Hale Collage

Spectropolarimetric Diagnostic Techniques

March 1 - 8, 2016

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Degree of polarization

The degree of polarization is defined as:

\[ p = \frac{Q^2 + U^2 + V^2}{I^2} \]

In a purely absorptive medium (no emission):

\[ \frac{d\vec{I}}{d\tau} = \mathbf{K}\vec{I} \]

It can be demonstrated that:

- If \( \eta_Q = \eta_U = \eta_V = 0 \) unpolarized light remains unpolarized.
- If \( \eta_I = \) constant, the degree of polarization, \( p \), increases asymptotically.

Absorption and dispersion processes are non-depolarizing. Depolarizing effects can only appear through emission processes.
Today

- Symmetry properties of the Stokes profiles
  - Net Circular Polarization
- Longitudinal magnetograms
- Quiet Sun magnetic fields
  - Unsigned magnetic flux density
  - Effects of spatial resolution
- Shortcomings of the Zeeman effect
- Scattering polarization and the Hanle effect
  - Second solar spectrum
Symmetry properties of the Stokes profiles

What happens to the propagation matrix when we do a change of variable: \((\lambda - \lambda_0) \rightarrow (\lambda_0 - \lambda)\)?

\[
K = \begin{pmatrix}
\eta_I & \eta_Q & \eta_U & \eta_V \\
\eta_Q & \eta_I & \rho V & -\rho U \\
\eta_U & -\rho V & \eta_I & \rho Q \\
\eta_V & \rho U & -\rho Q & \eta_I \\
\end{pmatrix} \quad \rightarrow \quad K' = \begin{pmatrix}
\eta_I & \eta_Q & \eta_U & -\eta_V \\
\eta_Q & \eta_I & \rho V & \rho U \\
\eta_U & -\rho V & \eta_I & -\rho Q \\
-\eta_V & -\rho U & \rho Q & \eta_I \\
\end{pmatrix}
\]

Voigt profile: 

\[
\phi_\alpha = \frac{1}{\sqrt{\pi}} \sum_i S_{\alpha,i} H(u_0 + u_{B,\alpha,i} - u_{LOS}, a)
\]

In the absence of velocity gradients, this transformation is a constant modification of \(K\) regarding the symmetry of wavelengths with respect to \((u_0 - u_{LOS})\).
Symmetry properties of the Stokes profiles

So the RTE transforms like this:

\[
\frac{d}{d\tau_c} \bar{I}(\lambda - \lambda_0) = K(\bar{I}(\lambda - \lambda_0) - \bar{S})
\]

\[
\frac{d}{d\tau_c} \bar{I}(\lambda - \lambda_0) = K'(\bar{I}(\lambda - \lambda) - \bar{S})
\]

\[
I(\lambda_0 - \lambda) = I(\lambda - \lambda_0)
\]

\[
Q(\lambda_0 - \lambda) = Q(\lambda - \lambda_0)
\]

\[
U(\lambda_0 - \lambda) = U(\lambda - \lambda_0)
\]

\[
V(\lambda_0 - \lambda) = -V(\lambda - \lambda_0)
\]

Stokes I, Q and U are symmetric with respect to their central wavelength, while Stokes V is antisymmetric.

from Orozco Suarez et al, 2006
Net Circular Polarization

Net circular polarization: integral of Stokes $V$ over the wavelength span ($W$) of the spectral line

\[
NCP = \int_W V_{\text{obs}}(\lambda)d\lambda
\]

It is zero in the absence of velocity gradients!

\[\text{blue} - \text{red} = 0\]
Net Circular Polarization

A fraction of the pixel is non-magnetic: no contribution to Stokes V

A fraction of the pixel is magnetized, with different field strengths, spatial extensions, plasma velocities… If we assume no velocity gradients, each Stokes V is still antisymmetric, but the sum might not.

However, the total NCP = 0!
Line Bisectors and Convective Blueshift

Line profiles

Bisectors

(from Asplund et al 2000)
Longitudinal magnetograms

Based on the weak-field approximation:

\[
\frac{\partial I(\lambda)}{\partial \lambda} = \frac{\partial I(\lambda)}{\partial \lambda}
\]

\[
V(\lambda) = -C g_{\text{eff}} B \cos \theta \frac{\partial I(\lambda)}{\partial \lambda}
\]

Magnetograph signal as a function of B\text{LOS} strength and inclination angle for a spectral line with geff = 2.5.

From Landi Degl’Innocenti & Landolfi “Polarization in spectral lines”
Longitudinal magnetograms

- Only valid for weak fields (no effect on Stokes I)
- Assumes magnetic field has no gradients along LOS
- Only gives the component of the magnetic field along the LOS

Yet line-of-sight magnetograms are by far the most popular measurement of magnetic fields in Solar Physics.

Magnetograms give us the net longitudinal component of the magnetic field, $B_z$, averaged over each resolution element.
Quiet Sun magnetic fields

(IMaX - SUNRISE, courtesy of V. Martínez Pillet)
Unsigned magnetic flux density

The unsigned magnetic flux density:

\[ \phi_B = \frac{\int_A |B_z| dA}{\int_A dA} \]

\( B_z = \text{component of the magnetic field vector projected along the LOS.} \)

In general:
\( B_z \neq B_{\text{radial}} \)
Effects of spatial resolution on Quiet Sun magnetic fields

How do measurements of unsigned magnetic flux change with increasing spatial resolution?

(from Sánchez Almeida & Martínez González 2011)
Shortcomings of the Zeeman Effect

The Zeeman Effect polarization signals cancel out when tangled magnetic fields are present at sub-pixel spatial scales.

\[ V_{\text{mac}} \uparrow \uparrow \]

Very weak magnetic fields do not produce measurable magnetic signals (when the Zeeman splitting is much smaller than the width of the spectral line).

\[ B = 100 \text{ G}, \theta = 0 \text{ deg} \]
Shortcomings of the Zeeman Effect

There is a 180 degree azimuth ambiguity, in the plane perpendicular to the LOS.

\[
\eta_Q = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} [\phi_{+1} + \phi_{-1}] \sin^2 \theta \cos 2\phi \right\}
\]

\[
\eta_U = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} [\phi_{+1} + \phi_{-1}] \sin^2 \theta \sin 2\phi \right\}
\]

When converting from LOS into a solar reference frame:

Anywhere but at disk center:
The azimuth ambiguity in the LOS reference frame translates into an inclination (with respect to the solar radial direction) and an azimuth ambiguity in the local solar frame.

Disambiguation methods are based on continuity and minimizing currents.
Mechanisms that produce polarization in spectral lines

- Anisotropy in the excitation mechanism of the atom
- Impact polarization
- Optical pumping
- External field breaking the axis of symmetry
- Electric field
- Magnetic field
Atomic polarization

Anisotropic illumination induces population imbalances between the magnetic energy sub-levels of an atom.

\[ \eta_Q = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} [\phi_{+1} + \phi_{-1}] \sin^2 \theta \cos 2\phi \right\} \]

If the number of \( \pi \)-transitions does not “compensate” the number of \( \sigma \)-transitions per unit volume and time, we will get a linear polarization signal.

Anisotropic illumination induces population imbalances between the magnetic energy sub-levels of an atom.
Atomic polarization

Fig. 4. Anisotropic illumination of the outer layers of a stellar atmosphere, indicating that the outgoing continuum radiation (which shows limb darkening) is predominantly vertical while the incoming radiation (which shows limb brightening) is predominantly horizontal. The figure also illustrates the type of anisotropic illumination experienced by atoms situated at a given height above the visible 'surface' of the star, including the polarization analysis of the scattered beam at $90^\circ$. The 'degree of anisotropy' of the incident field is quantified by $A = J_2 \theta^2 / J_0$, where $J_0$ is the familiar mean intensity and $J_2 \approx \iint d\Omega \, 4 \pi 1^2 \sqrt{2} (3 \mu^2 - 1) I_\nu$, $\mu = \cos \theta$, with $\theta$ the polar angle with respect to the Z-axis. The possible values of the 'anisotropy factor' $W = \sqrt{2} A$ vary between $W = -1/2$, for the limiting case of illumination by a purely horizontal radiation field without any azimuthal dependence (case b of Fig. 3), and $W = 1$ for purely vertical illumination (case of Fig. 3). It is important to point out that the larger the 'anisotropy factor' the larger the fractional atomic polarization that can be induced, and the larger the amplitude of the emergent linear polarization. We choose the positive direction for the Stokes-$Q$ parameter along the X-axis, i.e., along the perpendicular direction to the stellar vector through the observed point. The inset shows the wavelength dependence of the anisotropy factor corresponding to the center to limb variation of the observed solar continuum radiation. Note that in this case the maximum anisotropy factor occurs around 2800 $\AA$, i.e., very near the central wavelength of the $k$ line of Mg II, whose polarization may contain valuable information on the magnetic fields of the transition region from the chromosphere to the 10 $\times 10^6$ K solar coronal plasma.

In order to clarify that, depending on the scattering geometry, the Hanle effect can either destroy or create linear polarization in spectral lines, let us consider scattering processes in a $J_l = 0 \rightarrow J_u = 1$ transition for the following two geometries: $90^\circ$ scattering and forward scattering.

5.1. $90^\circ$ scattering

Figure 5 illustrates the $90^\circ$ scattering case, in the absence and in the presence of a magnetic field. For this geometry the largest polarization amplitude occurs for the zero field reference case, with the direction of the linear polarization as indicated in the top panel (i.e., perpendicular to the scattering plane). Chromosphere:

- non-frequent collisions
- anisotropic illumination: center-to-limb variation

(from Trujillo Bueno 2006)
Atomic polarization

- Unpolarized radiation
- Linearly polarized radiation
- Anisotropic radiation

Scattering scenario

Atoms
Atomic polarization

\[ \eta_Q = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} \left[ \phi_{+1} + \phi_{-1} \right] \sin^2 \theta \cos 2\phi \right\} \]

- \( M_U = 1 \)
- \( M_U = 0 \)
- \( M_U = -1 \)

\[ M_L = 0 \]

Anisotropic radiation

\[ B = 0 \]

Stokes Q

One lobe, not three!!

Stokes U
Atomic polarization and the Hanle Effect

\[ \begin{align*}
M_U &= 1 \\
M_U &= 0 \\
M_U &= -1 \\
M_L &= 0
\end{align*} \]

\[ B \neq 0 \text{ and inclined with respect to the axis of symmetry of the radiation} \]

Stokes Q

Stokes U

anisotropic radiation
The two lower panels illustrate what happens when the scattering processes take place in the presence of a magnetic field pointing (a) towards the observer (left panel) or (b) away from him/her (right panel). In both situations the polarization amplitude is reduced with respect to the previously discussed unmagnetized case. Moreover, the direction of the linear polarization is rotated with respect to the zero field case. Typically, this rotation is counterclockwise for case (a), but clockwise for case (b). Therefore, when opposite magnetic polarities coexist within the spatio-temporal resolution element of the observation the direction of the linear polarization is like in the top panel of Fig. 5, simply because the rotation effect cancels out. However, the polarization amplitude is indeed reduced with respect to the zero field reference case, which provides an "observable" that can be used for obtaining empirical information on hidden, mixed polarity fields at subresolution scales in the solar atmosphere (Stenflo, 1982; Trujillo Bueno et al., 2004).

Fig. 5. The 90° scattering case in the absence (top panel) and in the presence (bottom panels) of a deterministic magnetic field.

5.2. Forward scattering

Figure 6 illustrates the case of forward scattering, in the absence and in the presence of a magnetic field. In this geometry we have zero polarization for the unmagnetized reference case, while the largest linear polarization (oriented along the direction of the external magnetic field) is found for "sufficiently strong" fields (i.e., for a magnetic strength such that the ensuing Zeeman splitting is much larger than the level's natural width).

In other words, in the presence of an inclined magnetic field that breaks the symmetry of the scattering polarization problem, forward scattering processes can produce measurable linear polarization signals in spectral lines (Trujillo Bueno, 2001).

This occurs when the Landé factor, \( g_L \), of the transition 'superlevel' is positive, while the opposite behavior takes place if \( g_L < 0 \).

Atomic polarization and the Hanle Effect

The Hanle Effect can be sensitive to very weak magnetic fields, depending on the spectral line (from milligauss to hectogauss).

\[ B_H = 1.137 \times 10^{-7} / (t_{\text{life}} g_J) \]

It is also sensitive to tangled magnetic fields at sub-pixel scales, so it doesn’t cancel out as the Zeeman polarization signals would.

Hanle techniques suffer from a saturation effect, so there is an upper limit for the magnetic field strength sensitivity.

It has to be treated within the frame of the quantum theory of polarization.
The Second Solar Spectrum

(Gandorfer, 2001)
Next time

- Simple solutions to the RTE
  - Milne-Eddington approximation
  - LTE approximation

- The general inversion problem

- Spectral line inversion algorithms
  - Levenberg-Marquardt techniques
  - Principal Component Analysis