

Hale COLLAGE: Solar Observations Problem Set 2 (Due Feb. 18, 2016)

Please try to be neat when writing up answers. In cases where analytic calculations are called for, please show all intermediate steps, including any approximations you choose to make and any sketches you may need to illustrate what's what. In cases where you do numerical computations or plots, there's usually no need to submit your full source code. However, if you're having trouble getting an answer that looks realistic, feel free to submit the code so we can debug.

In several of the problems below, you will need to make use of the solar-disk intensity spectrum. I've collected observational data from a range of sources, interpolated them to a consistent wavelength grid, and put the resulting data file on this course's "supplementary material" page. The direct URL is: http://lasp.colorado.edu/~cranmer/ASTR_7500_2016/Handouts/solar_spectrum.dat

Note: The file is big (250,000 lines), so if you're using a default single-precision language like IDL, make sure to use "long integers" for array sizes.

1. Thomson scattering for a hot gas. In class we discussed that the K-corona spectrum has been "smeared out" from the line-filled Fraunhofer spectrum of the solar disk. Here you will perform that smearing and figure out it can be used to diagnose the properties of the corona.

- (a) If the process of Thomson scattering was isotropic and coherent in the frame of each scattered electron, then the photon redistribution function could be written as

$$\mathfrak{R}(\xi', \hat{\mathbf{n}}'; \xi, \hat{\mathbf{n}}) = \delta(\xi' - \xi)$$

where ξ' is the frequency of the incoming photon in the electron's frame and ξ is the frequency of the outgoing (scattered) photon in the electron's frame. Assuming the electrons obey a Maxwellian distribution, compute the mean, observer-frame redistribution function,

$$\langle \mathfrak{R} \rangle = \frac{\int d^3\mathbf{v} f(\mathbf{v}) \mathfrak{R}(\nu', \hat{\mathbf{n}}'; \nu, \hat{\mathbf{n}})}{\int d^3\mathbf{v} f(\mathbf{v})} \quad \text{where} \quad f(\mathbf{v}) = \frac{n}{\pi^{3/2} v_{\text{th}}^3} e^{-(|\mathbf{v}|/v_{\text{th}})^2}$$

and the electron thermal speed is given by $v_{\text{th}} = \sqrt{2k_{\text{B}}T/m_e}$.

In other words, show that

$$\boxed{\langle \mathfrak{R} \rangle = \frac{1}{\Delta\nu_{\text{D}}\sqrt{2\pi}} \exp\left[-\frac{(\nu - \nu')^2}{2(\Delta\nu_{\text{D}})^2}\right]} \quad \text{where} \quad \Delta\nu_{\text{D}} = \frac{\nu v_{\text{th}}}{c}$$

and the observer-frame frequencies are given as ν' and ν . They are specified (for each electron moving with velocity \mathbf{v}) by the Doppler shift formulae,

$$\xi' = \nu' - \frac{\nu}{c} \mathbf{v} \cdot \hat{\mathbf{n}}' \quad \xi = \nu - \frac{\nu}{c} \mathbf{v} \cdot \hat{\mathbf{n}} .$$

For simplicity, assume that each electron undergoes 90° scattering. This means that $\hat{\mathbf{n}}'$ is always oriented transversely to $\hat{\mathbf{n}}$. For convenience you can assume that they are given by any two of the (mutually orthogonal) x , y , or z unit vectors.

Hints: You will need to know how to manipulate the arguments of Dirac delta functions; i.e., to convert $\delta[g(x)]$ into a version that one can integrate over dx . Also, you might need to brush up on the algebraic procedure of "completing the square."

- (b) In lectures, we showed that the scattering source function S_ν can be written as an integral over all incoming frequencies and directions. Doing just the integral over frequency gives a smoothed intensity (i.e., a convolution between the spectrum and the redistribution function):

$$I_{\text{smooth},\nu} = \int d\nu' \langle \mathfrak{R} \rangle I_{\nu'} .$$

Numerically compute $I_{\text{smooth},\nu}$ for the provided high-resolution solar spectrum, and run it a few times for several different values of the coronal temperature:

$$T = \{0.5, 1, 2, 3\} \text{ MK} .$$

Note: Keep track of units! The spectrum was provided in “per unit wavelength” units (I_λ) that the observers tend to use most often, so you may have to convert. Once you’re done, keep the observers happy by converting *back* to per-wavelength units. Plot your $I_{\text{smooth},\lambda}$ curves for visible wavelengths between 3000 and 8000 Å.

- (c) You have a telescope and coronagraph that can observe the K-corona intensity to about 1% accuracy. However, you cannot observe the full spectrum. You are limited to making observations in only **two** narrow-band filters. Each filter has a width of about 1 Å, but you can tune their central wavelengths to whatever you want. What central wavelengths would you choose if you wanted to *measure the coronal temperature*? Provide any backup plots or calculations to justify your answer.

2. Thomson-scattered Lyman alpha. Consider the most prominent emission line in the solar spectrum: H I Ly α at 1216 Å. If you observe the off-limb K corona at ultraviolet wavelengths, you will see a “smeared” version of this line, too. The goal of this problem is to estimate the coronal electron temperature *and* electron density, given only two measurements of the specific intensity at an off-limb heliocentric observation radius (i.e., impact parameter) of $b = 3 R_\odot$. Those measurements give:

$$I_\lambda = 2.299 \times 10^3 \text{ photons/s/cm}^2/\text{sr}/\text{Å} \quad \text{at} \quad \lambda = 1230 \text{ Å}$$

$$I_\lambda = 1.742 \times 10^3 \text{ photons/s/cm}^2/\text{sr}/\text{Å} \quad \text{at} \quad \lambda = 1240 \text{ Å}$$

Feel free to use anything you like to estimate n_e and T_e at $r = 3 R_\odot$ (i.e., in the “plane of the sky” intercepted by the given line of sight), including all the work you did for problem 1 above. However, the following pieces of information may be useful:

- (a) In the lectures, we specified that the unpolarized intensity from a Thomson-scattered corona is given by

$$I(b) = \frac{3\sigma_T}{16\pi} I_\odot \int_{-\infty}^{+\infty} dx n_e [Y(r) - (b/r)^2 A(r)] .$$

In the above, you can think of I_\odot as the smoothed intensity spectrum from part (b) of problem 1. You can also approximate the A and Y functions as

$$A(r) \approx \left(\frac{R_\odot}{r}\right)^2 \quad \text{and} \quad Y(r) \approx 2 \left(\frac{R_\odot}{r}\right)^2 .$$

The nice thing is that the only temperature dependence in $I(b)$ is within I_\odot . Thus, if you look at the ratio of intensities at two different wavelengths, it doesn’t matter whether you use I_\odot or $I(b)$.

(b) Assume that the electron density obeys an inverse-square dependence on radial distance:

$$n_e(r) = n_0 \left(\frac{R_\odot}{r} \right)^2 .$$

(c) You may eventually find the following definite integral to be useful:

$$\int_1^\infty \frac{du}{u^3 \sqrt{u^2 - 1}} \left(2 - \frac{1}{u^2} \right) = \frac{5\pi}{16} \approx 1 .$$

3. Properties of the dusty F corona. From observations, we know that the visible-light intensity of the F corona is given roughly by

$$\frac{I_F}{I_\odot} \approx 2.65 \times 10^{-8} \left(\frac{R_\odot}{r} \right)^{2.22} .$$

Our goal here is to figure out if it really makes sense to think of the F corona as being due to *reflection* of solar light from a cloud of small (spherical) dust grains. Thus, let's make a model for I_F and plug in numbers for the same observation radius $b = 3 R_\odot$ used in problem 2.

- (a) Given the electron density function $n_0(R_\odot/r)^2$ that you used in problem 2, find an expression for the mass density $\rho_{\text{gas}}(r)$ for all gas in the solar corona. Assume the corona is only hydrogen and helium, with the latter having a number density fraction $n_{\text{He}}/n_{\text{H}} = 0.05$. What is ρ_{gas} at $b = 3 R_\odot$?
- (b) Astronomers who study dust often think in terms of a **dust-to-gas mass ratio** $\mathcal{M} = \rho_{\text{dust}}/\rho_{\text{gas}}$. Assume that all dust grains are solid spheres with equal radii a and internal densities $D = 1 \text{ g cm}^{-3}$. Also assume that $\boxed{\mathcal{M} = 1}$ (i.e., for a given volume of the corona, there are equal masses in gas and dust). Write a general expression for the dust number density n_{dust} as a function of radial distance.
- (c) The infallible source Wikipedia says that dust grains that make up the zodiacal light have diameters between 10 and 300 microns. Using this range, compute a range of possible values for n_{dust} at our observation distance of $b = 3 R_\odot$. Assume all dust grains are of identical size, but let a vary for the different cases.
- (d) Consider a single dust grain. If it is sitting a radial distance r from the Sun and a distance d from the observer (on Earth), show that the *phase angle* θ is given by

$$\cos \theta = \frac{r^2 + d^2 - (1 \text{ AU})^2}{2rd} .$$

Note that θ is the angle between two vectors, both centered on the grain, which point to the Sun and to the observer. Also, derive an expression for d in terms of the line-of-sight distance x and the observed impact parameter b .

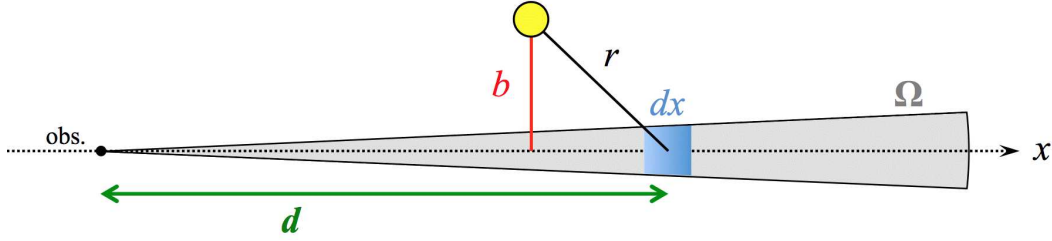
- (e) Consider sunlight reflecting off a single dust grain of radius a . Show that the flux observed at Earth is given by

$$F_1 = A \Phi(\theta) F_\odot \left(\frac{a R_\odot}{r d} \right)^2$$

where A is the albedo of the grain (assume $A = 0.1$), F_{\odot} is the outgoing flux at the solar photosphere, and $\Phi(\theta)$ is the *phase function* for reflection off a sphere. For $\theta \approx 0$ (“full moon”) $\Phi \approx 1$, then Φ decreases monotonically as θ increases (gibbous \rightarrow quarter \rightarrow crescent). Later, feel free to use the so-called Lambertian function

$$\Phi(\theta) = \frac{1}{\pi} [\sin \theta + (\pi - \theta) \cos \theta] .$$

- (f) The reflected photons we see at Earth come from a large number of dust grains, not just one. Consider an observation of a piece of the sky subtending a small solid angle Ω :



For the small volume element with length dx along the line of sight, demonstrate that the contribution to the observed flux is given by:

$$dF = F_1 n_{\text{dust}} \Omega d^2 dx .$$

- (g) Lastly, use all of the information above to numerically integrate over the full line of sight. Compute the observed flux from a radial distribution of dust grains as discussed above. Convert the flux to intensity (flux per unit solid angle) and divide by the solar-disk intensity to estimate I_F/I_{\odot} . Compute this quantity at the standard value of $b = 3 R_{\odot}$ for the full range of dust grain sized obtained from Wikipedia. Also feel free to compute a grid of observation heights and plot it alongside the observed ratio given above. Is any one grain size favored by comparison with observations?