1. Center-to-limb variation and angular resolution
Making the simplified assumption that the solar atmosphere only varies with depth the radiation I coming from a given point on the disk corresponds one-to-one to the radiation leaving the atmosphere under a given angle θ with the local normal of the atmosphere; hence measurement of the center-to-limb variation of the radiation allows us to measure the angular variation of I.

The angular diameter of the Sun is 30 arcmin (0.5 degree). Suppose that atmospheric seeing effects limitresolution to 1". Show that this sets a lower bound on the $\mu = \cos\theta$ for which we can infer $I(\mu)$ accurately, and determine this $\mu_{\rm min}$.

2. The Eddington–Barbier relation
Recall the equation of radiative transfer for a given source function $S(\tau)$ as function of vertical optical depth τ for a ray at an angle θ with the vertical of a plane-parallel atmosphere (i.e., one that varies only in the vertical direction).

a) Derive the integral form of the transfer equation, known as the formal solution for μ, where $\mu = \cos\theta$.

b) Obtain the formal solution for the emergent intensity I at the surface ($\tau = 0$) for a semi-infinite atmosphere, where optical depth goes to infinity.

c) Suppose that the source function in the above medium can be represented by a power-series expansion about the point $\overline{\tau}$; i.e.,

$$S(\tau) \approx S(\overline{\tau}) + S'(\overline{\tau})(\tau - \overline{\tau}) + 1/2S"(\overline{\tau})(\tau - \overline{\tau})^2. \tag{1}$$

Calculate the emergent intensity and show that the choice $\overline{\tau} = \mu$ is "optimum" in the sense that it *eliminates* the contribution of S' and *minimizes* the contribution of $S"$ to $I(0, \mu)$.

3. LTE Limit of the source function
Assume an atmic transition between two bound levels i, j with energy $E_j > E_i$.

a) Recall the expressions for the emission coefficients for spontaneous and stimulated and emission, respectively, and the coefficient for absorption of radiation, given the Einstein coefficients A_{ji}, B_{ji} and B_{ij}, the population numbers of the levels $n_{i,j}$, and the line-shape profiles φ_ν.

b) Write the expression for the line source function.

c) In thermodynamic equilibrium the ratio of the population numbers

n_j^*/n_i^* is given by the Boltzmann law. Using the relations between the three Einstein coefficients, show that for these values of the population numbers $n_{i,j}^*$ the source function reduces to the Planck function.

4. Scattering in a two-level atom

Consider a schematic atomic model consisting of only two bound levels j and i, where $E_j > E_i$.

a) Recall the expressions for the radiative rates R_{ji} and R_{ij}.

b) Write down the equation of statistical equilibrium for the level n_i, given the above radiative rates and the corresponding collisional rates C_{ji} and C_{ij}. Derive an expression for the ratio n_i/n_j.

c) Write down the expression for the line source function S_{ij} for the transition between the two levels, and using the detailed balance requirement for the collisional rates $C_{ij} = n_j^*/n_i^* C_{ji}$, show that this source function can be written in the form:

$$S_{ij} = (1-\epsilon)\overline{J} + \epsilon B, \tag{2}$$

where B is the Planck function, and $\overline{J}$ the integral of the angle-averaged mean intensity J times the profile shape φ_ν over frequncy ν.