Hale Collage

Spectropolarimetric Diagnostic Techniques

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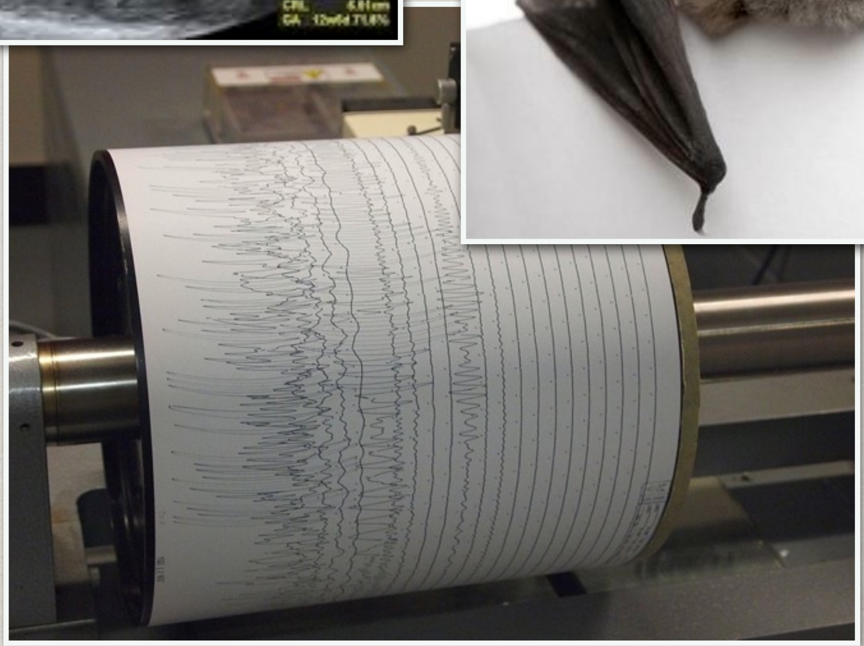
Today

- Spectropolarimetry and the remote sensing magnetic fields
- Polarization of light
- Polarization in spectral lines
- Zeeman Effect
  - Line profiles (absorption and dispersion)
  - Effective Lande factor
- Radiative transfer equation for polarized light
  - The propagation matrix
  - Physical dependencies
- Effects of the magnetic field on photospheric lines

Remote Sensing

"Remote sensing is the acquisition of information about an object without being in physical contact with it."



First “measurement” of magnetic fields on the Sun

ON THE PROBABLE EXISTENCE OF A MAGNETIC FIELD IN SUN-SPOTS[1]

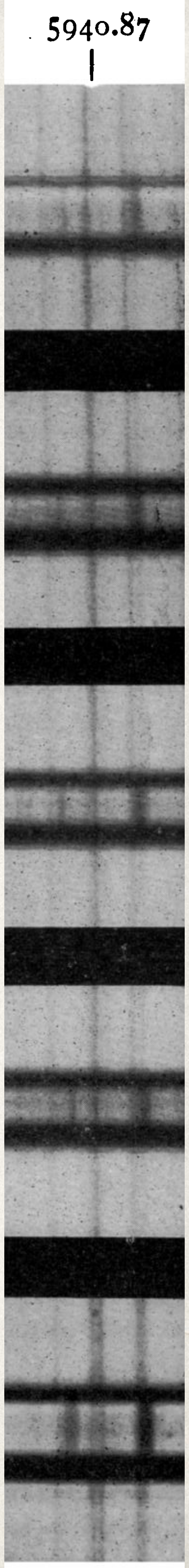
By GEORGE E. HALE

Soon after the discovery of the vortices associated with sun-spots, it occurred to me that if a preponderance of positive or negative ions or corpuscles could be supposed to exist in the rapidly revolving gases, a magnetic field, analogous to that observed by Rowland in the laboratory, should be the result. An equal number of positive and negative

Thanks to Zeeman’s discovery of the effect of magnetism on radiation it appeared that the detection of such a magnetic field should offer no great difficulty, provided it were sufficiently intense. When

CONCLUSION

Although the combined evidence presented in this paper seems to indicate the probable existence of a magnetic field in sun-spots, the weak points of the argument should be clearly recognized. Among these are the following:



Spectropolarimetry

***Spectropolarimetry* is the measurement of the distribution of energy and polarization of light as a function of frequency**

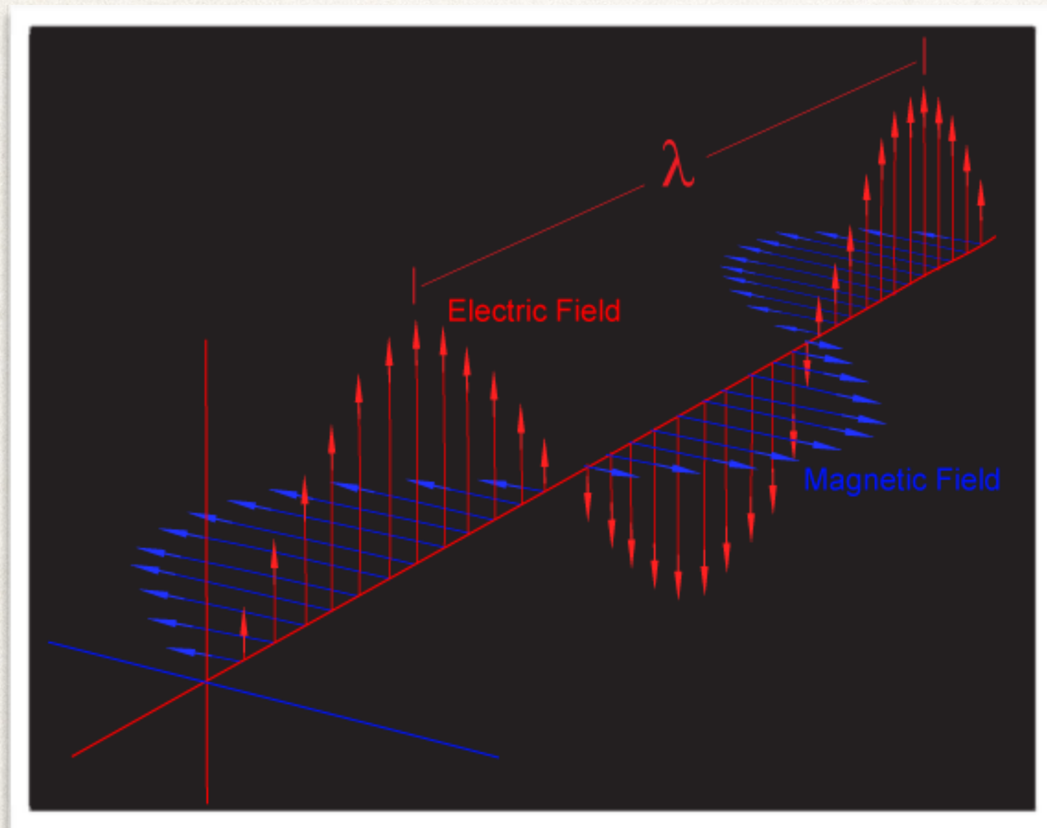
It is an incredibly powerful tool for **remote sensing** of **magnetic fields** in the Sun's atmosphere!

Polarization of light

$$E_x(t) = \varepsilon_x(t)\, e^{i\delta_x(t)}\, e^{-2\pi i \nu_0 t}$$

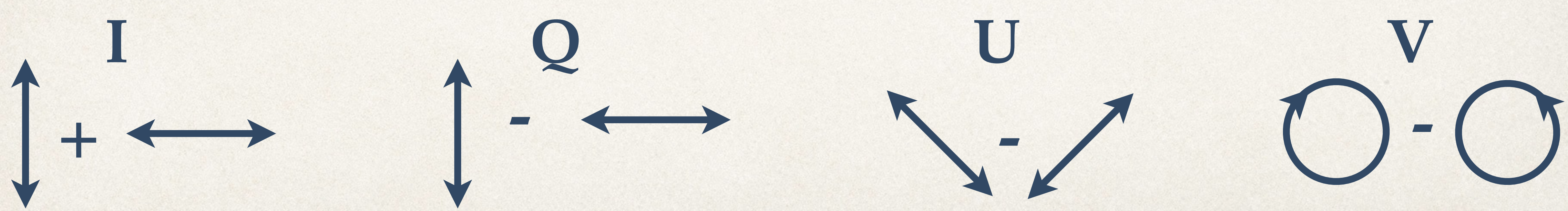
$$E_y(t) = \varepsilon_y(t)\, e^{i\delta_y(t)}\, e^{-2\pi i \nu_0 t}$$

$$E_z(t) = 0$$



$$\boxed{\begin{aligned} I &= \kappa\,(\langle \varepsilon_x^2\rangle + \langle \varepsilon_y^2\rangle) \\ Q &= \kappa\,(\langle \varepsilon_x^2\rangle - \langle \varepsilon_y^2\rangle) \\ U &= 2\kappa\,\langle \varepsilon_x\, \varepsilon_y \cos\delta(t)\rangle \\ V &= 2\kappa\,\langle \varepsilon_x\, \varepsilon_y \sin\delta(t)\rangle \end{aligned}}$$

where $\delta(t) = \delta_x(t) - \delta_y(t)$



Mechanisms that produce polarization in spectral lines

- Anisotropy in the excitation mechanism of the atom
    - Impact polarization
    - Optical pumping
- External field breaking the axis of symmetry
    - Electric field
    - Magnetic field

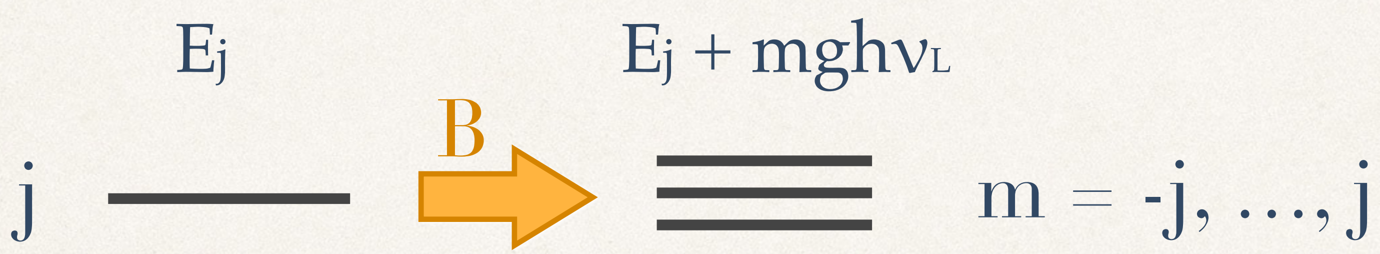
Zeeman Effect

Each energy level of an atom is characterized by a series of quantum numbers

j = total angular momentum
$m = -j, -j+1, \ldots, 0, \ldots, j$ (magnetic quantum number)

In the absence of magnetic field, $(2j+1)$ degenerate sublevels with energy E_j

In the presence of a magnetic field, an energy level with angular momentum j splits into $(2j+1)$ sublevels with energies $E_j + mgh\nu_L$.



where g is the Lande factor of the level, which in LS coupling:

$$g_{LS} = 3/2 + \{s(s+1) - l(l+1)\} \ / \ \{j\,(j+1)\}$$

and the Larmor frequency is given by:

$$\nu_L = e_0 B \ / \ (4\pi m_e c)$$

B magnetic field
e_0 electron charge
m_e mass of electron
c speed of light

Allowed transitions

Spectral lines are a consequence of electronic transitions between two levels (l and u) with energies $E_l < E_u$, which result in the absorption or emission of a photon.

Conservation of angular momentum requires that:

$$\Delta j \equiv j_u - j_l = 0, \pm 1$$
$$\text{with } j_u = j_l = 0 \text{ forbidden}$$

In the presence of a magnetic field, each j-level will split into (2j+1) components, producing several spectral lines, shifted in frequency (wavelength) by:

$$\Delta\nu_B = (m_u\, g_u - m_l\, g_l)\, \nu_L$$
$$\Delta\lambda_B = (m_l\, g_l - m_u\, g_u)\, \lambda_B$$

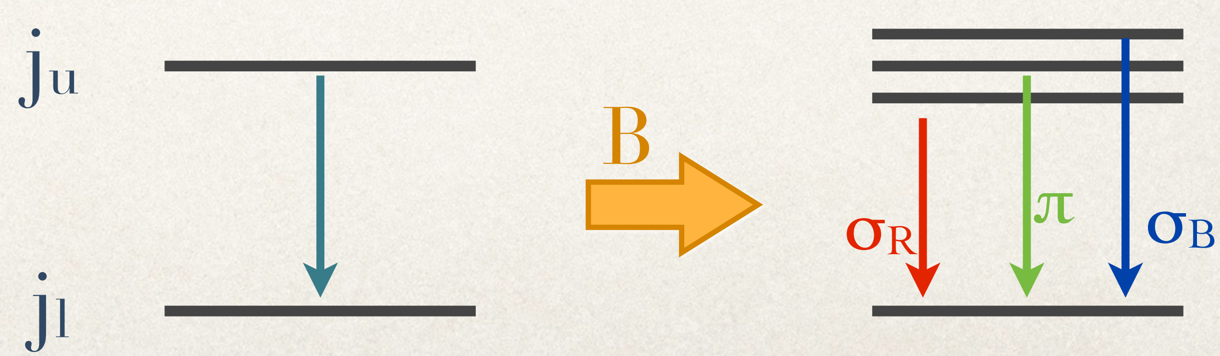
where: $\boxed{\lambda_B = 4.67 \times 10^{-13}\, \lambda_0^2\, B}$

B (gauss), λ_0 and λ_B (angstroms)

And the selection rules for electric dipole transitions require that:

$$\Delta m \equiv m_u - m_l = 0, \pm 1$$

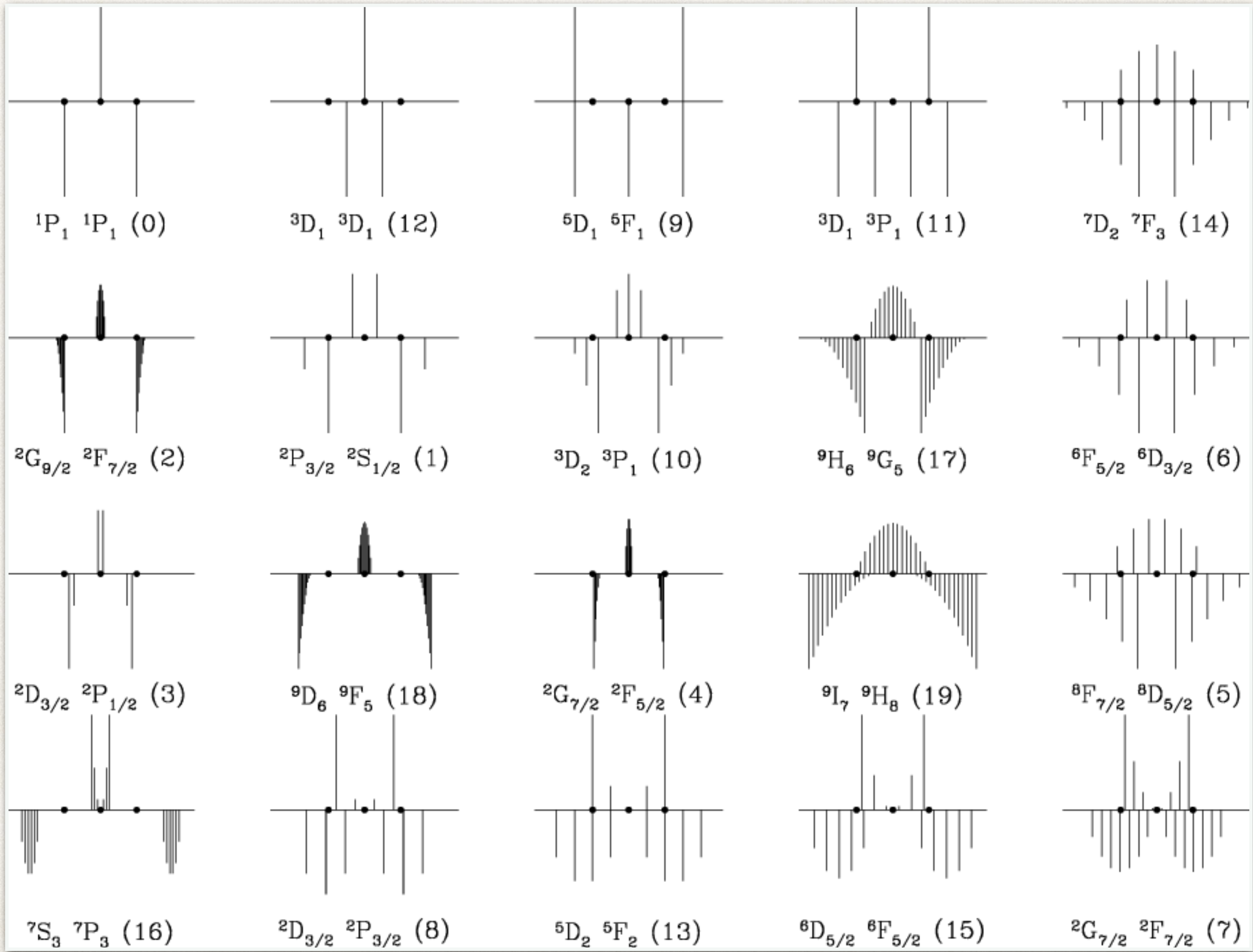
$\sigma_R \equiv \Delta m = +1$ $\quad \pi \equiv \Delta m = 0$ $\quad \sigma_B \equiv \Delta m = -1$



Zeeman Patterns

π components $\Delta m = 0$ (represented upwards)

σ components $\Delta m = \pm 1$ (represented downwards)



A&A 398, 411-421 (2003)

Absorption Profiles

The *absorption profiles* would be Lorentzian functions if the atoms were all at rest:

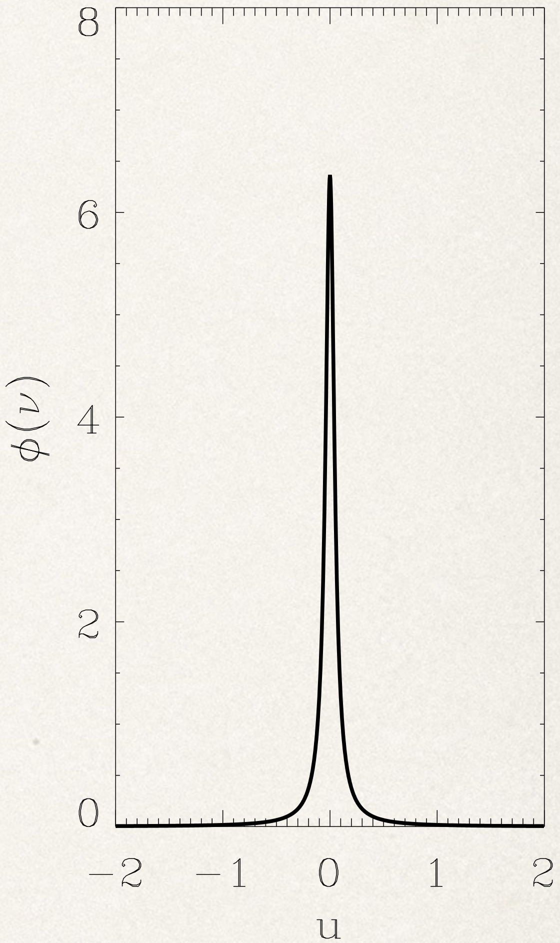
damping

$$\phi(u_0, a) = \frac{1}{\pi} \frac{a}{u_0^2 + a^2}$$

But the atoms have a distribution of velocities (let's assume Maxwellian). They will see the photons shifted in wavelength:

$$N(\lambda) = \frac{N}{\pi \Delta\lambda_D} e^{-(\Delta\lambda^2/\Delta\lambda_D^2)}$$

absorption profile



$$u = \frac{\lambda - \lambda_0}{\Delta\lambda_D}$$

Absorption Profiles

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$$N(\lambda) = \frac{N}{\pi \Delta\lambda_D} e^{-(\Delta\lambda^2/\Delta\lambda_D^2)}$$

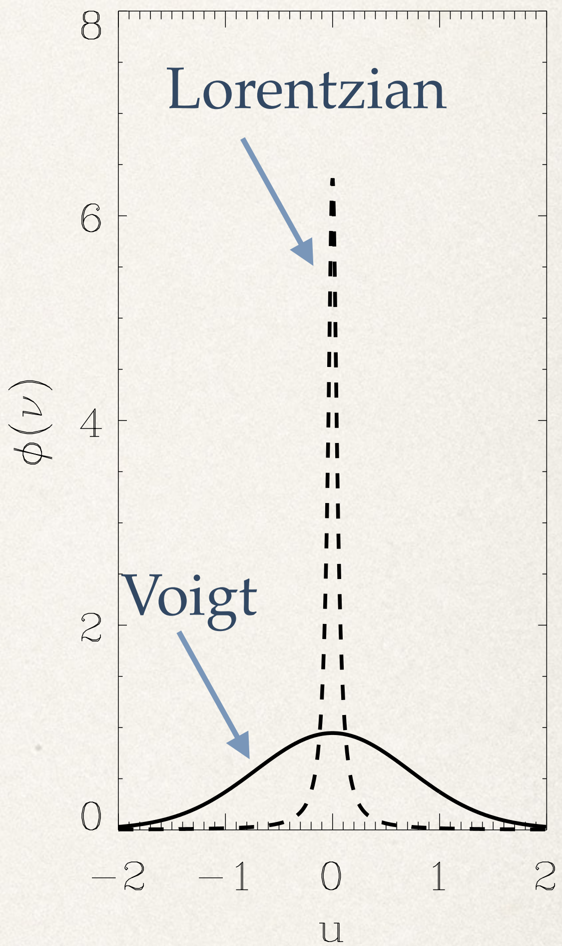
So the absorption profile becomes a Voigt function:

$$\phi(u_0, a) = \frac{1}{\sqrt{\pi}} H(u_0, a) = \frac{a}{\pi\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2} \frac{1}{(u-y)^2 + a^2} dy$$

Where $\Delta\lambda_D$ is the Doppler width in units of wavelength.

$$v_D = \sqrt{\frac{2kT}{m} + \xi_{\rm mic}^2} = \frac{\Delta\lambda_D c}{\lambda_0}$$

absorption profile



$$u = \frac{\lambda - \lambda_0}{\Delta\lambda_D}$$

Absorption Profiles

Unshifted absorption profile:

$$\phi(u_0, a) = \frac{1}{\sqrt{\pi}} H(u_0, a)$$

In the presence of magnetic field, each Zeeman component will be shifted by a quantity $\Delta\lambda = (m_l g_l - m_u g_u)\, \lambda_B$:

$$\phi = \frac{1}{\sqrt{\pi}} H(u_0 + u_B, a)$$

If there is a macroscopic LOS velocity there will be a Doppler shift of the entire Zeeman pattern

$$\phi = \frac{1}{\sqrt{\pi}} H(u_0 + u_B - u_{\mathrm{LOS}}, a)$$

Effective Lande factor

For some spectral lines we can use an approximation that substitutes the multiple-component Zeeman pattern for a simple Zeeman triplet.

By defining an effective Lande factor:

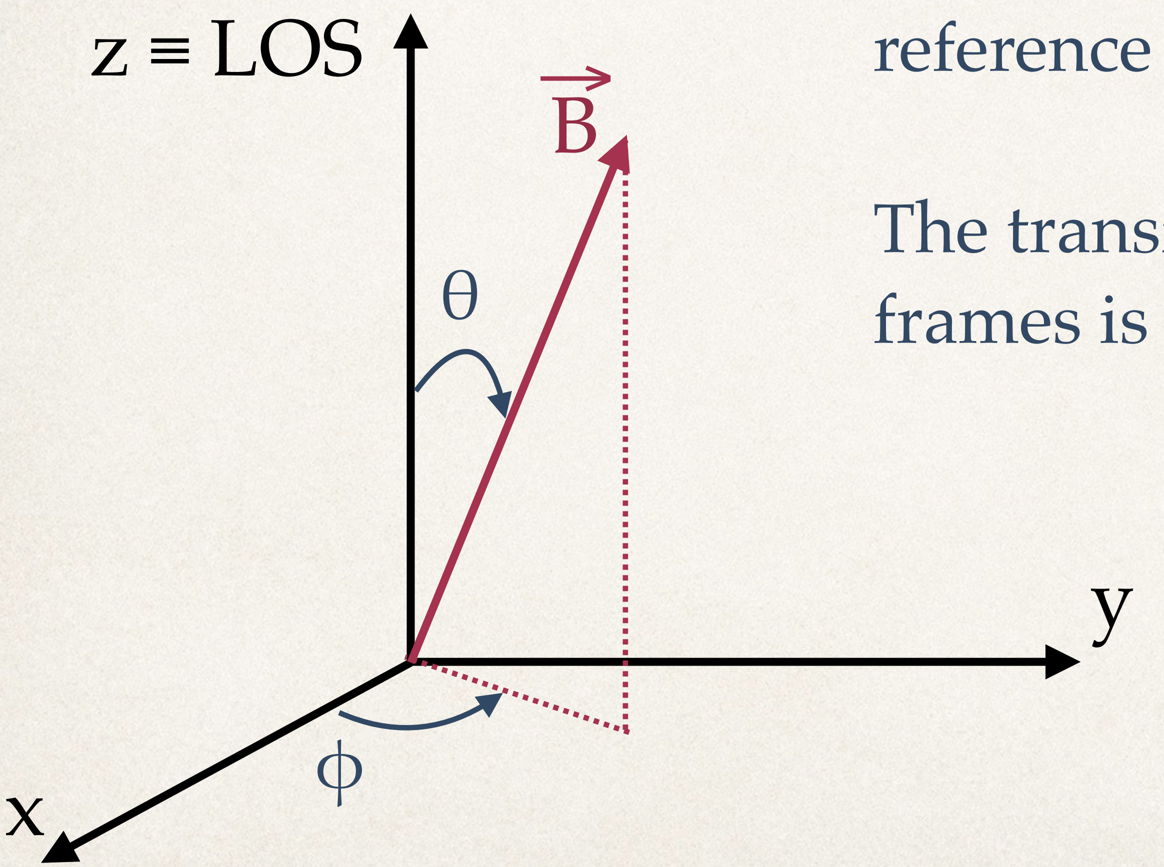
$$\mathrm{g_{eff}} = \frac{1}{2}(g_u - g_l) + \frac{1}{4}(g_u - g_l)[j_u(j_u + 1) - j_l(j_l + 1)]$$

The Voigt and Faraday profiles are approximated to those of an effective triplet, in which the Zeeman splitting is $\mathrm{g_{eff}}$ times that of a normal triplet:

Absorption (Voigt profile) $\quad \phi_\alpha = \frac{1}{\sqrt{\pi}} H(u_0 + \alpha \mathrm{g_{eff}} u_B - u_{LOS}, a) \quad$ with $\alpha = \Delta m = 0, \pm 1$

$\mathrm{g_{eff}}$ gives as an idea of the magnetic sensitivity of the transition!
If $\mathrm{g_{eff}} = 0$, the line won't split in the presence of a magnetic field.

Geometry



Because the magnetic field imposes an anisotropy in the medium through which the light propagates, it makes sense to pose the radiative transfer problem in the reference frame of B.

But, from the observer's point of view, because the EM radiation is transverse in nature, it makes sense to describe it (Stokes vector) in the LOS reference frame.

The transformation between the two reference frames is given by 2 angles, θ and ϕ.

Radiative transfer equation

Remember the unpolarized case!

$$\frac{dI}{dz} = -kI + j \longrightarrow \frac{dI}{d\tau} = -I + S$$

Polarized case:

$$\boxed{\frac{d\vec{I}}{d\tau_c} = \mathbf{K}(\vec{I} - \vec{S})}$$

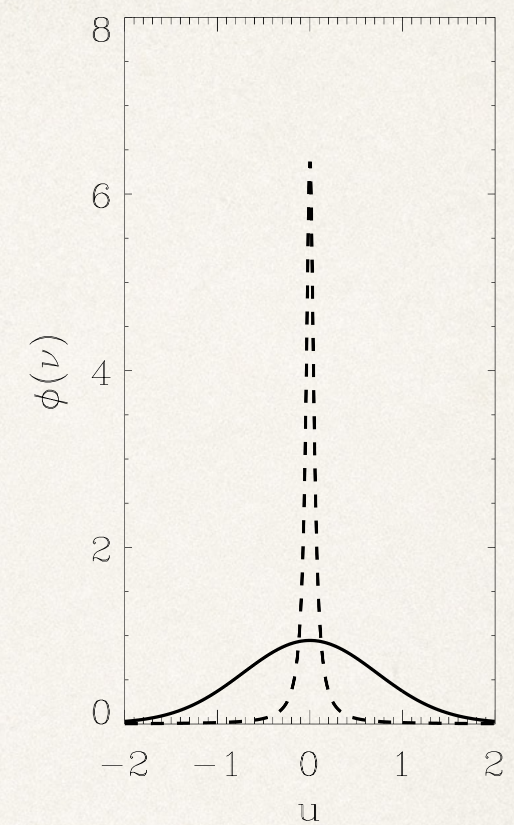
$$\mathbf{K} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$

$$\mathbf{K} = \underbrace{\begin{pmatrix} \eta_I & 0 & 0 & 0 \\ 0 & \eta_I & 0 & 0 \\ 0 & 0 & \eta_I & 0 \\ 0 & 0 & 0 & \eta_I \end{pmatrix}}_{\text{absorption}} + \underbrace{\begin{pmatrix} 0 & \eta_Q & \eta_U & \eta_V \\ \eta_Q & 0 & 0 & 0 \\ \eta_U & 0 & 0 & 0 \\ \eta_V & 0 & 0 & 0 \end{pmatrix}}_{\text{dichroism}} + \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_V & -\rho_U \\ 0 & -\rho_V & 0 & \rho_Q \\ 0 & \rho_U & -\rho_Q & 0 \end{pmatrix}}_{\text{dispersion}}$$

Absorption Profiles

The absorption profiles are Voigt functions:

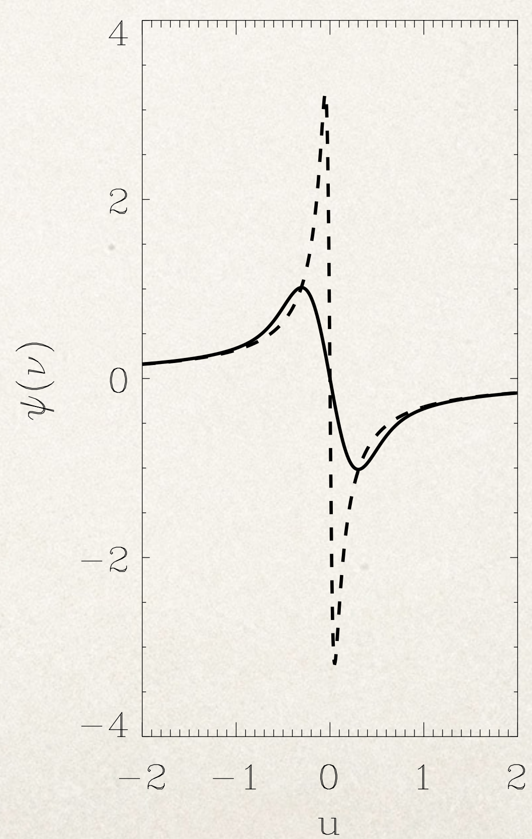
$$\phi(u_0, a) = \frac{1}{\sqrt{\pi}} H(u_0, a) = \frac{a}{\pi\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2} \frac{1}{(u-y)^2 + a^2} dy$$



Dispersion Profiles

The dispersion profiles are Faraday-Voigt functions:

$$\psi(u_0, a) = \frac{1}{\sqrt{\pi}} F(u_0, a) = \frac{1}{\pi\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-y^2} \frac{u-y}{(u-y)^2 + a^2} dy$$



Radiative transfer equation

$$\boxed{\frac{d\vec{I}}{d\tau_c} = \mathbf{K}(\vec{I} - \vec{S})}$$

$$\mathbf{K} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$

The elements of the propagation matrix:

$$\eta_I = 1 + \frac{\eta_0}{2}\{\phi_0 sin^2\theta + \frac{1}{2}[\phi_{+1} + \phi_{-1}](1 + cos^2\theta)\}$$

$$\eta_Q = \frac{\eta_0}{2}\{\phi_0 - \frac{1}{2}[\phi_{+1} + \phi_{-1}]sin^2\theta cos2\phi\}$$

$$\eta_U = \frac{\eta_0}{2}\{\phi_0 - \frac{1}{2}[\phi_{+1} + \phi_{-1}]sin^2\theta sin2\phi\}$$

$$\eta_V = \frac{\eta_0}{2}[\phi_{-1} - \phi_{+1}]cos\theta$$

$$\rho_Q = \frac{\eta_0}{2}\{\psi_0 - \frac{1}{2}[\psi_{+1} + \psi_{-1}]sin^2\theta cos2\phi\}$$

$$\rho_U = \frac{\eta_0}{2}\{\psi_0 - \frac{1}{2}[\psi_{+1} + \psi_{-1}]sin^2\theta sin2\phi\}$$

$$\rho_V = \frac{\eta_0}{2}[\psi_{-1} - \psi_{+1}]cos\theta$$

$$\phi_\alpha = \frac{1}{\sqrt{\pi}}\sum_i S_{\alpha,i}H(u_0 + u_{B,\alpha,i} - u_{LOS}, a)$$

$$\psi_\alpha = \frac{1}{\sqrt{\pi}}\sum_i S_{\alpha,i}F(u_0 + u_{B,\alpha,i} - u_{LOS}, a)$$

$\alpha = \Delta m = 0, \pm 1$

i = 1 … Zeeman components

$S_{\alpha,i} \equiv$ weights

$u_0 \equiv$ rest wavelength

$u_{B,\alpha,i} \equiv$ magnetic splitting

$u_{LOS} \equiv$ LOS bulk velocity

(in units of the Doppler broadening, $\Delta\lambda_D$)

Dependencies

*The propagation matrix, **K**, and the source function, $\vec{S}$, carry information about the thermodynamic, dynamic, magnetic, atomic and geometric properties of the medium.*

The optical depth scale, τ_c, depends on the continuum absorption coefficient, χ_{cont}, so also depends on the properties of the medium.

Thermodynamics plays a role in χ_{cont}, η_0, $B_\nu(T)$, $\Delta\lambda_D$ and the damping parameter, a.

Doppler shifts, through u_{LOS}, are due to macroscopic motions of the plasma.

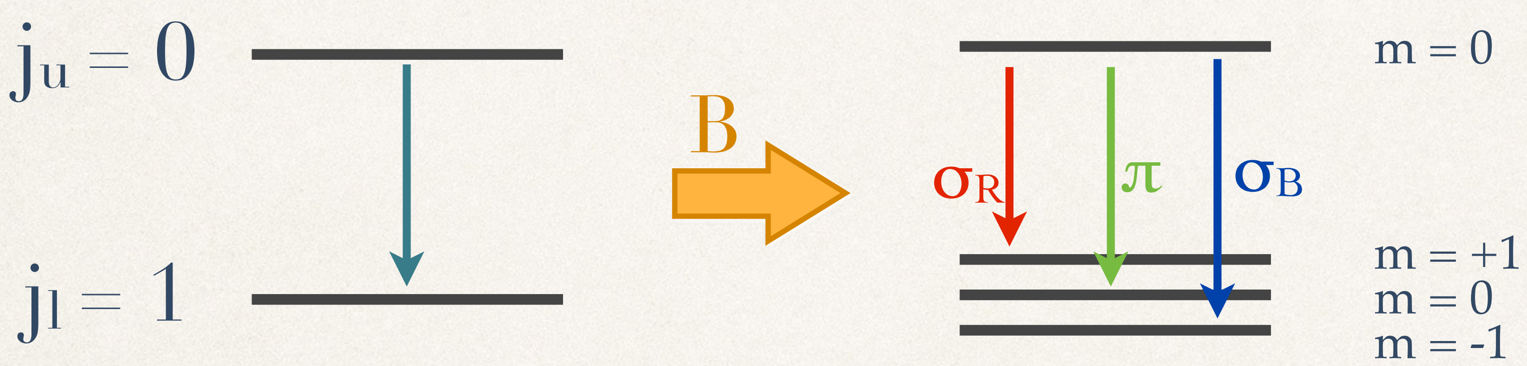
Atomic parameters (oscillator strength and quantum numbers), enter: η_0, ϕ_α, ψ_α.

The Zeeman splitting depends on the field strength.

The elements of the propagation matrix depend on the relative geometry of the LOS with the magnetic field vector (θ and ϕ).

Fe I @ 6302.5 Å line

	Spectral term	Angular momentum	Lande factor
Upper Level	$^5D_0$	J=0	0
Lower Level	$^5P_1$	J=1	2.487



$$\Delta\lambda_B = (m_l\, g_l - m_u\, g_u)\, \lambda_B$$

Simple triplet!

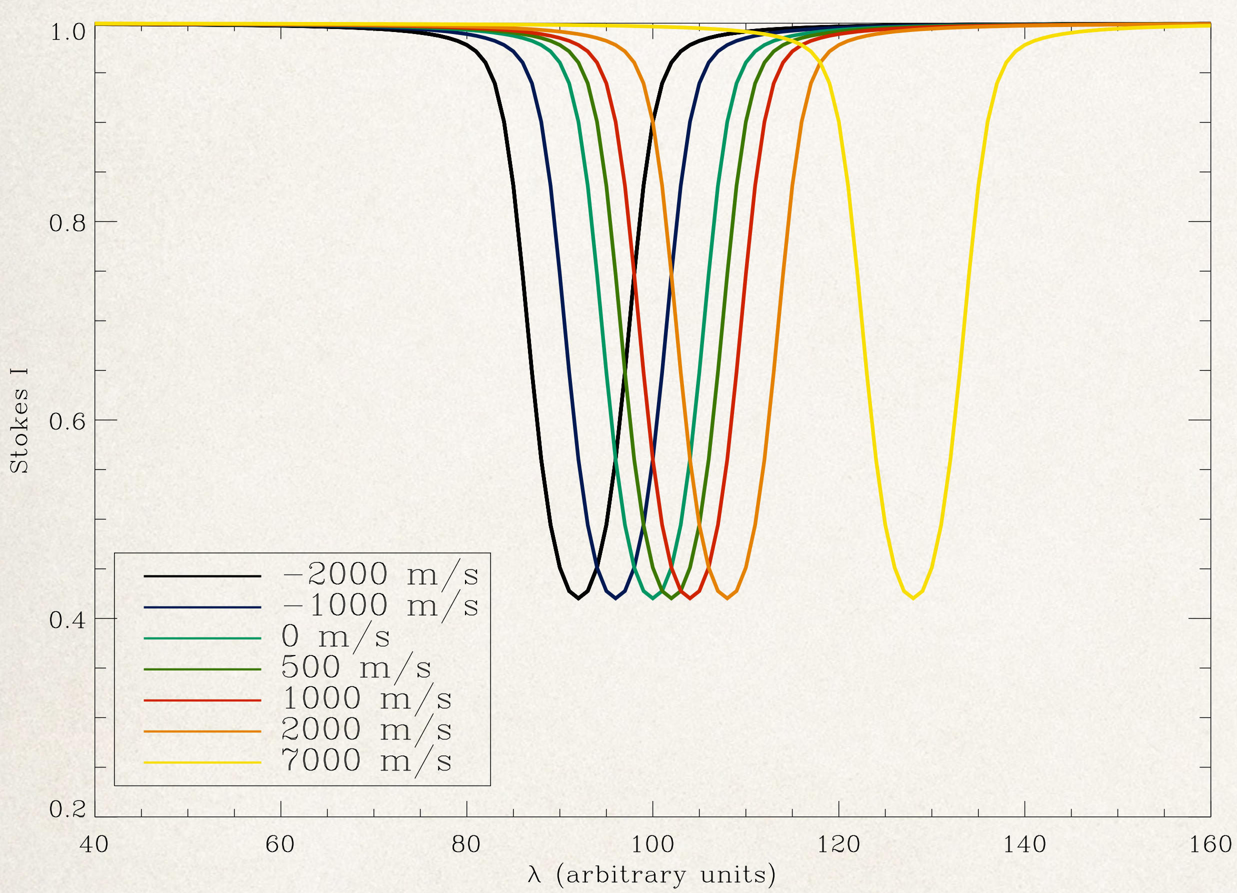
Effective Lande factor: 2.487

Intensity profiles

<u>Doppler shift</u>

B = 0 G

v = -2, -1, 0, 0.5, 1, 2, 7 km/s

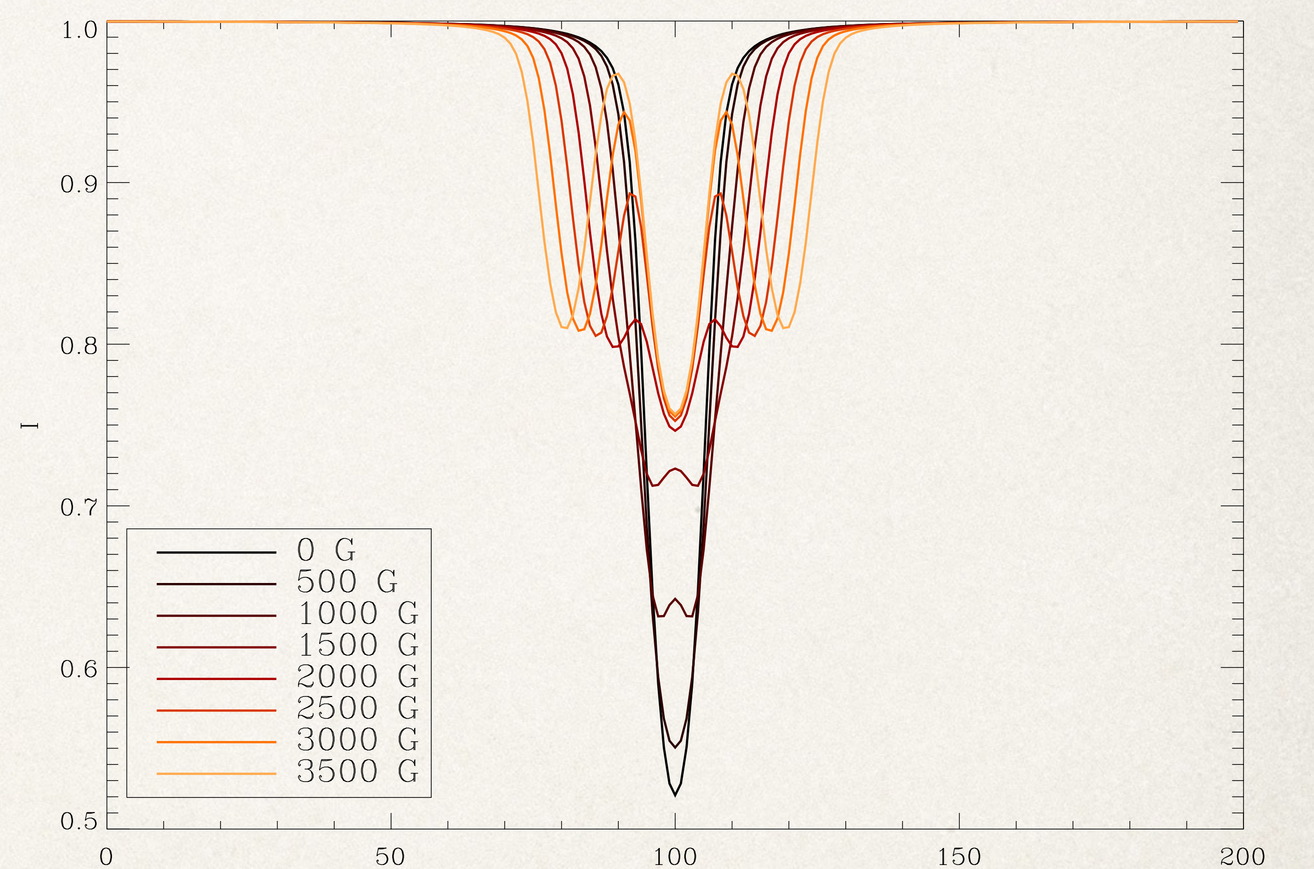


$\boxed{\Delta\lambda/\lambda = v/c}$

<u>Magnetic field strength</u>

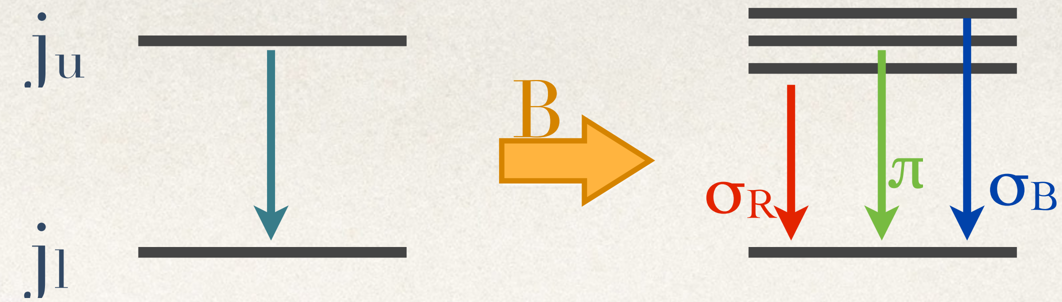
B = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5 kG

v = 0 km/s



$\boxed{\lambda_B = 4.67 \times 10^{-13}\, \lambda_0^2\, B}$

Polarization profiles:

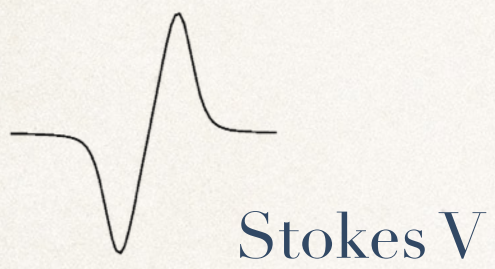


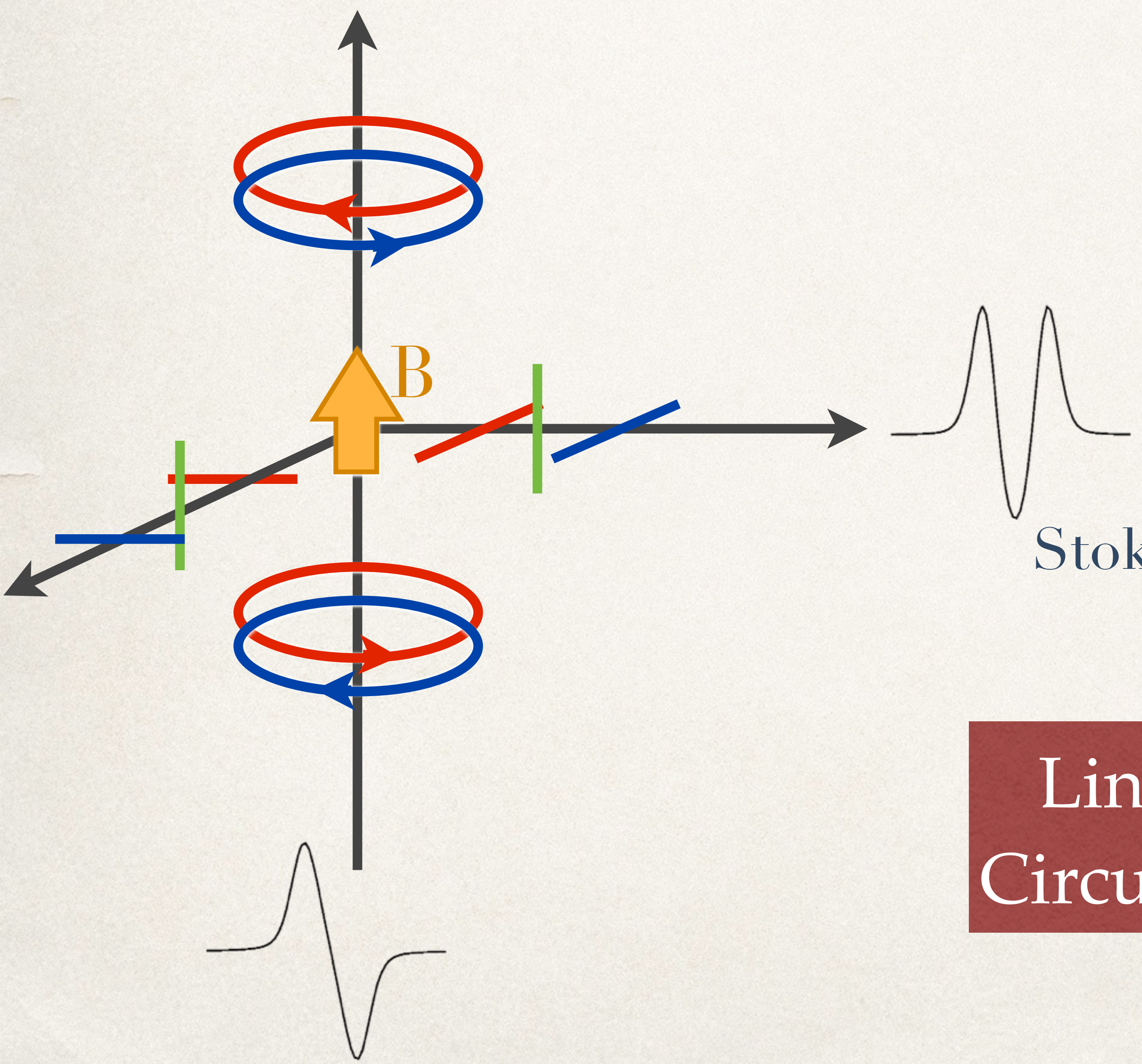
$$\eta_V = \frac{\eta_0}{2}[\phi_{-1} - \phi_{+1}]cos\theta$$

LOS || magnetic field:

The π-component is seen head on.

Only σ-components: Stokes V





LOS $\perp$ to magnetic field:

Projection of σ-components = line.

Only linearly polarized components:

Stokes Q or U

$$\eta_Q = \frac{\eta_0}{2}\{\phi_0 - \frac{1}{2}[\phi_{+1} + \phi_{-1}]sin^2\theta cos2\phi\}$$

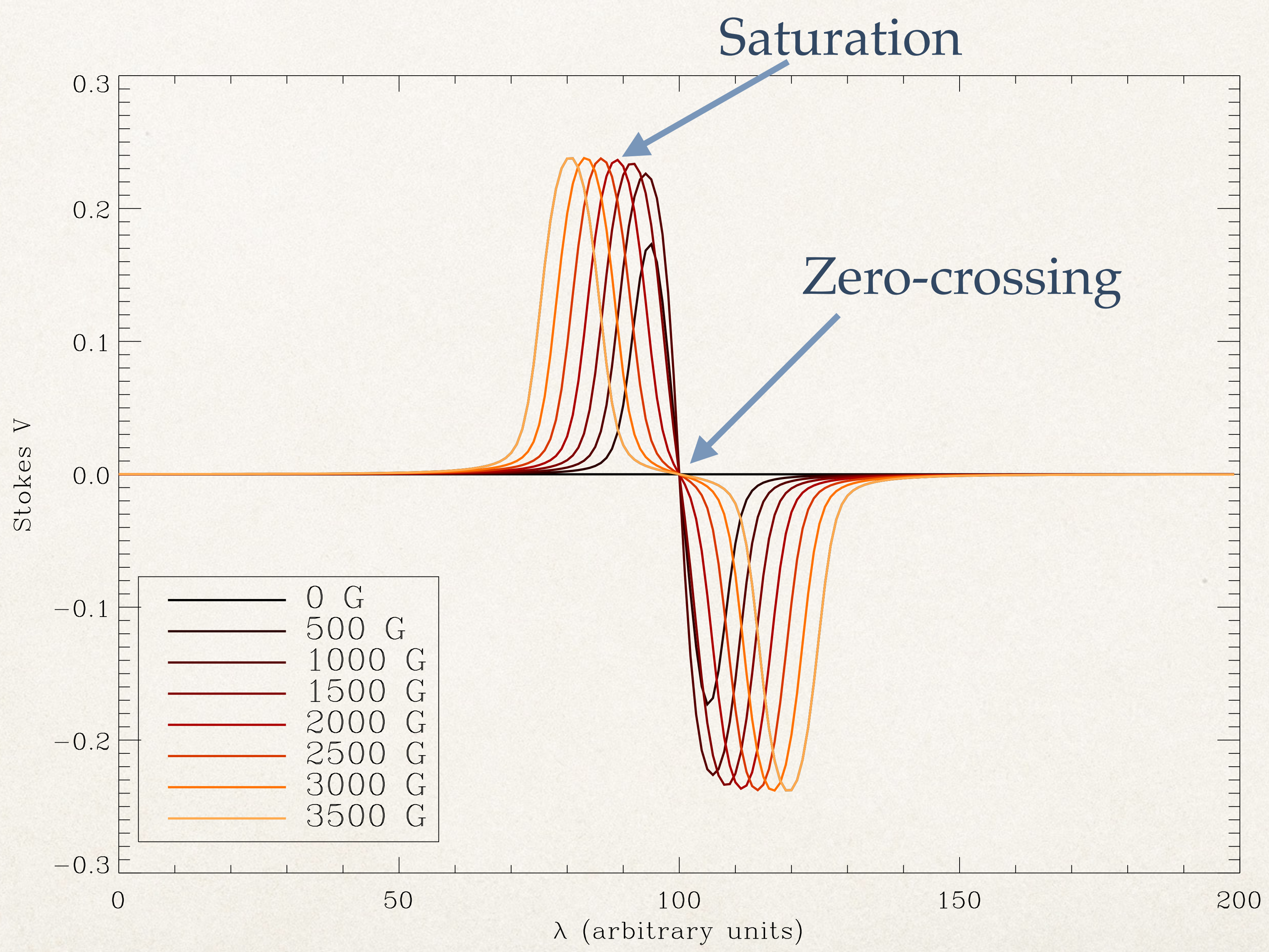
Linear polarization - transverse field

Circular polarization - longitudinal field

Magnetic field strength

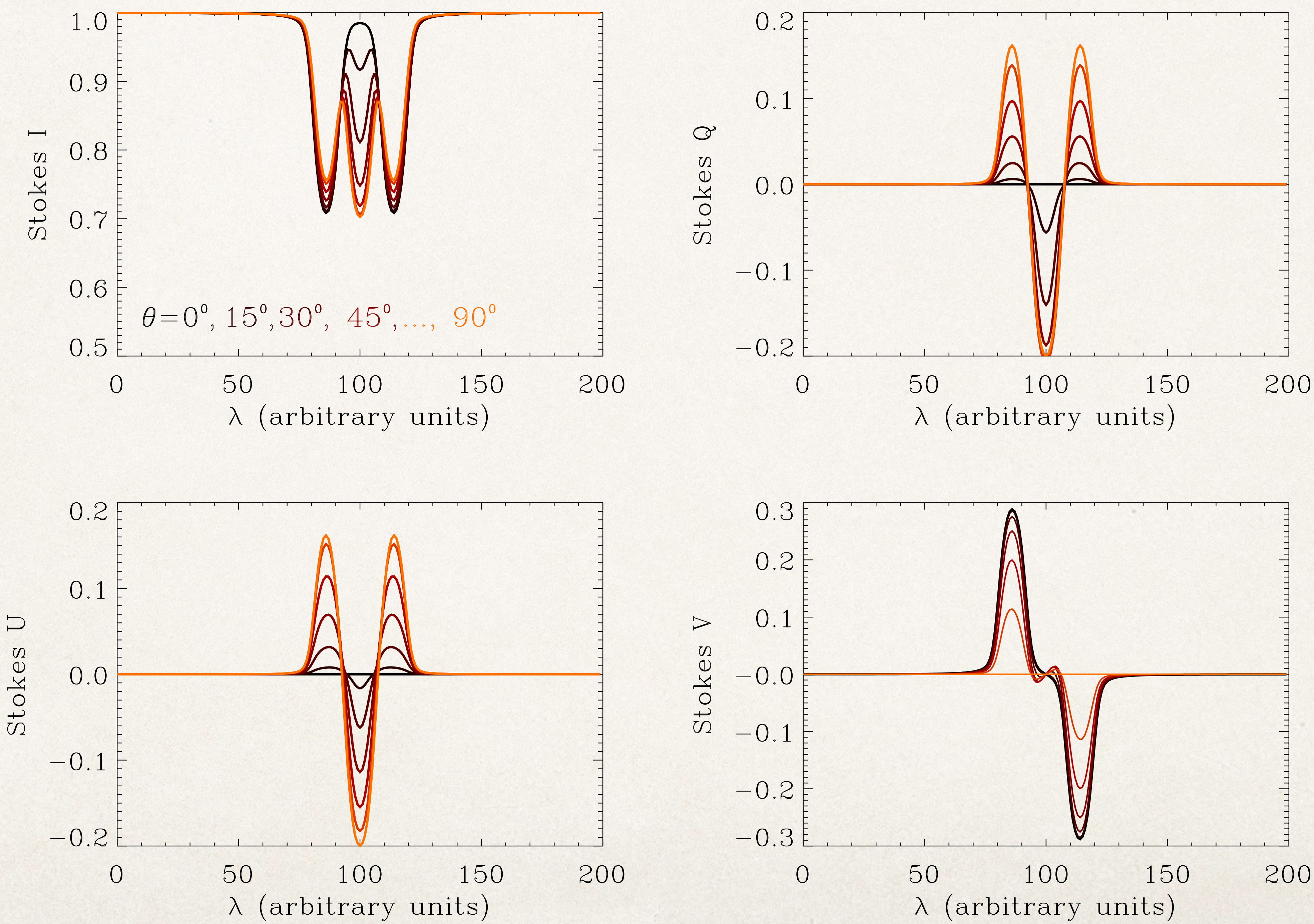
Inclination angle: θ = 0 (magnetic field ⫽ LOS)

Magnetic field strength, B = 0, 500, 1000,… 3500 gauss

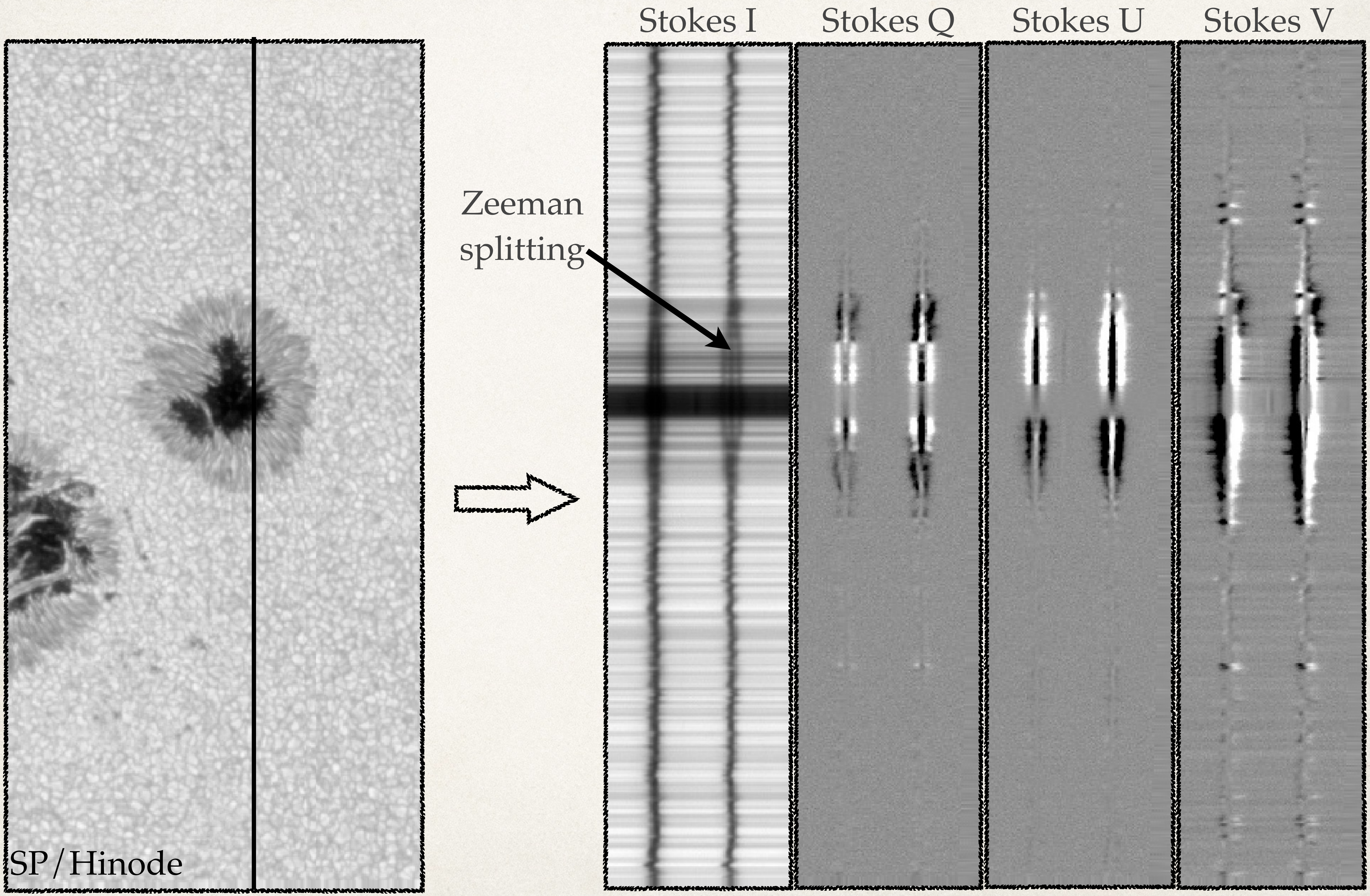


Magnetic field inclination

Inclination angle: θ = 0, 15, 30, … 90 degrees
Magnetic field strength, B = 2500 gauss
Azimuth angle: ϕ= 22.5 degrees



What the data look like



Next time

- Symmetry properties of the Stokes profiles
  - Net Circular Polarization
- Longitudinal magnetograms
- Quiet Sun magnetic fields
  - Unsigned magnetic flux density
  - Effects of spatial resolution
- Shortcomings of the Zeeman effect
- Scattering polarization and the Hanle effect
  - Second solar spectrum