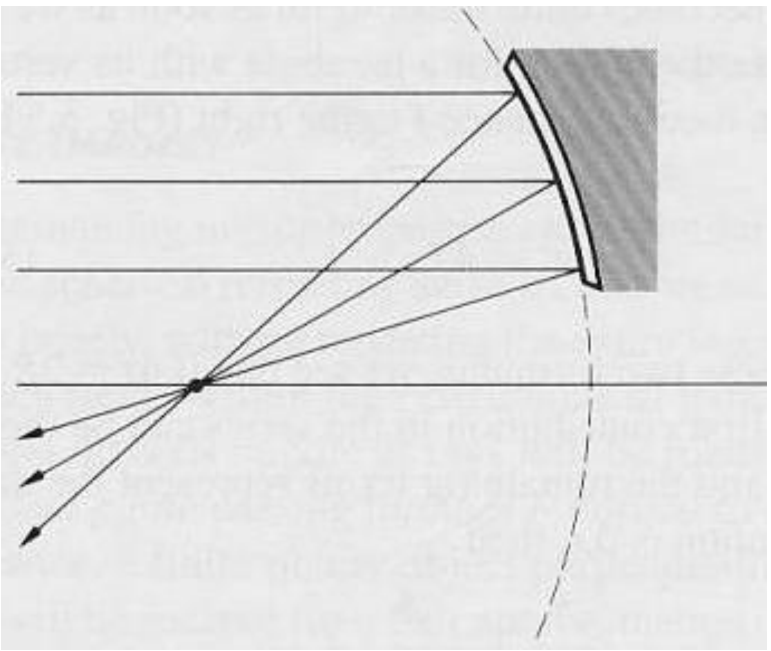


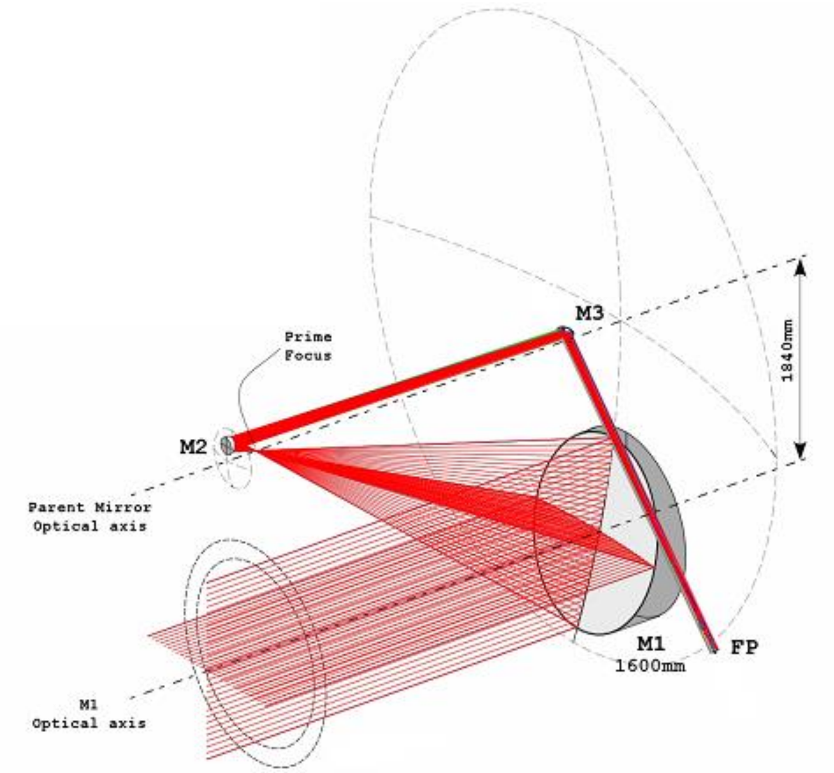
$$\frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$$

$$x_0 x_i = f^2$$

$$M_T \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_0} = -\frac{x_i}{f} = -\frac{f}{x_0}$$

On-axis and off-axis mirrors

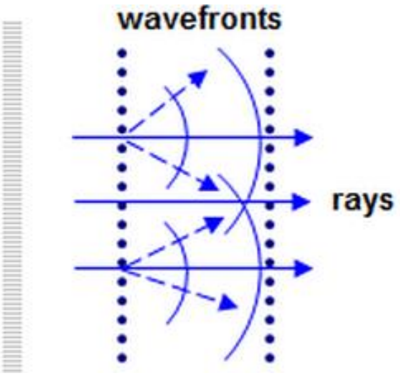


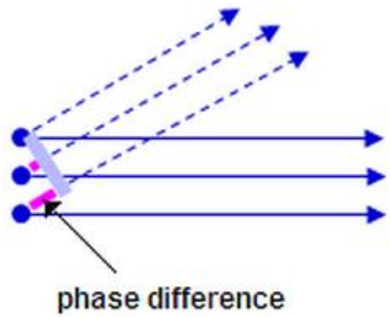


3 Physical Optics

- ***Geometrical optics (or Ray optics)***
- ***Physical optics (or Wave optics)***
  - ***Huygens's principle:*** each point on a wavefront can be envisaged as a source of secondary spherical wavelets.
  - ***Fresnel:*** E-field at any point can be constructed as the superposition of all "incoming" wave-fronts.
  - ***Kirchhoff:*** Above is a direct consequence of the wave equation deviable from Maxwell's equations.

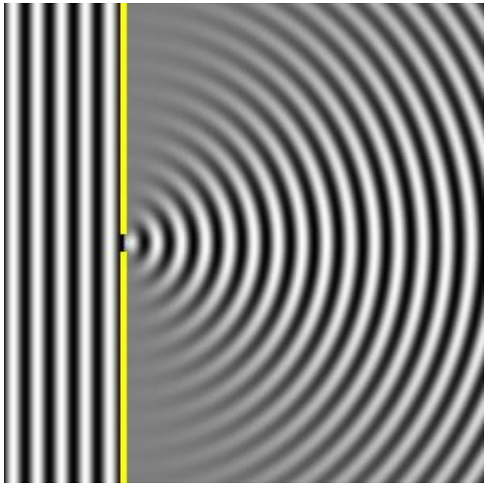
  - ***Superposition of waves***
  - ***Interference***
  - ***Polarization***
  - ***Diffraction***

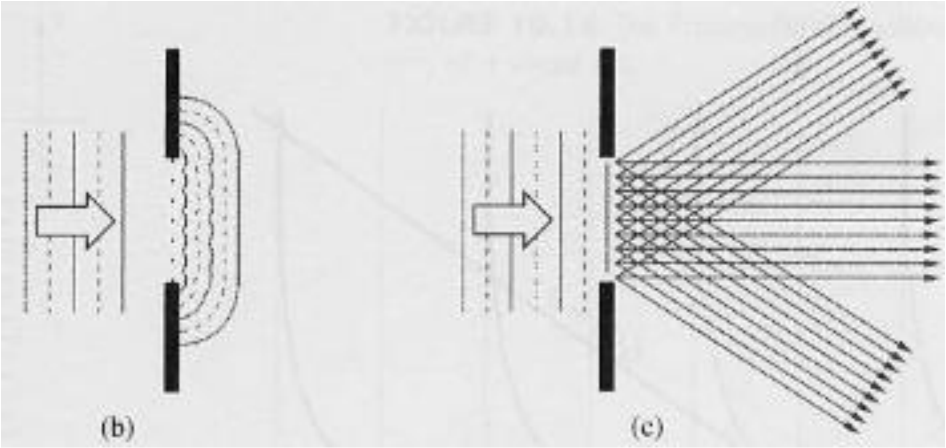


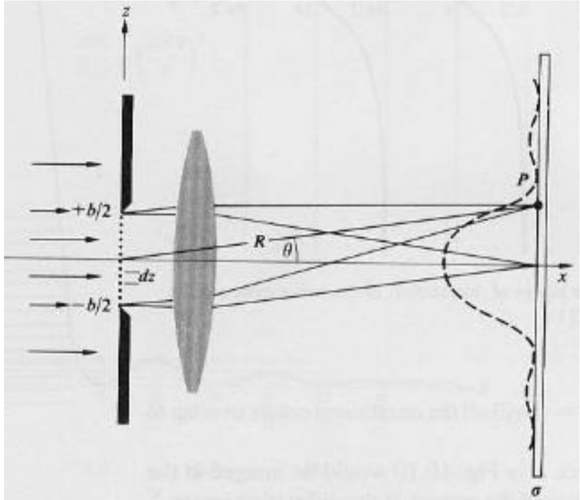


3.1 Diffraction Effects

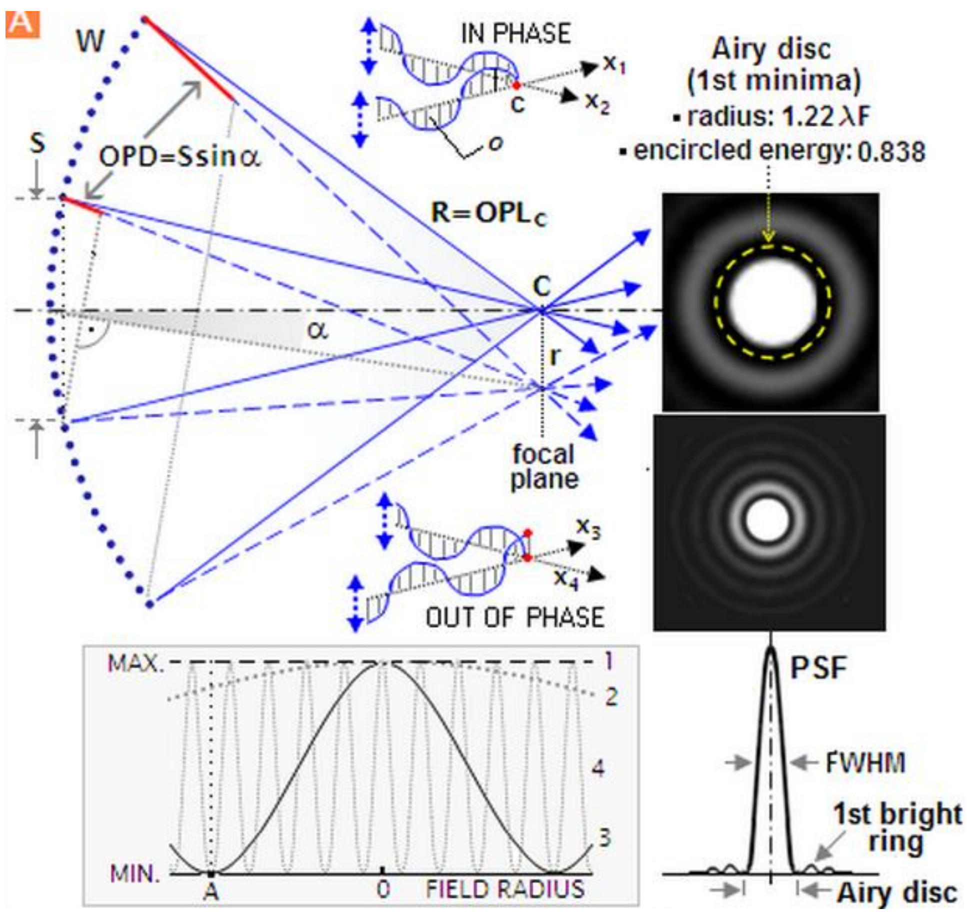
- *Diffraction refers to various phenomena which occur when a wave encounters an obstacle*
- *Light diffraction is a spreading of light as it passes the edge of an opaque body*
- *In a telescope, diffraction happens at the edge of the PM aperture and at any obstacle within the aperture, such as the SM and its supporting vanes*

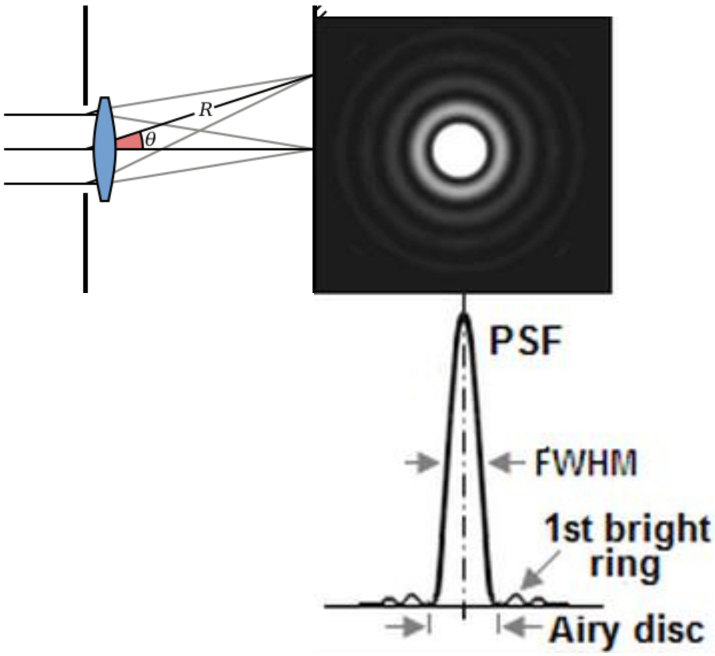


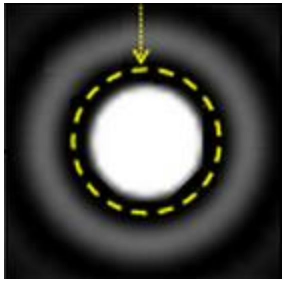




Telescope Imaging

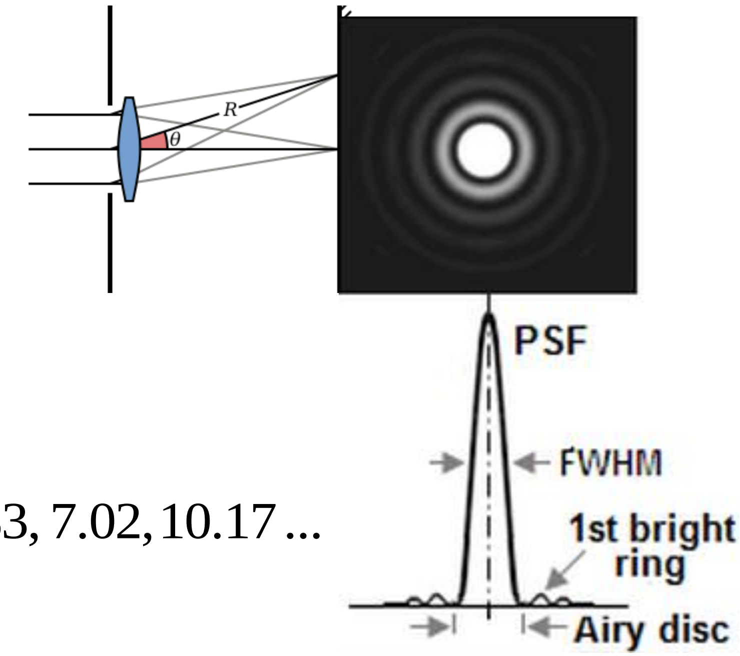






Airy Disc

- Point spread function

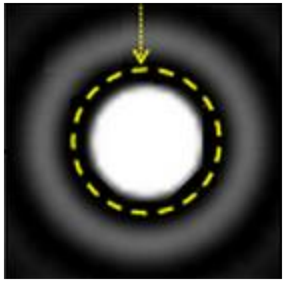


$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 = I_0 \left(\frac{2J_1(x)}{x} \right)^2$$

$$x = ka\sin\theta = \frac{2\pi a}{\lambda}\frac{q}{R} \qquad J_1(x) = 0 \text{ at } x \approx 3.83, 7.02, 10.17 \ldots$$

$$\sin\theta \approx \frac{3.83}{ka} = \frac{3.83\lambda}{2\pi a} = 1.22\frac{\lambda}{2a} = 1.22\frac{\lambda}{d}$$

$$q_1 = R\sin\theta \approx 1.22R\frac{\lambda}{D} \approx 1.22 f \frac{\lambda}{D} \qquad \theta_1 = \frac{q_1}{f} = 1.22\frac{\lambda}{D}$$

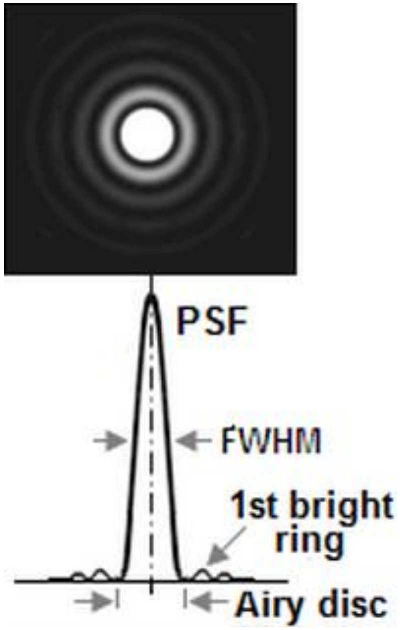


Point Spread Function

- **Simple case (ideal case): diffraction-limited telescopes**
  - *unobstructed circular aperture*
  - *no aberrations, perfect surfaces, no dusts*
  - *monochromatic light*
- **Airy function**

$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 = I_0 \left(\frac{2J_1(x)}{x} \right)^2$$

$$x = ka\sin\theta = \frac{2\pi a}{\lambda}\frac{q}{R}$$

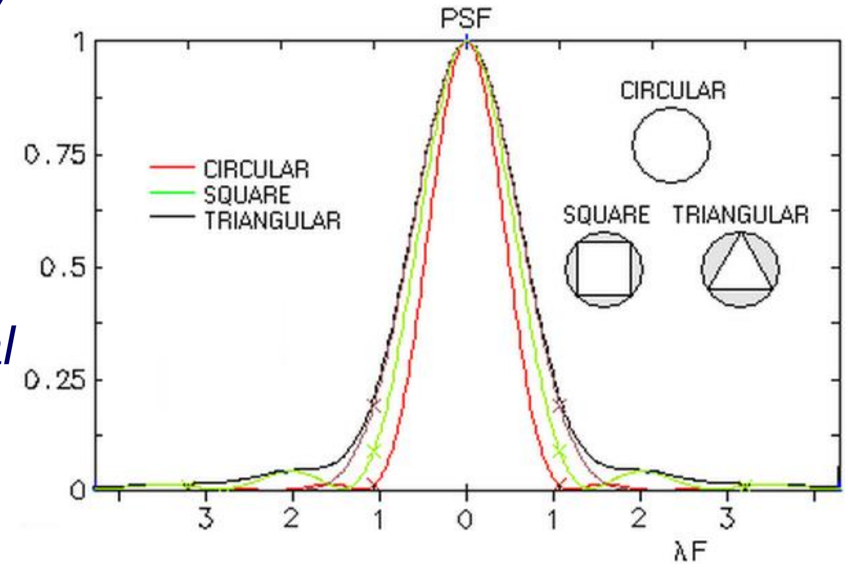


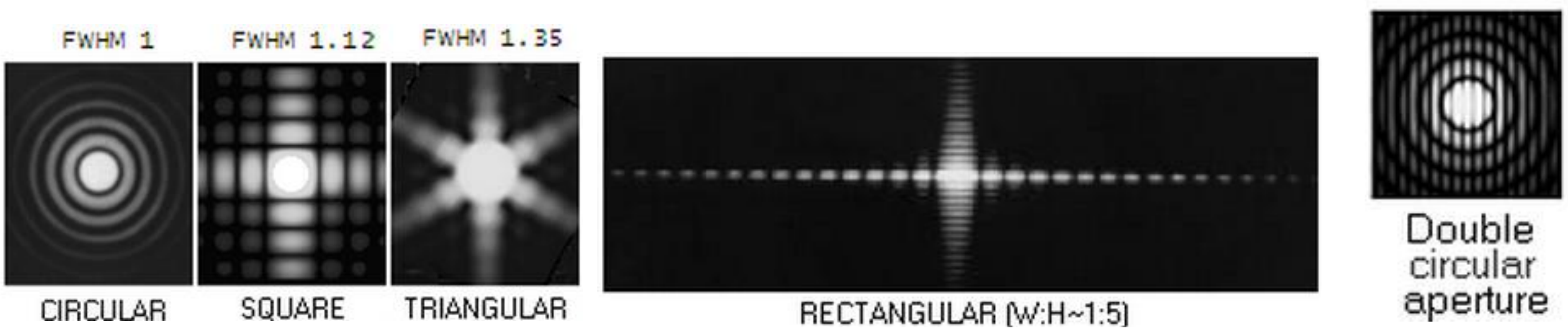
- **Encircled energy**

	1st		2nd		3rd		4th		5th		6th		7th		8th		9th	
	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min	max	min
r	0	1.22	1.63	2.23	2.68	3.24	3.70	4.24	4.71	5.24	5.72	6.24	6.72	7.25	7.73	8.25	8.73	9.25
I	1	0	0.0175	0	0.0042	0	0.0016	0	0.0008	0	0.0004	0	0.0003	0	0.0002	0	0.0001	0
EE	0	0.838	0.867	0.910	0.922	0.938	0.944	0.952	0.957	0.961	0.964	0.968	0.970	0.972	0.974	0.975	0.977	0.978

Point Spread Function

- PSF is the distribution of light intensity in the image of a point source
- Ideal case: diffraction-limited telescopes – Airy function
- PSF is a function of the shape of the aperture and obstructions, geometrical optical aberrations, and diffraction effects due to dust and defects on the optics surfaces



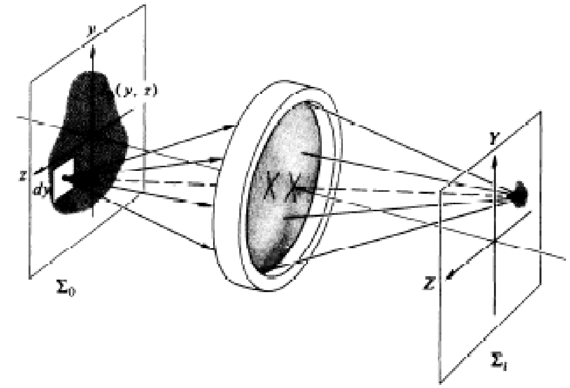


3.2 Fourier Optics: PSF, MTF

- ***Point Spread Function (PSF):*** *the distribution of light intensity in the image of a point source*
- ***Linear System (spatial domain):***

$$I_i(Y,Z) = \iint I_0(y,z)S(Y-y,Z-z)dydz$$

$$= I_0(y,z) \otimes S(y,z)$$



- ***Frequency domain:***

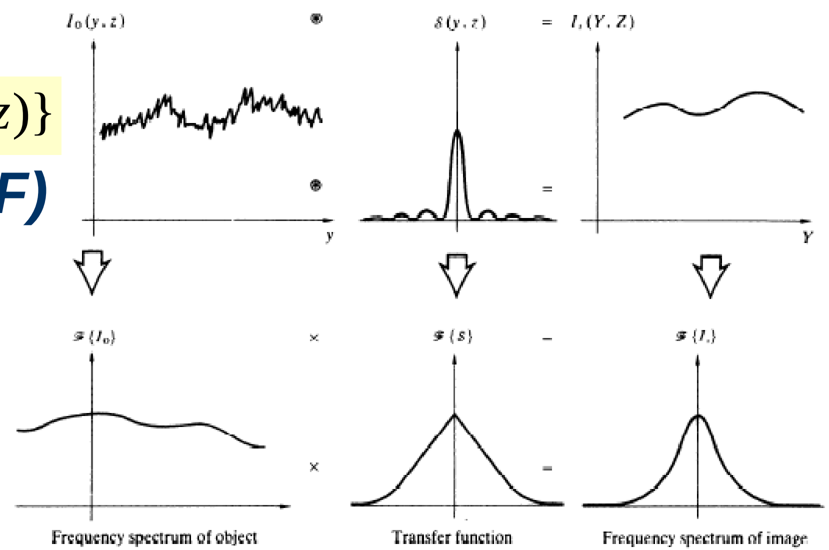
$$FT\{I_i(Y,Z)\} = FT\{I_0(y,z)\} \bullet FT\{S(y,z)\}$$

- ***Optical Transfer Function (OTF)***

$$OTF = FT\{S(y,z)\}$$

- ***Source Function***

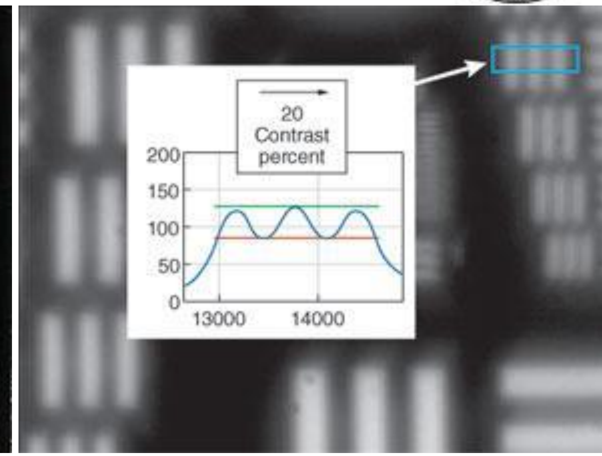
$$I_0(y,z) = FT^{-1}\left[\frac{FT\{I_i(Y,Z)\}}{OTF}\right]$$



Modulation Transfer Function

- ***Image Contrast:***

$$Modulation = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$



- ***MTF:*** *the ratio of the image modulation to the object modulation at all spatial frequency*

$$OTF(k_y,k_z) = MTF(k_y,k_z) \cdot PTF(k_y,k_z)$$

$$MTF(k_y,k_z) = |OTF| = \frac{|FT\{I_i(y,z)\}|}{|FT\{I_0(y,z)\}|}$$